Aircraft Dynamic Model Description

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Nomenclature

α	=	angle of attack, rad
β	=	aerodynamic sideslip angle, rad
$C_D,\ C_Y,\ C_L$	=	drag, lateral and lift forces coefficients
$C_l,\ C_m,\ C_n$	=	roll, pitch and yaw moments coefficients
$\bar{c}$	=	aerodynamic mean chord, m
$\delta a,\ \delta e,\ \delta r$	=	ailerons, elevators and rudder deflections, rad
EPR	=	exhaust pressure ratio
F	=	resulting force vector acting on aircraft body, N
$F_g,\ F_a,\ F_{eng}$	=	gravity, aerodynamic and engine forces, N
g	=	gravitational field intensity near the earth's surface, m/s^2
H_{LG}	=	landing gear height, m
I	=	aircraft tensor of inertia, kg/m^2
$I_{\{xx,yy,zz\}}$	=	moments of inertia on $\{x_b,\ y_b,\ z_b\}$ axes, kg/m^2
$I_{\{xy,xz,yz\}}$	=	products of inertia on $\{x_b,\ y_b,\ z_b\}$ axes, kg/m^2
λ	=	coefficient of the ground effect
M	=	resulting moment vector acting on aircraft body, N.m
m	=	aircraft total mass, kg
Ω	=	aircraft angular velocity vector, rad/s
$p,\ q,\ r$	=	aircraft angular X, Y, and Z velocity components with respect to ground and expressed in body axes, rad/s
Φ	=	euler angles vector, rad
$\phi,\ \theta,\ \psi$	=	aircraft euler angles components, rad
Q	=	quaternion vector from Euler angles
$q_{s\rightarrow b}$	=	quaternion vector from angle of attack
q_d	=	dynamic pressure, $kg/(m.s^2)$
$q_0,\ q_1,\ q_2,\ q_3$	=	quaternion components based on Euler angles
$q_4,\ q_5$	=	quaternion components based on angle of attack

$T(Q)$ = matrix transformation from angular velocity to quaternion rates

$R_{b \to v}$ = reference system rotation matrix from body axis to earth-linked vertical frame

$R_{s \to b}$ = reference system rotation matrix from wind axis to body axis

S = principal wing surface, m^2

V = aircraft velocity vector, m/s

V_a = true airspeed, m/s

$u,\ v,\ w$ = aircraft X, Y, and Z velocity components with respect to ground and expressed in body axes, m/s

x_A = X component of aerodynamic aircraft center position, m

$x_b,\ y_b,\ z_b$ = components of body axis frame, m

$x_v,\ y_v,\ z_v$ = components of earth-linked axis frame, m

$x_w,\ y_w,\ z_w$ = components of wind axis frame, m

W = wind velocity vector, m/s

z_{eng} = Z component of engine gravity center in body axis, m

I. Aircraft Model

A. Quaternions on Aircraft Dynamic Model

As stated on [1, 2], quaternions are widely used as attitude representation parameter of rigid bodies such as spacecrafts. This is due because that quaternion has some advantages such as no singularity and computationally less intense compared with Euler angles. For the aircraft dynamic model, the quaternions are used to give a coordinate transformation on the first hand from earth-linked vertical frame to body axis frame with respect to the gravity center of the aircraft and vice versa. On the second hand, the quaternions will provide the transformation between aircraft wind axes and aircraft body axes. For better comprehension, it seems important to define the three references systems.

Body axis frame, $[x_b, \quad y_b, \quad z_b]$: with the center of gravity as origin and where the axis are fixed to the aircraft structure. The x-axis is the longitudinal aircraft axis which the direction is forward, the y-axis is perpendicular to the aircraft plane of symmetry and the z-axis perpendicular to the other two and oriented downward.

Earth-Linked axis frame, $[\, x_v, \quad y_v, \quad z_v \,]$: it moves with the aircraft and its origin is also the center of gravity. But its axis rest parallels with respect the earth surface. The x-axis is oriented to the North, the y-axis to the East and z-axis perpendicular to the other two and oriented downward.

Wind axis frame, $[\, x_w, \quad y_w, \quad z_w \,]$: This is a particular Body axis frame, because the x-axis is aligned with Airspeed Velocity vector which is always tangent to the trajectory. The y-axis is perpendicular to x-axis on the same plane that x_b and y_b and z-axis is perpendicular to the other two and oriented downward.

For a good consistency of the aircraft attitude, the parameters should be reduce to the same reference system on each equation. This is the reason why it is important to express all the quaternion transformation coordinate matrix. On [1, 2], the transformation matrix from body axis to earth-linked frame, is represented by

$$R_{b\to v}(Q(t)) = \begin{pmatrix} q_0^2+q_1^2-q_2^2-q_3^2 & 2(q_1q_2-q_3q_0) & 2(q_1q_3+q_2q_0) \\ 2(q_1q_2+q_3q_0) & q_0^2-q_1^2+q_2^2-q_3^2 & 2(q_2q_3-q_1q_0) \\ 2(q_1q_3-q_2q_0) & 2(q_2q_3+q_1q_0) & q_0^2-q_1^2-q_2^2+q_3^2 \end{pmatrix} \tag{1}$$

Where $Q(t) = [\, q_0, \quad q_1, \quad q_2, \quad q_3 \,]^T$ is the quaternion 4 x 1 vector. $R_{b\to v}(Q(t))$ is an orthonormal matrix. Therefore, the transpose would be the same as its inverse transformation. That means that $R_{v\to b}(Q(t)) = R_{b\to v}(Q(t))^T$.

It is important to remark that the quaternions are physic meaningless, they are only an effective computational method. For visualize the aircraft attitude is usefull to convert the quaternion on Euler angles as it show in [1, 2].

$$\Phi = \begin{pmatrix} \phi(t) \\ \theta(t) \\ \psi(t) \end{pmatrix} = \begin{pmatrix} atan(\frac{2(q_2q_3+q_0q_1)}{q_3^2-q_2^2-q_1^2+q_0^2}) \\ -asin(2(q_1q_3-q_0q_2)) \\ atan(\frac{2.(q_2.q_1+q_0.q_3)}{-q_3^2-q_2^2+q_1^2+q_0^2}) \end{pmatrix} \tag{2}$$

Where ϕ, θ and ψ are the angles formed by the rotations of the x_b, y_b and z_b axis respectively.

As stated in [4], the aircraft dynamic model is given by the force and moment equations and kinematics equations. A spacecraft kinematic equation is proposed by Diebel in [1] which gives the dependency of the time derivative of its relative orientation in space from the angular rate.

$$\dot{Q}(t) = \begin{pmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q1 \\ -q_2 & q_1 & q_0 \end{pmatrix} \Omega(t) = \frac{1}{2} T(Q(t))\Omega(t) \tag{3}$$

Where $\Omega(t) = [\,p(t), \quad q(t), \quad r(t)\,]^T$ is the vector with the components of the angular velocity on the body reference frame. Note that $T(Q(t))$ is going to be the matrix transformation.

Another fact to consider using quaternion form, is to represent the components from wind axis to body-axis frame. On this model, consider that aerodynamic sideslip angle ($\beta(t)$) is null, in other words, only there is going to introduce a rotation around y-axis that create an angle $\alpha(t)$ called angle-of-attack. This hypothesis is assumed because the model is based on Onera benchmark [3] for its after validation. This outcomes by the following transformation matrix.

$$R_{s \to b}(\alpha(t)) = \begin{pmatrix} cos\alpha(t) & 0 & -sin\alpha(t) \\ 0 & 1 & 0 \\ sin\alpha(t) & 0 & cos\alpha(t) \end{pmatrix} \tag{4}$$

On [1, 2], is defined the method to make quaternions from a rotation angle. It can be calculated as follows

$$q_{s \to b} = \begin{pmatrix} cos\frac{-\alpha(t)}{2} \\ \|\, \vec{e} \,\| \; sin\frac{-\alpha(t)}{2} \end{pmatrix} = \begin{pmatrix} cos\frac{-\alpha(t)}{2} \\ 0 \\ sin\frac{-\alpha(t)}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} q_4 \\ 0 \\ q_5 \\ 0 \end{pmatrix} \tag{5}$$

Where $\|\, \vec{e} \,\|$ is the vector of the direction of the system rotation, that on this case it is $\vec{e} = [\,0, \quad 1, \quad 0\,]^T$. Finally the transformation matrix from wind-axis to body axis frame based on quaternions would be

$$R_{s\to b}(q_{s\to b}) = \begin{pmatrix} q_4^2 - q_5^2 & 0 & 2q_4q_5 \\ 0 & q_4^2 + q_5^2 & 0 \\ -2q_4q_5 & 0 & q_4^2 - q_5^2 \end{pmatrix} \tag{6}$$

B. Aircraft Dynamic Model

In Klein and Morelli [5], is detailed that the aircraft dynamic model is given by the forces and the kinematics equations. The 12^{th} order aircraft model may thus be summarized as follows

$$\begin{cases} \dot{V}(t) &= \frac{1}{m}F(t) - \Omega(t) \times V(t) \\ \dot{\Omega}(t) &= I^{-1}(M(t) - \Omega(t) \times I\Omega(t)) \\ \dot{Q}(t) &= \frac{1}{2}T(Q(t))\Omega(t) \\ \dot{X}(t) &= R_{b\to v}(Q(t))V(t) \end{cases} \tag{7}$$

In equation (13), m is the total mass of the system. Given the body reference frame, $V(t) = [\,u(t), \quad v(t), \quad w(t)\,]^T$ is the vector of the translational velocity, I is the matrix of inertia where I_{xx}, I_{yy}, I_{zz} are the moments of inertia and I_{xz} is a product of inertia. The products of inertia I_{xy} and I_{yz} related to the longitudinal plane ($Y_b = 0$), are both null because of the aircraft's symmetry with respect to this plane. F and M are the resulting vector of the aircraft forces and moments respectively.

1. Forces and Moments

The forces applied to the aircraft can be decomposed into three terms (engines thrust, gravity and aerodynamic forces): $F(t) = F_{eng}(t) + F_g(t) + F_a(t)$. All of them should be expressed in the body-axis frame. It is assumed that the thrust is aligned with the longitudinal axis.

$$F_{eng}(t) = \begin{pmatrix} T(t) \\ 0 \\ 0 \end{pmatrix} \tag{8}$$

Where T is the engines total thrust. The gravity forces are on earth-linked axis frame, to project it in the body-axis frame, it should be

$$F_g(t) = R_{v\to b}(Q(t)) \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} = mg \begin{pmatrix} 2(q_1q_3 - q_2q_0) \\ 2(q_2q_3 + q_1q_0) \\ q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{pmatrix} \tag{9}$$

Where g is the gravitational field intensity near the Earth's surface. The aerodynamic forces are initially expressed in the wind-axis frame which is derived to the body-axis as follows.

$$F_a(t) = q_dSR_{s\to b}(q_{s\to b})C = q_dS \begin{pmatrix} q_4^2 - q_5^2 & 0 & 2q_4q_5 \\ 0 & q_4^2 + q_5^2 & 0 \\ -2q_4q_5 & 0 & q_4^2 - q_5^2 \end{pmatrix} \begin{pmatrix} C_X \\ C_Y \\ C_Z \end{pmatrix} \tag{10}$$

The aerodynamic coefficients C_X, C_Y and C_Z are functions of the system variables, q_d is the dynamic pressure and S is the aerodynamic reference surface.

The moment about the center of gravity of the aircraft, result from the engines and aerodynamic actions as $M(t) = M_{eng} + M_a$. Reminding that this model case is based on A310 aircraft, its engines are turbofans. That is the reason why there is going to be assume that there are two engines delivering the same thrust and the distance of the midpoint of the motors to the gravity center would be null along the y-axis and x-axis. Only it will be considered that the engines are located below the center of gravity at z_{eng}. The moment resulting from the thrust is:

$$M_{eng}(t) = GE \times F_{eng} = \begin{pmatrix} 0 \\ z_{eng}.F_x(t) \\ 0 \end{pmatrix} \tag{11}$$

On this equation, GE means the distance vector between the center of the gravity "G" and the center of the gravity of the engines "E". The aerodynamic moment exhibit two terms. The first one contains the main distribution and is directly proportional to the moment coefficients C_l, C_m and C_n, that correspond about roll, pitch and yaw moments. The second term is the moment resulting from the aerodynamic forces Fa which are applied to a point A (aerodynamic center) which possibly differs from the center of gravity.

$$M_a(t) = q_d S\bar{c}\begin{pmatrix} C_l \\ C_m \\ C_n \end{pmatrix} + GA \times F_a \tag{12}$$

On Eq. (18) $\bar{c}$ is the aerodynamic mean chord and GA is the distance vector between the center of the gravity and the aerodynamic center, where it going to be assumed that the distance component is only about the x-axis called x_a.

2. Aircraft Aerodynamic Model

In the Civilian Aircraft Landing Challenge [3] there is an interesting aerodynamic coefficients definition. Concern to the aerodynamic forces, C_L (Lift coefficient), C_Y (lateral coefficient) and C_D (drag coefficient) are given by:

$$\begin{aligned} C_L &= C_{L0} + C_{L\alpha}\alpha(t) + \tfrac{\bar{c}}{V_a(t)} C_{Lq} q(t) + C_{L\delta_e}\delta_e + C_{LH} e^{-\lambda_L H_{LG}(t)} \\ C_Y &= C_{Y\beta}\beta(t) + C_{Yr}\delta_r \\ C_D &= C_{D0} + C_{D\alpha}\alpha(t) + C_{D\alpha_2}\alpha(t)^2 \end{aligned} \tag{13}$$

Similarly, the moment coefficients about x, y and z axes are given by:

$$\begin{aligned} C_l &= C_{l\beta}\beta(t) + \tfrac{\bar{c}}{V_a(t)}(C_{lp}p(t) + (C_{lr0} + C_{lr\alpha}\alpha(t))r(t)) + C_{l\delta_a}\delta_a + C_{\delta_r}\delta_r \\ C_m &= C_{m0} + C_{m\alpha}\alpha(t) + \tfrac{\bar{c}}{V_a(t)} C_{mq} q(t) + C_{m\delta_e}\delta_e + (C_{mH0} + C_{mH\alpha}\alpha(t))e^{-\lambda_m H_{LG}(t)} \\ C_n &= (C_{n\beta_0} + C_{n\beta_\alpha}\alpha(t))\beta(t) + \tfrac{\bar{c}}{V_a(t)}(C_{nr}r + (C_{np0} + C_{np\alpha}\alpha(t))p(t)) + C_{n\delta_a}\delta_a + C_{n\delta_r}\delta_r \end{aligned} \tag{14}$$

Note that in C_L and C_m equations, the term which depends of the height of the main landing gear above runway ($H_{LG}(t)$) describes the ground effect. δ_e, δ_a and δ_r represents the deflections of the control surface of the elevators, ailerons and rudder respectively. See also that these equations are functions of angular rate variables (p(t), q(t), r(t)), sideslip angle ($\beta(t)$), the angle-of-attack ($\alpha(t)$) and the true airspeed ($V_a(t)$). This airspeed come from its vector module $V_a(t) = \|\vec{V}_a\|$. This vector is clearly affected by wind denoted W and expressed in a earth-linked axis frame. The airspeed vector V_a in body-axis coordinates is thus obtained as:

$$\vec{V}_a = V(t) - R_{v\to b}(Q)W \tag{15}$$

Where W_x, W_y and W_z are the components of the wind vector and $V_{ax}(t)$, $V_{ay}(t)$ and $V_{az}(t)$ are the components of the airspeed vector. On the other hand, the $\alpha(t)$ and $\beta(t)$ variables can be calculated as:

$$\alpha(t) = arctan(\frac{V_{az}(t)}{V_{ax}(t)}) \;\; and \;\; \beta(t) = arcsin(\frac{V_{ay}(t)}{V_a(t)}) \tag{16}$$

Finally the relation between C_L, C_D and C_X, C_Z is:

$$\begin{aligned} C_X &= -C_D \\ C_Z &= -C_L \end{aligned} \tag{17}$$

II. Toolbox Inputs

A. Mass and Geometry

As the aircraft dynamic model has been described, one of the parameters to define it is mass and geometry aircraft characteristics. For this reason, the parameters summarized in table 1 should be inserted to the model before the creation of the fuzzy model.

B. Aerodynamic Coefficients

In this window all the aerodynamic coefficients should be introduced. All of these parameters are described in **Nomenclature** and **B.2** sections.

C. Atmosphere

The input parameters description of the toolbox are defined in the table 2

D. Engines

In this section of the input parameters, there is two remarks.

Remark 1: The effective thrust F_{eng} at given altitude is approximated by a lineal function of the Exhaust Pressure Ratio (EPR).

$$F_{eng} = A_{eng} EPR + B_{eng} \tag{18}$$

Where the coefficients depend on the temperature ratio T/T_0.

Remark 2: The EPR is considered the engine actuator. That is the reason for consider its dynamics that are approximated by magnitude and rate limited first order filters. Whose characteristics are summarized in table 3

E. Actuators

In this tab, the rest of the actuators parameters (Aileron, Elevator and Rudder) should be introduced. Their dynamics are analogue to the EPR dynamics and there are summarized in table 4

Variable	Units	Description	Variable	Units	Description
Sref	m^2	Wind surface area	Lref	m	Aircraft length
Mass	kg	Aircraft mass	Ixx	$kg.m^2$	See nomenclature
Iyy	$kg.m^2$	See nomenclature	Izz	$kg.m^2$	See nomenclature
Ixz	$kg.m^2$	See nomenclature	XCG	m	Position X of the gravity center
dxg	m	distance x of the engine to the CG	dze	m	distance z of the engine to the CG

Table 1 Mass and Geometry toolbox inputs

Variable	Units	Description
T0	K	Temperature at sea level
rho_0	kg/m^3	Air density at sea level
alt	m	Aircraft current altitude

Table 2 Atmosphere toolbox inputs

Variable	Units	Description	Variable	Units	Description
a_{tau}	s	Aileron time-constant	a_{ML}	deg	Aileron Upper and Lower limit
a_{RL}	deg/s	Aileron rate-limit	e_{tau}	s	Elevator time-constant
e_{ML}	deg	Elevator Upper and Lower limit	e_{RL}	deg/s	Elevator rate-limit
r_{tau}	s	Rudder time-constant	r_{ML}	deg	Rudder Upper and Lower limit
r_{RL}	deg/s	Rudder rate-limit			

Table 4 Actuators toolbox inputs

Variable	Units	Description	Variable	Units	Description
Ga	–	A_{eng} coefficient	Gb	-	B_{eng} coefficient
tau	s	Time-constant	UML	-	Upper-bound
LML	-	Lower-Bound	RL	-/s	Rate-Limit

Table 3 Engine toolbox inputs

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