



**INSTITUTE OF INFORMATION TECHNOLOGY
JAHANGIRNAGAR UNIVERSITY**

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Submitted To

Dr M Mahbubur Rahman
Associate Professor
Department of Physics
Jahangirnagar University

Submitted By

Md. Shakil Hossain
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IIT - 48 Batch JU

Q:1) Describe a platinum resistance thermometer. How would you calibrate and use it for measuring temperature of an object. Mention its advantage over a thermoelectric thermometer.

Ans to the question no- 01

The platinum resistance thermometer is based on the principle that the electrical resistance of a metallic wire is found to increase gradually and fairly uniformly with temperature over a wide range. Clausius was the first who gave a relation of the type given by

$$R_t = R_0 (1 + \alpha t) \quad \text{--- ①}$$

where R_0 and R_t are the resistances of the wire at 0°C and $t^\circ\text{C}$ respectively and α the co-efficient of increase of resistance of the wire with temperature whose value depends upon the nature and properties of the wire. According to equation ① the resistan-

of a wire should theoretically become zero at absolute zero. The relation is therefore not true for all ranges of temperature it does not represent experimental observation accurately even at ordinary temperature.

However able to prove that Pure platinum wire, free from alloy with silicon, carbon, tin or other impurities and not subjected to strain or rough usage, always possesses the same resistance of the same temperature. The variation of its resistance with temperature could be fairly represented by the relation

$$R_t = R_0 (1 + \alpha t + \beta t^2)$$

where α and β are constants for the material of the wire used. For pure platinum $\alpha = 3.99 \times 10^{-3}$ and $\beta = -5.8 \times 10^{-7}$

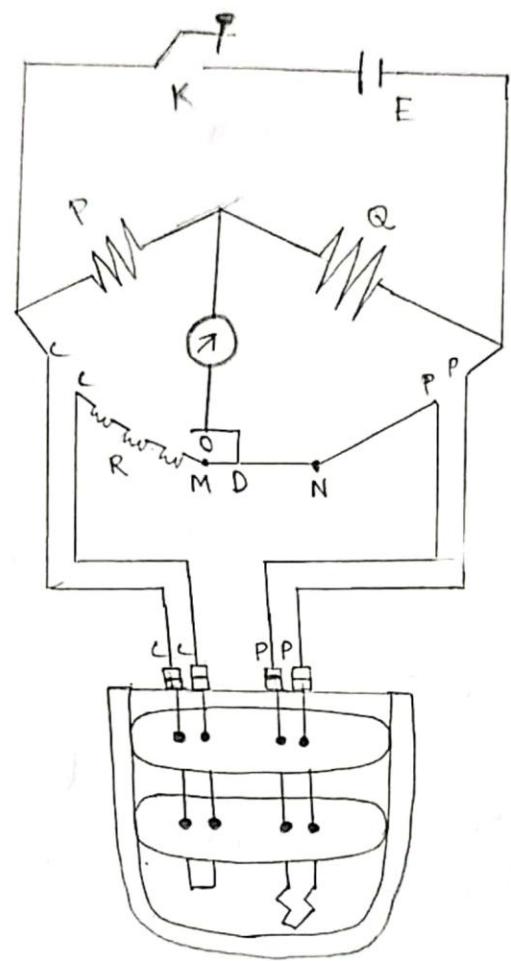
Working:

The resistance of the wound platinum coil of the platinum resistance thermometer can be measured with the help of a callender and Grahame's bridge. It is capable of measuring resistance to a high degree of accuracy upto $\frac{1}{10,000}$ of an ohm.

In Fig (1) P and a one equal resistance R is an adjustable resistance, capable of giving 10, 20, 50 etc units each unit being equal to 0.01 ohm.

The third arm R and the fourth arm S are connected to a stretched bridge wire MN, chosen for reason of its great uniformity. The arrangement of contact is adopted to eliminate thermo-electric forces.

For measuring resistance c.c and P.P are put in series with the third arm R and the fourth arm while T is connected to the galvanometer. To complete the circuits a battery



Since the relation arms P and Q are equal

$$R + \pi + q \cdot MD = R_f + \pi + S \cdot ND$$

where π is the resistance of either the platinum or the compensating leads. S is the resistance per cm of the wire MN

R_f is the resistance of the platinum wire at the tempat temperature of the bath.

$$\begin{aligned} \text{or, } R_f &= R + S(MD - ND) \\ &= R + S(OM + OD - ON + OD) \\ &= R + S \cdot 2 \times OD \quad [O \text{ being the midpoint of } \overline{MN}] \end{aligned}$$

Since,

$S = \frac{1}{2}$ of the unit which is equal to
0.01 ohm

$$R_f = R + 0.01 \times OD \text{ cm}$$

If D is to the right of 0.01 \times OD must be added to R, if to the left of 0, subtracted.

Thus the resistance R_f of the platinum wire of the thermometers at the unknown temperat

of the bath is determined accurately R_0 and R_{100} , the resistance of the wire at the ice-point and stream point.

Then, $t_p = \frac{R_t - R_0}{R_{100} - R_0} \times 100$

The true temperature t on the gas scale is obtained by using the relation.

$$(t - t_p) = \delta \left[\left(\frac{t}{100} \right)^2 - \frac{t}{100} \right]$$

Advantage:

- ① The temperature measurement through platinum resistance thermometer is easier as compared to the gas thermometer.
- ② Gives the precise reading of temperature.
- ③ quite sensitive.
- ④ The platinum has some resistance at the same temperature.
- ⑤ Has wide range from -200°C to 1200°C Celsius

Ams to the question no - 2

If two dissimilar metals are joined at two ends and if one of the junction is kept at higher temperature than the other, there is a temperature difference between the two junction, an electromotive force (e.m.f.) is set up. If the circuit is closed, this emf will cause a current to flow in the circuit as indicated by a galvanometer. This phenomenon was discovered by Seebeck, a Berlin physicist, and is known as Seebeck effect.

The two dissimilar metals joined at the two ends constitute what is known as a thermo-couple. And the electro-motive force developed is referred to as thermo electromotive force. Seebeck arranged different metal in a particular order such that if a thermo-couple be made by of any two of this series, known as Seebeck Series,

Seeback's original series has now been extended and is known as thermo-electric series. This follows:

Sb, Fe, Cd, Zn, Ag, Au, Rb, Mo, Cr, Sn, Pb, Hg, Cu, Pt, Co, Ni, Bi.

keeping the cold junction at 0°C , as the temperature of hot junction is gradually increased the emf developed at first goes on increasing, reaches a maximum and then begins to decrease until at a certain temperature the emf is reversed and goes on increasing in reversed direction without showing any sign of again coming to a maximum.

If the emf is plotted against the temperature of the hot junction, a curve OABD which is approximately parabolic in shape is obtained as shown in figure-a.

The temperature c at which emf. is maximum is called the neutral-temperature and its constant for the pair of metals.

The temperature B beyond which the emf is reversed is known as the temperature (of point) of inversion for the particular couple. If the cold junction be at any temperature higher than 0°C , the origin O is shifted to O' corresponding to the temperature O_p of the cold junction. The curve is now $O'AB'D$. So, that the neutral temperature remains unchanged but the temperature of reversal B' advances towards the neutral temperature by an amount B_q equal to O_p , the temperature of the cold junction. Thus while the temperature at which

maximum emf is produced is constant for the pair the temperature at which reversal takes place is variable, being as much above the neutral temperature as the cold junction is below it.

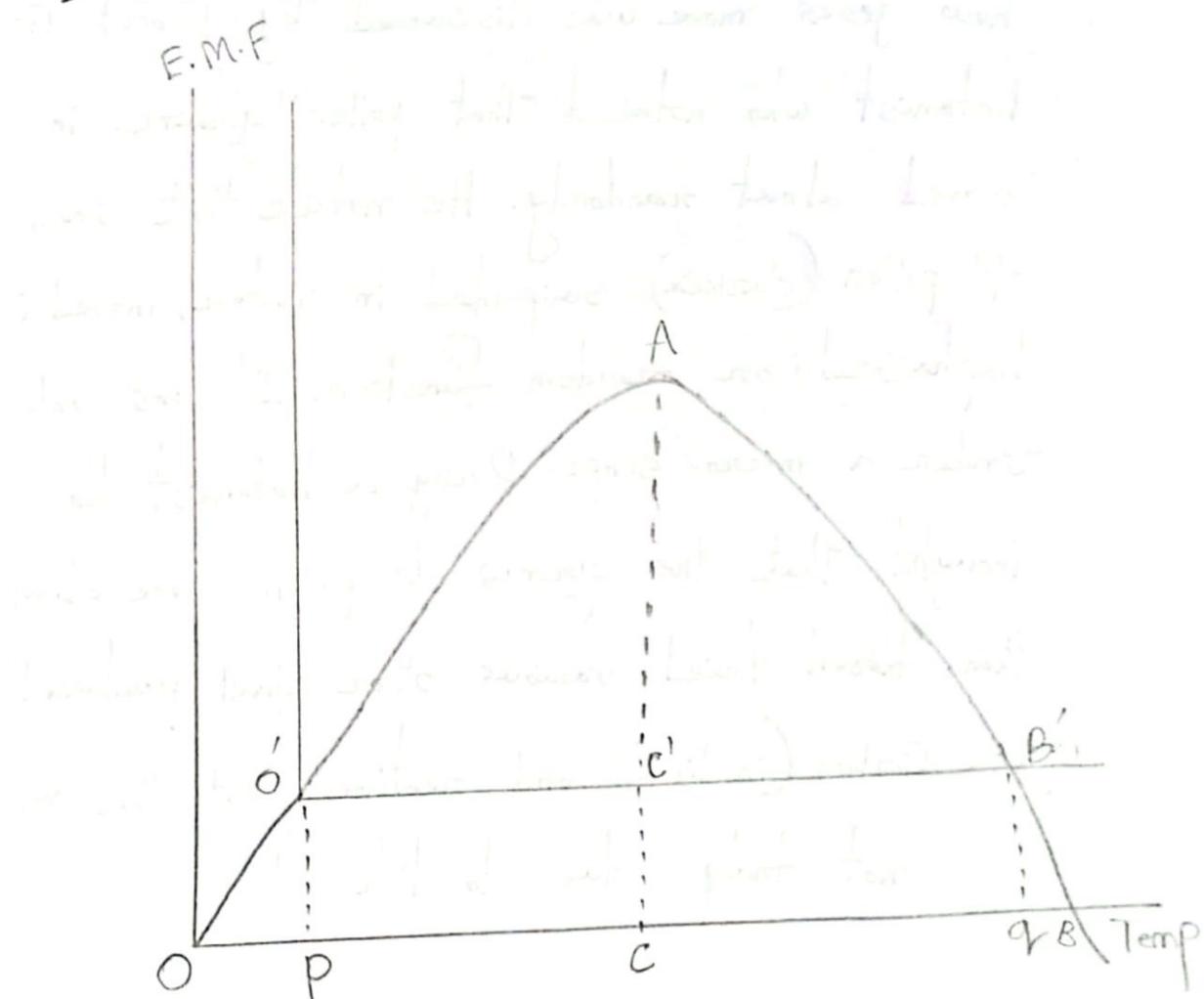
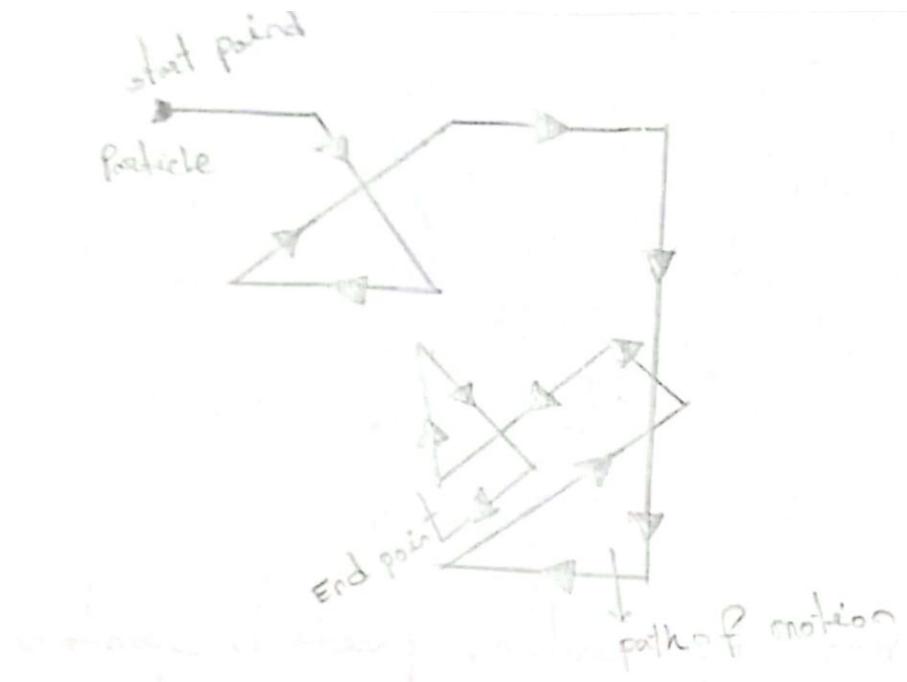


Figure-a

Q: 3) Define Brownian motion with its essential features. Explain the basis of Brownian motion

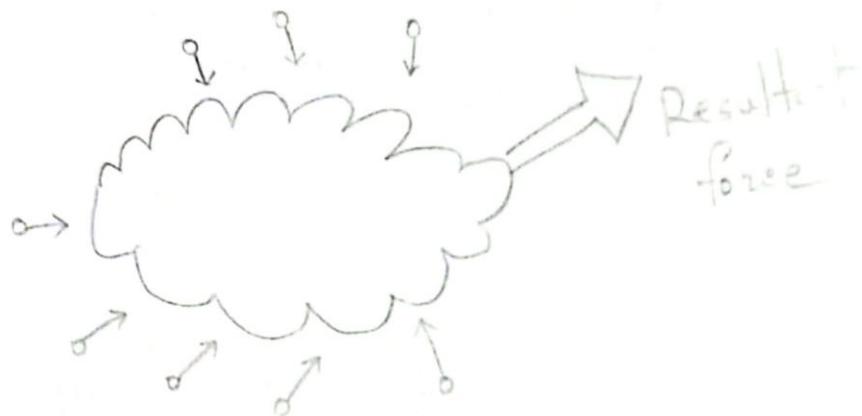
Ans to -the question no - 3

One of the first pieces of evidence for how gases move was discovered by Robert Brown a botanist who noticed that pollen grains in water moved about randomly. He notice that small grains of pollen (clarkia) suspended in water, moved in a haphazard or random function. It was only visible under a micro-scope. Being a botanist he logically thought that the grains of pollen were alive. He then tried various other small particles such as Carbon (in ink) and realized that the motion was not truly due to life.



- There is no appreciable displacement of the particle
- The particle travels more or less in straight lines
- The motion is random.

At any point of time the forces on the particles are uneven causing a net force and therefore acceleration in that direction. Their light mass means they change direction quickly and easily.



The non stop, random, parhelal, spontaneous and haphazard motion of a colloidal solution is called the Brownian motion.

Essential Features of Brownian motion :-

- ① The motion of each particle is completely irregular and random. No two particles are found to execute the same motion.
- ② The motion is continuous and takes place for ever.
- ③ The smaller particles appear to be more agitated than the larger ones.

- ④ The motion is independent of the nature of the suspended articles. Two particles of the same weight and size move equally fast the same temperature.
- ⑤ The motion becomes more conspicuous in a liquid of lower viscosity.
- ⑥ The motion is not modified due to the shaking of the containing vessel.

Basis of Brownian motion:

Brownian motion is due to the bombardment of the dispersed particles by molecules of the medium which are themselves in a continuous haphazard motion when the particles are sufficiently large the forces due to molecular impact is nearly balanced. This way, Brownian motion can not be observed with particles of large size. But when the size of the particles is

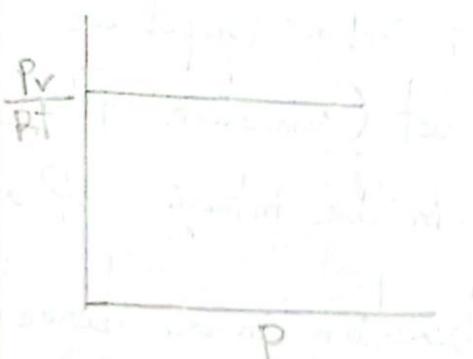
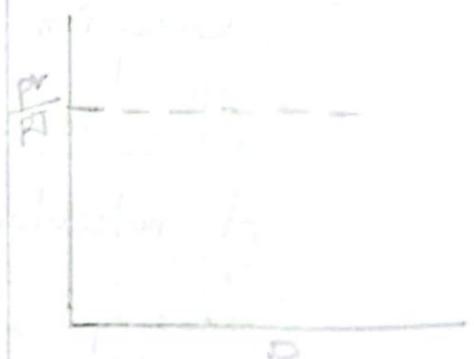
Very small each particle will be acted upon by a resultant unbalanced force and will consequently move in a haphazard manner in response to the magnitude and direction of this force. In other words, the particle move in response to the molecules of the liquid in completely haphazard manner. The motion of the molecules of a gas is similar to Brownian movement if suspended particles in colloidal solution. Thus, the laws of kinetic theory of gases are applicable to Brownian motion.

Q: 9) Distinguish between a real gas and an ideal gas. How is the ideal gas equation modified when mutual attraction between molecules and the finite size of the molecules are taken into consideration? obtain the relations among the Critical Constant and the Van der waal's constant.

Ans to the question no - 9

Distinguish between a real gas and an ideal gas is given below:

Topic of distinguish	ideal gas	real gas
1. Boyle's Law Charles's Law	Obey those gas laws,	Doesn't obey those gas laws
2. equation	$pV = nRT$	$\left(p + \frac{n^2}{V} \right) (V - nb) = nRT$

Topic of distinction	idealized gas	real gas
3. graph		
4. Volume of particle	negligible as compared to total volume of gas	not negligible
5. Pressure	Only pressure given by wall	More than ideal gas
6. Mean free path	straight	not straight
7. Attractive forces among the molecules	can be neglected in all temperature and pressure	can't be neglected under high pressure and low temperature

The ideal gas equation is modified based on:

- ① Attraction forces between gas particles/ correction of pressure (attraction effect).

⑥ Non-zero Volumes of gas particles.

Correction of volume (repulsive effect)

Attractive effect (connection of Pressure):

A molecule in the interior of a gas experiences forces of attraction in all direction and the resultant cohesive force is zero. A molecule near the walls of the two containers experiences a resultant force inwards for this reason, the observed pressure of the gas is less than that the actual pressure.

The correction for P depends on:

① The number of molecules striking unit area of the walls of the container / sec.

② The number of the molecules present in a given volume.

Both factors depend on the density of the gas.

Hence the correction for pressure,

$$p \propto p^2 \propto \frac{1}{V^2}$$

$$\text{or } p = \frac{a}{V^2}$$

a = Constant, V = Volume of the gas. The correct pressure of the gas,

$$p + P = p + \frac{a}{V^2}; [P = \text{observed pressure}]$$

Repulsive effect (correction for volume)

- ① Gas molecules behave like small, impenetrable spheres.
- ② Actual volume available for gas smaller than volume of container, V .
- ③ The molecules have the sphere of influence around them and due to this factor, the correction for volume b is approximately 9 times actual volume of the gas molecules.

The Correct Volume of gas = $V - b$.

Let the radius of one molecule be r .

(Figure.b). The Volume of the molecule,

$$n = \frac{4}{3} \pi (2r)^3 = 8n$$

Consider a container of volume V and the molecules are allowed to enter one by one the volume available for the first molecule =

for the second molecule = $V - s$

for the 3rd molecule = $V - 2s$

Volume available for the n th molecule

$$= V - (n-1)s$$

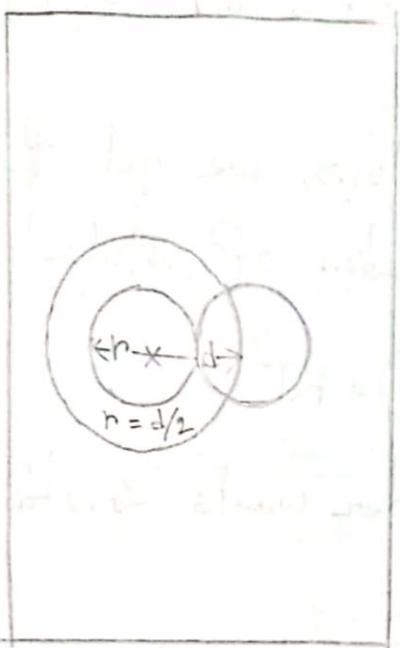


Figure - b

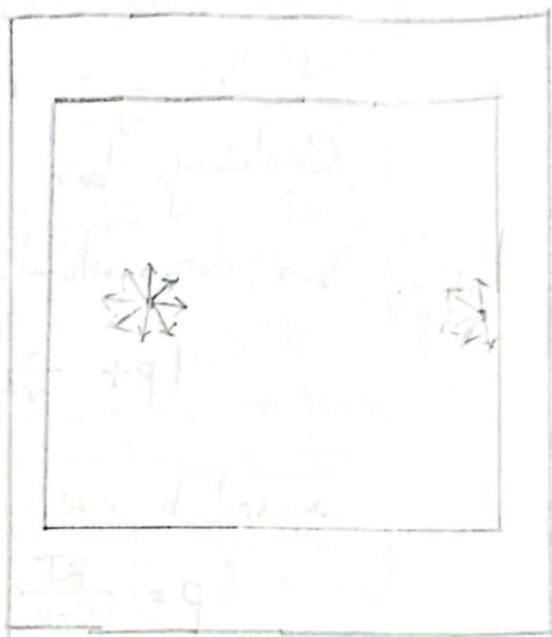


Figure - c

The average space available for each molecule

$$= \frac{V + (V-s) + (V-2s) + \dots + \{V + (n-1)s\}}{n}$$

$$= V - \frac{s}{n} \left\{ 1 + 2 + 3 + 4 + \dots + (n-1) \right\}$$

$$= V - \frac{s}{n} \cdot \frac{(n-1)n}{2} = V - \frac{ns}{2} + \frac{s}{2}$$

As the number molecule is very high, we can ignore the last term $\frac{s}{2}$.

Hence the space available for each molecule

$$= V - \frac{ns}{2} = V - \frac{n(8n)}{2} = V - 4(n^2)$$

$$= V - b,$$

where $b = qnm$, q times the actual volume of the gas molecule.

Combining both derivations, we get the Van der waal's equation of state

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

a and b are Van der waal's constants

$$p = \frac{RT}{v-b} - \frac{a}{v^2}$$

$\frac{a}{v^2}$ comes from interactions among gas molecules. b comes from volume of gas molecules. a and b are different for different gases.

Q:5) what is meant by entropy? show that the entropy of reversible process remains invariant and it gets increased in an irreversible process.

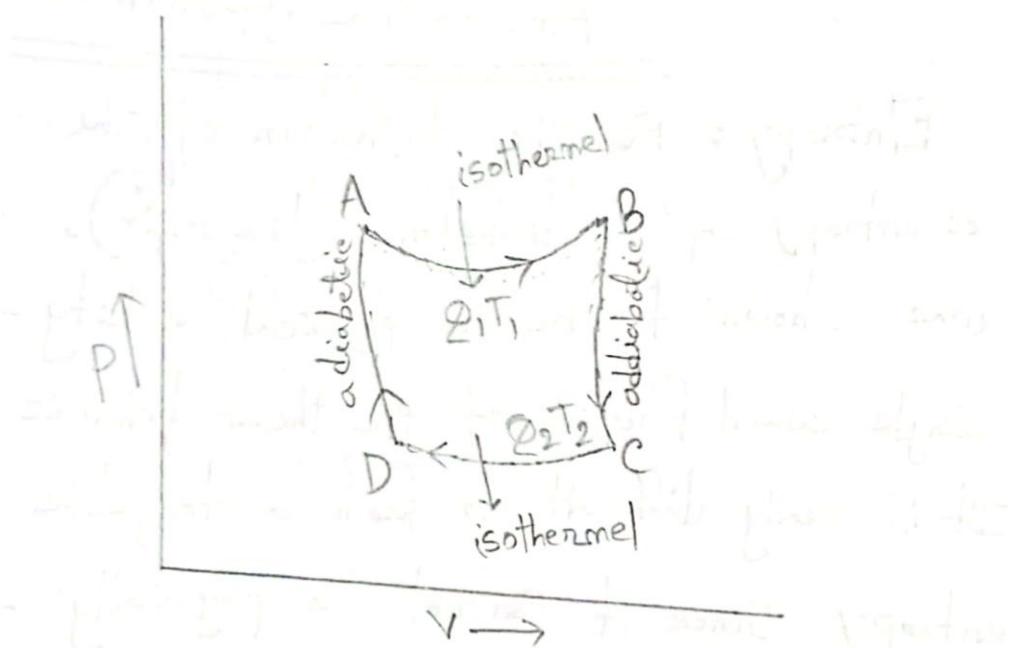
Ans to the question no - 5

Entropy: From the definition of the change of entropy of a substance ($ds = \frac{d\varnothing}{T}$), entropy was shown to be a physical entity - a definite single valued function of the thermodynamic variables. It is really difficult to form a tangible idea about entropy since it can not be physically felt like other physical quantities. Although there is nothing strictly physical to represent entity and not a physical illusion.

Entropy of a system remains constant in any reversible process. The entropy of the Universe tends toward a maximum. It represents the

the unavailability of energy. And because of all irreversible process, there is a continuous increase in the entropy of the Universe.

Reversible Process:



In this Process, a Carnot cycle, two isothermal and two adiabatic Process

In the adiabatic Processes there is neither absorption nor rejection of heat, no change in entropy of the two isothermal

Process, one is an expansion and other is compression. During expansion an amount of heat Q_1 is absorbed at $T_1^{\circ}\text{K}$. So, increase in entropy = $\frac{Q_1}{T_1}$, In the compression process an amount of heat Q_2 is rejected at $T_2\text{K}$. So, decrease in entropy = $\frac{Q_2}{T_2}$. Total change in entropy for whole reversible process,

$$\int ds = \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$$

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}; \quad [\text{for the reversible cycle}]$$

$$\therefore \int ds = \frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0$$

$$\Rightarrow s = \text{constant (shown)}$$

Irreversible process:

Let us consider, substance absorbs a quantity of heat Q_1 at T_1 and rejects Q_2 at T_2

Then the efficiency of the engine performing such a cycle is given by

$$\eta' = \frac{Q_1 - Q_2}{Q_1}$$

Let a reversible engine work between the same two temperature limits then its efficiency

$$\eta = \frac{T_1 - T_2}{T_1}$$

$\eta' < \eta$; [Carot's theorem]

$$\Rightarrow \frac{Q_1 - Q_2}{Q_1} < \frac{T_1 - T_2}{T_1}$$

$$\Rightarrow \eta = \frac{Q_2}{Q_1} < 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \frac{Q_2}{Q_1} > \frac{T_2}{T_1} \Rightarrow \frac{Q_2}{Q_1} > \frac{Q_1}{T_1}$$

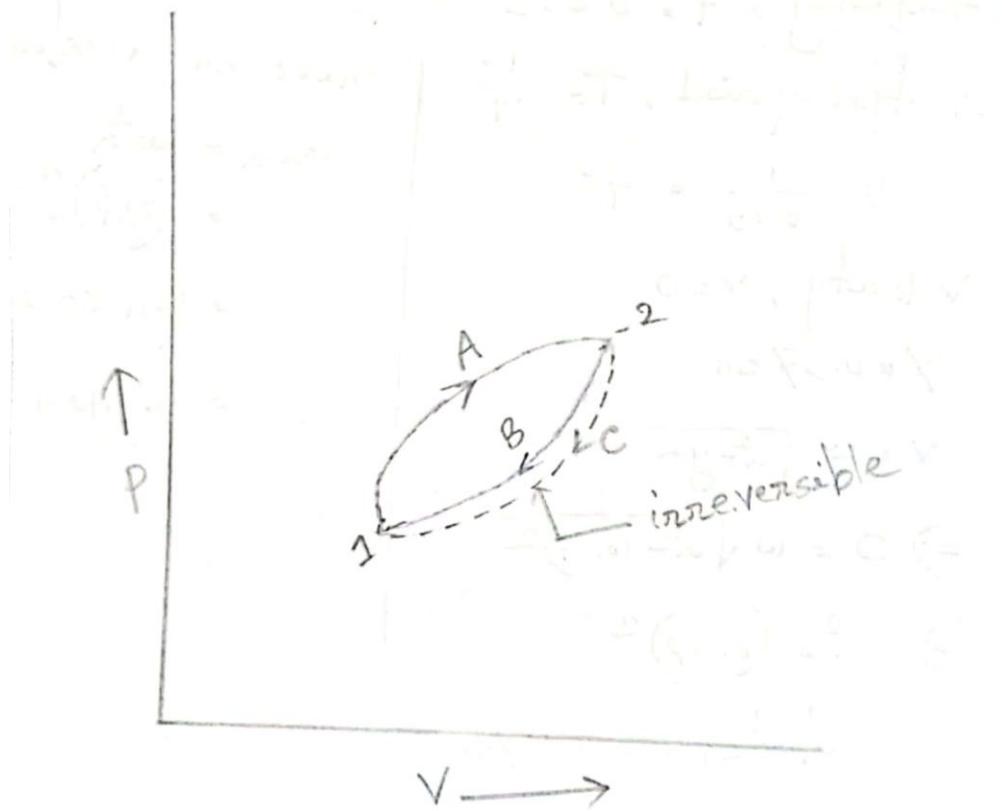
$$\Rightarrow \frac{Q_2}{T_2} - \frac{Q_1}{T_1} > 0$$

The source loses entropy = $\frac{\varrho_1}{T_1}$

The sink gains entropy = $\frac{\varrho_2}{T_2}$

Change in entropy = $\frac{\varrho_2}{T_2} - \frac{\varrho_1}{T_1}$

So, entropy increase in the irreversible process
(Showed)



Q: 6) $n=0$, $t=0$, displaced $y = 0.37\text{cm}$,
 zero velocity. If the frequency $0.25/\text{s}$ calculate
 Period, amplitude, max speed, maximum acceleration

Ans to the question no-6

frequency, $f = 0.25 \text{s}^{-1}$

\therefore time period, $T = \frac{1}{f}$

$$= \frac{1}{0.25} = 4\text{s}$$

Velocity, $v = 0$

$$Y = 0.37\text{cm}$$

$$v = \omega \sqrt{a^2 - y^2}$$

$$\Rightarrow 0 = \omega \sqrt{a^2 - (0.37)^2}$$

$$\Rightarrow a^2 = (0.37)^2$$

\therefore amplitude, $a = 0.37\text{cm}$

maximum speed

$$\begin{aligned} V_{\max} &= \omega \cdot a = 2\pi f \\ &= 2\pi \times 0.25 \times 0.37 \\ &= 0.5812 \text{ cm/s} \end{aligned}$$

maximum acceleration

$$\begin{aligned} a_{\max} &= \omega^2 a \\ &= (2\pi f)^2 \cdot a \\ &= (2\pi \times 0.25)^2 \times 0.37 \\ &= 0.9129 \text{ cm/s}^2 \end{aligned}$$

Q: 7) Calculate the mechanical energy of a particle executing SHM, and show that it remains conserved. Also show that its energy, on an average half kinetic and half potential.

Ans to the question no- 7

Mechanical energy of a particle executing SHM is sum of Potential energy (U) and Kinetic energy (K).

\therefore Mechanical energy, $E = K + U = \text{constant}$

As the displacement increase, U increase and K decrease and vice versa. The total energy

$E = K + U$ is conserved.

Let the displacement = y , mass = m , velocity = v .
potential energy, $U = \int_0^y F dy$

$$T = \text{mass} \times \text{acceleration} = m \times (-\omega^2 y) = -m\omega^2 y$$

$$\text{displacement, } y = a \sin(\omega t + \phi)$$

$$U = \int_0^y m\omega^2 y dy = m\omega^2 \int_0^y y dy = \frac{1}{2} m\omega^2 y^2$$

$$= \frac{1}{2} m \omega^2 a^2 \sin^2(\omega t + \phi)$$

$$= \frac{1}{2} k a^2 \sin^2(\omega t + \phi); [k = m \omega^2] \quad \text{--- ①}$$

Kinetic energy, $K = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{dy}{dt} \right)^2$

$$= \frac{1}{2} m \left[\omega a \cos(\omega t + \phi) \right]^2$$

$$= \frac{1}{2} m a^2 \omega^2 \cos^2(\omega t + \phi)$$

$$= \frac{1}{2} k a^2 \cos^2(\omega t + \phi); [k = m \omega^2] \quad \text{--- ②}$$

From equation ① and ②, both of V and K has a maximum value of $\frac{1}{2} k a^2$.

$$E = K + V = \frac{1}{2} k a^2 \sin^2(\omega t + \phi) + \frac{1}{2} k a^2 \cos^2(\omega t + \phi)$$

$$= \frac{1}{2} k a^2$$

The total energy is constant ($\frac{1}{2} k a^2$)

It's value is same as the maximum value of any one of the two forms of energy.

A maximum displacement, The kinetic energy is zero, but potential energy = $\frac{1}{2} k a^2$.

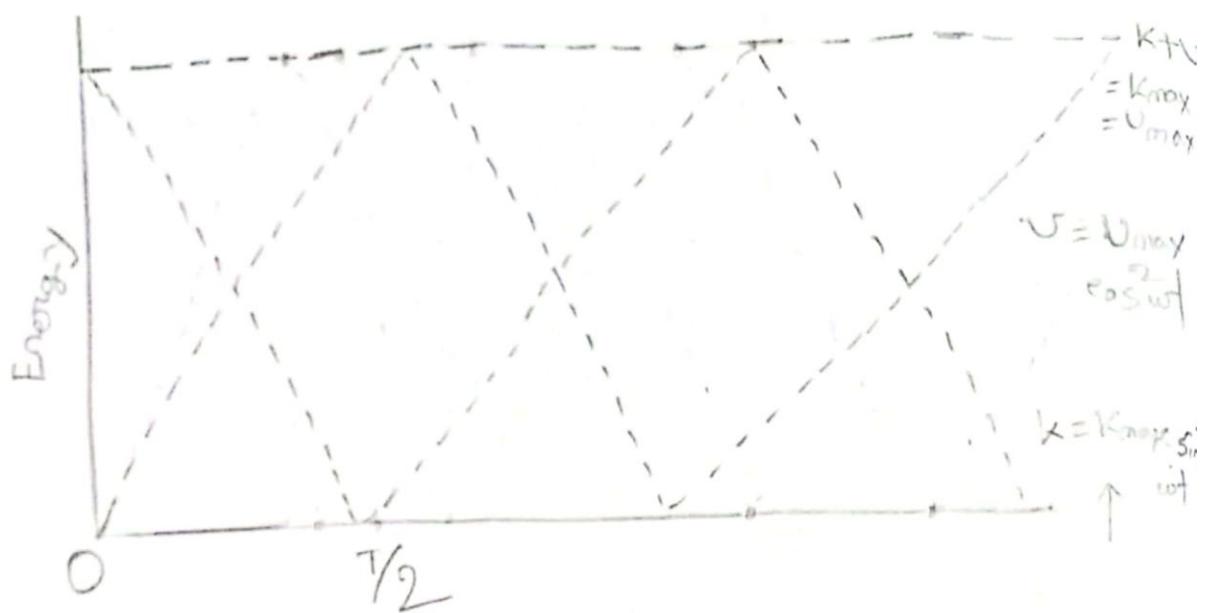


Figure - 1

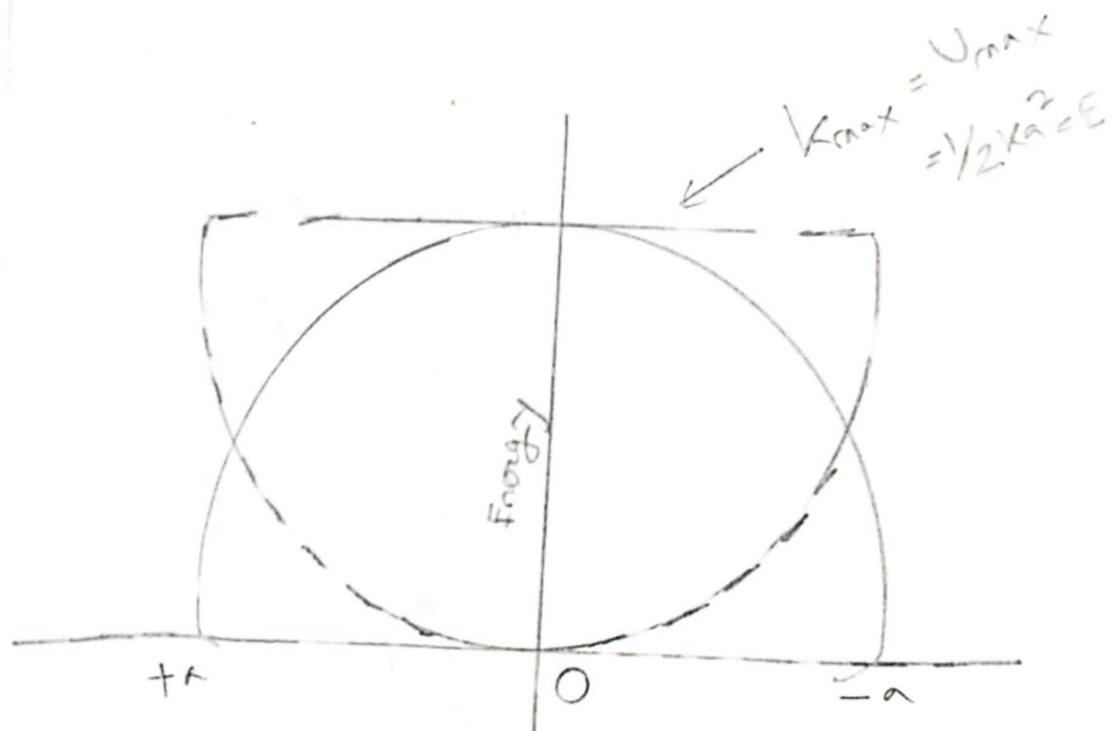


Fig - 2

At equilibrium Potential energy is zero, and kinetic energy is $\frac{1}{2}ka^2$. At any other position, K and U each contribute energy whose sum is always $\frac{1}{2}ka^2$.

Fig. 1 shows, Kinetic and potential energy and total energies as a function of time while

Fig. 2 the same energies are plotted as a function of displacement from the equilibrium position.

$$E \propto a^2$$

$$\text{Now } \frac{1}{2}ka^2 = \frac{1}{2}m\omega^2a^2 = \frac{1}{2}m\left(\frac{2\pi}{T}\right)^2 \cdot a^2$$

$$= \frac{2\pi^2 m a^2}{T^2}$$

$$E = 2\pi^2 m \cdot a^2 \cdot n^2 ; \quad \left[\text{Frequency, } n = \frac{1}{T} \right]$$

Thus total energy = maximum value of potential

energy = maximum value of kinetic energy

$$= \frac{1}{2}ka^2$$

$$= \frac{1}{2}m\omega^2a^2$$

$$= 2\pi^2 n^2 a^2 m$$

The total energy in Fig. 1 represented by upper horizontal line parallel to time axis and touching the total curves at point representing the maximum value of kinetic and potential energy. The upper horizontal line in Fig 2 which passes through two points of maximum displacement $+a$ and $-a$ on either side of mean position represents total energy. This line is also parallel to the displacement axis. Since the line representing total energy in either figure is a horizontal line, being parallel to either the time axis, it follows that energy of the particle executing SHM remains conserved throughout and its independent of time and displacement.

(Showed)

Average value of kinetic and potential energies:

Potential energy, $V = \frac{1}{2} m \omega^2 a^2 \sin^2(\omega t + \phi)$

So, average V over a complete cycle or a whole time period T .

$$\begin{aligned} &= \frac{1}{T} \int_0^T \frac{1}{2} m \omega^2 a^2 \sin^2(\omega t + \phi) dt \\ &= \frac{\frac{m \omega^2 a^2}{2}}{T} \int_0^T [1 - \cos 2(\omega t + \phi)] dt \\ &= \frac{m \omega^2 a^2}{2T} \left[\int_0^T dt - \int_0^T \cos^2(\omega t + \phi) dt \right] \end{aligned}$$

The average value of both a sine and a cosine function for a complete cycle or a whole time period T is zero. We therefore have average V of potential

$$= \frac{1}{2T} m \omega^2 a^2 [+]_0^T - 0$$

$$= \frac{1}{2T} m \omega^2 a^2 T = \frac{1}{2} m \omega^2 a^2$$

$$= \frac{1}{2} k a^2 ; [m \omega^2 = k]$$

Kinetic energy, $K = \frac{1}{2} m \omega^2 a^2 \cos^2(\omega t + \phi)$

The average K over a complete cycle or a whole time period T , as in the case of P.V., is given by

$$\begin{aligned} & \frac{1}{T} \int_0^T \frac{1}{2} m \omega^2 a^2 \cos^2(\omega t + \phi) dt \\ &= \frac{m \omega^2 a^2}{9T} \int_0^T 2 \cos^2(\omega t + \phi) dt \\ &= \frac{m \omega^2 a^2}{9T} \int_0^T [1 + \cos 2(\omega t + \phi)] dt \\ &= \frac{m \omega^2 a^2}{9T} \left[\int_0^T dt + \int_0^T \cos 2(\omega t + \phi) dt \right] \end{aligned}$$

Again average value of a Sine or Cosine function over a Complete cycle or a whole time period is zero. Hence average, $K = \frac{m \omega^2 a^2}{9T} [+]_0^T$

$$= \frac{m \omega^2 a^2}{9T} \cdot T = \frac{1}{9} m \omega^2 a^2 = \frac{1}{9} K a^2$$

Thus, average value of potential energy

= average value of kinetic energy

$$\therefore \frac{1}{9} m \omega^2 a^2 = \frac{1}{9} K a^2$$

= half of total energy

(Showed)

Ans to the question no - 08

Average kinetic energy of a Particle executing

SHM :

The displacement of a vibrating Particle is given by

$$y = a \sin(\omega t + \alpha)$$

$$v = \frac{dy}{dt} = a\omega \cos(\omega t + \alpha)$$

If m is the mass of the vibrating particle, the kinetic energy at any instant

$$= \frac{1}{2} mv^2$$

$$= \frac{1}{2} m a^2 \omega^2 \cos^2(\omega t + \alpha)$$

The average kinetic energy of the Particle in one Complete Vibration,

$$= \frac{1}{T} \int_0^T \frac{1}{2} m a^2 \omega^2 \cos^2(\omega t + \alpha) dt$$

$$= \frac{m a^2 \omega^2}{2T} \int_0^T 2 \cos^2(\omega t + \alpha) dt$$

$$= \frac{m a^2 \omega^2}{2T} \int_0^T [1 + \cos 2(\omega t + \alpha)] dt$$

$$= \frac{m a^2 \omega^2}{2T} \left[\int_0^T dt + \int_0^T \cos 2(\omega t + \alpha) dt \right]$$

The average value of a sine or cosine function over a complete cycle or a whole time period is zero. Hence,

$$\begin{aligned}\text{Average K.E.} &= \frac{m\omega^2 a^2}{2T} [t]_0^T \\ &= \frac{m\omega^2 a^2}{2T} \cdot T \\ &= \frac{1}{2} m\omega^2 a^2 \\ &= \frac{1}{2} k a^2\end{aligned}$$

So, the average kinetic energy of a particle executing SHM is $\frac{1}{2} k a^2 = \frac{1}{2} m\omega^2 a^2$.

Total energy of a particle executing SHM:

$$\begin{aligned}\text{potential energy, PE} &= \int y F dy \\ \text{Here, } F &= \text{mass} \times (\text{acceleration}) \\ &= m \times (-\omega^2 y) \\ &= -m\omega^2 y\end{aligned}$$

$$\begin{aligned}P.E. &= \int_0^y m\omega^2 y dy \\ &= m\omega^2 \int_0^y y dy \\ &= \frac{1}{2} m\omega^2 y^2\end{aligned}$$

The displacement of a particle executing SHM is -

$$y = a \sin(\omega t + \phi)$$

Hence, the PE is

$$PE = \frac{1}{2} m \omega^2 a^2 \sin^2(\omega t + \phi)$$

The kinetic energy of a particle executing SHM is

$$KE = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m \left(\frac{dy}{dt} \right)^2$$

$$= \frac{1}{2} m [a \omega \cos(\omega t + \phi)]^2$$

The total energy of a particle executing SHM can be obtained by adding potential and kinetic energies. That is

$$E = PE + KE$$

$$= \frac{1}{2} m \omega^2 a^2 \sin^2(\omega t + \phi) + \frac{1}{2} m \omega^2 a^2 \cos^2(\omega t + \phi)$$

$$= \frac{1}{2} m \omega^2 a^2 [\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$$

$$= \frac{1}{2} m \omega^2 a^2$$

$$= \frac{1}{2} k a^2$$

we can see that the total energy, is constant and has the value $\frac{1}{2}ka^2$. Thus the total energy of the particle is the same as the maximum value of any one of the two forms of energy. At the maximum displacement the kinetic energy is zero, but the potential energy has the value $\frac{1}{2}ka^2$. At the position of equilibrium the potential energy is zero, but the kinetic energy has the value $\frac{1}{2}ka^2$. At any other position the kinetic and potential energy each contribute energy whose sum is always $\frac{1}{2}ka^2$ which is constant. So, the Principle of Conservation of energy is obeyed by a harmonic oscillation.

Q:9: what are force, damped and forced vibration? A particle showing damped harmonic motion is subjected to an external periodic force. Establish the differential equation of motion of particle explain each terms and solve it to obtain an expression for the displacement of the oscillator.

Ans to the question no-9

Free Vibration: If an object such as the bob of a simple pendulum is displaced from its mean position, and then released, it will show simple harmonic motion. If there is no loss of energy due to friction or otherwise the pendulum will go on oscillating with the same time period and amplitude for any length of time without any damping or decay to oscillation. Such types of motion are referred as the undamped or free vibration.

Damped vibration: Generally a simple harmonic oscillator vibrates in a resisting medium such as air and energy is gradually dissipated in each vibration is overcoming the opposing resistive force.

As a result the amplitude of vibration gradually goes on decreasing with time and finally the oscillation die out. These kinds of vibrations are known as the damped vibration.

Free vibration: whenever a particle oscillates in a medium it gets damped i.e. its amplitude falls ~~expression~~ exponentially with time to zero due to dissipation of energy. If an external periodic force is applied to maintain the motion against the damping force, initially the amplitude of oscillation will increase, then decrease with time, because minus and again increase. Finally the particle will oscillate with frequency of the applied force at a constant amplitude as long as the force remains active, such vibrations of the body are called the forced vibrations.

Differential equation of damped harmonic motion:

A particle showing damped harmonic motion subjected to the following opposing force

- (i) The restoring force ($-ay$, where a is a force constant)
- (ii) Damping force ($-bv$, where b is the damping constant and v is the velocity of the object).

Thus, the differential equation of motion of the object showing simple harmonic motions in a damping medium is given as,

$$\begin{aligned} F &= -ay - bv \\ \Rightarrow m \frac{d^2y}{dt^2} &= -ay - b \frac{dy}{dt} \quad \left| \begin{array}{l} \text{where} \\ 2\lambda = b/m \\ \omega^2 = a/m \end{array} \right. \\ \Rightarrow \frac{d^2y}{dt^2} + 2\lambda \frac{dy}{dt} + \omega^2 y &= 0 \quad \textcircled{1} \end{aligned}$$

Eq. ① Is the differential equation of damped harmonic motion.

Eq ① which is a homogeneous linear type differential equation of the second must have at least one solution of the form $y = Ae^{kt}$ where A and k are arbitrary constant. Differentiating $y = Ae^{kt}$ with respect to time, we get,

$$\frac{dy}{dt} = kAe^{kt} \text{ and } \frac{d^2y}{dt^2} = k^2Ae^{kt}$$

Substituting these values in eq(1) we have

$$k^2Ae^{kt} \text{ and } \frac{d^2y}{dt^2} = k^2Ae^{kt}$$

$$k^2Ae^{kt} + 2\lambda kAe^{kt} + \omega^2Ae^{kt} = 0$$

Dividing throughout by Ae^{kt} , we know

$$k^2 + 2\lambda k + \omega^2 = 0 \quad \textcircled{2}$$

Equation $\textcircled{2}$ is clearly a quadratic equation in k , the solution of which is

$$k = -\lambda \pm \sqrt{\lambda^2 - \omega^2}$$

the differential equation $\textcircled{1}$ is therefore, satisfied by two values of y ,

$$y = Ae^{(-\lambda + \sqrt{\lambda^2 - \omega^2})t}$$

$$\text{and } y = Ae^{(-\lambda - \sqrt{\lambda^2 - \omega^2})t}$$

The equation being a linear one, the linear sum of the two linearly independent solution of the equation is the most general solution. Thus the general solution is

$$y = A_1 e^{(-\lambda + \sqrt{\lambda^2 - w^2})t} + A_2 e^{(\lambda - \sqrt{\lambda^2 - w^2})t} \quad (3)$$

where A_1 and A_2 are two arbitrary constant.

Equation (3) is the expression from the displacement of a demand harmonic oscillator.

Q: 10) what is meant by reverberation and reverberation time? Derive Sabine's reverberation formula.

Ans to the question no- 10

Reberberation: whenever a sound pulse is produced in a room or in an auditorium, a listener receives directly compressional sound waves from the source as well as sound waves from the wall and other materials present in the room. The quality of note received by the listener will be the combined effect on these two waves. There is also a time gap between direct wave and the waves received by successive reflection. Due to this the sound persists for sometime even after

the source has stopped. This persistence of sound is called reverberation.

Reverberation time: The time gap between the initial direct note and the reflected note upto the minimum audibility level is called reverberation time. It depends on the size of the room, on the auditorium, the nature of the reflecting material on the wall and the area of the reflected surfaces.

Sabine's reverberation formula:

The reverberation time, is the time that sound takes to fall to one millionth of its original intensity.

It is mathematically given as

$$It = I_0 \times 10^{-6} \text{ or } \frac{It}{I_0} = 10^{-6}$$

where I_0 = original intensity

It = decreased intensity

Average reverberation is about one second.

The exponential decrease in this intensity is due to the absorption of sound

Sound waves are absorbed by the surrounding so the intensity falls exponentially.

According to the definition of absorption co-efficient,

$$A = \frac{I_A}{I_0} \quad \textcircled{1}$$

where A = Absorption co-efficient

I_A = Absorbed intensity

I_0 = Original intensity

$$\therefore I_A = A \cdot I_0$$

$$\therefore I_R = I_0 - AI_0 \quad [I_R = \text{Reflected Intensity}] \\ = I_0(1-A) \quad \textcircled{2}$$

The sound waves is partially absorbed and partially reflected for a second absorption. So, the intensity of the first reflected wave is I_{R_1}

$$\therefore I_{R_1} = I_0(1-A)^2 \quad [\because A = \frac{I_A}{I_0(1-A)} \Rightarrow I_A = A(1-A)I_0]$$

Similarly,

$$I_{R_2} = I_0(1-A)^3$$

$$I_{R_3} = I_0(1-A)^4 \text{ and soon}$$

$$\begin{aligned} \text{So, } I_{R_2} &= I_0(1-A) - AI_0(1-A) \\ &= I_0(1-A)^2 \end{aligned}$$

Suppose there are n reflections before the sound becomes 10^{-6} times its original intensity

$$\therefore IR_N = I_0 (1-A)^n \quad \textcircled{3}$$

$$\therefore I_0 \times 10^{-6} = I_0 (1-A)^n$$

$$\therefore (1-A)^n = 10^{-6} \quad \textcircled{4}$$

taking loge on both sides,

$$n \log_e (1-A) = \log_e 10^{-6}$$

Hence, $A \ll 1$, so we can expand $\log_e(1-A)$ as follows,

$$\log_e (1-A) = -A - \frac{A^2}{2} - \frac{A^3}{3} \dots \quad \left[\begin{array}{l} \text{by series} \\ \text{expansion} \end{array} \right]$$

But as $A \ll 1$, we can neglect $A^2, A^3 \dots$ as

$$A^2 \approx A^3 \approx 0$$

$$\therefore n(-A) = \log_e 10^{-6}$$

$$\Rightarrow -nA = 2.303 \times (-6) \log_{10} 10$$

$$\therefore nA = 2.303 \times 6 = 13.82$$

$$\therefore A = \frac{1}{n} \times 13.82 \quad \textcircled{5}$$

If the number of reflections before the fall in intensity to 10^{-6} times is known

the absorption co-efficient can be calculated.

Now average distance travelled by source
or in other words, the difference between two
reflections is given by,

$$d = \frac{4V}{S} \quad \textcircled{6}$$

where, V = Volume of the room.

S = total surface area of the room

now velocity = $\frac{\text{dist}}{\text{time}} = \frac{d}{t} = \frac{4V}{St}$

$$\therefore t = \frac{4V}{\text{velocity} \times S}$$

let velocity = v

$$\therefore t = \frac{4V}{Sv} \quad \textcircled{7}$$

so, time n reflection time will be next i.e. $\frac{4nv}{Sv}$

$$\therefore t_n = \frac{4nv}{Sv}$$

$$\Rightarrow n = \frac{t_n sv}{4v} \quad \textcircled{8}$$

Substituting the value of n from above eqn to

(9) we get,

$$(1-A) \frac{t_n sv}{4v} = 10^{-6}$$

taking loge on both side, we get

$$\frac{t_n s u}{q v} \log_e (1-A) = -2.303 \times 6 \log_{10}^{10}$$

$$\therefore \log (1-A) \cdot t_n = \frac{-2.303 \times 6 \times q v}{s u} \quad \textcircled{9}$$

Expanding the l.h.s of above equation we have the following result,

$$\left\{ -A - \frac{A^2}{2} - \frac{A^3}{3} \dots \dots \right\} t_n = \frac{-2.303 \times 6 \times q v}{s u}$$

but $A \ll 1$, so $-A^2 \approx A^3 \approx 0$ so, we can neglect them

$$\begin{aligned} \therefore -A \cdot t_n &= \frac{-2.303 \times 6 \times q v}{s u} \\ \Rightarrow t_n &= \frac{-2.303 \times 6 \times q}{s u} \cdot \frac{5}{s \cdot A} \\ &\approx \frac{0.05 v}{s \cdot A} \quad [v = 1200] \end{aligned}$$

Here s is the total surface area and

A is the absorption co-efficient.

$$\therefore t_n = T = \frac{0.05 \times v}{\sum A_i s_i}$$

$$\therefore T = \frac{0.05 \times v}{\sum A_i s_i} \quad \textcircled{10}$$

THE END