4 Programming [30 pts]

In this problem, you will implement algorithm to analyze the behavior of *SP500* index over a period of time. For each week, we measure the price movement relative to the previous week and denote it using a binary variable (+1 indicates up and 1 indicates down). The price movements from week 1 (the week of January 5) to week 39 (the week of September 28) are plotted below.

Consider a Hidden Markov Model in which x_t denotes the economic state (good or bad) of week t and y_t denotes the price movement (up or down) of the *SP500* index. We assume that $x_{(t+1)} = x_t$ with probability 0.8, and $P_{(Y_t|X_t)}(y_t = +1|x_t = \text{good}) = P_{(Y_t|X_t)}(y_t = -1|x_t = \text{bad}) = q$. In addition, assume that $P_{(X_1)}(x_1 = \text{bad}) = 0.8$. Load the `sp500.mat`, implement the algorithm, briefly describe how you implement this and report the following :

(a) Assuming $q = 0.7$, plot $P_{(X_t|Y)}(x_t = \textbf{good}|y)$ for $t = 1, 2, ..., 39$. What is the probability that the economy is in a good state in the week of week 39. [15 pts]

(b) Repeat (a) for $q = 0.9$, and compare the result to that of (a). Explain your comparison in one or two sentences. [15 pts]