FIR Filter

A Finite Impulse Response filter is a type of digital filter used in signal processing to perform various filtering operation on digital signal. Unlike IIR filters, FIR filters are stable and have linear phase characteristics.

It has min phase distortion.

General equation, $y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \dots + b_N x(n-N)$

$y(n)$ → o/p at time index n

N – Filter order

$b_0, b_1, \dots b_N$ filter coefficient

Designing an FIR filter involves determining the values of the filter coefficient to achieve the desired filter characteristics. There are various methods to design FIR filter including window methods, freq. sampling methods, and optimal linear phase method ~~etc~~.

Some common applications

LPF, HPF, BPF for signal conditioning

Signal interpolation and decimation

Equalization of audio signal

Image processing such as image smoothing or edge detect

NB. Interpolation increasing the number of samples in signal to obtain a higher sampling rate, whereas decimation reduces the sampling rate

Disadvantage:

- Complex
- required more filter coefficient to be stored
- Long duration impulse response
- Narrow transition band

FIR filter requires more arithmetic operation

Advantages

- Sable
- Can be realized is both recursive or non-recursive
- exact linear phase
- Flexible
- low sensitivity to quantization noise
- efficiently realized in hardware

Impulse Response

Symmetric ~~Phase~~ representation FIR filter

$h(n) = h(M-1-n)$; $0 \leq n \leq M-1$ $M \to$ filter order

Let, $M=8$

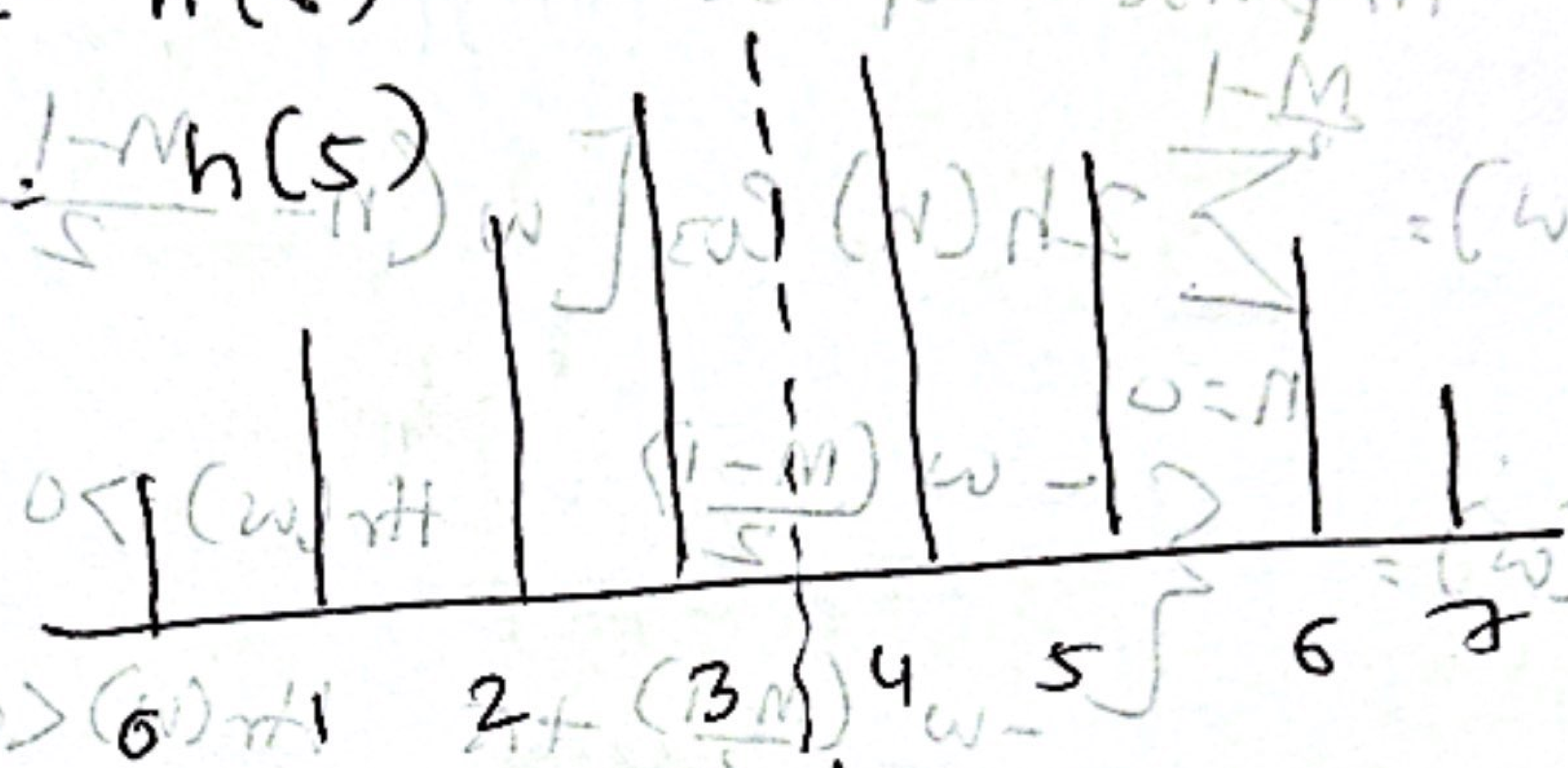
$h(n) = h(8-1-n), 0 \leq n \leq 7$

$n=0$ $h(0) = h(7)$

$n=1$ $h(1) = h(6)$

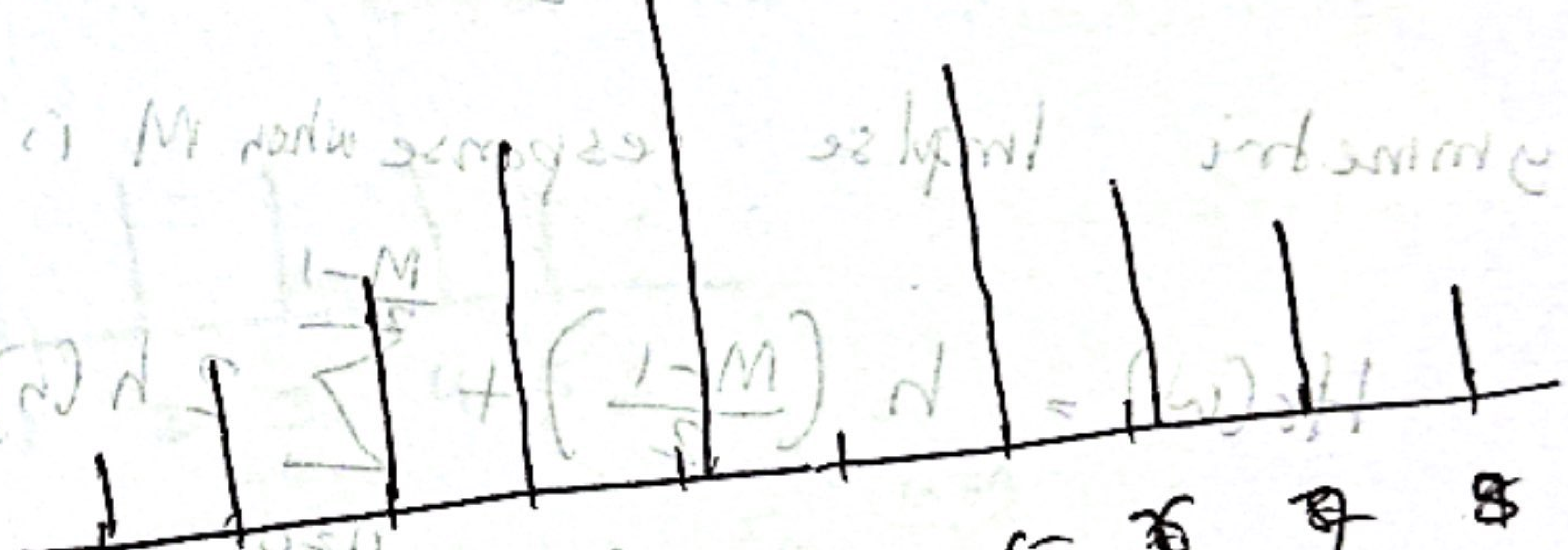
$n=2$ $h(2) = h(5)$

$n(3)$ $h(3) = h(4)$



Symmetric representation of FIR Filt.

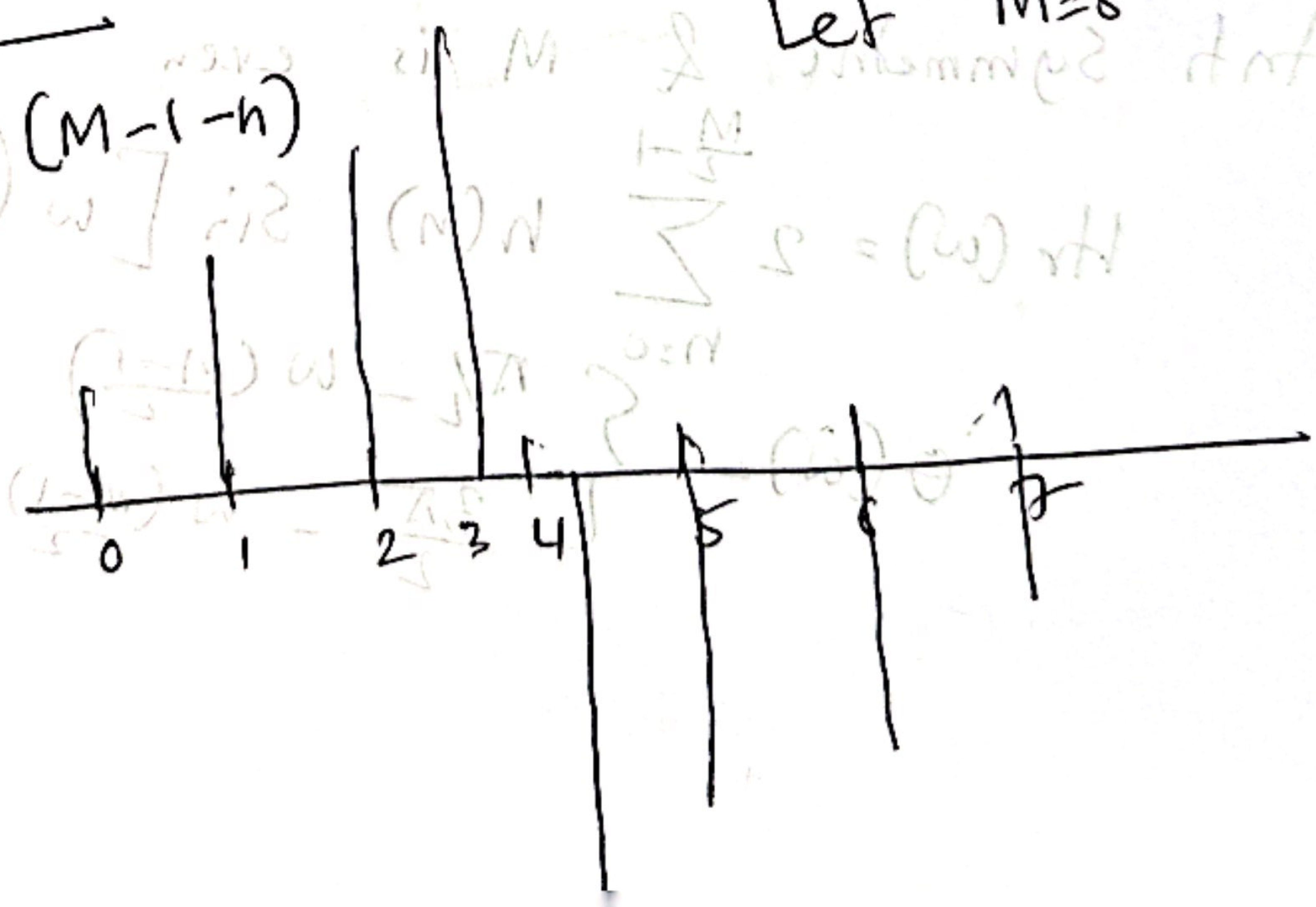
Let $M=9$



Anti symmetric

$h(n) = -h(M-1-n)$

Let $M=8$



Freq response

$$h(n) \xrightarrow{DTFT} H(\omega)$$

freq response of FIR filter can be given as

$$H(\omega) = \underbrace{H_r(\omega)}_{\text{real part of } H(\omega)} e^{-j\theta(\omega)}$$

(i) Symmetric Impulse response with M is even

$$H_r(\omega) = \sum_{n=0}^{\frac{M}{2}-1} 2h(n) \cos\left[\omega\left(n - \frac{M-1}{2}\right)\right]$$

$$\theta(\omega) = \begin{cases} -\omega\left(\frac{M-1}{2}\right) & H_r(\omega) > 0 \\ -\omega\left(\frac{M-1}{2}\right) + \pi & H_r(\omega) < 0 \end{cases}$$

(ii) Symmetric Impulse response when M is odd

$$H_r(\omega) = h\left(\frac{M-1}{2}\right) + \sum_{n=0}^{\frac{M-1}{2}} 2h(n) \cos\left[\omega\left(n - \frac{M+1}{2}\right)\right]$$

$$\theta(\omega) = \begin{cases} -\omega\left(\frac{M-1}{2}\right) & H_r(\omega) > 0 \\ -\omega\left(\frac{M-1}{2}\right) + \pi & H_r(\omega) < 0 \end{cases}$$

(iii) Anti Symmetric & M is even

$$H_r(\omega) = 2\sum_{n=0}^{\frac{M}{2}-1} h(n) \sin\left[\omega\left(\frac{M-1}{2} - n\right)\right]$$

$$\theta(\omega) = \begin{cases} \pi/2 - \omega\left(\frac{M-1}{2}\right) & H_r(\omega) > 0 \\ \frac{3\pi}{2} - \omega\left(\frac{M-1}{2}\right) & H_r(\omega) < 0 \end{cases}$$

Anti symmetric with M odd

$$H_r(\omega) = 2\sum_{n=0}^{\frac{M}{2}-1} h(n)\,\sin\left(\omega\left[\frac{M-1}{2}-n\right]\right)$$

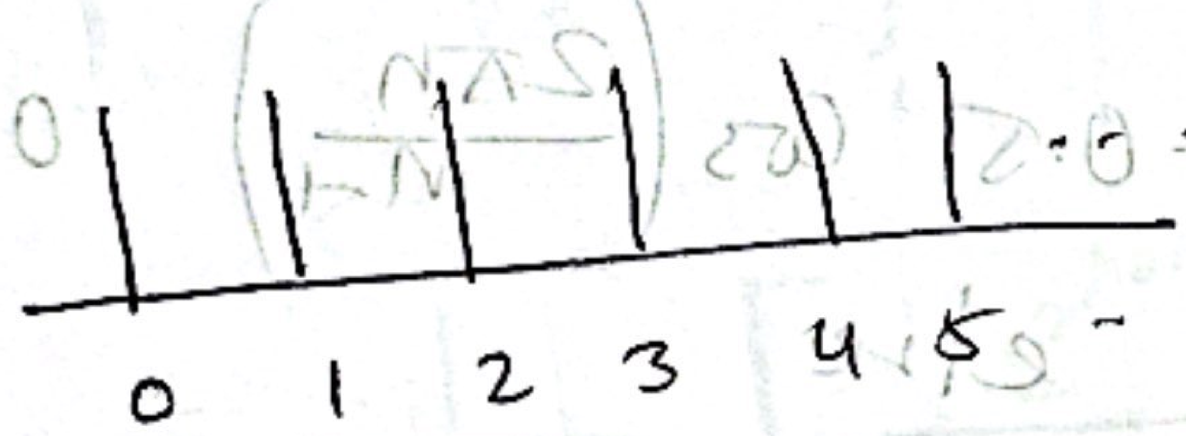
$$\theta(\omega) = \begin{cases} \pi/2 - \omega\left(\frac{M-1}{2}\right) & H_r(\omega) > 0 \\ \frac{3\pi}{2} - \omega\left(\frac{M-1}{2}\right) & H_r(\omega) < 0 \end{cases}$$

Designing of Linear Phase FIR Filter using windowing Method

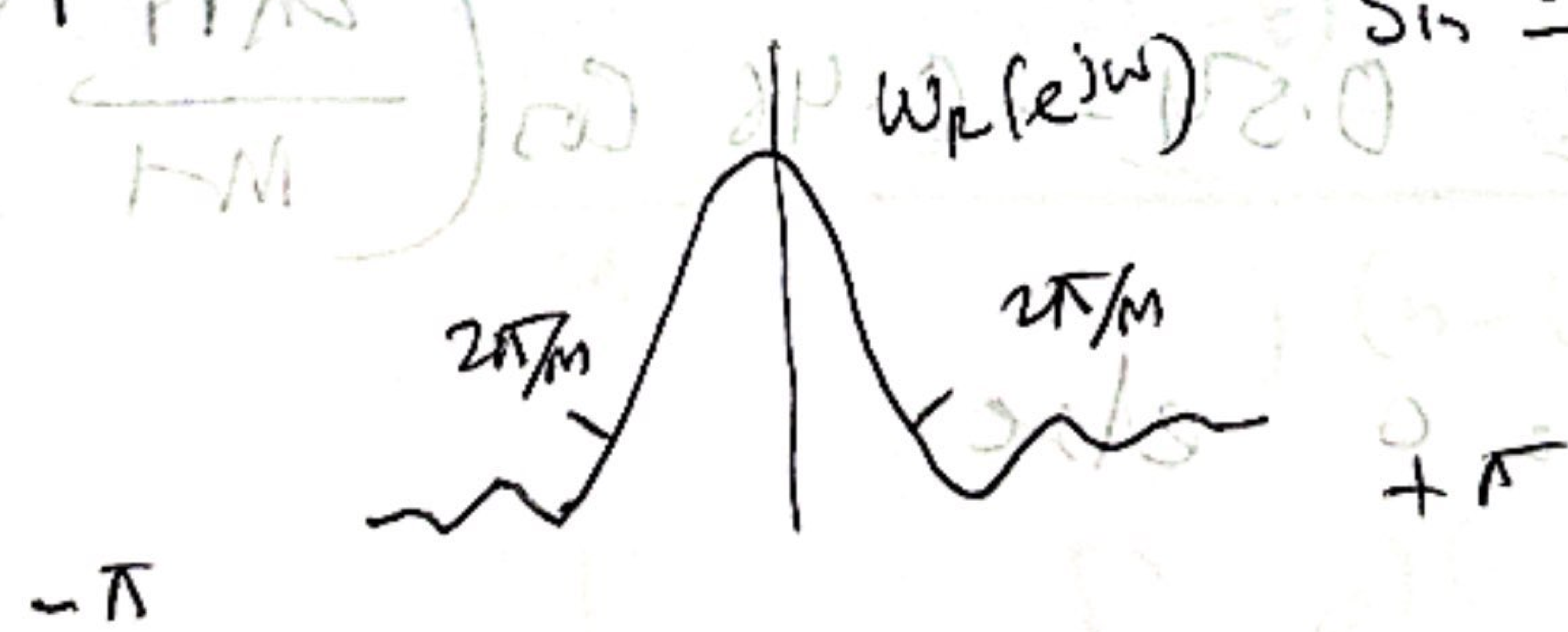
(a) Rectangular window

$$W_R(n) = \begin{cases} 1 & 0 \le n \le M-1 \\ 0 & \text{else} \end{cases}$$

$M = 6$



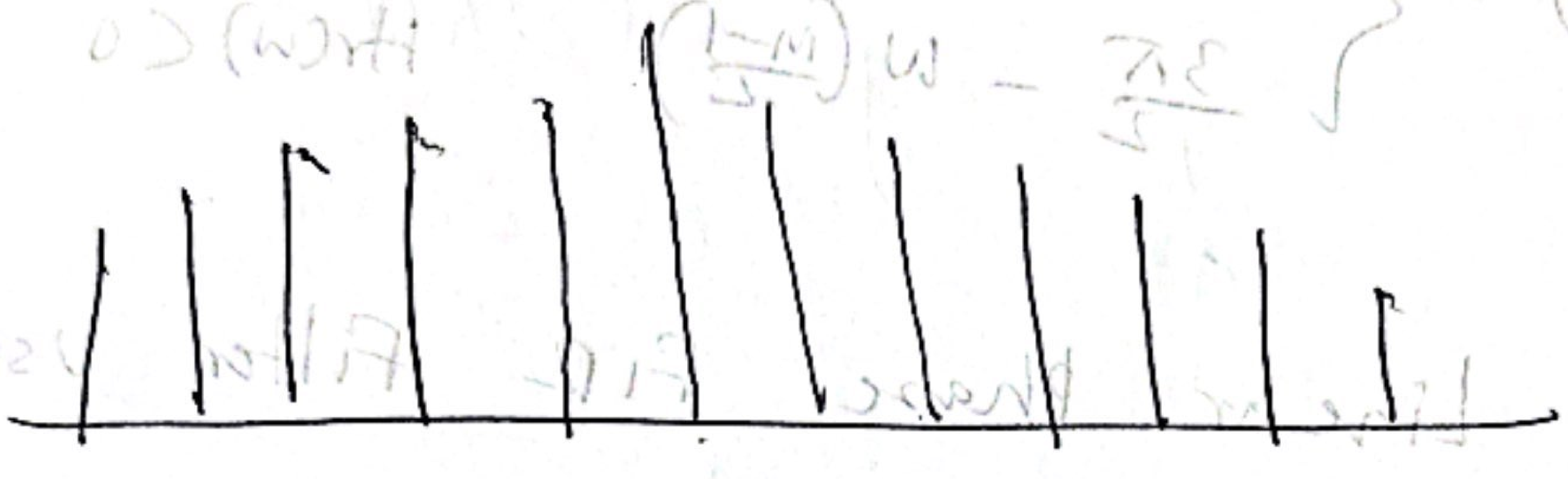
Spectrum $W_R(e^{jw}) = \dfrac{\sin \frac{wM}{2}}{\sin \frac{w}{2}}$



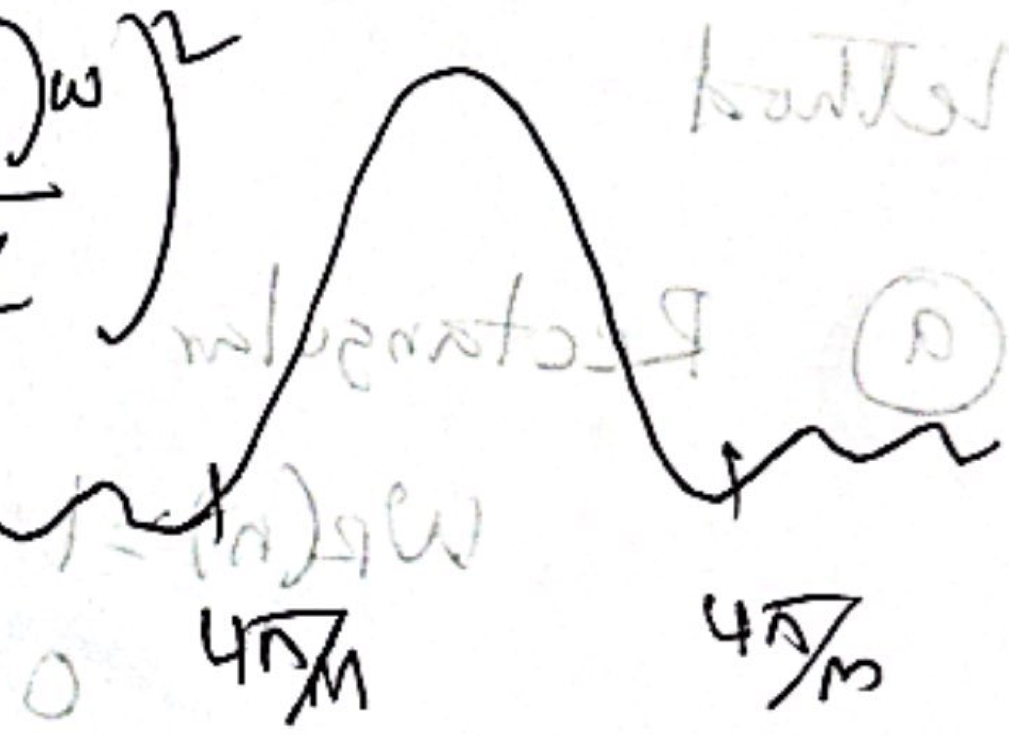
Bartlett window: (Transgular window)

$$W_B(n) = 1 - \frac{2\left[n - \left(\frac{M-1}{2}\right)\right]}{M-1} \qquad 0 \le n \le M-1$$

$= 0$



Spectrum $W_T(e^{j\omega}) = \left(\frac{\sin\left(\frac{M-1}{4}\right)\omega}{\sin \omega/2}\right)^2$



~~Hamming~~ Hanning window

$$W_H(n) = 0.5 - 0.5 \cos\left(\frac{2\pi n}{M-1}\right) \quad 0 \le n \le M-1$$

0 else

Hamming window

$$W_{HW}(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{M-1}\right) \quad 0 \le n \le M-1$$

$= 0$ else

Design the symmetric FIR LPF whose

$$H_d(\omega) = \begin{cases} e^{-j\omega t} & |\omega| \leq \omega_c \\ 0 & \text{otherwise} \end{cases}$$

with $M = 7$

$\omega_c = 1$ rad/sec

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega \quad \text{——} \ (1)$$

$$= \frac{1}{2\pi} \int_{-1}^{1} e^{-j\omega t} \cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-1}^{1} e^{j\omega(n-t)}\, d\omega$$

$$= \frac{1}{2\pi} \left[\frac{e^{j\omega(n-t)}}{j(n-t)} \right]$$

$$= \frac{1}{2\pi} \; \frac{e^{j(n-t)} - e^{-j(n-t)}}{j(n-t)}$$

$$= \frac{1}{\pi(n-t)} \left[\frac{e^{j(n-t)} - e^{-j(n-t)}}{2j} \right]$$

$$= \frac{\sin(n-t)}{\pi(n-t)} \qquad n \neq t$$

If $n = t$

$\cancel{hd(\omega)}$ $hd(n) \frac{1}{2\pi} \int_{-1}^{1} 1 \, d\omega = \frac{1}{\pi}$

$$hd(n) = \begin{cases} \frac{\sin(n-t)}{\pi(n-t)} & n \neq t \\ \frac{1}{\pi} & n = t \end{cases}$$

determine the value of t

$$h(n) = h(M-1-n)$$

$$h(n) = hd(n) \cdot w(n)$$

$$hd(n)\,w(n) = hd(M-1-n)\,w(n)$$

$$hd(n) = hd(M-1-n)$$

$$\frac{-\sin(n-t)}{-\pi(n-t)} = \frac{\sin(M-1-n-t)}{\pi(M-1-n-t)}$$

$$= \frac{\sin(-(n-t))}{\pi(-(n-t))} = \frac{\sin(M-1-n-t)}{\pi(M-1-n-t)}$$

$$-(n-t) = M-1-n-t$$

$$-n+t = M-1-n-t \qquad t = \frac{M-1}{2}$$

$$h_d(n) = \begin{cases} \dfrac{\sin\left(n - \frac{M-1}{2}\right)}{\pi \cdot \left(n - \frac{M-1}{2}\right)} & n \neq \frac{M-1}{2} \\ \dfrac{1}{\pi} & n = \frac{M-1}{2} \end{cases}$$

Since $M = 7$

$$h_d(n) = \begin{cases} \dfrac{\sin(n-3)}{\pi(n-3)} & n \neq 3 \\ \dfrac{1}{\pi} & n = 3 \end{cases}$$

$n = 0$ to 6