GROUP WORK PROJECT # 2 MScFE 652: PORTFOLIO MANAGEMENT
GROUP NUMBER: 4994

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Remember: Any group members who did **not contribute to the project should be given all zero (0) points for the collaboration grade on the GWP submission page.*

Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).	
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N/A

Step 1: Data collection

We meticulously curated a diverse set of securities to construct our portfolio. To ensure a well-rounded representation across industries, a variety of top corporations from different sectors are represented by the securities that were selected. The selected securities are as follows: Apple Inc. (AAPL), META Inc. (META), Alphabet Inc. (GOOGL), Amazon.com Inc. (AMZN), Exxon Mobil Corporation (XOM), Our selection criteria prioritized stocks with strong market presence, robust financial performance, and availability of recent news, headlines, and analyst reports. Using this method guarantees that we will have access to accurate and timely information to help us make decisions as we optimize our portfolio.

0.0.1 Daily returns

For a thorough analysis of the selected securities, we obtained daily returns data spanning approximately six months. We can collect more data points during this period than the required minimum of 100 data points for statistical significance. The daily returns of the stocks are shown in Fig. 1.

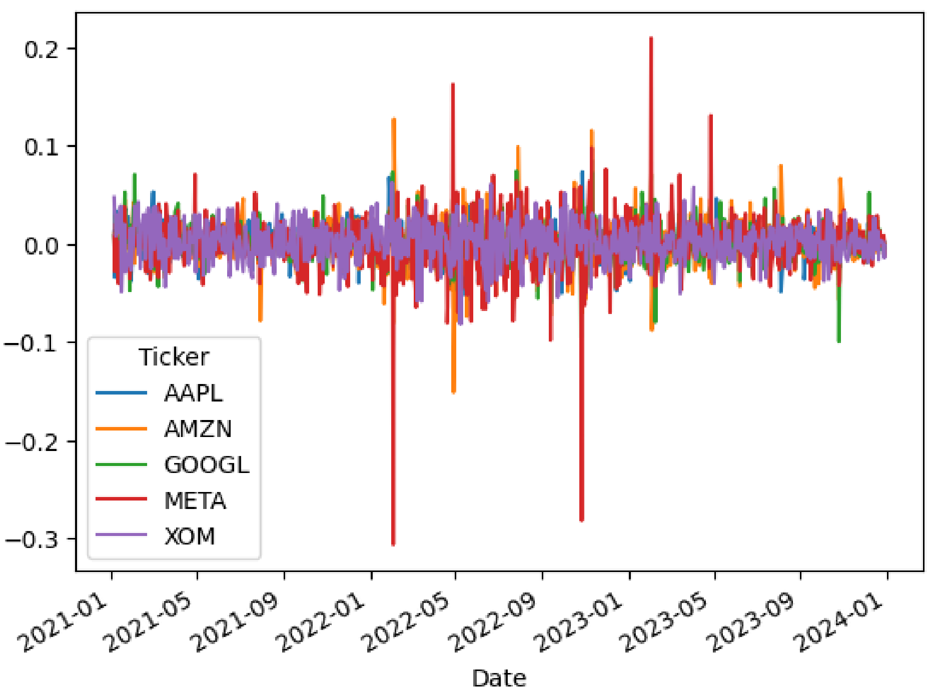

Fig. 1 – Daily returns

Covariance matrix

A key component of portfolio optimization is the covariance matrix, which makes it easier to evaluate the correlation and volatility of the chosen assets. The covariance matrix assists in risk mitigation and informs portfolio allocation decisions by providing a quantitative representation of the correlations between various assets in the portfolio. We generated the covariance matrix which is given in Fig. 2 to examine the co-movements of asset returns using the observed daily returns data. The resulting covariance matrix allows us to build a risk-adjusted and well-diversified investment portfolio by acting as a crucial input in the Markowitz portfolio optimization framework.

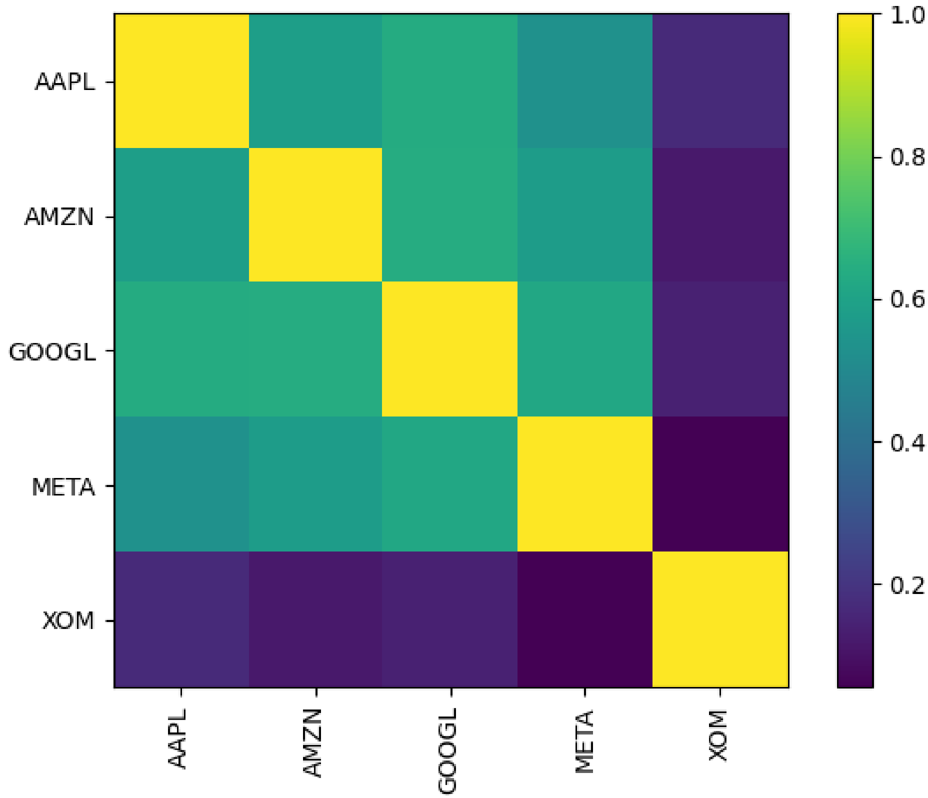

Fig. 2 – Covariance of asset returns

Step 2: Markowitz Optimization

To create an effective investment portfolio, we used the traditional Markowitz portfolio optimization method. By taking into account the covariance between asset returns and their respective expected returns, the Markowitz model seeks to optimize expected returns while minimizing portfolio risk. To find the best asset allocation for our investment portfolio, we applied the Markowitz portfolio optimization technique. To find the most effective asset combination that strikes a balance between risk and return, the optimization method involves taking into account the historical returns and covariances of the chosen securities. We developed and examined the weights given to every security in the portfolio as part of the optimization process. Understanding the composition and diversification of the portfolio requires an understanding of the weights, which indicate the percentage of the total investment allotted to each asset Markowitz (1991). Fig. 3 explains the weights of the portfolio using Markowitz optimization.

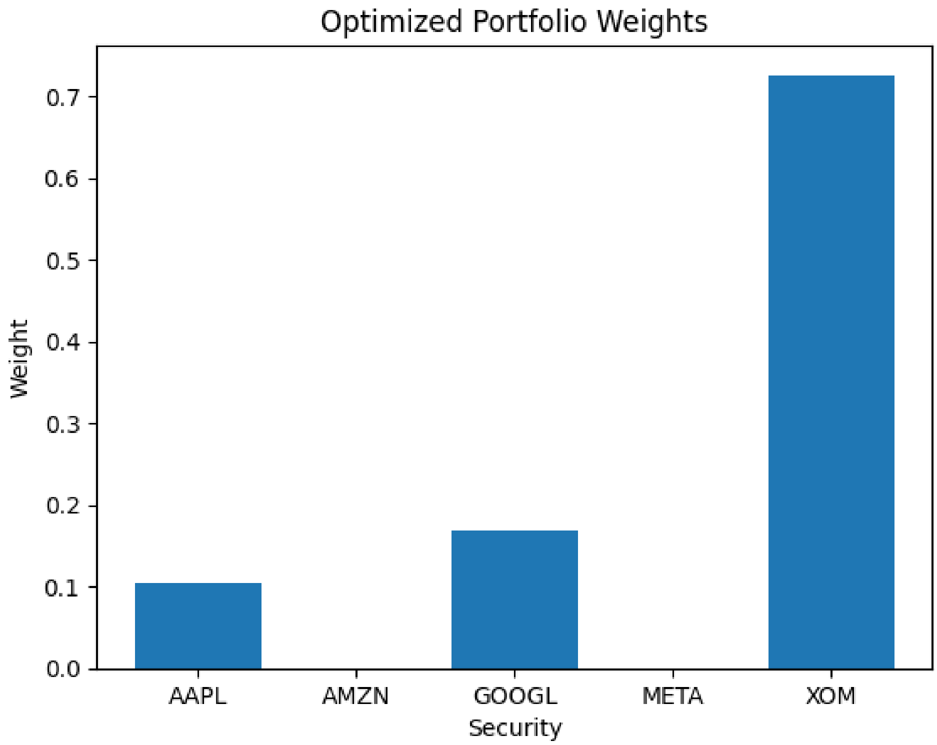

Fig. 3 – Markowitz optimized weights

Step 3: Random Strategy Optimization

A simple 1/N portfolio technique is used to allocate funds equally among all of the portfolio's assets. The performance of this simple investing strategy may be understood by calculating and visualizing the cumulative returns of the portfolio over time in Fig. 4, which is achieved by dispersing the initial investment equally. Using a different strategy, Student 2 analyzes portfolio performance and alternative situations using a Monte Carlo simulation. Random weights are applied to the assets through several simulations, allowing for the examination of the portfolio's worth over time and in various market scenarios. Plotting the mean portfolio value and standard deviation band provides a thorough understanding of possible outcomes. Student 3 uses a buy-and-hold approach, allocating the initial investment to each asset according to its starting price. Calculated and presented throughout the designated period, the portfolio's cumulative returns show the effectiveness of a passive investment strategy requiring nothing in the way of active management. Refer Fig. 5

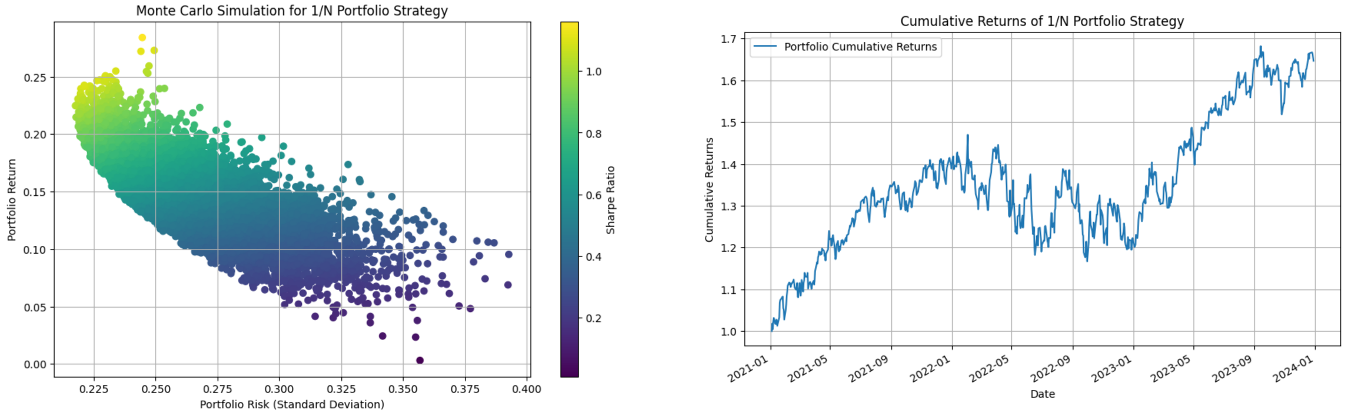

Fig. 4 – Montecarlo simulation and cumulative returns of $\frac{1}{N}$ portfolio strategy

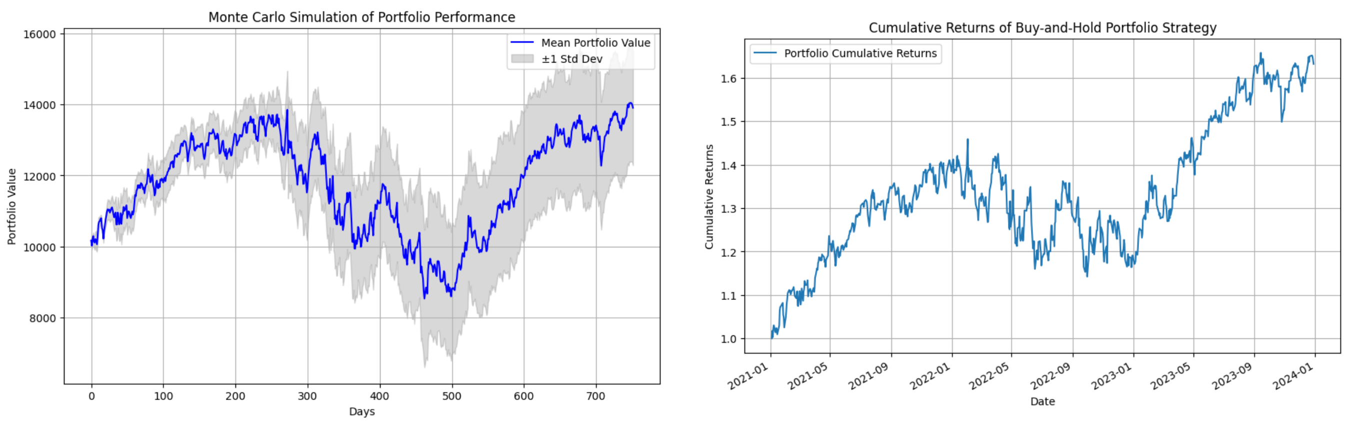

Fig. 5 – (Left) Monte Carlo Simulation of Portfolio Performance. (Right)Cumulative Return for Buy and Hold Portfolio Strategy

1/N Approach

- The 1/N approach distributes the investment among all of the portfolio's assets equally.
- The 1/N strategy's performance throughout the given time period was demonstrated by historical back-testing, which also highlighted the volatility and cumulative returns connected with this simple investing method.

The Markowitz Method

- The Markowitz strategy, on the other hand, uses portfolio optimization tools to figure out the best way to allocate assets based on trade-offs between risk and return.
- An optimal asset allocation was calculated using traditional Markowitz portfolio optimization, taking into account the covariance matrix and the projected returns of the chosen securities.

The 1/N method offers a straightforward, uniformly dispersed investment approach with the potential to improve portfolio diversification. The Markowitz approach, on the other hand, optimizes asset allocation based on past returns and risk characteristics in an effort to maximize the portfolio's Sharpe ratio.

Step 4: Black-Litterman

The Black-Litterman portfolio optimization employs a Bayesian methodology to merge investor perspectives with the market's equilibrium expected return vector (known as the prior distribution), resulting in a blended forecast of expected returns. The formula is given by: Walters et al. (2014)

$$E(R) = \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q\right]$$

Where:
$E(R)$ - Expected returns
Q - View vector
P - Picking matrix
Ω - Uncertainty matrix of views
Π - Prior expected returns (Implied equilibrium returns)
Σ - Covariance matrix
τ - Scalar

Views on the assets

Apple

On February 1, 2024, Apple unveiled its financial outcomes for the initial quarter of the fiscal year 2024, concluding on December 30, 2023. The company disclosed quarterly revenue amounting to \$111.6 billion, alongside quarterly earnings per share of \$2.18. As of February 2024, Apple's beta stands at 1.13. Given this data, we project a 10% absolute growth in Apple's stock.

META

Mark Zuckerberg's announcement on December 14, 2023, regarding the expansion of Threads into European markets is significant. Moreover, the potential inclusion of 448 million users to Instagram's reach in Europe underscores the importance of broadening Threads' accessibility. META remains actively involved in the metaverse domain, with promising prospects for leveraging generative AI. With a beta of 1.16, we hold the relative view that META will outperform AMZN by 5%.

Google

Google has been making significant strides in Artificial Intelligence (AI) and Machine Learning (ML), with 2024 poised to witness further groundbreaking advancements. These innovations are reshaping the landscape of search and digital marketing. Google reported revenue of $86.31 billion and earnings per share of $1.64. As of February 2024, Google's beta is at 1.06. Consequently, we foresee a 12% rise in GOOGLE's trajectory.

Exxon Mobil Corporation

Exxon Mobil Corporation (XOM) reported an annual EPS of $2.48 for 2023, excluding some items. This was higher than the Zacks Consensus Estimate of $2.21, but lower than the previous year's EPS of $3.40. The company's revenue for the fourth quarter of 2023 was $84,344 million, which was a decrease from the $95,429 million revenue of the same quarter in 2022. XOM's beta as of February 2024 was 0.97, indicating that the stock was slightly less volatile than the market. According to the ExxonMobil announces 2023 results report, XOM delivered a profit of $36 billion in 2023. This was the company's second-highest annual profit in a decade, despite the challenging energy landscape and lower oil prices. The company has also made strategic moves to enhance its portfolio, such as divesting its non-core assets, investing in low-carbon technologies, and expanding its presence in the Permian Basin. Therefore, we anticipate an 8% increase in Exxon Mobil's returns.

From the above views Q is a 4×1 vector given by:

$$Q = \begin{pmatrix} 0.1 \\ 0.05 \\ 0.12 \\ 0.08 \end{pmatrix}$$

and the picking matrix P is a 4×5 matrix since we have 4 views and 5 assets.

$$P = \begin{array}{c} \\ \text{view1} \\ \text{view2} \\ \text{view3} \\ \text{view4} \end{array} \begin{array}{c} \begin{array}{ccccc} AAPL & AMZN & GOOGL & META & XOM \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

Imblied Equilibrium excess returns

Market-implied equilibrium excess return is calculated using the following formula.

$$\Pi = \delta \Sigma \omega_{mkt}$$

where δ is the market-implied risk premium (risk-aversion parameter) and given by

$$\delta = \frac{R - R_f}{\sigma^2}$$

$R - R_f$ is excess return and σ^2 is variance, ω_{mkt} is the market-cap weights, and Σ is covariance matrix of asset returns which is shown in Fig. 2. Here market-cap weights ω_{mkt} are extracted from Yahoo Finance. We obtained the risk aversion parameter $\delta = 2.6$. Table 1 and Fig. 6 explains our prior returns Π.

Table 1 – Implied excess equilibrium return

Ticker	Prior
AAPL	0.189871
AMZN	0.241707
GOOGL	0.213238
META	0.283316
XOM	0.064231

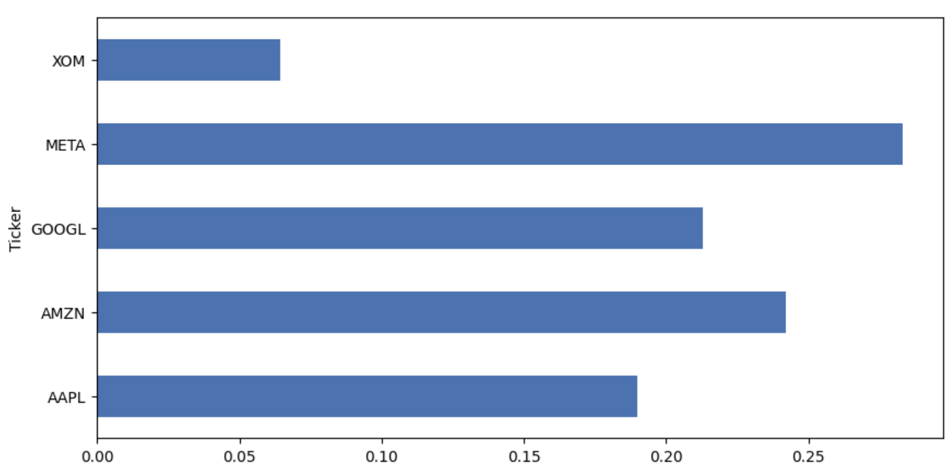

Fig. 6 – Implied excess equilibrium return

Uncertainity matrix of views

The uncertainty matrix Ω is obtained by the following formula:

$$\Omega = \tau P \Sigma P^T$$

We choose $\tau = 0.05$. The uncertainty matrix Ω of views on our portfolio is therefore a 4×4 diagonal matrix given by:

$$\Omega = \begin{pmatrix} 0.0039316 & 0. & 0. & 0. \\ 0. & 0.00782593 & 0. & 0. \\ 0. & 0. & 0.00506819 & 0. \\ 0. & 0. & 0. & 0.00460635 \end{pmatrix}$$

Posterior estimates of returns

The influence of the investor's perspective on the posterior return through Black-Litterman portfolio optimization can be discerned from Table 2. One can observe the difference between the implied return and the BL results from the table.

Table 2 – Comparison of prior and posterior estimates of returns

Ticker	Prior	BL return (μ_{BL})	Difference
AAPL	0.189871	0.136460	0.053411
AMZN	0.241707	0.186701	0.055006
GOOGL	0.213238	0.15495	0.058280
META	0.283316	0.227195	0.056121
XOM	0.064231	0.066278	-0.002047

The covariance matrix obtained from BL is shown in Fig. 7:

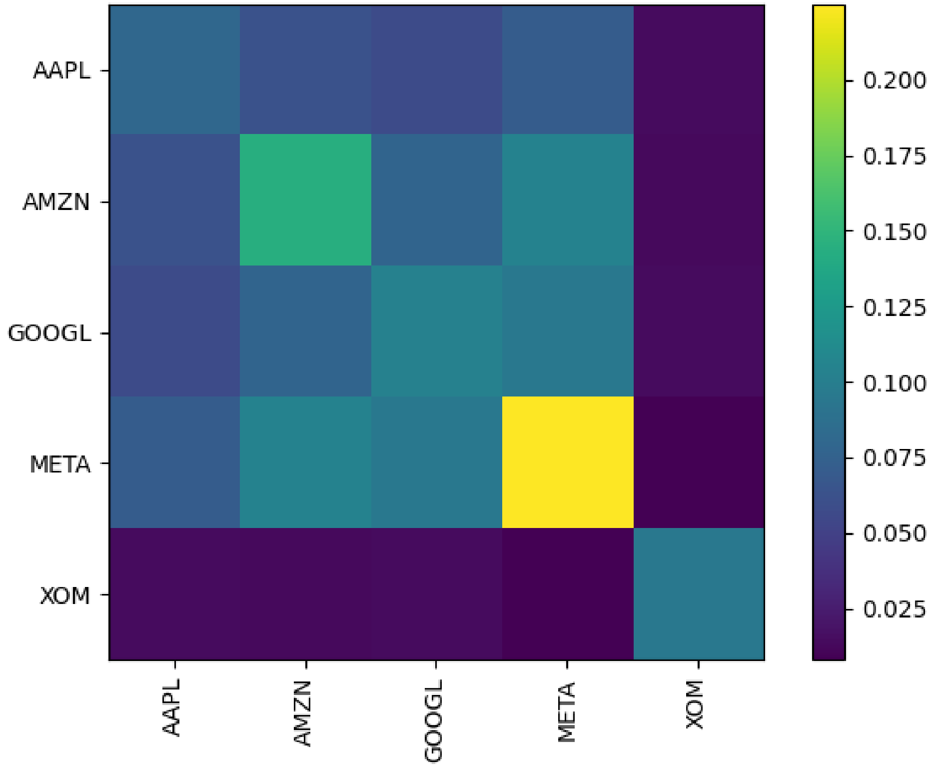

Fig. 7 – The covariance matrix associated with BL (Σ_{BL})

BL portfolio allocation

Now with BL posterior estimate μ_{BL} and BL covariance estimate Σ_{BL} we can solve our optimization problem to obtain the weight of BL portfolio optimization:

$$\begin{aligned} \min_{w} \quad & w^T\mu_{BL} - \tfrac{1}{2}w^T\Sigma_{BL}w \\ \text{s.t.} \quad & w^T\iota = 1 \\ & w_j \geq 0 \qquad\qquad j = 1, \dots, 5 \end{aligned}$$

The optimal weights are those that yield the highest Sharpe ratio. As a result, the optimal Black-Litterman portfolio optimization provides us with the allocation of asset weights as outlined in Table 3.

Table 3 – Optimal weights of BL

Ticker	weights
AAPL	0.17594
AMZN	0.24834
GOOGL	0.19808
META	0.30071
XOM	0.07693

Step 5: Kelly Criterion

The Kelly Criterion is a mathematical formula that helps with the optimal allocation of capital for a portfolio, based on the assets' expected returns and risks. The Kelly Criterion is implemented in three easy steps: Calculate Returns: Calculate the daily returns for each asset in the portfolio. Calculate Expected Returns and Risks: Calculate the average daily return and standard deviation of daily returns for each asset. Calculate Kelly Criterion Fraction: Use the Kelly Criterion formula to calculate the fraction of capital that should be allocated to each asset Thorp (1975). The Kelly Criterion formula for a single asset is:

$$f = \frac{bp - q}{b}$$

where:
f is the Kelly Criterion fraction
b is the winning amount
p is the probability of winning
q is the probability of losing (q = 1 p).

And for a portfolio of assets, the formula is:

$$f_i = \frac{r_i - r_f}{\sigma_i^2}$$

where:
f_i is the Kelly Criterion fraction to allocate to asset
r_i is the expected return of asset
r_f is the risk-free rate,
σ_i^2 is the variance of asset i's returns.

If the calculated Kelly Percentage for a stock is 0.167 or 17% of the available balance, it means that it would be wise to allocate 17% of the portfolio's capital to that stock. The Kelly Criterion helps prevent significant capital loss by providing a systematic approach to asset allocation, allowing investors to stay in the game even after a loss.

However, it's important to note that the Kelly Formula is most effective in situations where the odds are in favor of the investor and potential gains outweigh potential losses. For instance, suppose an investor has a portfolio of stocks with different probabilities of winning and losing, and different potential gains and losses. The investor can use the Kelly Criterion to calculate the optimal allocation of capital to each stock based on their perceived edge in each investment opportunity. As an example, consider a stock with a 45% probability of winning and a 55% probability of losing, with a potential gain of 150% and a potential loss of 100%. The investor applies the Kelly Formula to calculate the optimal allocation of capital to that stock:

$$\text{Kelly Percentage} = \frac{(0.45 \times 1.5) - (0.55 \times 1)}{1.5} = 0.083$$

This indicates that the investor should allocate 8% of the portfolio's capital to that stock, considering the perceived risks and rewards.

In our work, all individual equity stocks Fig. 9, Fig. 10 and Fig. 11(Left) weighted according to the Kelly Criterion always outperformed their historic returns, despite minor fluctuations, and including shorting periods.

Similarly, with the combined portfolio Fig. 11(right), the portfolio with allocations based on the Kelly Criterion outperformed the portfolio with equal weights, resulting in a higher final portfolio value (the value for the portfolio with allocations based on the Kelly Criterion is $143,199.05, while the final portfolio value for the portfolio with equal weights is $130,326.35). The Kelly Criterion allocates more capital to assets with higher expected returns and lower risks, while the equal weights portfolio allocates the same amount of capital to each asset regardless of their individual characteristics. The outperformance of the Kelly Criterion portfolio therefore suggests that the Kelly Criterion is an effective method for allocating capital in a multi-asset portfolio, as it takes into account the risk and return characteristics of each asset. It's important to note that the outperformance of the Kelly Criterion portfolio is based on historical data and may not necessarily hold true in the future. Additionally, the Kelly Criterion assumes that the future risk and return characteristics of the assets will be similar to their historical values, which may not always be the case.

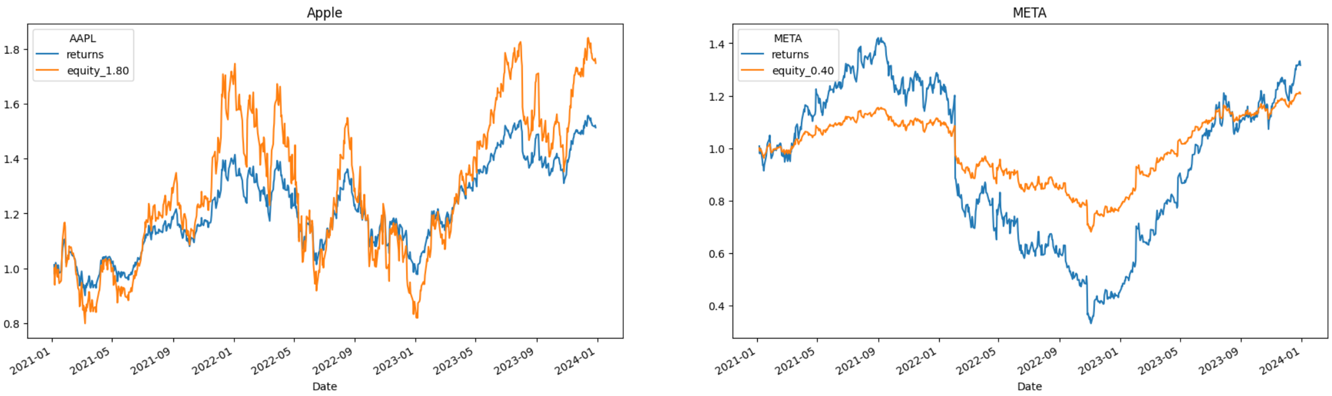

Fig. 9 – Back-testing using the Kelly criterion for Apple and META

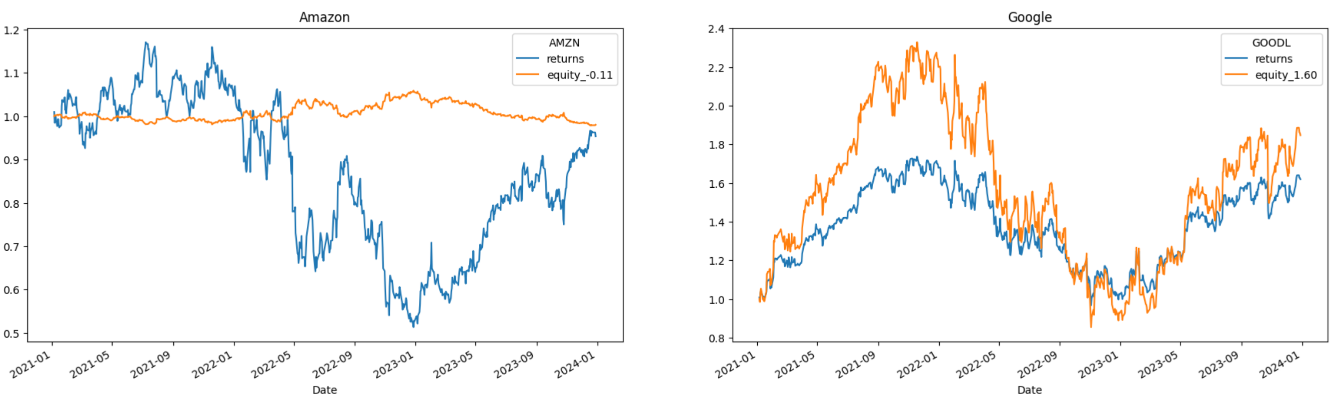

Fig. 10 – Back-testing using the Kelly criterion for Amazon(left) and Google (right)

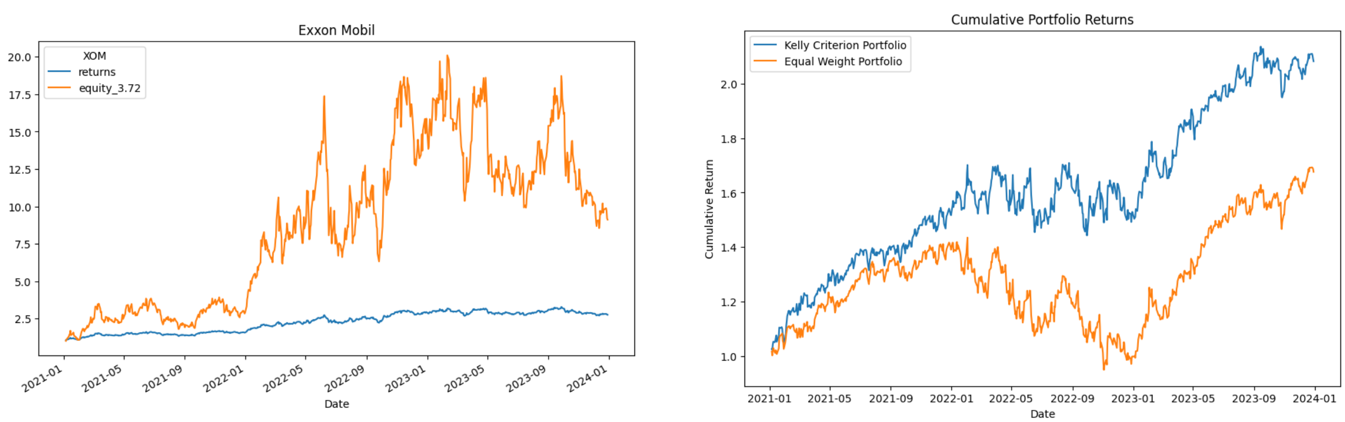

Fig. 11 – (Left) Back-testing using the Kelly criterion for XOM and (right) comparison of cumulative portfolio returns.

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