Definition of TGeoHMatrix

```
In[50]:= RM[phi_, tht_, psi_, x_, y_, z_] := {
           {Cos[tht] Cos[phi], -Cos[tht] Sin[phi], Sin[tht] , 0},
           {Sin[psi] Sin[tht] Cos[phi] + Cos[psi] Sin[phi],
            -Sin[psi] Sin[tht] Sin[phi] + Cos[psi] Cos[phi], -Cos[tht] Sin[psi], 0},
           {-Cos[psi] Sin[tht] Cos[phi] + Sin[psi] Sin[phi],
            Cos[psi] Sin[tht] Sin[phi] + Sin[psi] Cos[phi], Cos[tht] Cos[psi], 0},
           {x, y, z, 1}
          };
```

Definition of TGeoHMatrix multiplication: ML * MR

```
In[51]:= MPRD[ML_, MR_] := {
          {ML[[1, 1]] * MR[[1, 1]] + ML[[1, 2]] * MR[[2, 1]] + ML[[1, 3]] * MR[[3, 1]],
           ML[[1, 1]] * MR[[1, 2]] + ML[[1, 2]] * MR[[2, 2]] + ML[[1, 3]] * MR[[3, 2]],
           ML[[1, 1]] * MR[[1, 3]] + ML[[1, 2]] * MR[[2, 3]] + ML[[1, 3]] * MR[[3, 3]], 0},
          {ML[[2, 1]] * MR[[1, 1]] + ML[[2, 2]] * MR[[2, 1]] + ML[[2, 3]] * MR[[3, 1]],
           ML[[2, 1]] * MR[[1, 2]] + ML[[2, 2]] * MR[[2, 2]] + ML[[2, 3]] * MR[[3, 2]],
           ML[[2, 1]] * MR[[1, 3]] + ML[[2, 2]] * MR[[2, 3]] + ML[[2, 3]] * MR[[3, 3]], 0},
          {ML[[3, 1]] * MR[[1, 1]] + ML[[3, 2]] * MR[[2, 1]] + ML[[3, 3]] * MR[[3, 1]],
           ML[[3, 1]] * MR[[1, 2]] + ML[[3, 2]] * MR[[2, 2]] + ML[[3, 3]] * MR[[3, 2]],
           ML[[3, 1]] * MR[[1, 3]] + ML[[3, 2]] * MR[[2, 3]] + ML[[3, 3]] * MR[[3, 3]], 0},
          {ML[[4, 1]] + ML[[1, 1]] * MR[[4, 1]] + ML[[1, 2]] * MR[[4, 2]] + ML[[1, 3]] * MR[[4, 3]],
           ML[[4, 2]] + ML[[2, 1]] * MR[[4, 1]] + ML[[2, 2]] * MR[[4, 2]] + ML[[2, 3]] * MR[[4, 3]],
           ML[[4, 3]] + ML[[3, 1]] * MR[[4, 1]] +
            ML[[3, 2]] * MR[[4, 2]] + ML[[3, 3]] * MR[[4, 3]], 1}
         };
```

Unity matrix

```
In[52]:= U = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};
```

Transformation TGeoHMatrix matrices, usually we use RI = Inverse[R]

```
In[53]:= R = {{rd[0], rd[1], rd[2], 0}, {rd[3], rd[4], rd[5], 0},
             {rd[6], rd[7], rd[8], 0}, {td[0], td[1], td[2], 1}};
         RI = {{ri[0], ri[1], ri[2], 0}, {ri[3], ri[4], ri[5], 0},
             {ri[6], ri[7], ri[8], 0}, {ti[0], ti[1], ti[2], 1}};
```

Input parameters defining delta TGeoHMatrix : alignment increment. We assume that the parameters are so small
that cos(x)->1 and sin(x)->x approximation is valid

```
In[55]:= tau = RM[dphi, dtht, dpsi, dtx, dty, dtz];
         rule1 = {Cos[dphi] → 1, Cos[dpsi] → 1, Cos[dtht] → 1,
           Sin[dphi] → dphi, Sin[dpsi] → dpsi, Sin[dtht] → dtht};
         rule2 = {dphi * dpsi → 0, dpsi * dtht → 0, dphi * dtht → 0};
         tauS = tau /. rule1 /. rule2;
```

We need to compute transformation of delta matrix (tau) from its frame to another frame (vectors trasform as V = R*v) and take its
component linear in tau input paramets. The final aim is to have the sum of transformations of child volumes to be unity matrix in
their parent's frame, i.e. $\Sigma\, R_i \tau\, \mathrm{RInv}_i = \mathrm{I}$, hence we can require $\Sigma\, R_i (\tau\text{-I})\, \mathrm{RInv}_i = 0$.

In[59]:=
```
tauSU = tauS - U;
```

In[60]:=
```
TAUU = MPRD[R, MPRD[tauSU, RI]]
```

Out[60]=
```
{{rd[2] (-dtht ri[0] + dpsi ri[3]) +
    rd[1] (dphi ri[0] - dpsi ri[6]) + rd[0] (-dphi ri[3] + dtht ri[6]),
   rd[2] (-dtht ri[1] + dpsi ri[4]) + rd[1] (dphi ri[1] - dpsi ri[7]) +
    rd[0] (-dphi ri[4] + dtht ri[7]), rd[2] (-dtht ri[2] + dpsi ri[5]) +
    rd[1] (dphi ri[2] - dpsi ri[8]) + rd[0] (-dphi ri[5] + dtht ri[8]), 0},
  {rd[5] (-dtht ri[0] + dpsi ri[3]) + rd[4] (dphi ri[0] - dpsi ri[6]) +
    rd[3] (-dphi ri[3] + dtht ri[6]), rd[5] (-dtht ri[1] + dpsi ri[4]) +
    rd[4] (dphi ri[1] - dpsi ri[7]) + rd[3] (-dphi ri[4] + dtht ri[7]),
   rd[5] (-dtht ri[2] + dpsi ri[5]) + rd[4] (dphi ri[2] - dpsi ri[8]) +
    rd[3] (-dphi ri[5] + dtht ri[8]), 0}, {rd[8] (-dtht ri[0] + dpsi ri[3]) +
    rd[7] (dphi ri[0] - dpsi ri[6]) + rd[6] (-dphi ri[3] + dtht ri[6]),
   rd[8] (-dtht ri[1] + dpsi ri[4]) + rd[7] (dphi ri[1] - dpsi ri[7]) +
    rd[6] (-dphi ri[4] + dtht ri[7]), rd[8] (-dtht ri[2] + dpsi ri[5]) +
    rd[7] (dphi ri[2] - dpsi ri[8]) + rd[6] (-dphi ri[5] + dtht ri[8]), 0},
  {td[0] + rd[2] (dtz - dtht ti[0] + dpsi ti[1]) + rd[1] (dty + dphi ti[0] - dpsi ti[2]) +
    rd[0] (dtx - dphi ti[1] + dtht ti[2]), td[1] + rd[5] (dtz - dtht ti[0] + dpsi ti[1]) +
    rd[4] (dty + dphi ti[0] - dpsi ti[2]) + rd[3] (dtx - dphi ti[1] + dtht ti[2]),
   td[2] + rd[8] (dtz - dtht ti[0] + dpsi ti[1]) + rd[7] (dty + dphi ti[0] - dpsi ti[2]) +
    rd[6] (dtx - dphi ti[1] + dtht ti[2]), 1}}
```

In[61]:=
```
MatrixForm[TAUU]
```

Out[61]//MatrixForm=
```
⎛                rd[2] (-dtht ri[0] + dpsi ri[3]) + rd[1] (dphi ri[0] - dpsi ri[6]) + rd[0] (-dphi
⎜                rd[5] (-dtht ri[0] + dpsi ri[3]) + rd[4] (dphi ri[0] - dpsi ri[6]) + rd[3] (-dphi
⎜                rd[8] (-dtht ri[0] + dpsi ri[3]) + rd[7] (dphi ri[0] - dpsi ri[6]) + rd[6] (-dphi
⎝td[0] + rd[2] (dtz - dtht ti[0] + dpsi ti[1]) + rd[1] (dty + dphi ti[0] - dpsi ti[2]) + rd[0]
```

In[62]:=
```
MatrixForm[D[TAUU, dphi]]
```

Out[62]//MatrixForm=
```
⎛rd[1] ri[0] - rd[0] ri[3]  rd[1] ri[1] - rd[0] ri[4]  rd[1] ri[2] - rd[0] ri[5]  0⎞
⎜rd[4] ri[0] - rd[3] ri[3]  rd[4] ri[1] - rd[3] ri[4]  rd[4] ri[2] - rd[3] ri[5]  0⎟
⎜rd[7] ri[0] - rd[6] ri[3]  rd[7] ri[1] - rd[6] ri[4]  rd[7] ri[2] - rd[6] ri[5]  0⎟
⎝rd[1] ti[0] - rd[0] ti[1]  rd[4] ti[0] - rd[3] ti[1]  rd[7] ti[0] - rd[6] ti[1]  0⎠
```

In[63]:=
```
MatrixForm[D[TAUU, dpsi]]
```

Out[63]//MatrixForm=
```
⎛rd[2] ri[3] - rd[1] ri[6]  rd[2] ri[4] - rd[1] ri[7]  rd[2] ri[5] - rd[1] ri[8]  0⎞
⎜rd[5] ri[3] - rd[4] ri[6]  rd[5] ri[4] - rd[4] ri[7]  rd[5] ri[5] - rd[4] ri[8]  0⎟
⎜rd[8] ri[3] - rd[7] ri[6]  rd[8] ri[4] - rd[7] ri[7]  rd[8] ri[5] - rd[7] ri[8]  0⎟
⎝rd[2] ti[1] - rd[1] ti[2]  rd[5] ti[1] - rd[4] ti[2]  rd[8] ti[1] - rd[7] ti[2]  0⎠
```

In[64]:=
```
MatrixForm[D[TAUU, dtht]]
```

Out[64]//MatrixForm=
```
⎛-rd[2] ri[0] + rd[0] ri[6]  -rd[2] ri[1] + rd[0] ri[7]  -rd[2] ri[2] + rd[0] ri[8]  0⎞
⎜-rd[5] ri[0] + rd[3] ri[6]  -rd[5] ri[1] + rd[3] ri[7]  -rd[5] ri[2] + rd[3] ri[8]  0⎟
⎜-rd[8] ri[0] + rd[6] ri[6]  -rd[8] ri[1] + rd[6] ri[7]  -rd[8] ri[2] + rd[6] ri[8]  0⎟
⎝-rd[2] ti[0] + rd[0] ti[2]  -rd[5] ti[0] + rd[3] ti[2]  -rd[8] ti[0] + rd[6] ti[2]  0⎠
```

In[65]:= **MatrixForm[D[TAUU, dtx]]**

Out[65]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ rd[0] & rd[3] & rd[6] & 0 \end{pmatrix}$$

In[66]:= **MatrixForm[D[TAUU, dty]]**

Out[66]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ rd[1] & rd[4] & rd[7] & 0 \end{pmatrix}$$

In[67]:= **MatrixForm[D[TAUU, dtz]]**

Out[67]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ rd[2] & rd[5] & rd[8] & 0 \end{pmatrix}$$

In[68]:= **TextForm[D[TAUU, dphi]]**

Out[68]//TextForm=

```
{{rd[1] ri[0] - rd[0] ri[3], rd[1] ri[1] - rd[0] ri[4], rd[1] ri[2] - rd[0] ri[5],
   {rd[4] ri[0] - rd[3] ri[3], rd[4] ri[1] - rd[3] ri[4], rd[4] ri[2] - rd[3] ri[5]
   {rd[7] ri[0] - rd[6] ri[3], rd[7] ri[1] - rd[6] ri[4], rd[7] ri[2] - rd[6] ri[5]
   {rd[1] ti[0] - rd[0] ti[1], rd[4] ti[0] - rd[3] ti[1], rd[7] ti[0] - rd[6] ti[1]
```

In[69]:= **TextForm[D[TAUU, dtht]]**

Out[69]//TextForm=

```
{{-(rd[2] ri[0]) + rd[0] ri[6], -(rd[2] ri[1]) + rd[0] ri[7], -(rd[2] ri[2]) + rd[0
   {-(rd[5] ri[0]) + rd[3] ri[6], -(rd[5] ri[1]) + rd[3] ri[7], -(rd[5] ri[2]) + rd
   {-(rd[8] ri[0]) + rd[6] ri[6], -(rd[8] ri[1]) + rd[6] ri[7], -(rd[8] ri[2]) + rd
   {-(rd[2] ti[0]) + rd[0] ti[2], -(rd[5] ti[0]) + rd[3] ti[2], -(rd[8] ti[0]) + rd
```

In[70]:= **TextForm[D[TAUU, dpsi]]**

Out[70]//TextForm=

```
{{rd[2] ri[3] - rd[1] ri[6], rd[2] ri[4] - rd[1] ri[7], rd[2] ri[5] - rd[1] ri[8],
   {rd[5] ri[3] - rd[4] ri[6], rd[5] ri[4] - rd[4] ri[7], rd[5] ri[5] - rd[4] ri[8]
   {rd[8] ri[3] - rd[7] ri[6], rd[8] ri[4] - rd[7] ri[7], rd[8] ri[5] - rd[7] ri[8]
   {rd[2] ti[1] - rd[1] ti[2], rd[5] ti[1] - rd[4] ti[2], rd[8] ti[1] - rd[7] ti[2]
```

In[71]:= **TextForm[D[TAUU, dtx]]**

Out[71]//TextForm=

```
{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {rd[0], rd[3], rd[6], 0}}
```

In[72]:= **TextForm[D[TAUU, dty]]**

Out[72]//TextForm=

```
{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {rd[1], rd[4], rd[7], 0}}
```

In[73]:= **TextForm[D[TAUU, dtz]]**

Out[73]//TextForm=

```
{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {rd[2], rd[5], rd[8], 0}}
```