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# Personalising Crutch Geometries through Bayesian Optimisation

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**Riccardo Conci**

Department of Computer Science  
University of Cambridge  
rc667@cam.ac.uk

**Riccardo Ali**

Department of Computer Science  
University of Cambridge  
rma55@cam.ac.uk

**Deepro Choudhury**

Department of Computer Science  
University of Cambridge  
dc755@cam.ac.uk

**Sergio Rivera**

Department of Computer Science  
University of Cambridge  
sr2070@cam.ac.uk

## Abstract

Crutches are optimised for stable motion, but this safety comes at the cost of comfort and speed. In this paper, we employ Gaussian Processes (GPs) and Bayesian Optimisation (BO) as hypothesis generators to find better crutch configurations, which we validate on a physical prototype. We do so by defining a novel loss function indicating the quality of a crutch design which combines subjective metrics (joint pain, instability and effort) and the corresponding objective ones. Finally, we (1) use this methodology to build a more stable, less effortful and less painful personalised crutch design and (2) use the knowledge built by the GP through these experiments to enhance our understanding of the physical dynamics of crutching.

Code: [github.com/DeeproChoudhury/crutch-simulation](https://github.com/DeeproChoudhury/crutch-simulation)

## 1 Introduction and Motivation

Crutches are used worldwide to regain mobility following lower limb injuries [8], with an estimated 300,000 patients treated for ankle sprains in the UK each year alone [3]. Historically, there have been two main crutch designs: axillary and forearm (Lofstrand [6]), both of which have well-documented complications due to their design. These include brachial plexus neuropathies [10] and severe wrist pain [9]. Despite the need for improved designs across a diverse population, there has been little innovation over the past century in effectively mapping crutch geometries to personalised quantitative metrics [12].

Addressing the diverse needs and characteristics of crutch users presents a significant challenge, and mathematically deriving a model to capture such complexity is intractable. Furthermore, the scarcity of large datasets limits the application of conventional machine learning (ML) methods. However, Gaussian Processes (GPs) [11] inherently incorporate uncertainty in their predictions, making them suited to situations with limited data. Additionally, they allow for the integration of prior knowledge through kernels, and can be queried to create a better understanding of the data they model.

In this paper, we present a pioneering approach using GPs and Bayesian Optimisation (BO) for generating hypotheses about novel crutch geometries. We explore the following questions:

1. Can we use BO to rapidly discover an optimal crutch geometry tailored towards an individual, enabling us to move away from traditional one-size-fits-all approaches?
2. Can we interrogate the GP in the BO loop to give us previously unknown insights into crutching dynamics?

## 2 Methods

To address the first question, we set up an interface between a flexible crutch design that is used to extract crutching metrics, and the Gaussian Process BO model. A summary of the workflow for this experimental loop is shown in figure 1.

1. An initial crutch geometry (section 2.2) is chosen and run.
2. Qualitative and quantitative metrics are extracted and aggregated into a loss metric (sections 2.3, 2.4, and 2.5.1).
3. These are fed to the Bayesian optimiser with the crutch parameters. The BO suggests the next set of crutch parameters to experiment (section 2.5.3).
4. The new parameters are fed into a simulator that assesses the feasibility of the suggested crutch geometry (section 2.6).
5. If the suggested geometry is feasible, the experiment is run, and the loop is repeated.
6. After the BO loop is completed, the data is queried through Sensitivity Analysis (section 3.2.2)

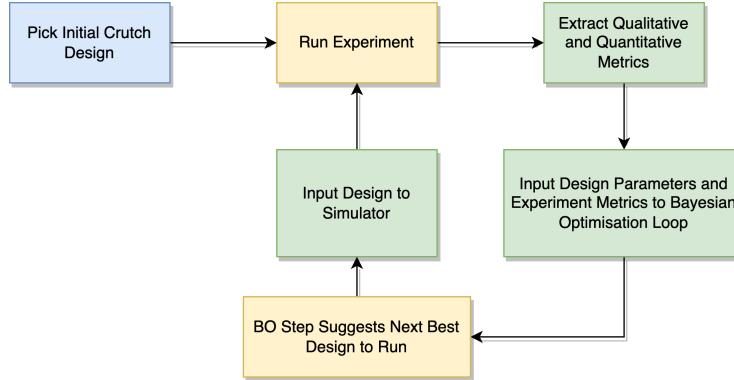


Figure 1: Experimental workflow.

### 2.1 Experimental design

Experiments were performed indoors. The user crutches 15 steps, then turns around and repeats this six times, for a total of 90 steps. Each set of 15 steps is a ‘bout’. A three-point crutching technique was used with the same leg completely non-weight bearing, as is shown in Figure 3. No specific rate or step length was set. All the experiments were conducted on author RC, to ensure consistency in the metrics gathered, especially the subjective ones (Section 2.4).

### 2.2 Crutch designs

To rapidly vary crutch parameters in our experiments, we collaborated with Dr. Christof Schwiening at the PDN department at the University of Cambridge to design and manufacture a flexible crutch. For comparison purposes, experiments were also run on the standard NHS (Lofstrand) elbow crutch and the Smart Crutch [1], which is currently on the market as a replacement to the standard NHS design. The flexible crutch design was improved by RC, enhancing its strength, stability and range of angles, with the help of Dr Schwiening. The crutches are visible in Appendix A. A simple way to understand the crutch parameters is through figure 2. The design is characterised by 3 angles:  $\alpha$ ,  $\beta$ ,  $\gamma$ , where  $\gamma$  is the vertical distance from the forearm contact to the vertical strut,  $\beta$  is the angle between the forearm contact point and the hand contact point (with  $0^\circ$  starting at the horizontal), and  $\alpha$  is the angle from a vertical line to the handle rod.

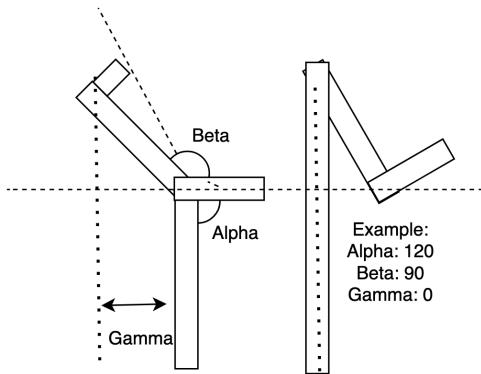


Figure 2: Alpha, Beta and Gamma across two designs.

### 2.3 Data acquisition

The Polar H10 heart rate monitor uses a direct current (DC) tri-axial accelerometer to detect acceleration in the anterior-posterior ( $Z$ ), vertical ( $X$ ) and mediolateral ( $Y$ ) directions. DC accelerometers are sensitive to static acceleration like gravity, such that the axis-aligned vertical ( $X$ ) acceleration will measure  $-1g$ . Due to the lack of an internal gyroscope, the accelerometer data is also affected by the device's tilt. Appendix B shows a clear diagram of this.

The open-source *FingerPulseLatency* software ([13]) was utilised to pair with the H10 polar monitor and extract and save the accelerometer and heart rate data during the experiments. An interactive step detection algorithm developed by Schwiening was used to identify each crutch step through the  $X + Z$  accelerometer traces. The steps detected match three-point crutching as shown in Figure 3.

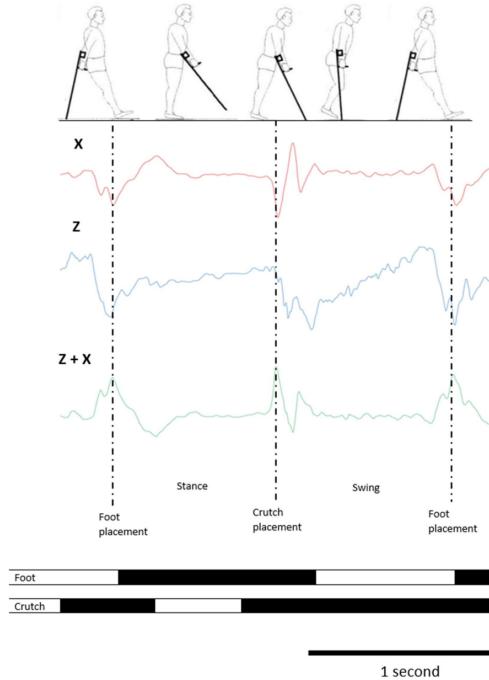


Figure 3: A crutching step where the cycle is divided into two phases as depicted by the peaks in  $Z + X$ . The bottom segments represent the periods when feet or crutches contact the ground (black strip). Illustration adapted from [5].

## 2.4 Metrics

The metrics for each experiment run include both subjective and objective quantitative indicators.

The objective metrics are derived from the accelerometer data, as in Appendix B, and from ECG signals. The accelerometer-based metrics each describe aspects of instability during crutch use. Firstly, we calculate the step frequency per bout  $i$  and average across them, denoted  $\mathbb{E}[\text{stepfreq}_i]$ . We would expect this metric to be high in better-performing crutches. Next, we compute the standard deviation of the step frequency for each bout  $i$  and sum these values. Given the small values, these are scaled by a factor of 20 in the loss, giving  $20 \sum_i [\sigma_{\text{stepfreq}_i}]$ .

The  $X$ ,  $Y$  and  $Z$  acceleration metrics are then used to extract metrics  $\Delta(X + Z)$ ,  $\Delta Y$  and  $|Y|$ . The metric  $\Delta(X + Z)$  is calculated by summing the  $X + Z$  acceleration in the last two bouts, then subtracting the summed  $X + Z$  acceleration values in the first two bouts. Through this change, we are able to detect if the  $X + Z$  acceleration stays constant or increases (suggesting greater user confidence), or decreases (suggesting gradual instability increase or fatigue). The  $X + Z$  are divided by their absolute mean to normalise.

$\Delta(Y)$  is calculated the same way as  $\Delta(X + Z)$ .  $Y$  is specifically the sideways motion, so a change should be more sensitive to instability than  $\Delta(X + Z)$ . Finally, we extract  $|Y|$  to mitigate that  $\Delta(Y)$  only looks at relative values, not absolute.  $\Delta(Y)$  is the normalised sum of the  $Y$  acceleration across all bouts. Finally, as shown in table 1, to align the magnitude of the metrics, we scale  $\Delta Y$ ,  $\Delta(X + Z)$ ,  $|Y|$  and  $\Delta \text{HR}$  by 100, 100, 3000 and 10 respectively, so that they carry similar contributions to the loss function. The motivation behind this processing is further described in Section 2.5.1. Finally, we consider Heart Rate (HR) change  $\Delta \text{HR}$  between the start and end of the crutching experiment.

Table 1: Metrics

Interpretation	Subjective metrics	Objective metrics	Weighting (for objective)
Instability	User instability score	Total step frequency variation	20
		Average Step Frequency	-1
		Change in Y Acceleration	1/300
		Change in X & Z Acceleration	1/100
		Total Y Acceleration	1/100
Effort	User effort score	Heart rate change	1/5
Pain	User pain score		

## 2.5 Modelling

We use a Gaussian Process (Section 2.5.2) to model an inaccessible loss function (Section 2.5.1) that informs the quality of a given crutch configuration. Because the loss function is inaccessible analytically, we run the experiments described in Section 2.1 to empirically find the loss associated with the suggested configuration. All the implementations are done with GPy [4] and GPyOpt [2].

### 2.5.1 Loss function

Our novel loss function  $\mathcal{L}$  formalises what a ‘good’ crutch is in a quantifiable and precise way. It takes a crutch configuration consisting of 3 angles  $\alpha, \beta, \gamma \in \mathbb{R}$  and outputs the weighted sum of the metrics described in Section 2.4 corresponding to that particular design.

$$\begin{aligned} \mathcal{L} = & \text{JointPain} + \text{Instability} + \text{Effort} + \frac{1}{100} \Delta(X + Z) + \frac{1}{100} \Delta Y + \frac{1}{3000} |Y| + \frac{1}{10} \Delta \text{HR} \\ & - \mathbb{E}[\text{stepfreq}_i] + 20 \sum_i [\sigma_{\text{stepfreq}_i}] \end{aligned}$$

Combining subjective and objective measures in the function is necessary for a well-rounded representation of the problem and a common practice for complex modelling (e.g. [15, 7]). The subjective metrics make the intrinsic variability of these across users explicit. Combining objective and subjective allows us to build crutch designs that are safe and validated with scientific rigour and ‘feel’ better and easier to use.

### 2.5.2 Gaussian Process Kernel

The Gaussian Process kernel allows us to input prior knowledge about the system to influence the posterior in small data settings. As such, we created the kernel  $K$  that is the product of a Matern  $\frac{5}{2}$  kernel, denoted  $M$ , and a white noise kernel  $W$  that acts only on the  $\beta$  angle with variance  $\sigma_W = 0.2$ , so that  $K := MW$ .

The first kernel,  $M$ , describes our prior assumption that similar crutch geometries will share a similar loss and that this loss will not be smooth. We set the kernel's *variance* to 1 to allow for variability inherent in the data and the measurements. As a prior, we set the kernel's *lengthscale*  $l$  to 3. When two angle configurations  $x$  and  $y$  are  $l = 3$  apart, i.e.  $\|x - y\|^2 = 3$ , their degree of correlation will be  $\sim 0.52$ , giving it a good correlation.

We multiply  $M$  by  $W$  acting only on  $\beta$  to take into account the fact that our physical prototype does not allow us to fix this angle with certainty. This variability comes from the fact that there is no proper handle to hold on to, which means that the hand's position could shift slightly during movement and is difficult to set initially. Hence, we scale the variance of  $\beta$  by  $\sigma_W = 1.1$  on the second entry of the kernel's diagonal to increase  $\beta$ 's inherent noise and variability.

### 2.5.3 Bayesian Optimisation loop

We employ BO as a hypothesis generator for better crutch geometries. Our exploration domain for sampling has the following bounds:  $\alpha$  from  $70^\circ$  to  $120^\circ$ ,  $\beta$  from  $90^\circ$  to  $145^\circ$  and  $\gamma$  from 0cm to 30cm. We impose a further constraint that  $\alpha + \beta \geq 190$ , as shorter combinations of angles are physically unusable. We set the acquisition function to *expected improvement* (EI), as it is a more exploitative strategy to help find an optimal design rapidly by refining promising designs found in our parameter space, rather than extensively exploring new and potentially less viable areas in the design space.

To initially guide our BO loop, we input 3 selected crutch designs as starting points. These 3 configurations correspond to the NHS crutches, the SmartCrutch, and the Opmo crutches. Opmo crutches are the best design created by the Cambridge Engineering Design team (unpublished). In scenarios where the BO-proposed geometries were not physically possible with the flexible crutch design, we adjusted them to the closest viable geometry and ran that instead.

## 2.6 Crutch Simulators

**SolveSpace:** We built our own simulated model of the flexible crutch design using the parametric 3D modelling simulator, SolveSpace. For each suggested geometry by the BO loop, the parameters were checked on SolveSpace for physical feasibility. The simulated crutch is visible in Appendix C.

**Unity:** To support the subjective metric for joint pain (Section 2.5.1), we attempted to build a simulation in Unity [14] to provide a simulated pressure at the hand grip site. This yielded a time series of pressure applied to each place during a stereotyped step. We planned to integrate this as a summary statistic in the loss function. However, we soon realised that the inherent limitations of the engine would introduce too much uncertainty to our data. This motivated further the need to run a hybrid model-hardware experimental loop as above. Appendix D shows a snapshot of the latest modelled crutch.

## 3 Results & Discussion

### 3.1 Bayesian Optimisation to discover better designs

Results from the BO loop are shown in Table 2. The top three rows show the crutch experiments that were given to the model as initial points. Remarkably, by the second run, the BO loop already found a geometry that is considerably better than the initial points. Then to explore the space it asks for a design that has a similar  $\alpha$  and  $\beta$ , but a different  $\gamma$ . This pattern repeated across multiple short BO runs. This BO loop was set up with the kernel described in the methods, with lengthscale  $l = 3$ . As shown in Figure 7, this was likely much too low. Despite this, the BO loop performed well and rapidly discovered a novel better geometry for the user.

Table 2: Bayesian Optimisation loop for crutch designs .

Crutch	Alpha (°)	Beta (°)	Gamma (cm)	Combined Subjective Loss	Combined Objective Loss	Total combined loss
NHS	100	95	3	10	36.8	46.8
Smart	140	140	0	11	43.7	54.7
Opmo	90	140	30	6	37.1	43.1
BO1	110	100	12	11	45.6	56.6
BO2	75	125	3	3	29.2	32.2
BO3	80	120	25	9	37.4	46.4

### 3.2 Knowledge extraction from data and models

Over the experimental period, a total of 12 different geometries were tested. These included 2 runs from the NHS crutches to give the GP a sense of the noise intrinsic to the data. To best extract the knowledge from this data, all experiments were aggregated and used for the following three sections, reviewing the correlation across metrics (3.2.1), sensitivity analysis (3.2.2) and finally through GP predictions across various hyperparameters (Section 3.2.3).

#### 3.2.1 Correlations

We run a pairwise correlation across the recorded metrics, the results of which are shown in figure 5. These correlations show some notable findings.

The parameter  $\alpha$ , which controls the angle of the handle from vertical and ranges from 70 to 120 degrees, has a correlation of 0.59 with subjective effort. A down-sloping handle means immediately that  $\beta$  has to be greater than 90 for the crutch to be physically possible. The wrist is in a stronger position, so even with increased pressure, it is less painful. However, pain is really explained by the combination of alpha with beta, and in this case, the higher the  $\beta$ , the less the subjective pain (-0.56 correlation). Subjective effort is also very strongly correlated (0.82) with the overall loss, which combines the full subjective and objective metrics. Effort, indeed, is the most encompassing of the metrics. Low effort is when a crutch is not painful, stable, and easy to use - exactly what we are aiming for.

Correlations with  $\gamma$  also show interesting results.  $\gamma$  is the vertical distance in cm from the forearm contact point to the vertical strut. It has positive correlations with step frequency (0.4), its variation (0.52) and  $\Delta(X + Z)$  (0.45). This can be explained by subplot b in Figure 4. When  $\gamma$  is 0, the forward moment in mid-swing is as extreme as it can get. The effect is that regardless of how one lands on the crutch the user will be rapidly pushed forward, decreasing the step variability. As  $\gamma$  increases, the user has less initial moment, so more control but also increased variability and the possibility to crutch faster and more smoothly, rather than ‘plant the crutch and fall forward’.

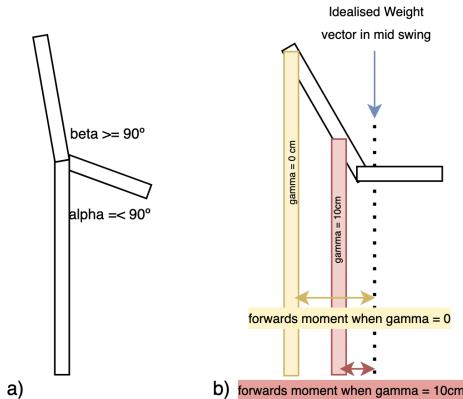


Figure 4: a)  $\alpha + \beta \geq 190$  b) Decreasing  $\gamma$  increases the forward moment

$\Delta Y$  is picked to assess the variability of instability during crutching.  $|Y|$  allows us to compare absolute values across crutches. Despite their similar intent, and their high correlation with subjective instability, they have a correlation of 0. The differences arise across step frequencies, variations and  $\Delta(X + Z)$ . With a lower  $\Delta Y$ , and a more stable crutch, the user can crutch quickly. Instead, the results for  $|Y|$  are more challenging.  $|Y|$  is strongly correlated with subjective instability (0.6). Yet it has a strong negative correlation with step frequency variation and  $\Delta X + Z$ , both intuitive indicators of instability. A possible explanation for this is that when a certain level of instability ( $|Y|$ ) is reached, the user changes the crutching style to be more prudent. However, this hypothesis would need further testing.

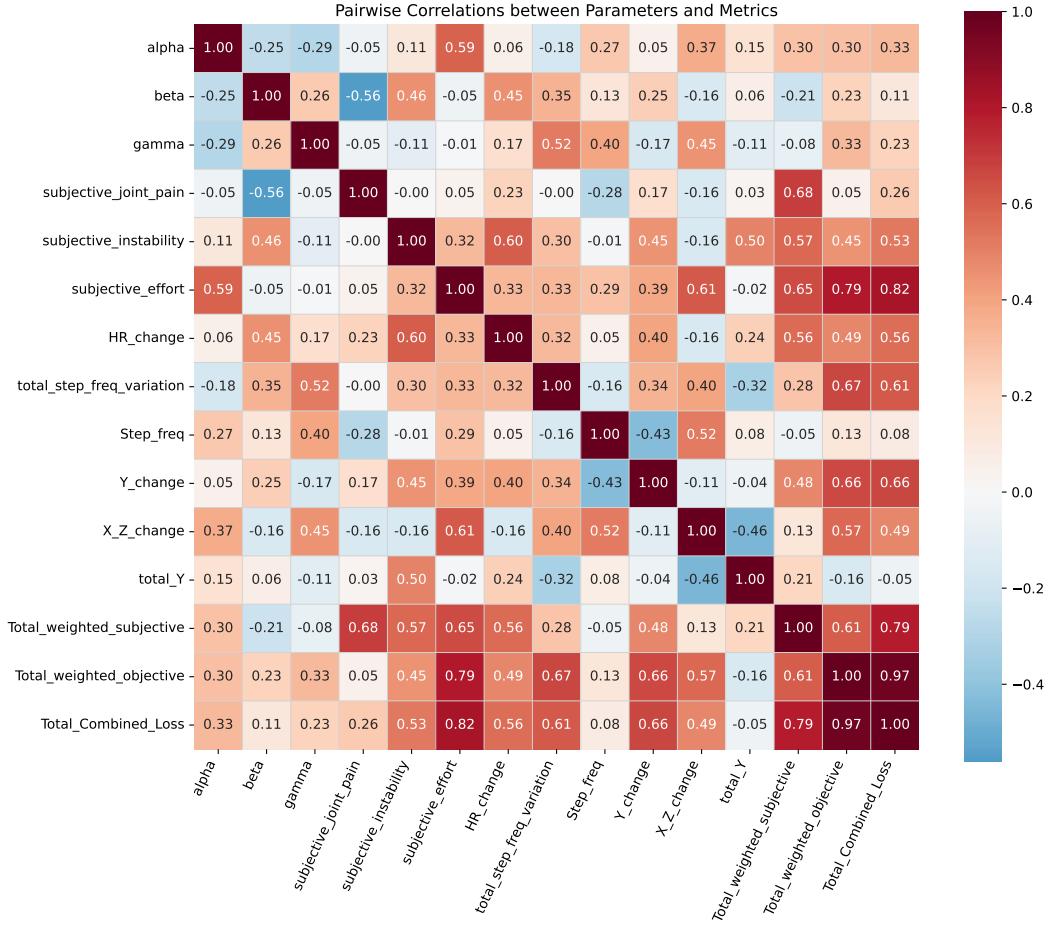


Figure 5: A pairwise correlation heatmap for each aspect and performance metric of the crutch

### 3.2.2 Sensitivity Analysis

The Sobol indices measure the effect of each parameter individually (S1), the effect of pairs of parameters (S2), and the total effect (ST) of each parameter on each individual loss (which includes interactions with all other parameters). We will focus our analysis on the individual contributions as they are the most interpretable.

We can observe that the angle  $\alpha$  *individually* is highly impactful on subjective joint pain, with an index of 0.49. This is because  $\alpha$  is largely responsible for determining the weight distribution between the forearm and the wrist, which is ultimately the source main source of joint pain.

We find that the angle  $\beta$  is the most impactful on instability, with a surprising 0.98 index. More investigation on the dynamics of crutching from an analytical point of view could shed light on this perhaps surprising result.

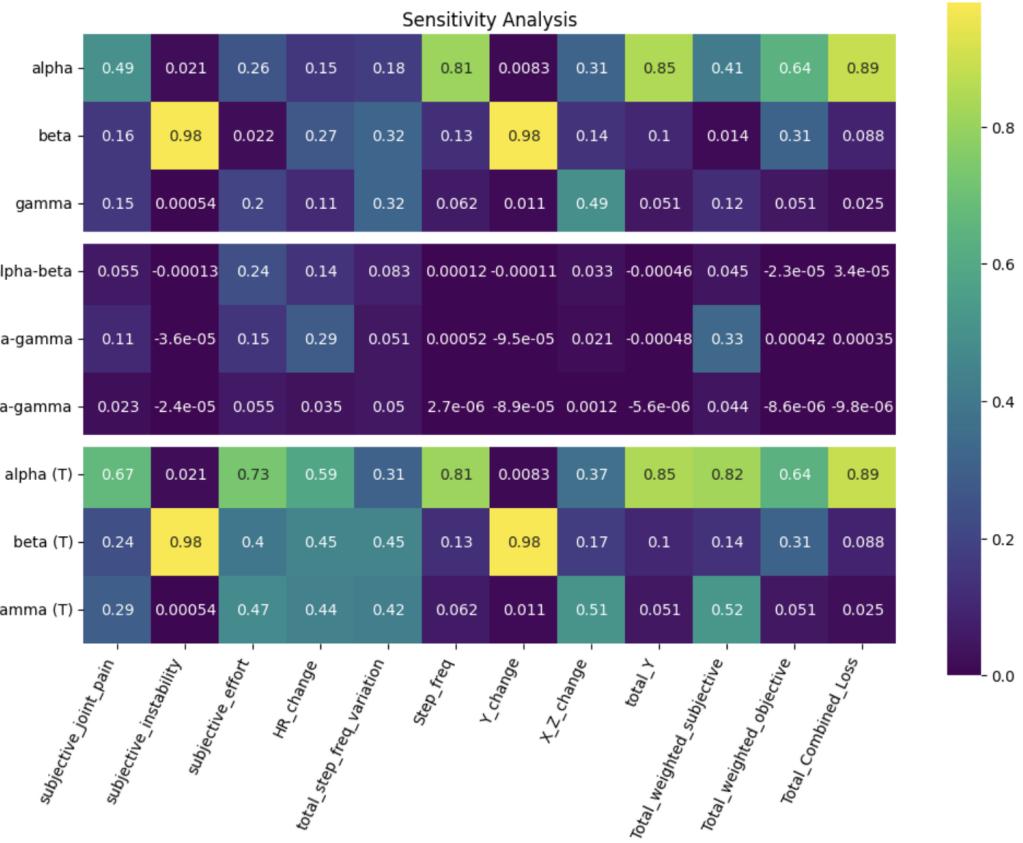


Figure 6: **Sensitivity Analysis** with Sobol Indices

As illustrated in Figure 4,  $\gamma$  is directly correlated to the forward moment, and hence is the one most responsible for  $\Delta(X + Z)$ .

By conducting these analyses we address the question (2) from the introduction. In particular, if a computational or analytical model of the dynamics of crutches were to be built, it should preserve these indices in the form of ‘dependencies’ between input and output variables. Hence, this analysis allows us to extract valuable insights into the physics of the problem by integrating the GP in a feedback loop as an effective hypotheses generator.

### 3.2.3 GP predictions

Figure 7 depicts the parameter space  $\alpha$ ,  $\beta$ , and  $\gamma$  through a three-dimensional plot. The interactive version of this plot can be found in the accompanying code. The left side of the figure presents two plots that show the predicted uncertainty for two different length scales as a posterior to the full experimental data. On the right, the plot shows the predicted means with a length scale of 15. The “cropped” space in the bottom right corners corresponds to the area of the parameter space representing impossible crutch configurations, as detailed in Section 2.5.2. The opaque circles represent the 13 experimental data points, with the colour indicating their loss. Finally, the brown circle indicates the crutch parameters that the Bayesian Optimization (BO) loop would suggest for the next experiment, given these 13 experiments and a length scale of 15.

The location this BO suggested is at  $\alpha = 85^\circ$ ,  $\beta = 130^\circ$ ,  $\gamma = 0$  cm. Given the data and lengthscale, its predicted uncertainty is also low. These parameters encompass the essence of what the GP has learned from all the experiments:  $\alpha$  should be less than  $90^\circ$ , forcing beta to be  $>110^\circ$ . This immediately strengthens the position of the wrist, allowing it to take more load. However, the story is more complicated. There is a large area of unexplored space around the higher values of alpha, especially

with middle values of beta and higher of  $\gamma$ . Unfortunately, this suggested location cannot be built on the flexible crutches we have currently.

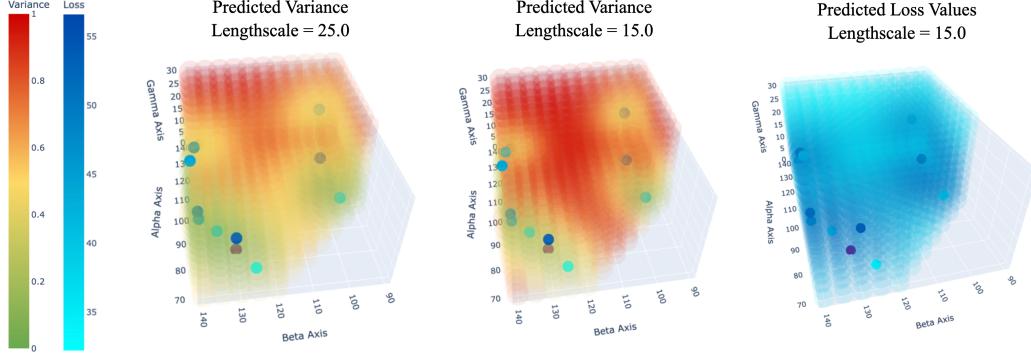


Figure 7: Variance fields for two different length scales and mean values for length scale = 15.

## 4 Limitations

Despite promising results, there are certain aspects of our method which we would like to improve given more time. Firstly, all experiments were run on a single user, one of the authors, potentially biasing the results. Secondly, the BO loop was run with a combination of Matern  $\frac{5}{2}$  with a lengthscale of 3 and a White kernel, but different choices for these parameters may yield better results. Despite this, the methods provided successfully discovered better and personalised crutches for the user and have been used to increase our understanding of crutching dynamics.

**Future work** The most exciting aspect of this work is the potential to scale across people. Currently, despite twelve distinct geometries, it is not yet clear what is the *optimal* geometry for this paper’s user. However, as more data is gathered, transfers promise new individuals with similar characteristics to previous users to more rapidly discover their own personal crutch geometry. There are various challenges to this. Firstly, the crutching method and quality have to be such that the data platform can extract useful and relevant metrics. In particular, the quality and material of the crutch should be comparable across different users, as well as their skill in using crutches. This is likely to be very heterogeneous. Secondly, subjective metrics likely do not transfer well and may need to be separated, perhaps by employing multiple GPs. However, as a simple test, the current GP model could be used as the starting point for an individual with similar height, weight, etc. to assess the quality of its suggestions based on the knowledge derived from the previous experiments.

As noted previously, a subsection of the possible configurations is not possible with the current flexible crutch configuration. The next step forward would be to enhance the design to fully reach across the parameter space.

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## A Flexible Crutches



Figure 8: Flexible crutches used to run experiments

## B Accelerometer

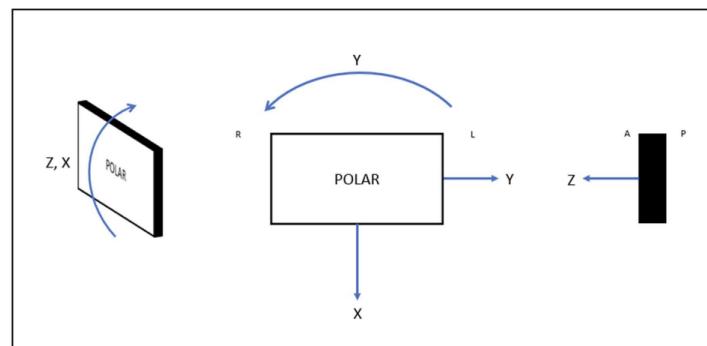


Figure 9: **Properties of Tri-axial DC accelerometer.** The X, Y and Z axes of the polar H10 respond positively to downward, right-left and forward acceleration respectively. These axes are also affected by lean as demonstrated by the curved arrows. R=Right, L=Left, A=Anterior, P=posterior. Image from Jonathan Bennett thesis: "Forearm crutch design: designing a platform for physiological testing" - supervised by R Conci in 2022.

## C SolveSpace simulator

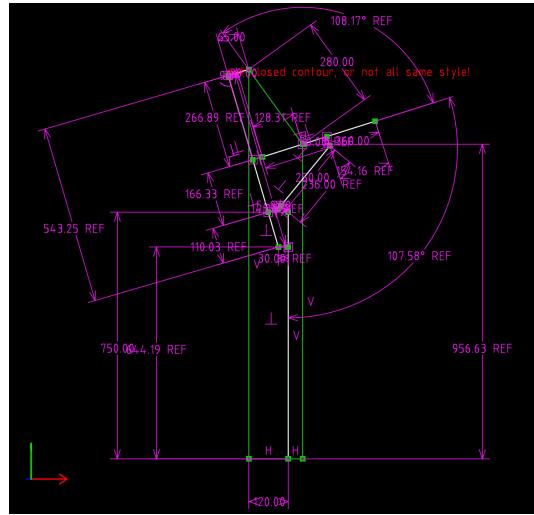


Figure 10: SolveSpace simulator to check feasibility of BO proposed design.

## D Unity simulator

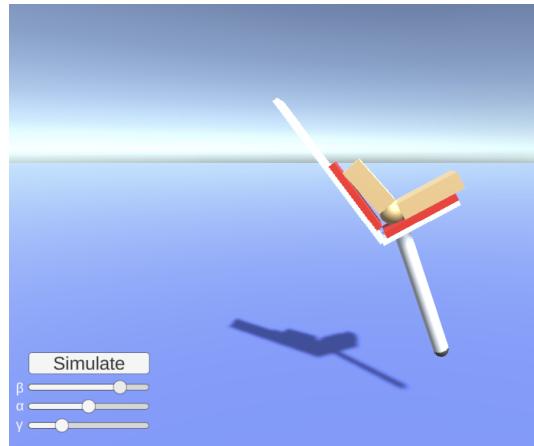


Figure 11: Simulator environment we built in Unity.