# FIR Filter

A Finite Impulse Response filter is a type of digital filter used in signal processing to perform various filtering operation on digital signal. Unlike IIR filters, FIR filters are stable and have linear phase characteristics. It has min phase distortion.

General equation, $y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \dots + b_N x(n-N)$

$\downarrow$

o/p at time index n

N - Filter order

$b_0\ b_1 \dots b_N$ filter coefficient

Designing an FIR filter involves determining the values of the filter coefficient to achieve the desired filter characteristics. There are various methods to design FIR filter including window methods, freq. sampling methods, and optimal linear phase method etc.

Some common applications

LPF, HPF, BPF for signal conditioning

Signal interpolation and decimation

Equalization of audio signal

Image processing such as image smoothing or edge detection

NB. Interpolation increasing the number of samples in signal to obtain a higher sampling rate. whereas decimation reduces the sampling rate

Disadvantage:

- Complex
- required more filter coefficient to be stored
- Long duration impulse response
- Narrow transition band

FIR filter requires more arithmetic operation

Advantages

- Sable
- Can be realized in both recursive or non-recursive
- exact linear phase
- Flexible
- low sensitivity to quantization noise
- efficiently realized in hardware

Impulse Response representation

Symmetric Phase FIR filter

$h(n) = h(M-1-n)$ ; $0 \le n \le M-1$ $M \to$ filter order

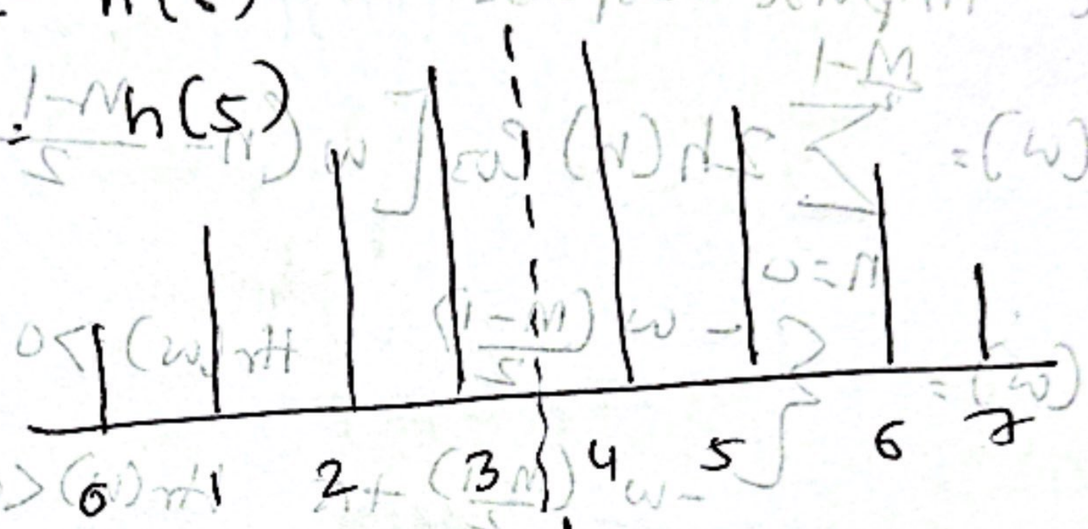
Let, $M = 8$

$h(n) = h(8-1-n), 0 \le n \le 7$

$n=0$ $h(0) = h(7)$ $h(3) = h(3) = h(4)$

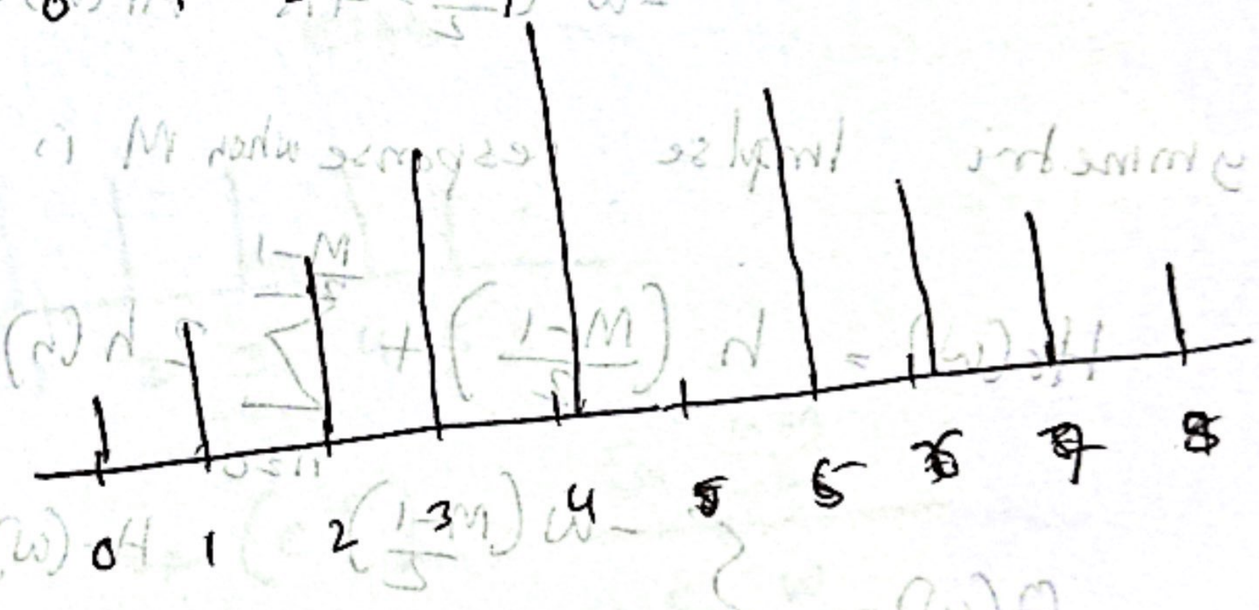
$n=1$ $h(1) = h(6)$

$n=2$ $h(2) = h(5)$




Symmetric representation of FIR filter

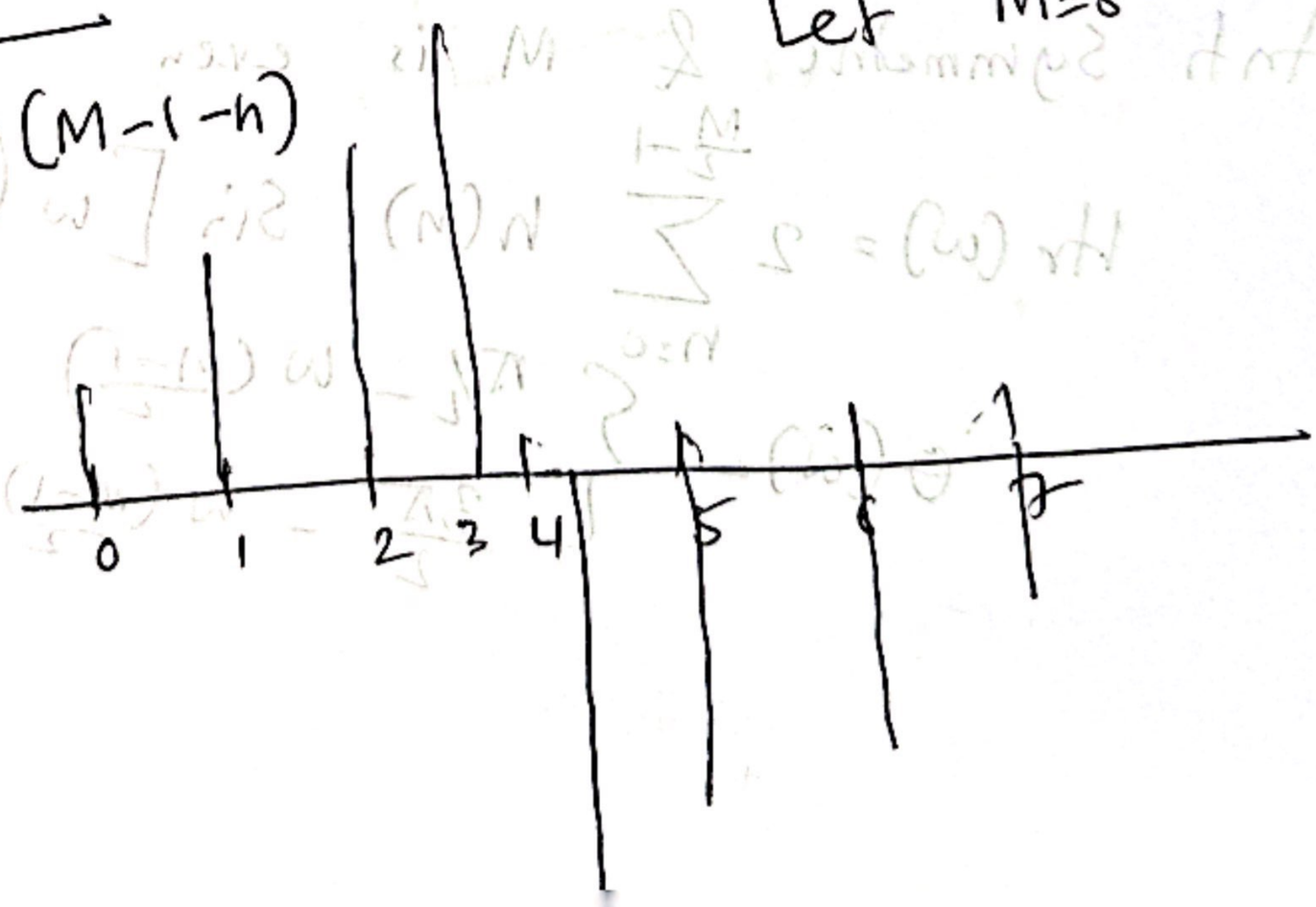
Let $M = 9$




Anti symmetric

Let $M=8$

$h(n) = -h(M-1-n)$



## Freq response

$$h(n) \xrightarrow{DTFT} H(\omega)$$

freq response of FIR filter can be given as

$$H(\omega) = \underbrace{H_r(\omega)}_{\text{real part of } H(\omega)} e^{-j\theta(\omega)}$$

(i) Symmetric Impulse response with M is even

$$H_r(\omega) = \sum_{n=0}^{\frac{M}{2}-1} 2h(n) \cos\left[\omega\left(n - \frac{M-1}{2}\right)\right]$$

$$\theta(\omega) = \begin{cases} -\omega\left(\frac{M-1}{2}\right) & H_r(\omega) > 0 \\ -\omega\left(\frac{M-1}{2}\right) + \pi & H_r(\omega) < 0 \end{cases}$$

(ii) Symmetric Impulse response when M is odd

$$H_r(\omega) = h\left(\frac{M-1}{2}\right) + \sum_{n=0}^{\frac{M-1}{2}} 2h(n) \cos\left[\omega\left(n - \frac{M+1}{2}\right)\right]$$

$$\theta(\omega) = \begin{cases} -\omega\left(\frac{M-1}{2}\right) & H_r(\omega) > 0 \\ -\omega\left(\frac{M-1}{2}\right) + \pi & H_r(\omega) < 0 \end{cases}$$

(iii) Anti Symmetric & M is even

$$H_r(\omega) = 2\sum_{n=0}^{\frac{M}{2}-1} h(n) \sin\left[\omega\left(\frac{M-1}{2} - n\right)\right]$$

$$\theta(\omega) = \begin{cases} \pi/2 - \omega\left(\frac{M-1}{2}\right) & H_r(\omega) > 0 \\ \frac{3\pi}{2} - \omega\left(\frac{M-1}{2}\right) & H_r(\omega) < 0 \end{cases}$$

Anti symmetric with M odd

$$H_r(\omega) = 2\sum_{n=0}^{\frac{M}{2}-1} h(n)\sin\left(\omega\left[\frac{M-1}{2}-n\right]\right)$$

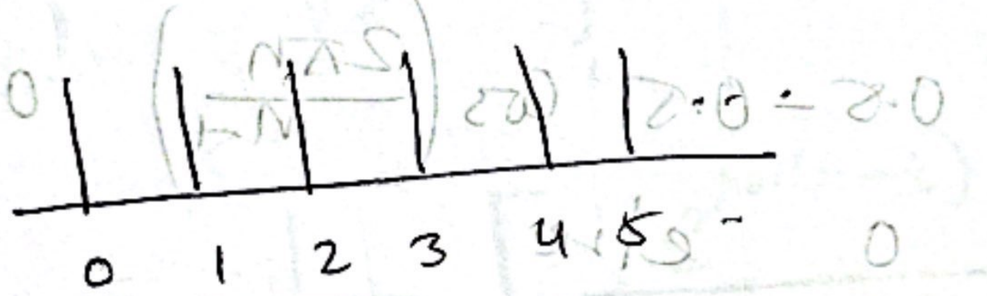
$$\theta(\omega) = \begin{cases} \pi/2 - \omega\left(\frac{M-1}{2}\right) & H_r(\omega) > 0 \\ \frac{3\pi}{2} - \omega\left(\frac{M-1}{2}\right) & H_r(\omega) < 0 \end{cases}$$

Designing of Linear phase FIR Filter using windowing Method

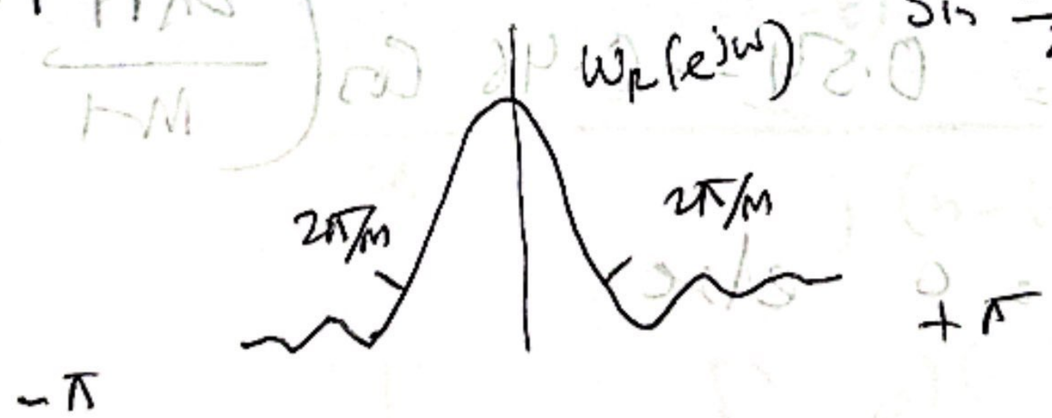
(a) Rectangular window

$$w_R(n) = 1 \quad 0 \le n \le M-1$$
$$0 \quad \text{else}$$

M=6



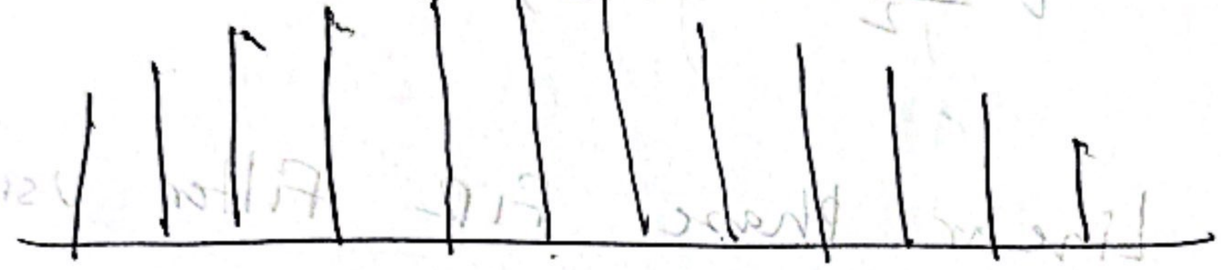

Spectrum $W_R(e^{j\omega}) = \dfrac{\sin\frac{\omega M}{2}}{\sin\frac{\omega}{2}}$



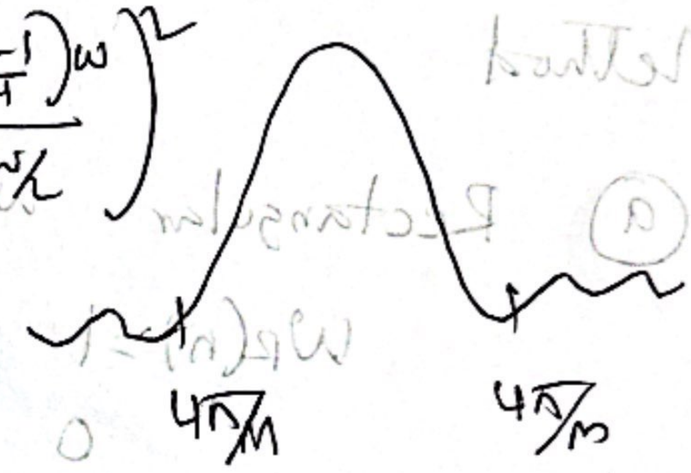
Bartlett window: (Triangular window)

$$W_B(n) = 1 - \frac{2\left[n - \left(\frac{M-1}{2}\right)\right]}{M-1} \qquad 0 \le n \le M-1$$

$$= 0$$



Spectrum $W_T(e^{j\omega}) = \left(\frac{\sin\left(\frac{M-1}{4}\right)\omega}{\sin \omega/2}\right)^2$




~~Hamming~~ Hanning window

$$W_H(n) = 0.5 - 0.5 \cos\left(\frac{2\pi n}{M-1}\right) \qquad 0 \le n \le M-1$$

$$0 \quad \text{else}$$

Hamming window

$$W_{HW}(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{M-1}\right) \qquad 0 \le n \le M-1$$

$$= 0 \quad \text{else}$$

Design the symmetric FIR LPF whose

$$H_d(\omega) = \begin{cases} e^{-j\omega t} & |\omega| \le \omega_c \\ 0 & \text{otherwise} \end{cases}$$

with $M=7$, $\omega_c = 1$ rad/sec

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega \quad \text{—— (1)}$$

$$= \frac{1}{2\pi}\int_{-1}^{1} e^{-j\omega t} \cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-1}^{1} e^{j\omega(n-t)}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(n-t)}}{j(n-t)}\right]$$

$$= \frac{1}{2\pi}\,\frac{e^{j(n-t)} - e^{-j(n-t)}}{j(n-t)}$$

$$= \frac{1}{\pi(n-t)}\left[\frac{e^{j(n-t)} - e^{-j(n-t)}}{2j}\right]$$

$$= \frac{\sin(n-t)}{\pi(n-t)} \qquad n \neq t$$

If $n = t$

$$\cancel{h_d(n)} \quad h_d(n) \frac{1}{2\pi} \int_{-1}^{1} 1 \, d\omega = \frac{1}{\pi}$$

$$h_d(n) = \begin{cases} \dfrac{\sin(n-t)}{\pi(n-t)} & n \neq t \\ \dfrac{1}{\pi} & n = t \end{cases}$$

determine the value of t

$$h(n) = h(M-1-n)$$

$$h(n) = h_d(n) \cdot w(n)$$

$$h_d(n) w(n) = h_d(M-1-n) \, w(n)$$

$$h_d(n) = h_d(M-1-n)$$

$$\frac{-\sin(n-t)}{-\pi(n-t)} = \frac{\sin(M-1-n-t)}{\pi(M-1-n-t)}$$

$$= \frac{\sin(-(n-t))}{\pi(-(n-t))} = \frac{\sin(M-1-n-t)}{\pi(M-1-n-t)}$$

$$-(n-t) = M-1-n-t$$

$$-n+t = M-1-n-t \qquad t = \frac{M-1}{2}$$

$$h_d(n) = \begin{cases} \dfrac{\sin\left(n - \frac{M-1}{2}\right)}{\pi \cdot \left(n - \frac{M-1}{2}\right)} & n \neq \frac{M-1}{2} \\ \dfrac{1}{\pi} & n = \frac{M-1}{2} \end{cases}$$

Since $M = 7$

$$h_d(n) = \begin{cases} \dfrac{\sin(n-3)}{\pi(n-3)} & n \neq 3 \\ \dfrac{1}{\pi} & n = 3 \end{cases}$$

$n = 0$ to $6$

Design the symmetric FIR LPF whose

$$H_d(\omega) = \begin{cases} e^{-j\omega t} & |\omega| \le \omega_c \\ 0 & \text{otherwise} \end{cases}$$

with $M=7$, $\omega_c = 1$ rad/sec

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega \quad \text{—— (1)}$$

$$= \frac{1}{2\pi}\int_{-1}^{1} e^{-j\omega t} \cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-1}^{1} e^{j\omega(n-t)}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(n-t)}}{j(n-t)}\right]$$

$$= \frac{1}{2\pi}\,\frac{e^{j(n-t)} - e^{-j(n-t)}}{j(n-t)}$$

$$= \frac{1}{\pi(n-t)}\left[\frac{e^{j(n-t)} - e^{-j(n-t)}}{2j}\right]$$

$$= \frac{\sin(n-t)}{\pi(n-t)} \qquad n \neq t$$

If $n = t$

$$\cancel{hd(n)} \quad hd(n) \frac{1}{2\pi} \int_{-1}^{1} 1 \, d\omega = \frac{1}{\pi}$$

$$hd(n) = \begin{cases} \dfrac{\sin(n-t)}{\pi(n-t)} & n \neq t \\ \dfrac{1}{\pi} & n = t \end{cases}$$

determine the value of t

$$h(n) = h(M-1-n)$$

$$h(n) = hd(n) \, w(n)$$

$$hd(n) \, w(n) = hd(M-1-n) \, w(n)$$

$$hd(n) = hd(M-1-n)$$

$$\frac{-\sin(n-t)}{-\pi(n-t)} = \frac{\sin(M-1-n-t)}{\pi(M-1-n-t)}$$

$$= \frac{\sin(-(n-t))}{\pi(-(n-t))} = \frac{\sin(M-1-n-t)}{\pi(M-1-n-t)}$$

$$-(n-t) = M-1-n-t$$

$$-n+t = M-1-n-t \qquad t = \frac{M-1}{2}$$

$$h_d(n) = \begin{cases} \dfrac{\sin\left(n - \frac{M-1}{2}\right)}{\pi\left(n - \frac{M-1}{2}\right)} & n \neq \frac{M-1}{2} \\ \dfrac{1}{\pi} & n = \frac{M-1}{2} \end{cases}$$

Since $M = 7$

$$h_d(n) = \begin{cases} \dfrac{\sin(n-3)}{\pi(n-3)} & n \neq 3 \\ \dfrac{1}{\pi} & n = 3 \end{cases}$$

$n = 0$ to $6$

$n=0$ $h_d(0) = 0.01497$

1 $h_d(1) = 0.14472$

2 $h_d(2) = 0.26786$

3 $h_d(3) = 1/\pi$

4 $h_d(4)$ $0.26786$

5 $h_d(5) = 0.14472$

6 $h_d(6) = 0.01497$

$h(n) = h_d(n) \cdot w(n)$

$w_n(n) = \begin{cases} 1 & 0 \le n \le 6 \\ 0 & \text{other} \end{cases}$

$h(n) = h_d(n)$ ; $0 \le n \le$

$= 0$ otherwise

$h(0) = 0.1497$

$h(1) = 0.14472$

$h(2) = 0.267888$

~~$h(2)$~~ $=$

$h(3) = 1/\pi$

$h(4) = 0.26786$

$h(5) = 0.14472$

$h(6) = 0.1497$

Linear phase

Design the symmetric FIR Filter using windows

$H_d(\omega) \rightarrow$ desired freq response

$h_d(n) \rightarrow$ " Sample "

$$H_d(\omega) = \sum_{n=0}^{\infty} h_d(n) e^{-j\omega n} \quad \text{—— (1)}$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n} d\omega \quad \text{—— (2)}$$

Example

Rectangular Window

$$W_R(n) = \begin{cases} 1 & n = 0, 1, 2 \ldots M-1 \\ 0 & \text{otherwise} \end{cases}$$

$$h(n) = h_d(n) W_R(n) \quad \text{—— (3)}$$

$$h(n) = \begin{cases} h_d(n) & n = 0, 1, 2 \ldots M-1 \\ 0 & \text{otherwise} \end{cases}$$

General eq$^n$ of linear phase response

$$h(n) = h_d(n) W(n)$$

$$H(\omega) = H_d(\omega) * W(\omega) \quad \text{—— (4)}$$

Design the symmetric FIR LPF

$$H_d(\omega) = \begin{cases} e^{-j\omega\tau} & |\omega| \le \omega_c \\ 0 & \end{cases}$$

$M=7$ $\omega_c = 1$ User rect window

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega \quad \text{— (1)}$$

$$H_d(\omega) = \begin{cases} e^{-j\omega\tau} & -1 \le \omega \le 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{— (2)}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-1}^{1} e^{-j\omega\tau}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(n-\tau)}}{j(n-\tau)}\right]_{-1}^{1}$$

$$= \frac{1}{2\pi}\,\frac{e^{j(n-\tau)} - e^{-j(n-\tau)}}{j(n-\tau)}$$

$$= \frac{1}{\pi(n-\tau)}\sin(n-\tau)$$

$$= \operatorname{sinc}(n-\tau) \quad n \ne \tau \quad \text{— (3)}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-1}^{1} 1\, d\omega = \frac{1}{\pi}$$

$$h_d(\tau) = \begin{cases} \dfrac{\sin(n-\tau)}{\pi(n-\tau)} & n \neq \tau \\ \dfrac{1}{\pi} & n = \tau \end{cases}$$

$$h(n) = h(M-1-n)$$

$$h(n) = h_d(n)\, w(n)$$

$$h_d(n)\, w(n) = h_d($$

Design the Symmetric FIR Filter

$$H_d(\omega) = \begin{cases} e^{-j\omega\tau} & |\omega| \le \omega_c \\ 0 & \text{otherwise} \end{cases} \qquad M=7 \quad \omega_c = 1$$

Hanning Window

| n | hd |
|---|---|
| 0 | 0.1497 |
| 1 | 0.14472 |
| 2 | 0.26785 |
| 3 | 1/π |
| 4 | 0.26785 |
| 5 | 0.14472 |
| 6 | 0.1497 |

Hanning Window

$$W(n) = 0.5\left[1 - \cos\left(\frac{2\pi n}{M-1}\right)\right]$$

$M=7$

$$W(n) = 0.5\left[1 - \cos\left(\frac{2\pi n}{6}\right)\right]$$

$$= 0.5\left[1 - \cos\left(\frac{\pi n}{3}\right)\right]$$

$\boxed{\text{Kaul}}$

$$0.5(1 - \cos(\pi n/3))$$

$n=0 \quad W(0) = 0$

$n=1 \quad W(1) = 0.25$

$n=2 \quad W(2) = 0.75$

$n=3 \quad W(3) = 1$

$n=4 \quad W(4) = 1$

$n=5 \quad W(5) =$

$n=6 \quad W(6) =$

$$h(n) = h_d(n) \cdot W(n)$$

$h(0) = 0$

$h(1) = 0.03618$

$h(2) = 0.20089$

$h(3) = 1/5$

$h(4) = 0.20089$

$h(5) = 0.03618$

$h(6) = 0$

Determine the filter coefficient $h_d(n)$ for the desired freq response of a low pass filter given by

$$H_d(e^{jw}) = \begin{cases} \cancel{e^{-jw}}\, e^{-j2w} & -\pi/4 \le w \le \pi/4 \\ 0 & \pi/4 \le |w| \le \pi \end{cases}$$

if we defined new filter co-efficient by $h_d(n) \cdot w(n)$ where $w(n) = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & \text{else} \end{cases}$ Determine $h(n)$ and also the frequency response $H(e^{jw})$ and compare with $H_d(e^{jw})$. Determine $H(e^{jw})$ using the Hanning window.

$$\cancel{h_d(n) = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-jw} \cdot e^{-j}}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(w)\, e^{jwn}\, dw = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-j2w} \cdot e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{jw(n-2)}\, dw = \frac{1}{2\pi} \left. \frac{e^{jw(n-2)}}{j(n-2)} \right|_{-\pi/4}^{\pi/4}$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\pi/4(n-2)} - e^{-j\pi/4(n-2)}}{j(n-2)}\right]$$

$$= \frac{1}{\pi(n-2)} \cdot \sin \pi/4 (n-2) \qquad n \ne 2$$

If $n=2$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} 1 \, dw = \frac{1}{4}$$

$$h_d(n) = \begin{cases} \dfrac{\sin \frac{\pi}{4}(n-2)}{\pi(n-2)} & n \neq 2 \\ \dfrac{1}{4} & n = 2 \end{cases}$$

<u>$h(n)$</u>

$$h(n) = h_d(n)\, w(n) \qquad 0 \le n \le 4$$

$$h(0) = h_d(0) = 0.159091$$

$$h(1) = h_d(1) = 0.224989$$

$$h(2) = h_d(2) = 1/4$$

$$h(3) = h_d(3) = 0.224989$$

$$h(4) = h_d(4) = 0.159091$$

$M = 5$ as $0 \leq n \leq 4$

↳ odd

$$H(\omega) = e^{-j\omega\left(\frac{M-1}{2}\right)} \left\{ h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{\frac{M-3}{2}} h(n) \cos \omega\left(n - \frac{M-1}{2}\right) \right]$$

$$= e^{-j2\omega} \left[ h(2) + 2\, h(n) \cos \omega(n-2) \right]$$

$$= e^{-j2\omega} \left[ h(2) + 2\, h(0) \cos \omega(-2) + 2h(1) \cos \omega(-1) \right]$$

$$= e^{-j2\omega} \left[ 0.25 + 2 \times 0.159091 \cos 2\omega + 2 \times 0.224989 \cos \omega \right]$$

$$= e^{-j2\omega} \left[ 0.25 + 0.318 \cos 2\omega + 0.45 \cos \omega \right]$$

$$|H(\omega)| = 0.25 + 0.318 \cos 2\omega + 0.45 \cos \omega$$

$$\angle H(\omega) = -2\omega \; ; \quad |H(\omega)| > 0$$

$$= -2\omega + \pi \quad |H(\omega)| < 0$$

<u>$H(e^{j\omega})$ with $H_d(e^{j\omega})$</u>

Not Same

Obtain $H(e^{jw})$ using Hamming window

Hamming window $w(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{M-1}\right)$ ; $0 \le n \le M$

$$w(n) = 0.54 - 0.46 \cos\left(\frac{\pi n}{2}\right) \qquad 0 \le n \le 4$$

$n=0$ $w(0) = 0.08$

$n=1$ $w(1) = 0.54$

$n=2$ $w(2) = 1$

$n=3$ $w(3) = 0.54$

$n=4$ $w(4) = 0.08$

$h(n) = h_d(n) \cdot w(n)$

$h(0) = 0.01273$

$h(1) = 0.12149$

$h(2) = 0.25$

$h(3) = 0.12149$

$h(4) = 0.01273$

$M=5$

$$\cancel{H(w) = e^{-j2w}\left[2\sum h(n) \cos w(n-2)\right]}$$

$$H(w) = e^{-j2w}\left[h(2) + 2\sum_{n=0}^{1} h(n) \cos w(n-2)\right]$$

$$= e^{-j2w}\left[0.25 + 0.02544 \cos 2w + 0.243 \cos w\right]$$

Design a FIR Linear phase Filter using Kaiser window to meet the following specification.

$$0.99 \le |H(e^{j\omega})| \le 1.01 \qquad 0 \le |\omega| \; 0.19\pi$$

$$|H(e^{j\omega})| \le 0.01 \qquad 0.21\pi \le |\omega| \le \pi$$

$$\underbrace{1 - 0.01}_{\delta_1} \le |H(e^{j\omega})| \le \underbrace{1 + 0.01}_{\delta_2} \qquad \rightarrow 0 \le |\omega| \le 0.19\pi \qquad \cancel{0.21\pi \le |\omega| \le \pi}$$

$$\underbrace{0.21\pi \le |\omega| \le \pi}_{\omega_p}$$

$$|H(e^{j\omega})| \le 0.01 \qquad \underbrace{0.21\pi \le |\omega| \le \pi}_{\omega_s}$$

$$\underbrace{\delta_1 = 0.01 \qquad \delta_2 = 0.01}_{\text{ripple factor}} \qquad \omega_p = 0.19\pi \qquad \omega_s = 0.21\pi$$

Pass band edge

Stop band edge

$$\Delta\omega = \omega_s - \omega_p = 0.21\pi - 0.19\pi = 0.02\pi$$

$$\delta = \min(\delta_1, \delta_2) = 0.01$$

Attenuation $A = -20\log_{10}\delta = -20\log_{10} 0.01 = 40$

Cutoff freq $\omega_c = \dfrac{\omega_p + \omega_s}{2} = \dfrac{0.19\pi + 0.21\pi}{2}$

$= 0.2\pi$

Shape parameter

Obtain $\beta$ and $M$

$$\beta = \begin{cases} 0.1102(A - 8.7) & A > 50 \\ 0.5842(A-21) + 0.07886(A-21) & 21 \le A \le 50 \\ 0 & A < 21 \end{cases}$$

$$\beta = 0.5842(40-21)^{0.4} + 0.07886(40-21) = 3.375$$

$$M = \frac{A-8}{2.285\Delta\omega} = \frac{40-8}{2.285(0.02\pi)} = 223$$

Design of linear phase FIR filter using freq sampling method.

Desired freq response $H_d(\omega)$

This freq response is sampled at M points

Thus $\omega = \frac{2\pi}{M} \cdot k \qquad K = 0, 1, \ldots M-1$

Discrete Forier Transform

$H(k) = H_d(\omega) \; ; \quad K = 0, 1, 2, \ldots M-1$

$$H(k) = H_d\left(\frac{2\pi}{M} k\right)$$

$H(k) \rightarrow$ M point DFT

Take IDFT of $H(k)$ to get $h(n)$

$h(n) \rightarrow$ unit sample response of FIR Filter

If $M \rightarrow$ odd $\quad h(n) = \frac{1}{M}\left[H(0) + 2\sum_{k=1}^{\frac{M-1}{2}} Re\{H(k) e^{j\frac{2\pi}{N}kn}\}\right]$

If $M \rightarrow$ even $\quad h(n) = \frac{1}{M}\left[H(0) + 2\sum_{k=1}^{\frac{M}{2}-1} Re\{H(k)\, e^{-j\frac{2\pi}{N}kn}\}\right]$

$$\boxed{\text{Window } H_d(\omega) \xrightarrow{\text{IDFT}} \underset{\text{infinite}}{h_d(n)} \xrightarrow{\text{Finite}} \overbrace{h_d(n) \cdot w(n)}^{h(n)}}$$

Example

$$H_d(e^{jw}) = \begin{cases} e^{-j(N-1)w/2} & 0 \le |w| \le \pi/2 \\ 0 & \pi/2 \le w \le \pi \end{cases}$$

$M = N = 7$ use sampling approach

(I) <u>Desired freq response</u>

$$N = 7 \quad H_d(e^{jw}) = \begin{cases} e^{-j3w} & 0 \le |w| \le \pi/2 \\ 0 & \pi/2 \le |w| \le \pi \end{cases}$$

$$w = \frac{2\pi k}{M} = \frac{2\pi k}{7} \qquad k = 0, 1, 2 \ldots$$

6

(II) <u>Sample $H_d(e^{jw})$</u>

$$H_d(e^{jw}) = \begin{cases} e^{-j\frac{6\pi k}{7}} & 0 \le \left|\frac{2\pi k}{7}\right| \le \pi/2 \\ 0 & \pi/2 \le \left|\frac{2\pi k}{7}\right| \le \pi \end{cases}$$

$$H_d(k) = \begin{cases} e^{-j\frac{6\pi k}{7}} & 0 \le |k| \le 7/4 \\ 0 & 7/4 \le |k| \le 7/2 \end{cases}$$

(III) $M = N = 7$ odd

$$h(n) = \frac{1}{M}\left[H(0) + 2\sum_{k=1}^{\frac{M-1}{2}} \mathrm{Re}\left\{H(k)\, e^{j\frac{2\pi}{N}kn}\right\}\right]$$

$$\frac{M-1}{2} = \frac{7-1}{2} = 3 \qquad H(0) = 1$$

$$= \frac{1}{7}\left[1 + 2\sum_{k=1}^{3} \mathrm{Re}\left\{e^{-j2\pi k(3-n)/7}\right\}\right]$$

$$h(n) = \frac{1}{7}\left[1 + 2\sum_{k=1} \cos\left(\frac{2\pi k(3-n)}{7}\right)\right]$$

$n = 0, 1, \ldots 6$

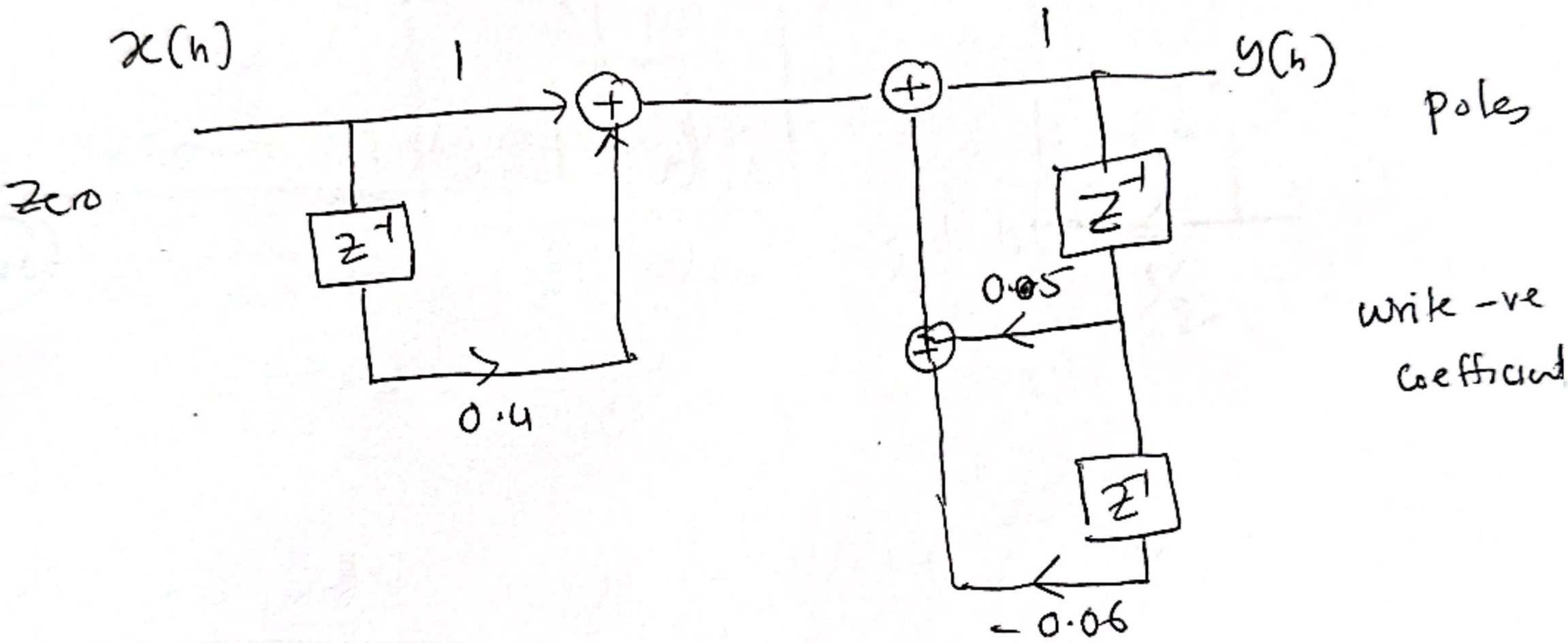
## Direct form structure

$$H(z) = \frac{1 + 0.4z^{-1}}{1 - 0.5z^{-1} + 0.06z^{-2}}$$

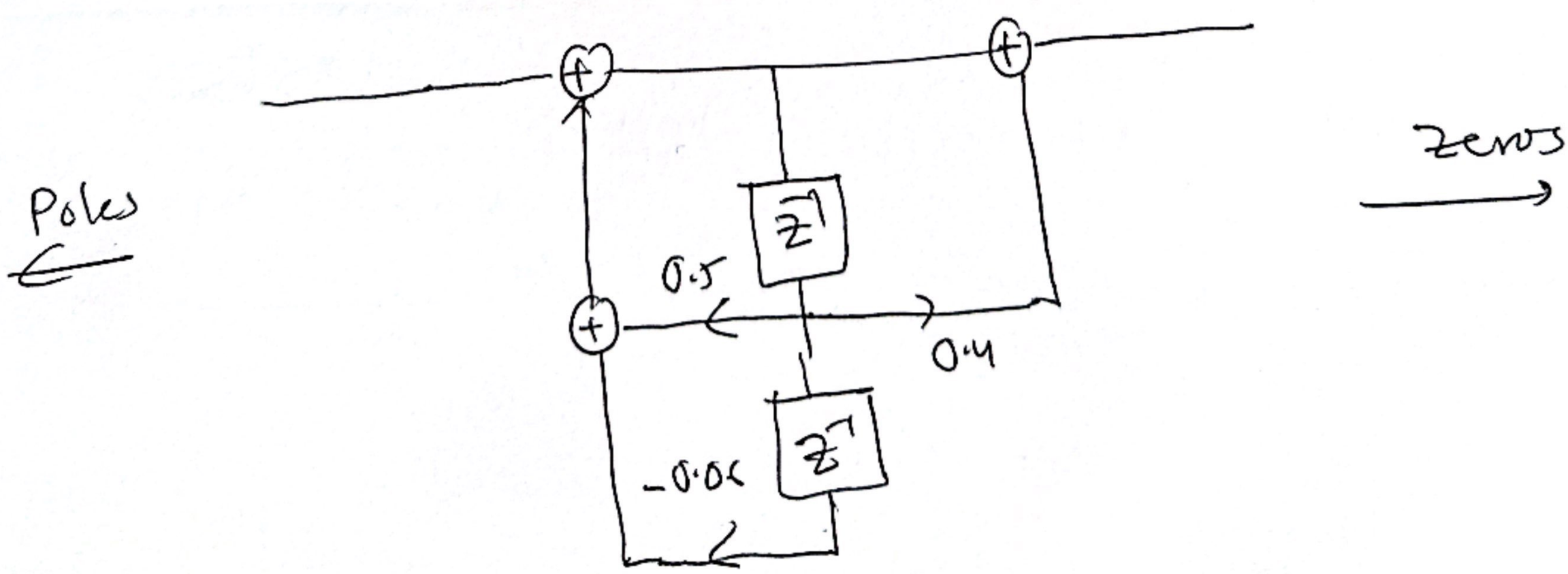
— Zeros

— Poles

### Direct form I

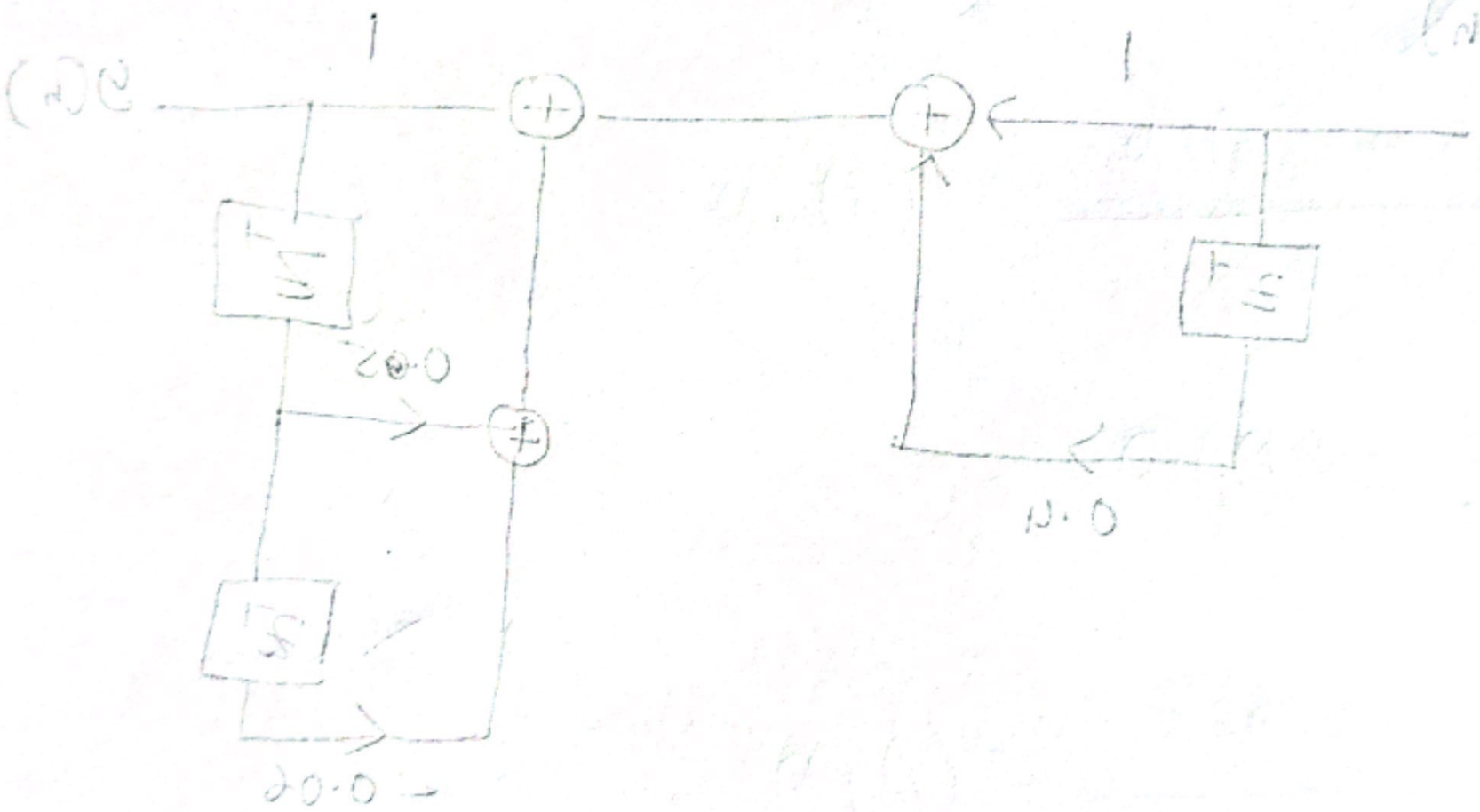



### Direct form II



Solve

$$H(z) = \frac{z^{-1} - 3z^{-2}}{10 + 4z^{-1} + 4.5z^{-2} - 0.5z^{-3}}$$

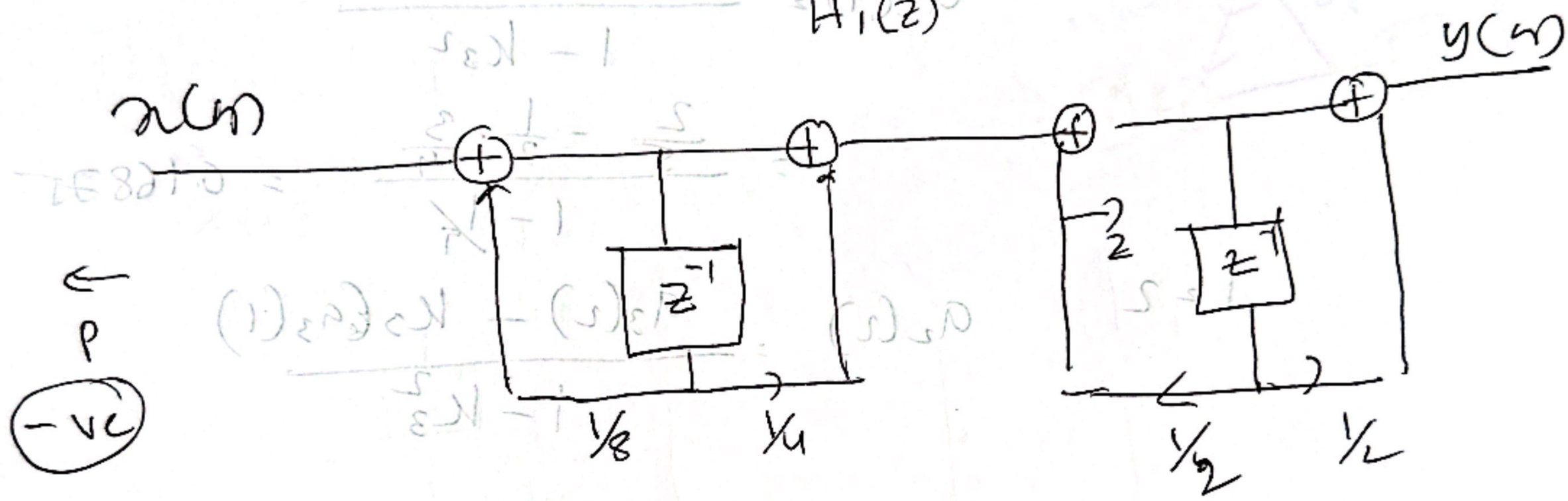


$$H(z) = H_1(z)\, H_2(z)$$

Cascade form structure:

$$H(z) = \frac{1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}{1-\frac{5}{8}z^{-1}+\frac{1}{16}z^{-2}} = \frac{\left(1+\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)}{\left(1-\frac{1}{8}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}$$

$$= \underbrace{\frac{\left(1+\frac{1}{4}z^{-1}\right)}{\left(1-\frac{1}{8}z^{-1}\right)}}_{H_1(z)} \cdot \underbrace{\frac{\left(1+\frac{1}{2}z^{-1}\right)}{\left(1-\frac{1}{2}z^{-1}\right)}}_{H_2(z)}$$




Solve

$$H(z) = \frac{\left(1+\frac{1}{2}z^{-1}\right)}{\left(1-\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2}\right)\left(1+\frac{1}{5}z^{-1}+\frac{1}{6}z^{-1}\right)}$$

Lattice Representation

$$H(z) = \frac{1}{1 + \frac{2}{5} z^{-1} + \frac{3}{4} z^{-2} + \frac{1}{3} z^{-3}}$$

$m = 3$ $\quad a_3(0) = 1 \quad a_3(1) = 2/5 + a_3(2) = 3/4$

$a_3(3) = 1/3 = k_3$

$m = 3$

$i = 1$

$$a_2(1) = \frac{a_3(1) - k_3 a_3(2)}{1 - k_3^2}$$

$$= \frac{\frac{2}{5} - \frac{1}{3} \cdot \frac{3}{4}}{1 - 1/9} = 0.16875$$

$i = 2$

$$a_2(2) = \frac{a_3(2) - k_3 a_3(1)}{1 - k_3^2}$$

$$= 0.69375 = k_2$$

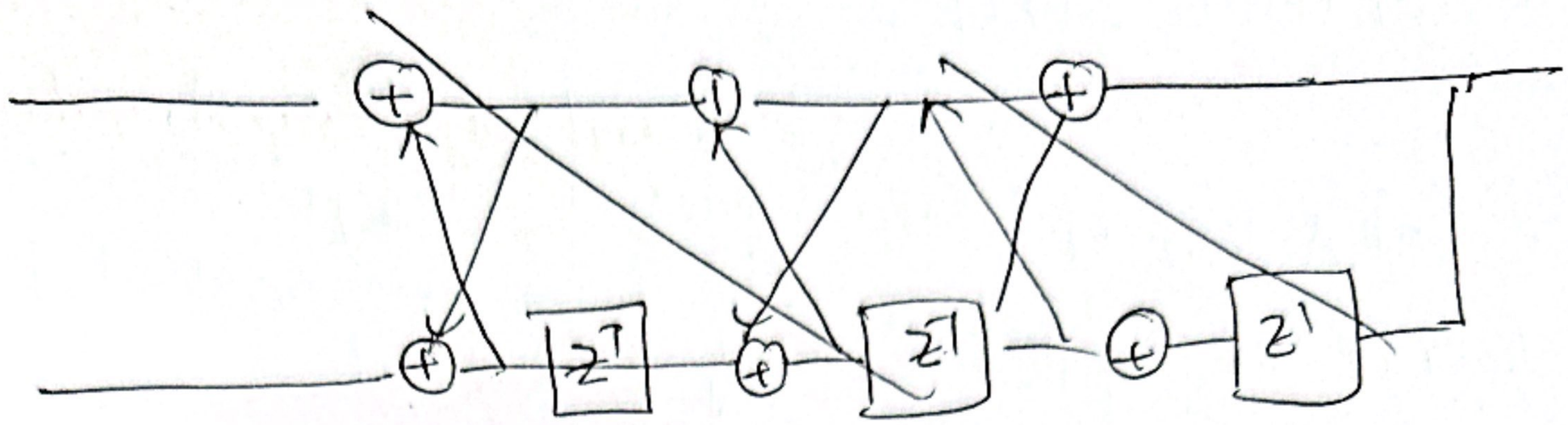
$m = 2$

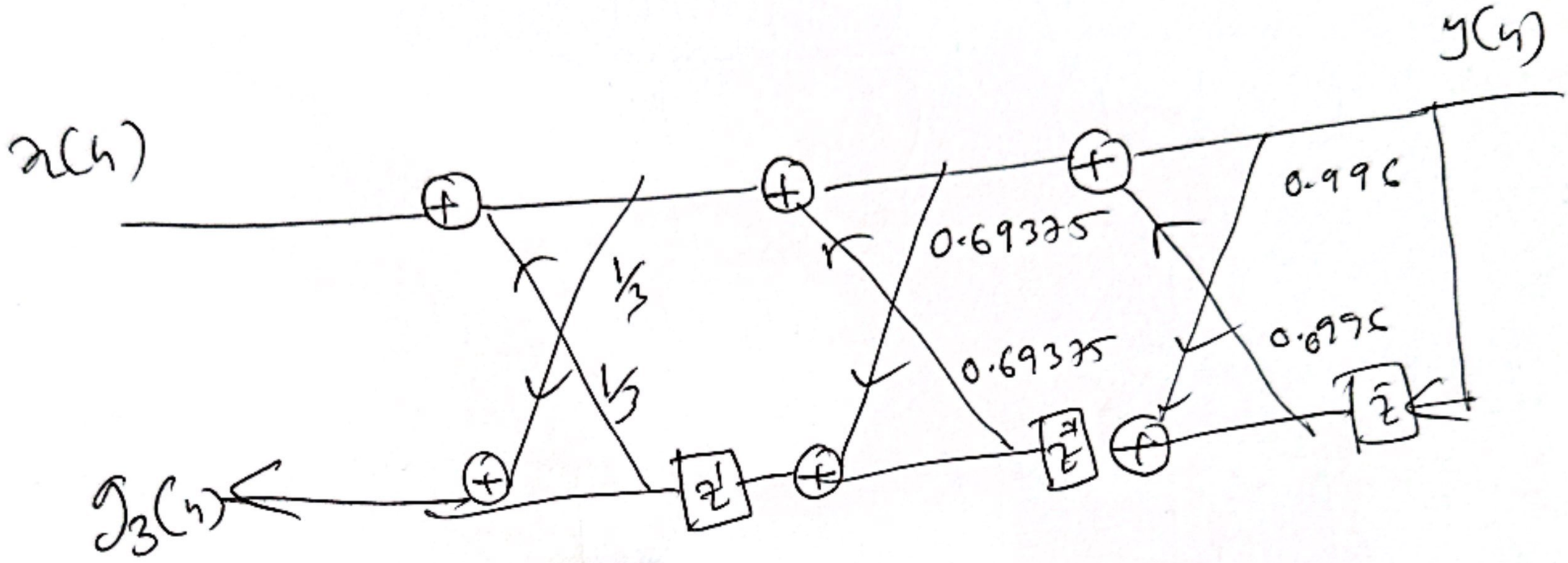
$i = 3$

$$a_1(1) = \frac{a_2(1) - k_2 a_2(1)}{1 - k_2^2}$$

$$= \frac{0.16875 - (0.69875)(0.16875)}{1 - (0.69375)^2}$$

$$= 0.0976 = k_1$$





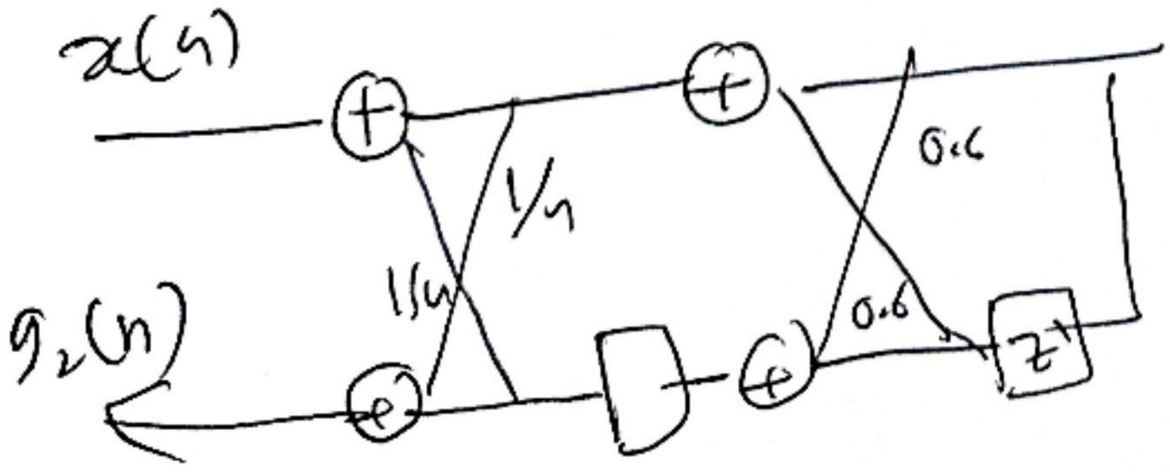

② $$H(z) = \frac{1+2z^{-1}}{1+\frac{3}{4}z^{-1}+\frac{1}{4}z^{-2}}$$

$a_2(0)=1$ $\qquad$ $a_2(2)=\frac{1}{4}=k_3$

$a_2(1)=3/4$

$m=2$

$i=1$ $\qquad$ $$a_1(1) = \frac{a_2(1) - k_2 a_2(1)}{1-k_2^2}$$

$$= \frac{3/4 - 1/4\ 3/4}{1-(1/4)^2}$$

$k_1 = a_1(1) = 0.6$