Institut Eurécom
Digital Communications

Final Examination

Date: February 6th, 2015

Duration: 2 hours

Chooose and complete any 3 out of the following questions. All questions will be graded equally. All documents are allowed.

1 Problem 1

Consider an ideal QPSK system with a received signal

$$y = \sqrt{E_s}x + z$$

where x is a symbol from a QPSK alphabet and z is AWGN with variance N_0. In many radio system the receiver electronics induces a distortion termed *IQ Imbalance* whereby the quadrature mixer in the receiver does not perform and ideal complex multiplication (i.e. by sine and cosine signal generators) to generate the complex baseband signal. Instead the phase difference between the two signals is $\pi/2 + \alpha$ where α is an unknown imbalance that the receiver must estimate or ignore. This distortion results in the approximate channel model

$$\mathrm{Re}(y) = \sqrt{E_s}\mathrm{Re}(x) + \mathrm{Re}(z), \mathrm{Im}(y) = \sqrt{E_s}(\mathrm{Im}(x) + \alpha\mathrm{Re}(x)) + \mathrm{Re}(z),$$

where some of the energy from the imaginary component "spills" into the real component.

1. Give an upper-bound or exact expression for the symbol error rate as a function of α under the assumption that the maximum-likelihood receiver for $\alpha = 0$ is used (i.e. neglecting the imbalance). In other words, the decision regions remain the four quadrants of the real-imaginary plane.
2. What would the ML receiver be in the case of perfect knowledge of α?

2 Problem 2

Consider the following $N-$dimensional detection problem

$$\mathbf{y} = \sqrt{2E_s}\mathbf{h}x + \mathbf{z}$$

where $\mathbf{y}$ is an N-dimensional observation column vector, $\mathbf{h}$ is an N-dimensional zero-mean real Gaussian column vector with covariance matrix $\mathbf{K}_h = \mathrm{E}\mathbf{h}\mathbf{h}^t$, x is an equally-likely information bit which takes on the value 0 when the bit to be transmitted is zero and 1 when the bit to be transmitted is one, and z is an N-dimensional zero-mean white Gaussian column vector (i.e. diagonal covariance matrix) with variance $N_0/2$ in each component. The value of $\mathbf{h}$ is assumed to be unknown to the receiver which observes $\mathbf{y}$ and thus this is a form of non-coherent detection, although more than just phases are unknown to the receiver!

1. What are the likelihood functions under the two hypotheses (i.e. 0 and 1) as a function of K_h.
2. What is the maximum-likelihood detection rule?
3. Since K_h is a covariance matrix, it admits the following diagonalization $K_h = U\Lambda U^t$, where U is an N-dimensional unitary matrix, and Λ is an N-dimensional diagonal matrix with positive entries (eigenvalues). Suppose we create the transformed observation $\mathbf{y}' = U^t\mathbf{y}$ (this is known as a Karhunen-Loève Transform, or decorrelating transform). Show first that the transformed problem is

$$\mathbf{y}' = \sqrt{2E_s}\mathbf{h}'x + \mathbf{z}'$$

where $\mathbf{h}'$ has $\mathrm{E}h_ih_j = \lambda_i\delta_{ij}$, where $\delta_{ij} = 1$, if $i = j$, and $\delta_{ij} = 0$, $i \neq j$. In other words, the covariance matrix of $\mathbf{h}'$ is $K_{h'} = \Lambda$. Next show that the ML detection rule in terms of $\mathbf{y}'$ is

$$\text{choose } 1 \text{ if} : \sum_{i=1}^{N} \frac{\lambda_i}{\lambda_i + \frac{N_0}{2E\lambda_i}} y_i'^2 \geq \sum_{i=1}^{N} \ln\left(1 + \frac{2E}{N_0}\lambda_i\right) \tag{1}$$

3 Problem 3

Consider a receiver for OFDM system to be designed on a wireless channel. The sampling rate of the system is 40 Ms/s. The number of carriers per OFDM symbol is denoted $N_c = 256$. The length of the cyclic-prefix is $N_p = 16$. The number of useful carriers (i.e. those that are non-zero) is $N_u = 208$. One of the zeroed carriers is in the DC component (i.e. position 0 in the frequency-domain).

1. What is the occupied bandwidth for the chosen system parameters?
2. What is the maximum channel duration that the system can cope with and explain in words what effect a longer channel would have on the system performance?
3. Assuming we use 64-QAM modulation what is the spectral-efficiency of the system (spectral efficiency is measured in bits/s/Hz)?
4. In such an OFDM system, in our Lab session we plotted the channel output on a partciular carrier position over time (OFDM symbols), explain why we saw a rotated 64-QAM constellation, whose amplitude and phase depends on the carrier index

4 Problem 4

A BPSK (2-AM) signal with symbol energy E_s is generated using a square-pulse of duration T seconds,

$$p_T(t) = \begin{cases} \sqrt{\frac{1}{T}}, t \in [0, T) \\ 0, t \notin [0, T). \end{cases}$$

It is transmitted across a dispersive channel $h(t) = h_0\delta(t) + h_1\delta(t - T) + h_2\delta(t - 1.5T)$ yielding the received signal

$$r(t) = \sqrt{E_s} \sum_n a_n p(t - nT) * h(t) + z(t)$$

where a_n is the BPSK information sequence (i.e. $a_n \in \{-1, 1\}$).

1. What is the autocorrelation sequence (g_n) of the cascaded channel $p_T(t) * h(t)$.
2. How many states does the correspoding state-space representation (Ungerboeck form) have?

3. Draw the trellis
4. What is the maximum-likelihood update rule in the Viterbi algorithm for this example?