Friction Factors and Drag Coefficients

Several equations that we have seen have included terms to represent dissipation of energy due to the viscous nature of fluid flow. For example, in the energy balance, the dissipation function Φ represents rate of viscous dissipation in units of energy/(volume time),

$$\rho \frac{\mathrm{D}u}{\mathrm{D}t} = \frac{\partial}{\partial x_i}\left(k \frac{\partial T}{\partial x_i}\right) + \dot{q} - p\,(\nabla \bullet \mathbf{v}) + \Phi \tag{1}$$

In the extended version of the Bernoulli Equation from fluid mechanics

$$\frac{v_2{}^2 - v_1{}^2}{2} + g(z_2 - z_1) + \frac{p_2 - p_1}{\rho} = -W_\mathrm{f} - W_\mathrm{S} \tag{2}$$

the W_f term represents work done against frictional forces per mass of fluid, in units of energy/mass, as it flows from point 1 to point 2 along a streamline (Note: strictly speaking, equation (2) holds for steady, incompressible flow along a streamline. However, for flows that are irrotational ($\nabla \times \mathbf{v} = \mathbf{0}$), equation (2) holds between any two points in the flow.) W_S represents shaft work per mass. In the earlier handout on dimensional analysis we used Φ to calculate W_f. In this handout, we will take a closer look at how to perform calculations of W_f in various flow geometries starting with the favorite, a piece of straight pipe.

Friction Factors for Internal Flow.
We recall from an earlier handout that, for pipe flow, W_f was the amount of mechanical energy dissipated to internal energy per unit mass of fluid flowing from location 1 to 2 in the below figure. W_f is often used to define a "head loss" H_L,

$$W_\mathrm{f} = gH_\mathrm{L} \tag{3}$$

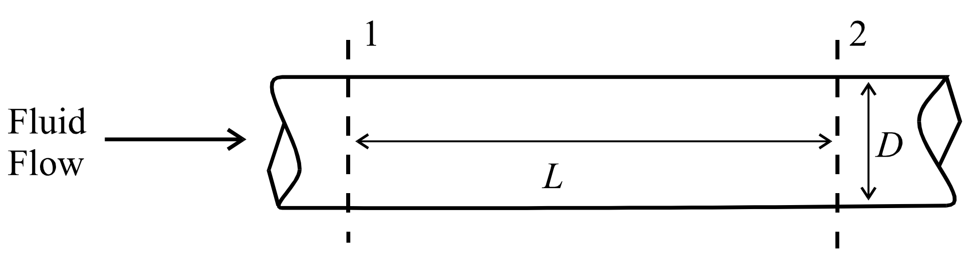


Figure 1.

Multiplying H_L by density of the fluid being pumped and g yields the pressure drop needed to overcome the viscous resistance represented by W_f and keep the fluid flowing at a steady pace. This conclusion can be deduced by careful consideration of equations (2) and (3).

The **friction factor** f for pipe flow is defined as,

$$f = \frac{W_f}{\frac{1}{2}V_o^2}\left(\frac{D}{L}\right) \tag{4}$$

where V_o is the average fluid velocity (V_o = volumetric flowrate / pipe cross-sectional area). Note that f is dimensionless. Therefore, measurements of f on a model system as a function of relevant dimensionless variables (typically, these variables are Reynolds number Re and a dimensionless wall surface roughness referred to as e^*) can be used to create correlations for determining W_f by looking up f for geometrically and dynamically similar systems and then using a re-arranged equation (4)

$$W_f = \frac{f\,V_o^2}{2}\left(\frac{L}{D}\right) \tag{5}$$

For laminar, steady state, incompressible, constant viscosity Newtonian parabolic flow in a pipe we previously calculated f to be

$$f = 64\,\mu / (\rho V_o D) = 64 / Re \tag{6}$$

Equation (6) gives the friction factor f for cylindrical (circular cross-section) pipes. If the value of the Reynolds number is known, f can be calculated from (6) and the viscous dissipation W_f follows from (5). Some textbooks may define f a little differently, resulting in a numerical prefactor in (6) other than 64 (a prefactor of 16 is common). As long as one is consistent about how f and Re are defined no difficulties will arise.

What happens when the flow is turbulent ($Re > 2300$)? Using dimensional analysis and the Buckingham Pi Theorem, with f as the dependent variable and the set of parameters ρ, μ, V_o, D, and a characteristic pipe wall roughness lengthscale e, we could show that f should obey

$$f = f\,(Re, e^*) \tag{7}$$

In equation (7), $e^* = e/D$ is a dimensionless roughness of the pipe wall. In general, f for turbulent flow cannot be calculated analytically. Rather, f has been determined experimentally as a function of Re and e^* for a model set of pipes (see figure on next page). On the left side of the figure, the flow is laminar and the correlation is given by the analytical relationship in equation (6). At about $Re = 2300$ a laminar to turbulent transition typically starts to set in. When Re reaches about 3500, a well developed turbulent flow is usually present – in this region the correlation was determined experimentally.

Given the pipe roughness e^* and the Reynolds number, the value of f can be read from the figure and used to calculate W_f or head loss H_L using expressions given above. If the only parameters relevant to pipe flow are indeed represented by the set ρ, μ, V_o, D, and e, then f must be the same for two *different* pipe flows provided both flows are characterized by the same value of Re and the same value of e^*. This is a direct consequence of dynamic similarity.

In practice, the details of pipe wall roughness often are not adequately described by a single parameter e^*. Based on the specific type of pipe, a pipe manufacturer might provide a more detailed set of measurements for f.

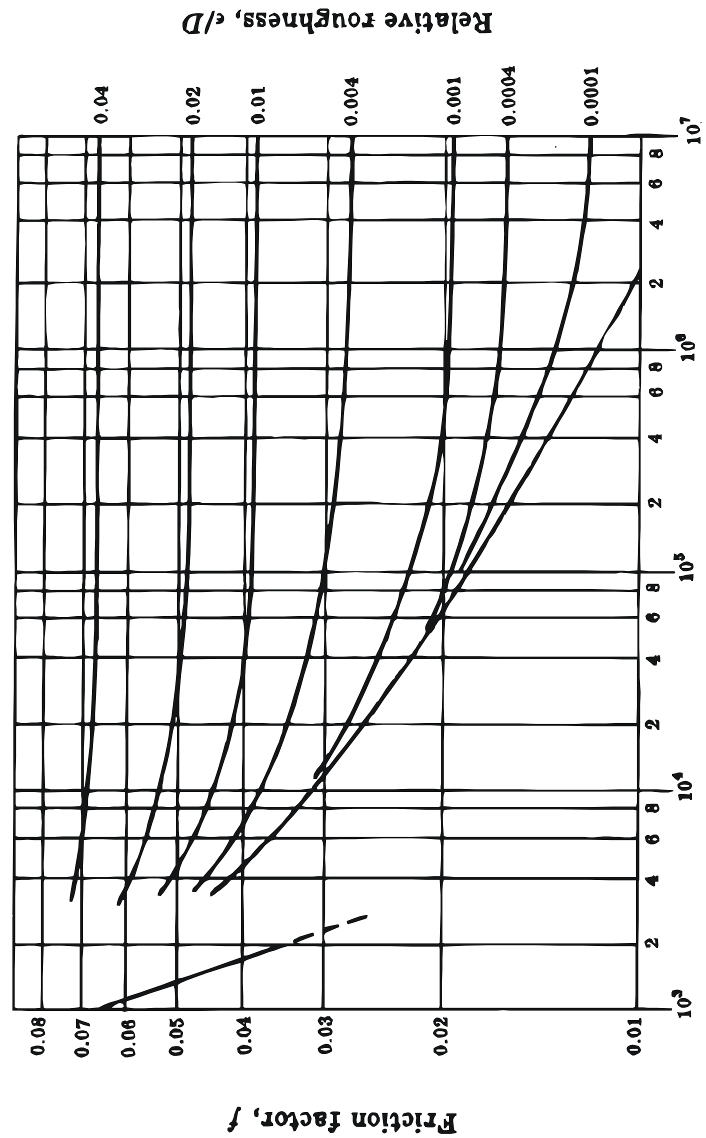


Fig. 5-21. **Friction factors for flow in pipes.**

If a pipe is not circular, f from the above figure can still be used as a rough approximation for the friction factor. For a pipe with a cross-section other than circular, the effective pipe diameter D' is defined as

$$D' = 4\,A/P_w \qquad (8)$$

where A is the cross-sectional area available for flow and P_w is the so-called "**wetted perimeter**." The wetted perimeter is simply the perimeter of the area in which the fluid is in direct contact with the confining solid walls. The ratio A/P_w is also referred to as the "**hydraulic radius**" R_H. For a circular pipe $A = \pi R^2$ and $P_w = 2\pi R$ (assuming that the pipe is completely filled with fluid), so that $D' = 2R$ as expected. For a square pipe with length B for each side, $A = B^2$ and $P_w = 4B$, so that $D' = B$. The value of D' is used to calculate the Reynolds number and e^*, and these two quantities are then used to read off f from the figure. Since the procedure is only approximate, if precision is critical it would be better to measure the frictional dissipation directly.

Loss Coefficients for Flow Through Valves and Fittings. In the previous section, we discussed frictional losses for flow through a straight length of pipe. What happens if the fluid passes through a valve of some type, or through an elbow (bend) in the pipe, or through a region where sudden expansion or contraction occur (Figure 2)?

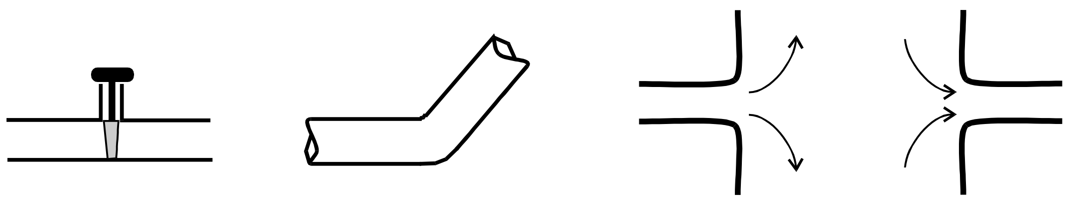


Figure 2.

All such features with which a flowing, viscous fluid interacts give rise to frictional losses. The losses are difficult to estimate analytically, and are typically measured experimentally and then tabulated. Usually the losses are expressed in terms of **loss coefficients** K defined by

$$K = W_f / (V_o{}^2/2) \tag{9}$$

where W_f is the frictional dissipation of mechanical energy to internal energy associated with the passage of a unit mass of fluid through the fitting. The physical meaning of W_f is same as earlier, but with the fluid flowing through the fitting rather than a straight piece of pipe. V_o is the average fluid velocity through the fitting. Note that, in contrast to the friction factor f in equation (4), the ratio D/L is not used in the definition of K since it would be constant for a given type of fitting and thus is absorbed into the definition for K. Several approximate K values for high Re flows are shown in the below table.

Valves, Fittings and Piping	K
globe valve (wide open)	10.0
gate valve (wide open)	0.19
90^o elbow	0..90
45^o elbow	0.42
sharp-edged entrance to circular pipe	0.50
rounded entrance to circular pipe	0.25
sudden expansion	$(1 - A_1/A_2)^2$; A_1 upstream area, A_2 downstream area

Why are the K values listed constant, rather than presented as a function of the Reynolds number as with the friction factor f ? Certainly, the loss coefficients K will vary with Re, roughness of fitting components, and possibly other parameters. However, at sufficiently high Re, $Re > 10^5$ or so, the loss coefficients depend only weakly on Re and so are approximated as constant in the above table. A similarly weakened dependence also arises for f, as can be seen in the earlier plot.

Drag Coefficients for External Flow. For external flows, it is often necessary to determine the drag force on an object in contact with a flowing fluid. This problem can be difficult to solve directly with the conservation equations, and so the required correlations are developed experimentally. However, there are also important cases where analytical results have been obtained. Some of these will be mentioned below.

The dimensionless quantity that expresses the magnitude of the drag force for external flows is the **drag coefficient** C_D, defined by

$$C_D = \frac{F_f}{A\frac{\rho V_o^2}{2}} \tag{10}$$

where F_f is the total drag force acting on the object, A is a reference area, and V_o is a reference velocity. F_f contains both form drag (arising from normal stresses exerted by the fluid on the object) and viscous drag (arising from shear stresses exerted by the fluid on the object) - it is the total drag force. For flow over a flat plate, A is the surface area of the plate. Otherwise, A is most often the projected area of the object normal to the direction in which the fluid is flowing. For instance, for a sphere $A = \pi R^2$ where R is the sphere radius. For a cylinder of radius R and length L, oriented with its axis perpendicular to the direction of flow (Figure 3), $A = 2RL$. Since C_D is dimensionless, it can only depend on other dimensionless groups (such as the Reynolds number). These dimensionless groups can be identified through dimensional analysis of the applicable conservation and other laws, or through the Buckingham Pi Theorem. Below are some examples of drag coefficient correlations.

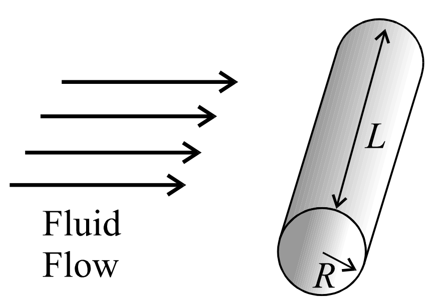


Figure 3.

Flow Past a Sphere. A classical example in transport phenomena is the derivation of the drag force on a sphere in the so-called "creeping flow" approximation, characterized by *Re* < 0.1. This analytical result for an incompressible, laminar, constant viscosity Newtonian flow leads to

$$F_f = 6\pi\mu R V_o \tag{11}$$

Inserting equation (11) into (10), and using $A = \pi R^2$ and $R = D/2$, where D is the sphere diameter

$$C_D = 24\ \mu / (\rho D V_o) = 24 / Re \tag{12}$$

Equation (12) gives the drag coefficient for creeping flow around a smooth sphere, and was derived theoretically.

For values of *Re* greater than about 0.1, charts are available that plot the drag coefficient as a function of the Reynolds number. One such chart is reproduced on the next page. For *Re* less than 1, the Stoke's Law (i.e. creeping flow) formula holds increasingly well as *Re* decreases. At such low *Re* values, the flow around the sphere is laminar.

For *Re* values greater than 1, the information on the plot comes primarily from experimental measurements. As the Reynolds number increases past 1, the total drag increases faster than linearly with V_o. This results in the upward deviation from the asymptotic creeping flow prediction, which just goes linearly with V_o (equation (11)). The growth in total drag is primarily due to increasing form drag, which dominates over the skin friction contribution as *Re* becomes larger. The boundary layer region near the surface of the sphere is still laminar at this stage. With *Re* increasing past ~ 50, a boundary layer begins to separate from the surface, Figure 4. The turbulent wake which forms behind the sphere is a region of low pressure, and is responsible for the prominent increase in form drag. At sufficiently high *Re* ($1000 < Re < 10^5$), a regime in which the drag force goes approximately as V_o^2 is observed. This regime is sometimes termed the Newton's Law region (see plot). Here, the boundary layer region before the point of separation is *still* laminar. As *Re* increases yet further, the boundary layer region becomes turbulent and a sudden drop in the drag force (and the drag coefficient) occurs. This drop takes place at about $Re = 10^5$. The reason for the drop in drag is that, as a result of the boundary layer region becoming turbulent, the point of separation moves back along the sphere surface, Figure 5. This shift in the point of separation decreases the size of the low pressure wake region, thereby decreasing the magnitude of the form drag.

If the sphere surface is roughened, as in the case of golf balls or baseballs, the laminar to turbulent transition of the boundary layer and the accompanying reduction in the drag force will take place at lower *Re*. Form drag can also be reduced by gradually tapering the rear portion of a body, referred to as streamlining. An airplane wing is a good example of a streamlined body. However, streamlining increases the surface area of the body, contributing to an increase in its skin friction (viscous) drag. Eventually, a decrease in form drag will be more than offset by an increase in viscous drag. The minimization of the total drag force therefore reflects a compromise between such competing requirements.

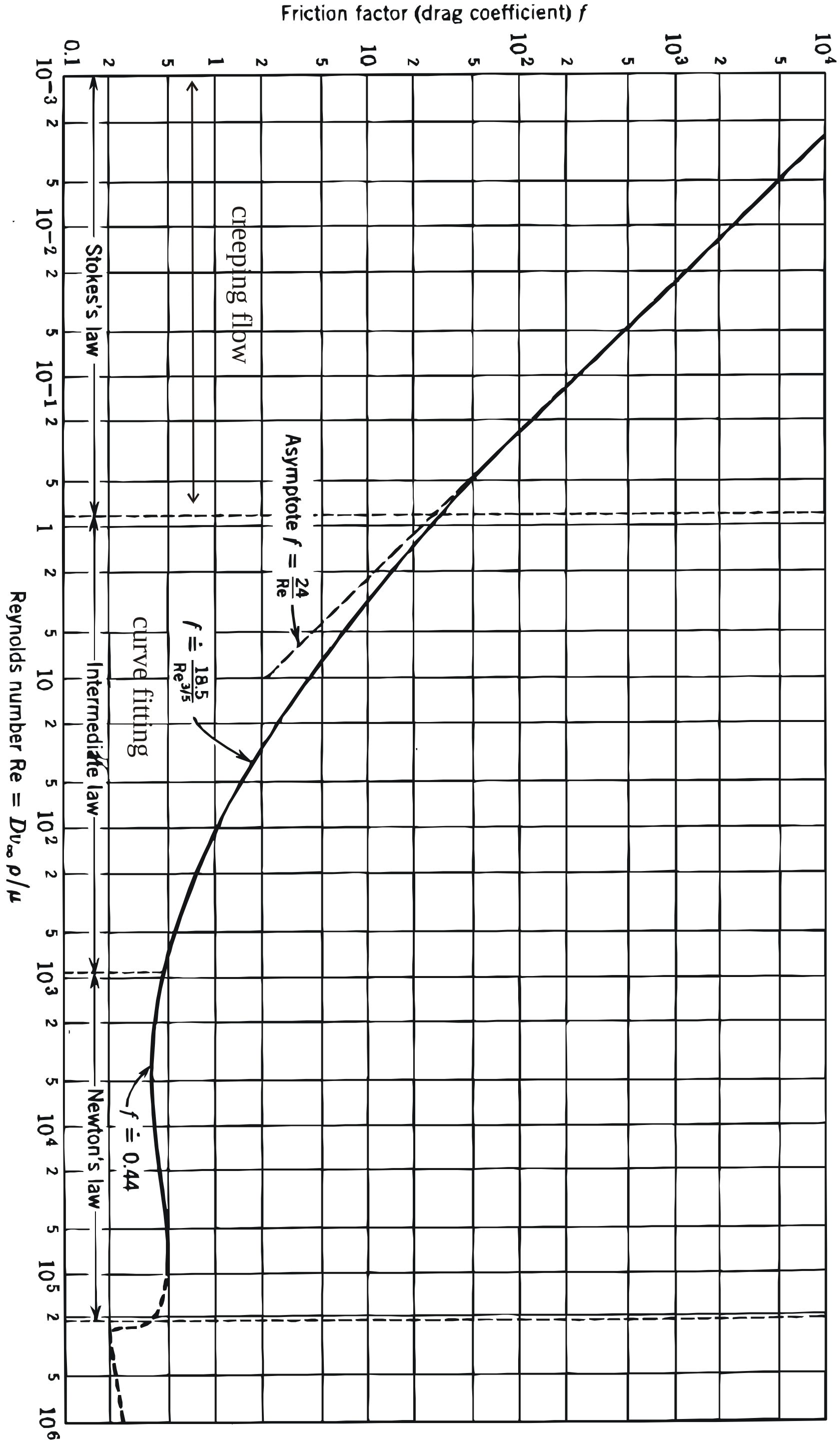
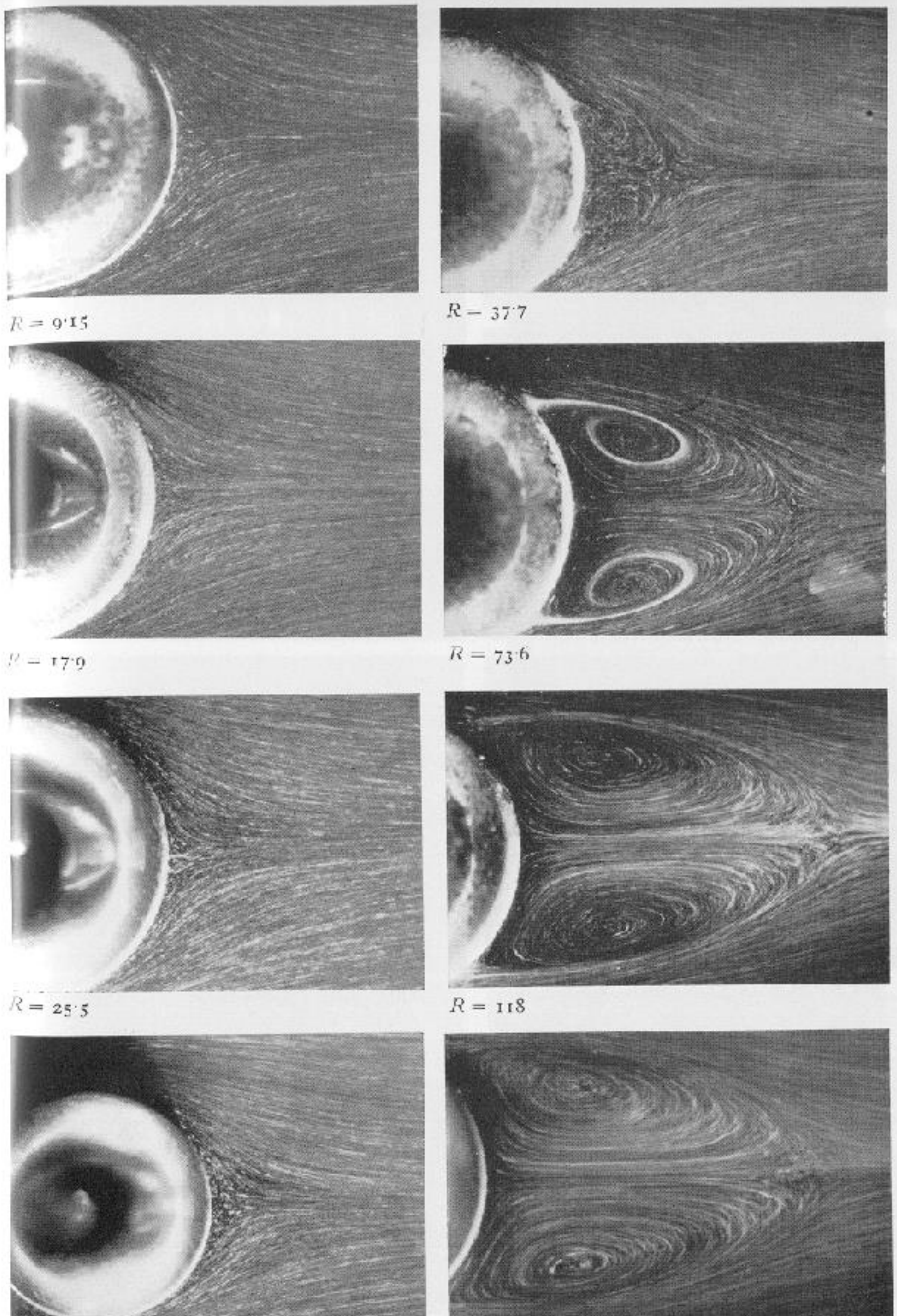


Fig. 6.3-1. Friction factor (or drag coefficient) for spheres moving relative to a fluid with a velocity v_∞. See definition of f in Eq. 6.1-5. [Curve taken from C. E. Lapple, "Dust and Mist Collection," in *Chemical Engineers' Handbook* (ed. by J. H. Perry), McGraw-Hill, New York (1950), Third Edition, p. 1018.]

PLATE 3



$R = 9{\cdot}15$, $R = 37{\cdot}7$, $R = 17{\cdot}9$, $R = 73{\cdot}6$, $R = 25{\cdot}5$, $R = 118$, $R = 26{\cdot}8$, $R = 133$

Figure 4.12.8. Streamlines, in an axial plane, of steady flow (from left to right) past a sphere of radius a (from Taneda 1956b); $R = 2aU/\nu$.

Figure 4

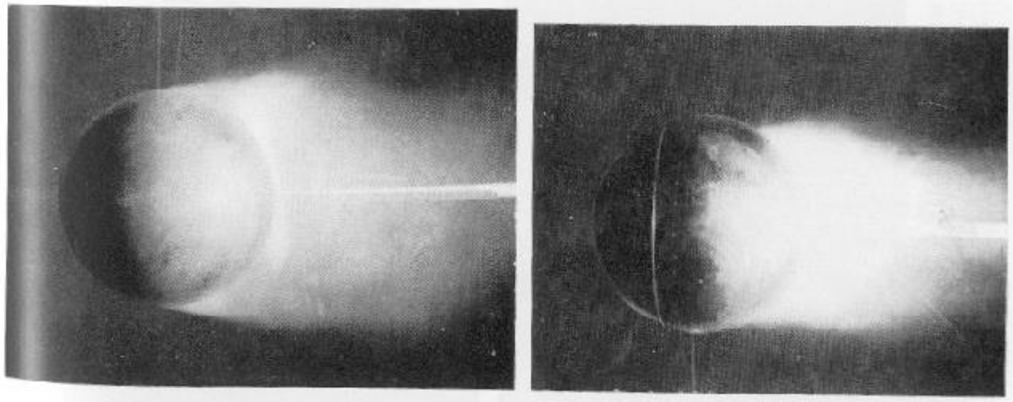


Figure 5.11.7. Smoke released at the rear of a sphere in a stream flowing from left to right. The second photograph shows the effect of disturbing the boundary layer with a wire. (From Wieselsberger 1914.)

Figure 5

Flow Past a Cylinder. The preceding discussion of flow past a sphere pointed out several typical features of external flow. For external flow over a cylinder the features observed in the drag coefficient curve are very similar and share similar origins. The drag coefficient for incompressible Newtonian flow past a smooth, circular cylinder can be found in most texts on fluid mechanics.

Flow Past a Flat Plate. This type of external flow is typically modeled in the context of a boundary layer. For incompressible, Newtonian, laminar flow past a flat plate oriented parallel to the flow, it is found that

$$C_D = 1.328 / Re_L^{1/2} \tag{13}$$

where

$$Re_L = \rho V_o L/\mu \tag{14}$$

In equation (14), L is the length of the plate (the "length" is the extent of the plate along the direction of flow). The reference area A is defined as the surface area of the plate, $A = WL$, where W is the width of the plate. The above expression is for laminar flows, and would change for a turbulent flow past a flat plate. The flow is assumed to take place along both sides (top and bottom) of the flat plate, so C_D accounts for drag exerted on both the top and bottom plate surfaces.