RLC Circuits and Band Pass Filter

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Electronic filters play an important rule on remote communication based on electronic signal, e.g. radio technology. In this report, we demonstrate the electronic filters. We checked that the elimination/extraction of desired bandwidth can be achieved by constructing each filters with passive components.

I. BACKGROUND

A. Electronic filters

Since the invention of remote communication via electronic signal, it became crucial for everyday life. In order to receive desired information from mixture of various signals, one must have an ability of eliminating/extracting specified bandwidth of wave frequency.

In this report, we demonstrate the electronic filters built with passive components: resistor, inductor, and capacitor. First, we establish the background theory on filters. Then, we demonstrate electrical circuits for each filters and analyze the characteristic of them. In specific manner, we aimed to observe the ability of eliminating/extracting specified bandwidth from LC notch filter/active filter and band pass filter, respectively. Finally, we discuss about Thevenin/Norton theorem and wheatstone bridge, as required in our lab module manual.

B. RC filter

The RC filter can be constructed with series connection of resistor and capacitor, as shown in figure 9, 10. Let's say that capacitor is charged with $q(t)$ in time t. Then, Kirchhoff's law on voltage gives

$$R\dot{q} + \frac{q}{C} = V_{\text{in}} \sin \omega t. \tag{1}$$

Through some calculation, we get the amplitude of voltage applied to capacitor and resistor as

$$V_{\text{C}} = \frac{1/\omega C}{\sqrt{R^2 + (1/\omega C)^2}} \cdot V_{\text{in}}. \tag{2}$$

$$V_{\text{R}} = \frac{R}{\sqrt{R^2 + (1/\omega C)^2}} \cdot V_{\text{in}}, \tag{3}$$

We may observe that $V_{\text{R}}^2 + V_{\text{C}}^2 = V_{\text{in}}^2$ holds. Moreover, when we define bandwidth(critical) frequency f_c as

$$f_c = 1/2\pi RC, \tag{4}$$

then $V_{\text{R}} > V_{\text{C}}$ for $f > f_c$, and $V_{\text{R}} < V_{\text{C}}$ for $f < f_c$. By exploiting this characteristic, we use V_{C} and V_{R} as low-pass filter and high-pass filter, respectively.

Low-pass filter works as an differentiator for $f \ll f_c$ with triangle-shaped input wave signal, and works as an integrator for $f \gg f_c$ with square-shaped input wave signal.

C. RLC circuit and resonance

In this report, we will address two kinds of RLC circuits – LC active/notch filter. Each of circuits are shown in figure 11, 12.

1. LC notch filter

Let's denote the charge of capacitor as q. Kirchhoff's law on voltage gives

$$L\ddot{q} + R\dot{q} + \frac{q}{C} = V_{\text{in}} \sin \omega t, \tag{5}$$

which is in identical format with mechanical damped oscillator,

$$m\ddot{x} + b\dot{x} + kx = F_0 \sin \omega t. \tag{6}$$

We already have a solution for (6) as[1]

$$x_p(t) = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2}} \sin(\omega t + \delta), \tag{7}$$

where

$$\delta = \arctan\left(\frac{\omega_0^2 - \omega^2}{2\omega\beta}\right), \beta = \frac{b}{2m}, \omega_0 = \sqrt{\frac{k}{m}}. \tag{8}$$

Now, we can use (7) as a solution for RLC circuit by substituting $\beta = R/2L$ and $\omega_0 = 2\pi f_0 = \sqrt{1/LC}$. The complementary solution of (7) vanishes after $t \gg 1/\beta$.

Next, let's consider the resonance of the oscillator. Through some calculation, we can derive that x's resonance and $\dot{x}$'s resonance occurs in ω_R, ω_E, respectively:

$$\omega_R = \sqrt{\omega_0^2 - 2\beta^2}, \tag{9}$$

$$\omega_E = \omega_0. \tag{10}$$

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Now we define Q-factor. A Q-factor describes the degree of damping with a definition

$$Q \equiv \frac{\omega_R}{2\beta}, \tag{11}$$

where large Q-factor represents weak damping and small Q-factor represents strong damping. When the damping force is small enough($\beta \ll \omega_0$), we may calculate the Q-factor as

$$Q \simeq \frac{\omega_0}{\Delta\omega} \tag{12}$$

where $\Delta\omega$ is the interval between two frequencies which make oscillator's amplitude as $1/\sqrt{2}$ of the maximum amplitude.

2. Active filter

Let's denote the charge of capacitor as q and electrical current of inductor as I. Kirchoff's law on current shows that the amount of current flowing through resistor is $I + dq/dt$. Then, Kirchoff's law on voltage gives

$$L\frac{dI}{dt} = \frac{q}{C}, \tag{13}$$

$$V_{\text{in}} \sin\omega t = R\left(I + \frac{dq}{dt}\right) + \frac{q}{C}. \tag{14}$$

Now we get

$$\ddot{I} + \frac{1}{RC}\dot{I} + \frac{1}{LC}I = \frac{V_{\text{in}}}{RLC}\sin\omega t. \tag{15}$$

Here, equation (15) has identical format with (6), therefore we may get solution for I as equation (7) while $\beta = 1/2RC$ and $\omega_0 = \sqrt{1/LC}$.

To sum up, we can conclude that

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \begin{cases} \frac{\left|1/LC - \omega^2\right|}{\sqrt{(1/LC-\omega^2)^2 + (\omega^2 R/L)^2}} & \text{LC notch filter,} \\ \frac{\omega/RC}{\sqrt{(1/LC-\omega^2)^2+(\omega/RC)^2}} & \text{Active filter.} \end{cases} \tag{16}$$

D. Band pass filter

The band pass filter passes desired interval of frequency from input signal. By combining low-pass filter and high-pass filter, we can construct the band pass filter as shown in figure 13. Convolution of (2) and (3) gives

$$V_{\text{out}} = V_{\text{in}} \cdot \frac{1/\omega C_1}{\sqrt{R_1^2 + (1/\omega C_1)^2}} \cdot \frac{R_2}{\sqrt{R_2^2 + (1/\omega C_2)^2}}. \tag{17}$$

Here, the output signal's frequency will range between bandwidth frequency f_c of two filters. Note that low-pass filter's f_c must be smaller than high-pass fiiter's.

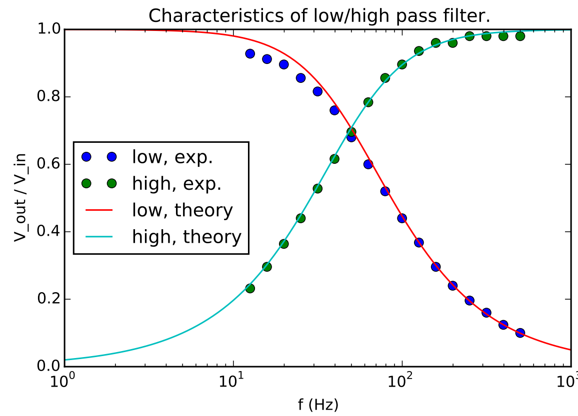

FIG. 1. $V_{\text{out}}/V_{\text{in}}$ of low pass filter and high pass filter. Here, x-axis represents log-scaled frequency and y-axis represents output voltage. Theoretical value of each plot is derived from (2) and (3). *exp.* is an abbreviation of 'experiment'.

Moreover, it is notable that band-pass filter's output signal can be significantly smaller than input. Through some calculation with (17), we can get the maximal value of $V_{\text{out}}/V_{\text{in}}$ as

$$\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)_{\text{max}} = \left[1 + 4\left(\frac{R_1C_1}{R_2C_2}\right) + \left(\frac{R_1C_1}{R_2C_2}\right)^2\right]^{-1/2} \tag{18}$$

Here, we can approximate $V_{\text{out}}/V_{\text{in}}$ as

$$V_{\text{out}}/V_{\text{in}} \simeq R_2C_2/R_1C_1. \tag{19}$$

when $R_1C_1/R_2C_2 \gg 1$.

II. EXPERIMENT & ANALYSES

Before we start, we shall clarify information of our experimental devices. Refer to table I on appendix. Moreover, throughout this experiment, we utilized an AC power from function generator represented as

$$V = V_{\text{in}} \sin\omega t = V_{\text{in}} \sin(2\pi f t), \tag{20}$$

where $V_{\text{in}} = 5.0\,\text{V}$, unless otherwise noted.

A. RC filter

In order to check the low/high pass filtering characteristic of RC circuit, we conducted two kinds of experiment. The RC filters were designed as shown in figure 9, 10. A single resistor with $R = 6.8 \times 10^4\,\Omega$ and a single capacitor with $C = 4.7 \times 10^{-8}\,\text{F}$ was connected in serial.

We measured the amplitude of output signal, V_{out}, while varying the oscillating frequency of input signal from 12.6 Hz to 500 Hz in log scale. Initially, we aimed to

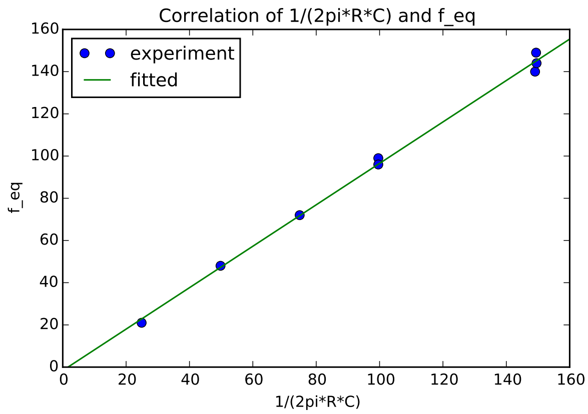

FIG. 2. Correlation of $1/(2\pi RC)$ and f_c from (4). The equation of fitted line is $y = -1.6 + 0.98x$ with $r^2 = 0.997$.

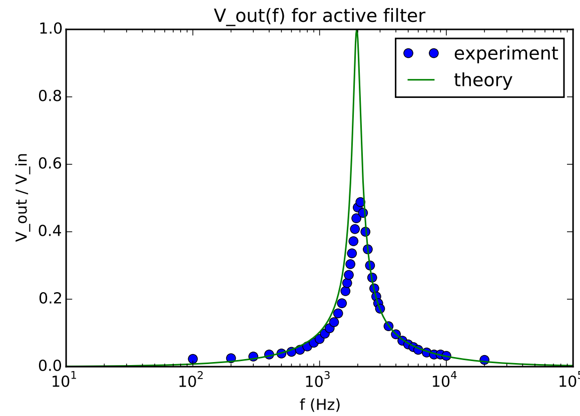

FIG. 3. V_{out}/vin of LC active filter. Theoretical values are calculated from equation (16). We could observe that output signal gets 'active' when $f \simeq f_0$.

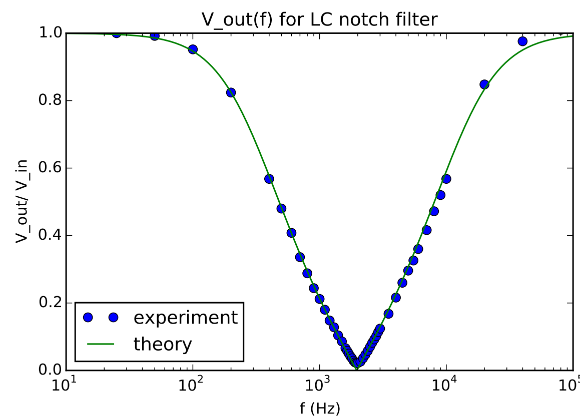

FIG. 4. $V_{\mathrm{out}}/V_{\mathrm{in}}$ of LC notch filter. Theoretical values are calculated from equation (16). We could observe that output signal gets 'notched' when $f \simeq f_0$

observe the V_{out} in low input frequency such as $f = 5\,\mathrm{Hz}$, but the function generator didn't worked properly at that frequency, therefore we abandoned them.

Both results on low/high pass filters are shown in figure 1. For each filter, theoretical value was calculated from (2) and (3) for low/high pass filter, respectively.

It could be observed that V_{out} converges to V_{in} when $f \to 0$ for low pass filter, and $f \to \infty$ for high pass filter. Moreover, V_{out} was significantly low when $f \to \infty$ for low pass filter, and $f \to 0$ for high pass filter.

Next, we checked the f_c, which was defined as (4). We aimed to measure the frequency of input signal where V_C equals V_R, while varying the LC value by connecting additional resistors and capacitors in serial/parallel. To check $V_C = V_R$, we adjusted the frequency of input signal to make $V_C = (1/\sqrt{2})V_{\mathrm{in}}$.

The result is shown in figure 2. Linear regression of data points in graph gives slope 0.98 with y-intersection -1.6 and r-squared value 0.996, which indicates that (4) holds for given circumstances.

Moreover, we observed the differentiator/integrator characteristic of low-pass filter. First, we gave square-shaped wave with $f \ll f_c$. As shown in figure 18, output signal repeatedly increasing and decreasing for a same amount in constant time ratio, showing integrator characteristic. Next, we gave triangle-shaped wave with $f \gg f_c$. As shown in figure 19, output signal was square-shaped, showing positive/negative constant voltage repeatedly with frequency f. Each output signal represented the slope of input signal, thus representing differentiator characteristic.

B. Resonance circuits

Next, we designed two kinds of circuit consisted of resistor, inductor, and capacitor. One was serial connection of them – LC active filter(figure 11), and another was parallel connection of inductor and conductor, connected with resistor in serial – LC notch filter(figure 12).

Circuits were built in breadboard and images are shown in 17. Then we measured the amplitude of output, V_{out}, while varying input frequency from $25\,\mathrm{Hz}$ to $8.0 \times 10^4\,\mathrm{Hz}$. Especially, we wrote down more data points around resonance frequency f_0. In order to make $f_0 = 1/2\pi\sqrt{LC} \simeq 2.0\,\mathrm{kHz}$, we have chosen $L = 32.8\,\mathrm{mH}$ and $C = 200\,\mathrm{nF}$, forming $f_0 = 1.97 \times 10^3\,\mathrm{Hz}$. Resistor of $R = 2.7\,\mathrm{k\Omega}$ was used to prevent excessive current.

Results are shown in figure 3, 4. For each filter, we calculated theoretical values with equation (16).

First, we could observe that output signal of LC active filter peaks around f_0. However, the peak didn't reach $V_{\mathrm{out}}/V_{\mathrm{in}} = 1.0$; the authors attribute this error to the leakage current on inductor's internal resistance.

Next, we could observe that output signal of LC notch filter plunges around f_0. We could explain this phenomenon by focusing on serial connection of inductor and capacitor – the summing signal of them vanishes since

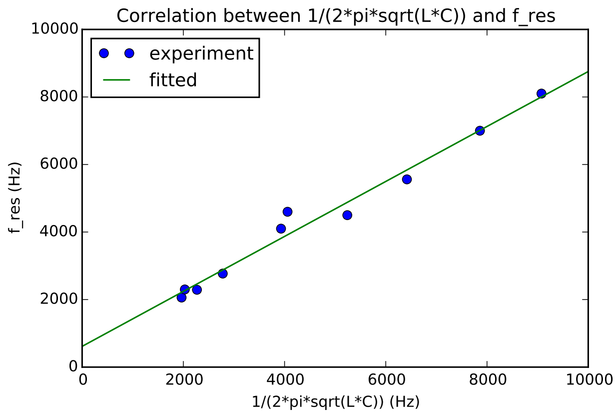

FIG. 5. Correlation of $1/2\pi\sqrt{LC}$ and $f_{\rm res}$ from (21). The equation of fitted line is $y = (6.2\times10^2\,\mathrm{Hz}) + 0.81x$ with $r^2 = 0.978$.

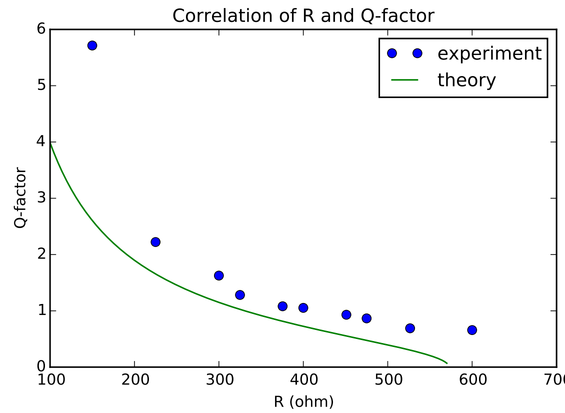

FIG. 6. Q-factor with respect to R. Note that the system shouldn't be in resonance for $R = 600\,\Omega$, but still it does.

$1/\omega L = \omega C$ holds for $f \simeq f_0$.

C. RLC circuit

In this section, we studied more on LC notch filter. We connected $R = 300\,\Omega$, $L = 32.8\times10^{-3}\,\mathrm{H}$, and $C = 2.0\times10^{-7}\,\mathrm{F}$ in series to construct a RLC circuit discussed in section I C 1, unless otherwise noted.

First, we aimed to check that the amplitude of output signal peaks at the resonant frequency,

$$f_{\rm res} = 1/2\pi\sqrt{LC}. \tag{21}$$

We varied the value of LC to check the by connecting additional inductors/capacitors in serial or parallel. For each LC value, we could get $f_{\rm res}$ by measuring the amplitude of output signal with respect to the frequency of input signal. As shown in figure 5, we could check that $f_{\rm res}$ depends on $1/2\pi\sqrt{LC}$ linearly. Though the linear model fitted well, we could observe that y-intersection is not close to 0. Moreover, the slope of fitted line was slightly smaller than 1. We can attribute this mismatch of equation (21)and figure 5 to some additional capacitance, e.g. the capacitance of BNC cable that we used.

Next, we aimed to observe the Q-factor's dependence on R. We initially aimed to vary the R from tiny value as $10\,\Omega$ to maximal value for resonance, but failed since input signal was unstable for small R values. For every R values ranging from $150\,\Omega$ to $600\,\Omega$, we varied f to find the width of resonance's peak. In concise manner, we found two f value that gives $V_{\rm out} = V_{\rm in}/\sqrt{2}$.

The result had significant error for small R values, as shown in figure 6. We explain those error in three ways. First, we attribute the first error to the mechanical limitations of our function generator. When R is too small, the resonance gets too huge that function generator can't provide efficient amount of energy. Second, when R is too large, then (12) fails, generally. Third, we can say that there were some mismatch between our L and C value since the system shouldn't have oscillate at $R = 600\,\Omega$, but it did. As $Q \to 0$ for such maximal R, the error rate spiked for large R values.

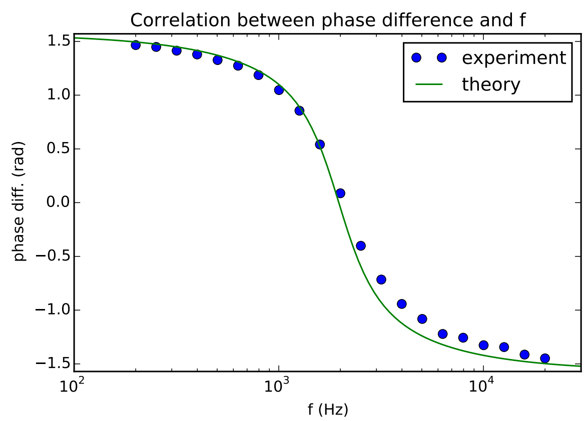

FIG. 7. Phase difference between input/output signal with respect to f. Consistent errors on $f > f_{\rm res}$ was notable.

Finally, we measured the phase difference between input/output signal, which was shown in equation (8). To plot $\delta(f)$, we varied f from $200\,\mathrm{Hz}$ to $2.0\times10^4\,\mathrm{Hz}$ in logarithmic scale. As a result, we got figure 7, which was consistent with (8), while some errors were notable for $f > f_{\rm res}$. We attribute this error to the frequency dependence of resistance of a metal resistor we've used. As f increases, R increase, making $\beta = R/2L$ smaller. This makes the absolute value of δ smaller, thus the error for $f > f_{\rm res}$ is explained.

D. Band pass filter

Finally, we demonstrated the band pass filter by combining low-pass filter and high-pass filter. Our circuit is shown in figure 8, where used $R_1 = 2.7\times10^3\,\Omega$,

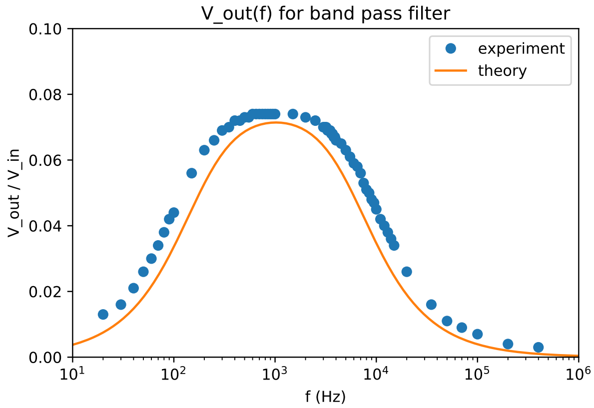

FIG. 8. Characteristic of band pass filter. Note that the maximal y-value of this plot is not 1.

$C_1 = 3.0 \times 10^{-7}\,\mathrm{F}$, $R_2 = 3.0 \times 10^2\,\Omega$, $C_2 = 1.0 \times 10^{-7}\,\mathrm{F}$.

We should consider that the bandwidth frequency of low-pass filter must be lower than high-pass filter's one. The bandwidth frequency of each filter is

$$f_{\mathrm{low}} = 1/2\pi R_1 C_1 = 1.96 \times 10^2\,\mathrm{Hz}, \tag{22}$$

$$f_{\mathrm{high}} = 1/2\pi R_2 C_2 = 5.30 \times 10^3\,\mathrm{Hz}. \tag{23}$$

Here, the filter will pass the signal with frequency $f_{\mathrm{low}} \leq f \leq f_{\mathrm{high}}$.

The resulting output voltage with input's frequency varying from 20 Hz to 4.0×10^5 Hz is shown in figure 8. We could check that the magnitude of output signal is approximately $R_2C_2/R_1C_1 \simeq 0.04$ times smaller than input, which is consistent with (18). Though there were slight difference between theoretical/experimental value, it is acceptable since it can be attributed to an error of resistance/capacitance value.

III. CONCLUSION

In this report, we established the theory on electric filters and demonstrated the ability of eliminating/extracting signal of specified bandwidth. Elimination was shown in LC notch filter and extraction was shown in LC active filter & band pass filter.

Specifically, we could find that strength of the signal significantly weakens when it passes through the band pass filter. To achieve the functionality of 'filter', it should pass the signal in original strength for desired frequency, e.g. RC low pass filter. Therefore, more work should be done on re-amplifying the signal or find more appropriate filters to create band pass filter which passes the desired signal in original strength.

IV. EXTRA DISCUSSION

A. Thevenin's theorem

Thevenin's theorem states that any electrical circuit consisted of resistances and power sources can be replaced by a single with serial connection of a single voltage source and a single resistor, as shown in figure 15.

B. Norton's theorem

Norton's theorem resembles Thevenin's theorem, but it utilizes parallel connection of a single current source with a single resistor, instead of a single voltage source, as shown in figure 16.

C. Wheatstone bridge

Wheatstone bridge (figure 14) is an circuit designed for measuring the resistance of unknown resistor. Here, we have two resistors R_1, R_3 and one variable resistor R_2. When

$$R_1 R_x = R_2 R_3 \tag{24}$$

holds, there will be no current flowing through the ammeter in middle of the circuit. By adjusting the resistance of variable resistor R_2, we can check that there's no current flowing through the middle, thus (24) holds. Therefore, we can calculate R_x. This wheatstone bridge can be also applied on measuring the capacitance or inductance of capacitor or inductor, by substituting the reactance of each components to equation (24).

ACKNOWLEDGMENTS

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V. REFERENCES

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Appendix A: Circuit diagrams

FIG. 9. Low pass filter. See section I B for details.

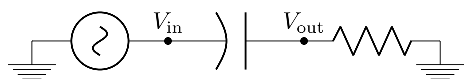

FIG. 10. High pass filter. See section I B for details.

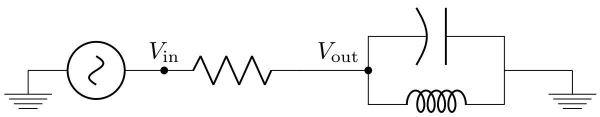

FIG. 11. LC active filter. See section I C 2 for details.

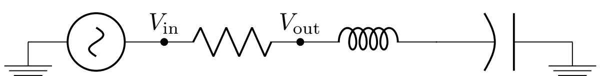

FIG. 12. LC notch filter. See section I C 1 for details.

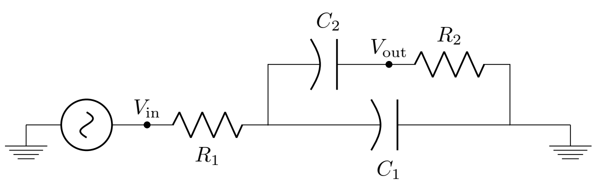

FIG. 13. Band pass filter. Only signal within specified frequency bandwidth can pass through this filter.

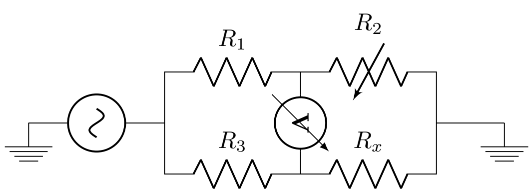

FIG. 14. Wheatstone bridge. Here, the variabel resistor changes its value to satisfy (24).

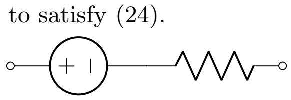

FIG. 15. Thevenin equivalent circuit. The left part indicates a single voltage source.

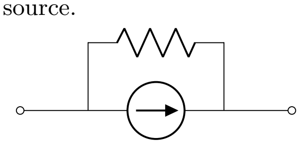

FIG. 16. Norton equivalent circuit. The lower part of parallel connection indicates a single current source.

Appendix B: Breadboard images: section II B

We show our breadboard images of LC active filter and LC notch filter, as required in our lab module manual.

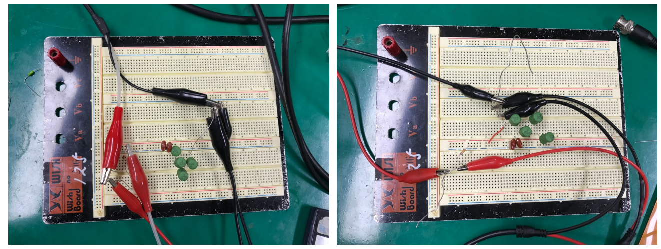

FIG. 17. Breadboard image and artwork of LC active filter(left) and LC notch filter(right).

Appendix C: Screenshot of oscilloscope

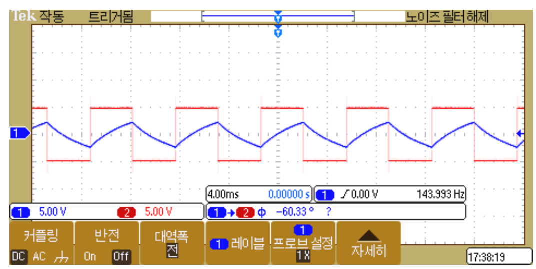

FIG. 18. Integrator characteristic appears is low-pass filter for $f \ll f_c$ with square-shaped wave input signal. Square-shaped wave with $V_{\text{pp}} = 10\,\text{V}$ and $f = 150\,\text{Hz}$ was used for input signal.

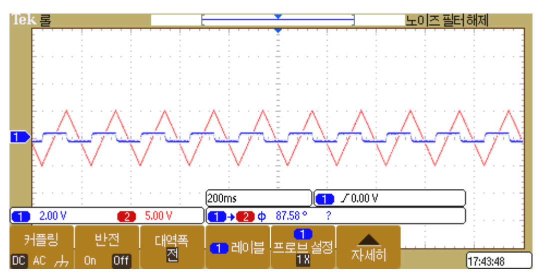

FIG. 19. Differentiator characteristic appears in low-pass filter for $f \gg f_c$ with triangle-shaped wave input signal. Triangle shaped wave with $V_{\text{pp}} = 10\,\text{V}$ and $f = 5\,\text{Hz}$ was used for input signal.

Appendix D: Info. on experimental devices

Here, we clarify the devices that we've used to conduct our experiments.

TABLE I. Information on experimental devices.

Device	model name / info
Function generator	Agilent 33250A
Oscilloscope	Tektronix DPO 2024
Resistor	Metal resistors
Inductor	(no info. to provide)
Capacitor	Polyester capacitors