Design the symmetric FIR LPF whose

$$H_d(\omega) = \begin{cases} e^{-j\omega t} & |\omega| \le \omega_c \\ 0 & \text{otherwise} \end{cases} \qquad \text{with } M = 7, \quad \omega_c = 1 \text{ rad/sec}$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega \quad \text{—— ①}$$

$$\cancel{H_d(\omega) = \{ e^{j}}$$

$$= \frac{1}{2\pi} \int_{-1}^{1} e^{-j\omega t} \cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-1}^{1} e^{j\omega(n-t)}\, d\omega$$

$$= \frac{1}{2\pi} \left[\frac{e^{j\omega(n-t)}}{j(n-t)} \right]$$

$$= \frac{1}{2\pi} \frac{e^{j(n-t)} - e^{-j(n-t)}}{j(n-t)}$$

$$= \frac{1}{\pi(n-t)} \left[\frac{e^{j(n-t)} - e^{-j(n-t)}}{2j} \right]$$

$$= \frac{\sin(n-t)}{\pi(n-t)} \qquad n \ne t$$

If $n = t$

$\cancel{hd(n)}$ $hd(n) \frac{1}{2\pi}\int_{-1}^{1} 1\, dw = \frac{1}{\pi}$

$$hd(n) = \begin{cases} \frac{\sin(n-t)}{\pi(n-t)} & n \neq t \\ \frac{1}{\pi} & n = t \end{cases}$$

determine the value of t

$$h(n) = h(M-1-n)$$

$$h(n) = hd(n)\, w(n)$$

$$hd(n)\, w(n) = hd(M-1-n)\, w(n)$$

$$hd(n) = hd(M-1-n)$$

$$\frac{-\sin(n-t)}{-\pi(n-t)} = \frac{\sin(M-1-n-t)}{\pi(M-1-n-t)}$$

$$= \frac{\sin(-(n-t))}{\pi(-(n-t))} = \frac{\sin(M-1-n-t)}{\pi(M-1-n-t)}$$

$$-(n-t) = M-1-n-t$$

$$-n+t = M-1-n-t \qquad t = \frac{M-1}{2}$$

$$h_d(n) = \begin{cases} \dfrac{\sin\left(n - \frac{M-1}{2}\right)}{\pi \cdot \left(n - \frac{M-1}{2}\right)} & n \neq \frac{M-1}{2} \\ \dfrac{1}{\pi} & n = \frac{M-1}{2} \end{cases}$$

Since $M = 7$

$$h_d(n) = \begin{cases} \dfrac{\sin(n-3)}{\pi(n-3)} & n \neq 3 \\ \dfrac{1}{\pi} & n = 3 \end{cases}$$

$n = 0$ to 6

$n=0$ $h_d(0) = 0.01497$

1 $h_d(1) = 0.14472$

2 $h_d(2) = 0.26786$

3 $h_d(3) = 1/\pi$

4 $h_d(4)$ 0.26786

5 $h_d(5) = 0.14472$

6 $h_d(6) = 0.01497$

$h(n) = h_d(n) \cdot w(n)$

$w_n(n) = \begin{cases} 1 & 0 \le n \le 6 \\ 0 & \text{other} \end{cases}$

$h(n) = h_d(n)$; $0 \le n \le$

$= 0$ otherwise

$h(0) = 0.1497$

$h(1) = 0.14472$

$h(2) = 0.26788$

$\cancel{h(2)} =$

$h(3) = 1/\pi$

$h(4) = 0.26788$

$h(5) = 0.14472$

$h(6) = 0.1497$

Linear phase

Design the ~~symmetric~~ FIR Filter using window

$H_d(\omega) \rightarrow$ desired freq response

$h_d(n) \rightarrow$ " Sample s

$$H_d(\omega) = \sum_{n=0}^{\infty} h_d(n) e^{-j\omega n} \quad \text{— (1)}$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n} \, d\omega \quad \text{— (2)}$$

Example

Rectangular Window

$$W_R(n) = \begin{cases} 1 & n = 0, 1, 2 \ldots M-1 \\ 0 & \text{otherwise} \end{cases}$$

$$h(n) = h_d(n) \, W_R(n) \quad \text{— (3)}$$

$$h(n) = \begin{cases} h_d(n) & n = 0, 1, 2 \ldots M-1 \\ 0 & \text{otherwise} \end{cases}$$

General eq^n of linear phase response

$$h(n) = h_d(n) \, W(n)$$

$$H(\omega) = H_d(\omega) * W(\omega) \quad \text{— (4)}$$

Design the symmetric FIR LPF

$$H_d(\omega) = \begin{cases} e^{-j\omega\tau} & |\omega| \le \omega_c \\ 0 & \end{cases} \qquad M=7 \quad \omega_c=1$$

User rect window

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega \quad \text{—} ①$$

$$H_d(\omega) = \begin{cases} e^{-j\omega\tau} & -1 \le \omega \le 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{—} ②$$

$$h_d(n) = \frac{1}{2\pi}\int_{-1}^{1} e^{-j\omega\tau}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(n-\tau)}}{j(n-\tau)}\right]_{-1}^{1}$$

$$= \frac{1}{2\pi}\, \frac{e^{j(n-\tau)} - e^{-j(n-\tau)}}{j(n-\tau)}$$

$$= \frac{1}{\pi(n-\tau)} \sin(n-\tau)$$

$$= \operatorname{Sinc}(n-\tau) \qquad n \ne \tau \quad \text{—} ③$$

$$h_d(n) = \frac{1}{2\pi}\int_{-1}^{1} 1\, d\omega = \frac{1}{\pi}$$

$$h_d(\tau) = \begin{cases} \dfrac{\sin(n-\tau)}{\pi(n-\tau)} & n \neq \tau \\ \dfrac{1}{\pi} & n = \tau \end{cases}$$

$$h(n) = h(M-1-n)$$

$$h(n) = h_d(n)\,\omega(n)$$

$$h_d(n)\,\omega(n) = h_d($$

Design the Symmetric FIR Filter

$$Hd(w) = \begin{cases} e^{-jw3} & |w| \le w_c \\ 0 & \text{otherwise} \end{cases} \quad M=7 \quad w_c=1$$

Hanning Window

n	hd
0	0.1497
1	0.14472
2	0.26785
3	$1/\pi$
4	0.26785
5	0.14472
6	0.1497

Hanning Window

$W(n) = 0.5\left[1 - \cos\left(\frac{2\pi n}{M-1}\right)\right]$

$M=7$

$W(n) = 0.5\left[1 - \cos\left(\frac{2\pi n}{6}\right)\right]$

$= 0.5\left[1 - \cos\left(\frac{\pi n}{3}\right)\right]$

$\boxed{\text{Rad}}$

$0.57(1 - \cos(\pi \cdot n/3))$

$n=0 \quad W(0) = 0$

$n=1 \quad W(1) = 0.25$

$n=2 \quad W(2) = 0.75$

$n=3 \quad W(3) = 1$

$n=4 \quad W(4) =$

$n=5 \quad W(5) =$

$n=6 \quad W(6) =$

$h(n) = h_d(n) \cdot W(n)$

$h(0) = 0$

$h(1) = 0.03618$

$h(2) = 0.20089$

$h(3) = 1/5$

$h(4) = 0.20089$

$h(5) = 0.03618$

$h(6) = 0$

Determine the filter coefficient $h_d(n)$ for the desired freq response of a low pass filter given by

$$H_d(e^{jw}) = \begin{cases} e^{-j2w} & -\pi/4 \le w \le \pi/4 \\ 0 & \pi/4 \le |w| \le \pi \end{cases}$$

If we defined new filter co-efficient by $h_d(n) \cdot w(n)$ where $w(n) = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & \text{else} \end{cases}$ Determine $h(n)$ and also the frequeny response $H(e^{jw})$ and compare with $H_d(e^{jw})$. Determine $H(e^{jw})$ using the Hanning window.

$$\cancel{h_d(n) = \frac{1}{2\pi}\int^{\pi/4} e^{-jw} \cdot e^{-j}}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{jw})\, e^{jwn}\, dw = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-j2w} \cdot e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{jw(n-2)}\, dw = \frac{1}{2\pi}\left[\frac{e^{jw(n-2)}}{j(n-2)}\right]_{-\pi/4}^{\pi/4}$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\pi/4(n-2)} - e^{-j\pi/4(n-2)}}{j(n-2)}\right]$$

$$= \frac{1}{\pi(n-2)} \cdot \sin \pi/4(n-2) \qquad n \ne 2$$

If $n=2$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} 1\, d\omega = \frac{1}{4}$$

$$h_d(n) = \begin{cases} \dfrac{\sin\frac{\pi}{4}(n-2)}{\pi(n-2)} & n \neq 2 \\ \dfrac{1}{4} & n = 2 \end{cases}$$

$h(n)$

$h(n) = h_d(n)\, w(n) \qquad 0 \leq n \leq 4$

$h(0) = h_d(0) = 0.159091$

$h(1) = h_d(1) = 0.224989$

$h(2) = h_d(2) = 1/4$

$h(3) = h_d(3) = 0.224989$

$h(4) = h_d(4) = 0.159091$

$M = 5$ as $n \leq n \leq 4$

$\hookrightarrow$ odd

$$H(\omega) = e^{-j\omega\left(\frac{M-1}{2}\right)} \left\{ h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{\frac{M-3}{2}} h(n) \cos\omega\left(n - \frac{M-1}{2}\right)\right]$$

$$= e^{-j2\omega}\left[h(2) + 2h(n)\cos\omega(n-2)\right]$$

$$= e^{-j2\omega}\left[h(2) + 2h(0)\cos\omega(-2) + 2h(1)\cos\omega(-1)\right]$$

$$= e^{-j2\omega}\left[0.25 + 2\times 0.159091\cos 2\omega + 2\times 0.224989\cos\omega\right]$$

$$= e^{-j2\omega}\left[0.25 + 0.318\cos 2\omega + 0.45\cos\omega\right]$$

$$|H(\omega)| = 0.25 + 0.318\cos 2\omega + 0.45\cos\omega$$

$$\angle H(\omega) = -2\omega; \quad |H(\omega)| > 0$$

$$= -2\omega + \pi \quad |H(\omega)| < 0$$

$H(e^{j\omega})$ with $H_d(e^{j\omega})$

Not same

Obtain $H(e^{jw})$ using Hamming window

Hamming window $w(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{M-1}\right)$; $0 \le n \le M$

$w(n) = 0.54 - 0.46 \cos\left(\frac{\pi n}{2}\right)$ $0 \le n \le 4$

~~$h_d(n)$~~

$n=0$ $w(0) = 0.08$

$n=1$ $w(1) = 0.54$

$n=2$ $w(2) = 1$

$n=3$ $w(3) = 0.54$

$n=4$ $w(4) = 0.08$

$h(n) = h_d(n) \cdot w(n)$

$h(0) = 0.01273$

$h(1) = 0.12149$

$h(2) = 0.25$

$h(3) = 0.12149$

$h(4) = 0.01273$

$M=5$

$\cancel{H(\omega) = e^{-j2w}\left[2\sum h(n) \cos w(n-2)\right]}$

$$H(\omega) = e^{-j2w}\left[h(2) + 2\sum_{n=0}^{1} h(n) \cos w(n-2)\right]$$

$$= e^{-j2w}\left[0.25 + 0.02546 \cos 2w + 0.243 \cos w\right]$$

Design a FIR Linear phase Filter using Kaiser window to meet the following specification.

$$0.99 \leq |H(e^{j\omega})| \leq 1.01 \qquad 0 \leq |\omega| \; 0.19\pi$$

$$|H(e^{j\omega})| \leq 0.01 \qquad 0.21\pi \leq |\omega| \leq \pi$$

$$\underbrace{1 - 0.01}_{\delta_1} \leq \| H(e^{j\omega})| \leq 1 + \underbrace{0.01}_{\delta_2} \qquad \underbrace{0 \leq |\omega| \leq 0.19\pi}_{\omega_p} \qquad \cancel{0.21\pi \leq |\omega| \leq \pi}$$

$$|H(e^{j\omega})| \leq 0.01 \qquad \underbrace{0.21\pi \leq |\omega| \leq \pi}_{\omega_s}$$

$$\underbrace{\delta_1 = 0.01 \qquad \delta_2 = 0.01}_{\text{ripple factor}} \qquad \underbrace{\omega_p = 0.19\pi}_{\text{Pass band edge}} \qquad \underbrace{\omega_s = 0.21\pi}_{\text{Stop band edge}}$$

$$\Delta\omega = \omega_s - \omega_p = 0.21\pi - 0.19\pi = 0.02\pi$$

$$\delta = \min(\delta_1, \delta_2) = 0.01$$

Attenuation $A = -20\log_{10}\delta = -20\log_{10} 0.01 = 40$

Cutoff freq $\omega_c = \dfrac{\omega_p + \omega_s}{2} = \dfrac{0.19\pi + 0.21\pi}{2}$

$= 0.2\pi$

Shape parameter
↓
Obtain β and M

$$\beta = \begin{cases} 0.1102(A - 8.7) & A > 50 \\ .5842(A-21) + 0.07886(A-21) & 21 \leq A \leq 50 \\ 0 & A < 21 \end{cases}$$

$$\beta = 0.5842(40-21)^{0.4} + 0.07886(40-21) = 3.395$$

$$M = \frac{A-8}{2.285\Delta\omega} = \frac{40-8}{2.285(0.02\pi)} = 223$$

Design of linear phase FIR filter using freq sampling method.

Desired freq response $H_d(\omega)$

This freq response is sampled at M points

Thus $\omega = \frac{2\pi}{M} \cdot k \qquad k = 0, 1, \dots, M-1$

Discrete Forier Transform

$H(k) = H_d(\omega) \; ; \; k = 0, 1, 2, \dots M-1$

$$H(k) = H_d\left(\frac{2\pi}{M} \cdot k\right)$$

$H(k) \rightarrow M$ point DFT

Take IDFT of $H(k)$ to get $h(n)$

$h(n) \rightarrow$ unit sample response of FIR Filter

If $M \rightarrow$ odd $\quad h(n) = \frac{1}{M}\left[H(0) + 2\sum_{k=1}^{\frac{M-1}{2}} Re\left\{H(k) e^{j\frac{2\pi}{N}kn}\right\}\right]$

If $M \rightarrow$ even $\quad h(n) = \frac{1}{M}\left[H(0) + 2\sum_{k=1}^{\frac{M}{2}-1} Re\left\{H(k) e^{-j\frac{2\pi}{N}kn}\right\}\right]$

$$\boxed{\text{Window} \quad H_d(\omega) \xrightarrow{\text{IDFT}} \underset{\text{Infinite}}{h_d(n)} \xrightarrow{\text{Finite}} \overbrace{h_d(n) \cdot w(n)}^{h(n)}}$$

Example

$$H_d(e^{j\omega}) = \begin{cases} e^{-j(N-1)\omega/2} & 0 \le |\omega| \le \pi/2 \\ 0 & \pi/2 \le \omega \le \pi \end{cases}$$

M = N = 7 Use sampling approach

(I) Desired freq response

$$N = 7 \quad H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & 0 \le |\omega| \le \pi/2 \\ 0 & \pi/2 \le |\omega| \le \pi \end{cases}$$

$$\omega = \frac{2\pi k}{M} = \frac{2\pi k}{7} \qquad k = 0, 1, 2 \ldots 6$$

(II) Sample $H_d(e^{j\omega})$

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\frac{6\pi k}{7}} & 0 \le \left|\frac{2\pi k}{7}\right| \le \pi/2 \\ 0 & \pi/2 \le \left|\frac{2\pi k}{7}\right| \le \pi \end{cases}$$

$$H_d(k) = \begin{cases} e^{-j\frac{6\pi k}{7}} & 0 \le |k| \le 7/4 \\ 0 & 7/4 \le |k| \le 7/2 \end{cases}$$

(III) M = N = 7 odd

$$h(n) = \frac{1}{M}\left[H(0) + 2\sum_{k=1}^{\frac{M-1}{2}} \mathrm{Re}\left\{H(k)\, e^{j\frac{2\pi}{N}kn}\right\}\right]$$

$$\frac{M-1}{2} = \frac{7-1}{2} = 3 \qquad H(0) = 1$$

$$= \frac{1}{7}\left[1 + 2\sum_{k=1}^{3} \mathrm{Re}\left\{e^{-j2\pi k(3-n)/7}\right\}\right]$$

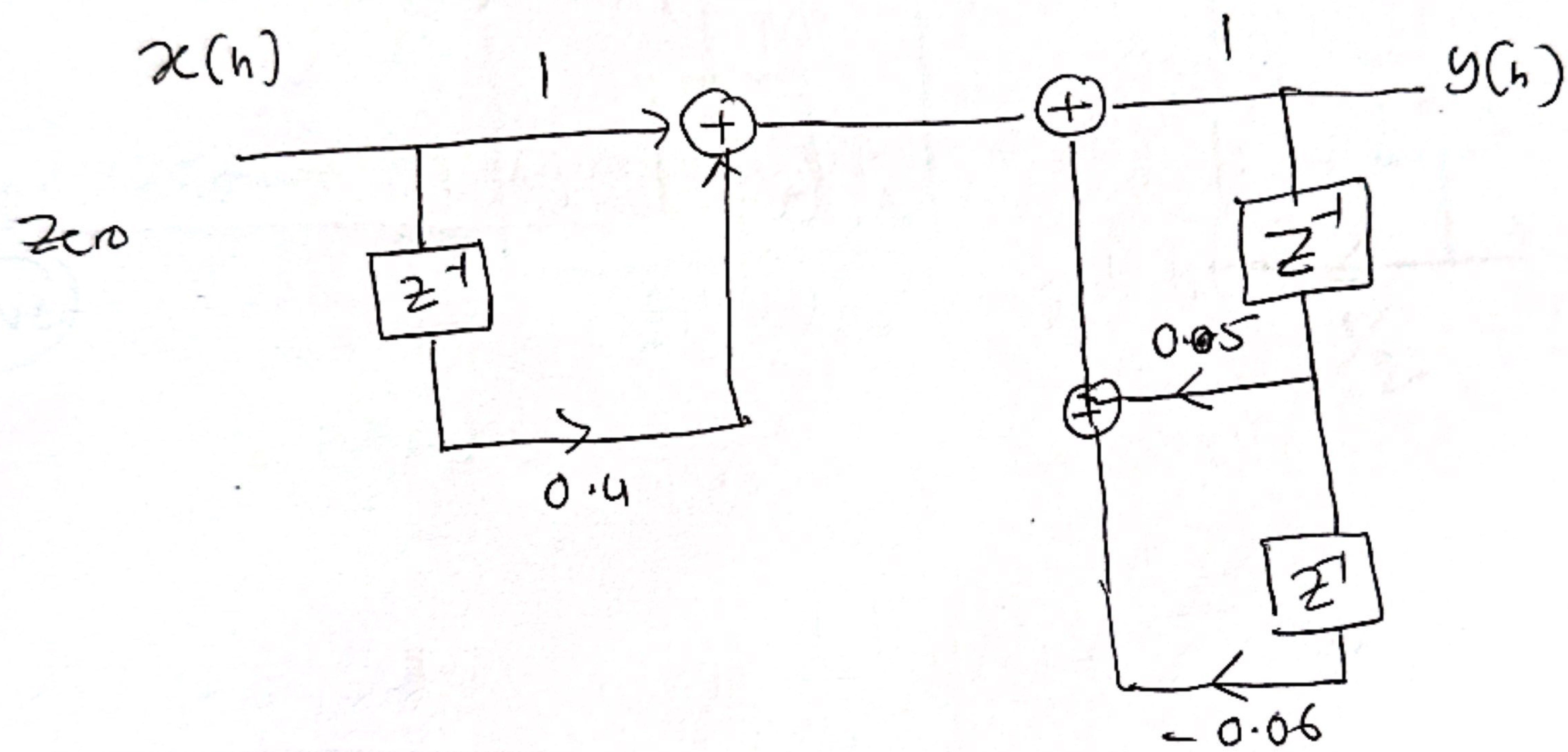
$$h(n) = \frac{1}{7}\left[1 + 2\sum_{k=1} \cos\left(\frac{2\pi k(3-n)}{7}\right)\right]$$

$n = 0, 1, \ldots 6$

Direct form Structure

$$H(z) = \frac{1 + 0.4 z^{-1}}{1 - 0.5 z^{-1} + 0.06 z^{-2}}$$ — Zeros — Poles

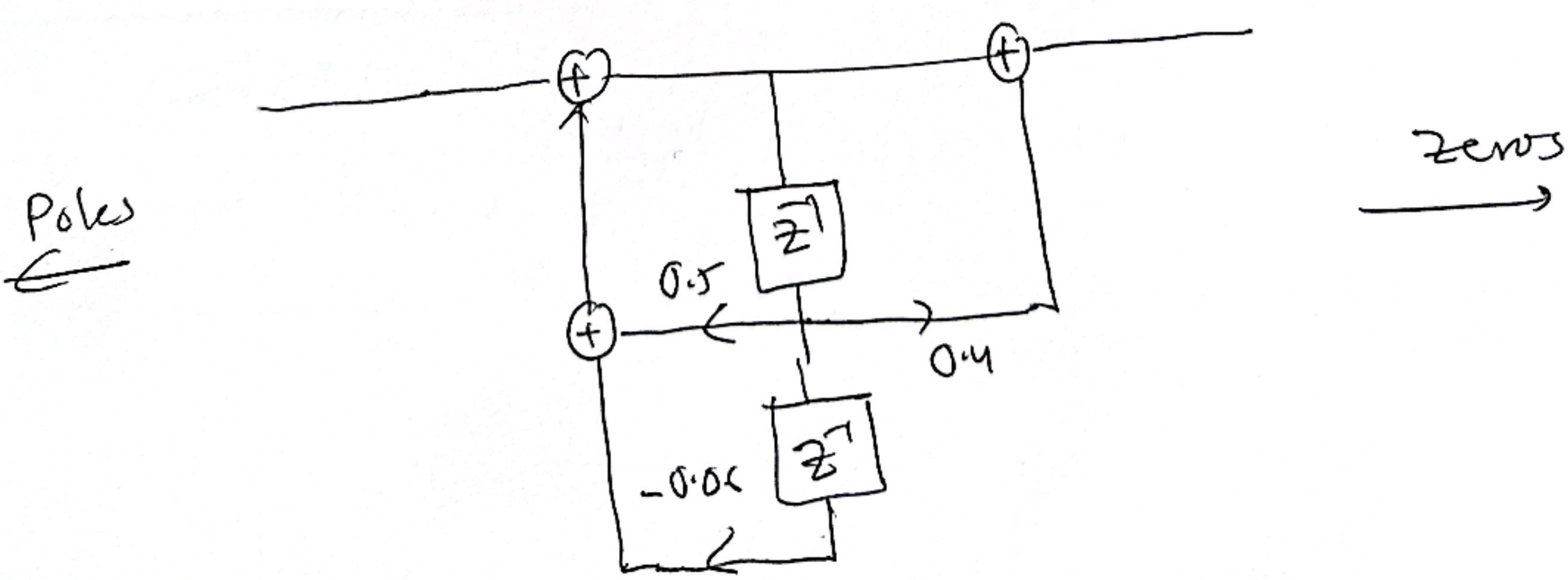
Direct form I



poles

write -ve coefficient

Direct form II



Solve

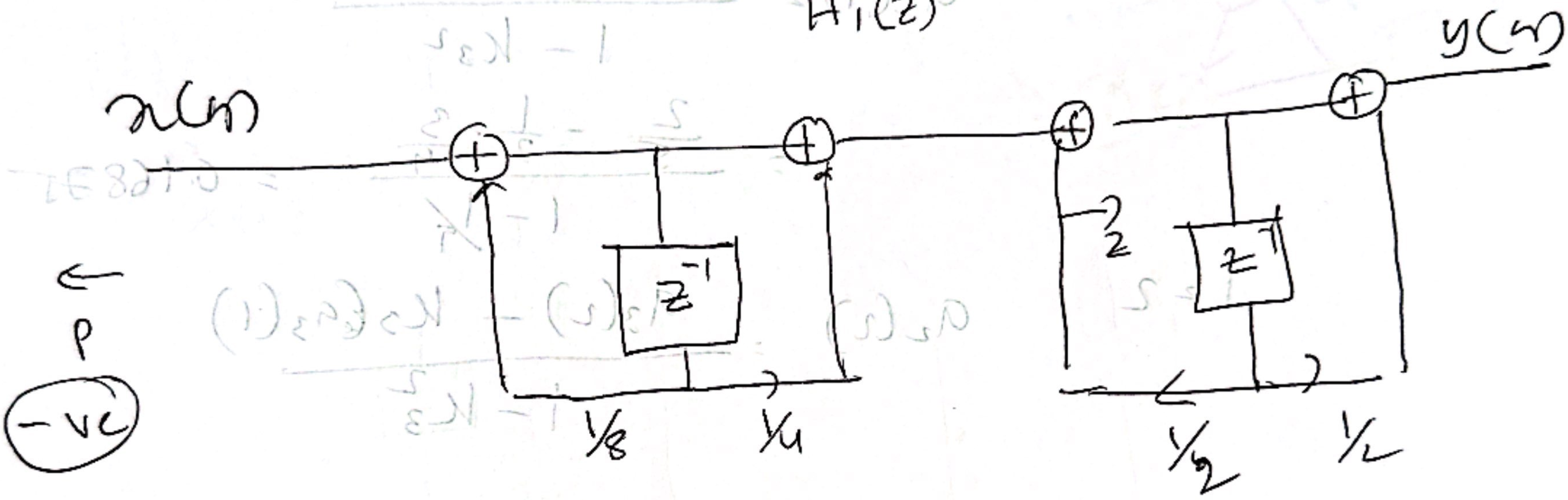
$$H(z) = \frac{z^{-1} - 3z^{-2}}{10 + 4z^{-1} + 4.5z^{-2} - 0.5z^{-3}}$$

$$H(z) = H_1(z)\, H_2(z)$$

Cascade form structure :

$$H(z) = \frac{1 + \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}{1 - \frac{5}{8} z^{-1} + \frac{1}{16} z^{-2}} = \frac{\left(1 + \frac{1}{4} z^{-1}\right)\left(1 + \frac{1}{2} z^{-1}\right)}{\left(1 - \frac{1}{8} z^{-1}\right)\left(1 - \frac{1}{2} z^{-1}\right)}$$

$$= \underbrace{\frac{\left(1 + \frac{1}{4} z^{-1}\right)}{\left(1 - \frac{1}{8} z^{-1}\right)}}_{H_1(z)} \cdot \underbrace{\frac{\left(1 + \frac{1}{2} z^{-1}\right)}{\left(1 - \frac{1}{2} z^{-1}\right)}}_{H_2(z)}$$



Solve

$$H(z) = \frac{\left(1 + \frac{1}{2} z^{-1}\right)}{\left(1 - \frac{1}{4} z^{-1} + \frac{1}{2} z^{-2}\right)\left(1 + \frac{1}{5} z^{-1} + \frac{1}{6} z^{-1}\right)}$$

Lattice Representation

$$H(z) = \frac{1}{1 + \frac{2}{5} z^{-1} + \frac{3}{4} z^{-2} + \frac{1}{3} z^{-3}}$$

$m = 3$, $a_3(0) = 1$, $a_3(1) = 2/5$, $a_3(2) = 3/4$

$a_3(3) = 1/3 = k_3$

$m=3$

$i = 1$

$$a_2(1) = \frac{a_3(1) - k_3\, a_3(2)}{1 - k_3^2}$$

$$= \frac{\frac{2}{5} - \frac{1}{3} \cdot \frac{3}{4}}{1 - \frac{1}{4}} = 0.16875$$

$i = 2$

$$a_2(2) = \frac{a_3(2) - k_3\, a_3(1)}{1 - k_3^2}$$

$$= 0.69375 = k_2$$

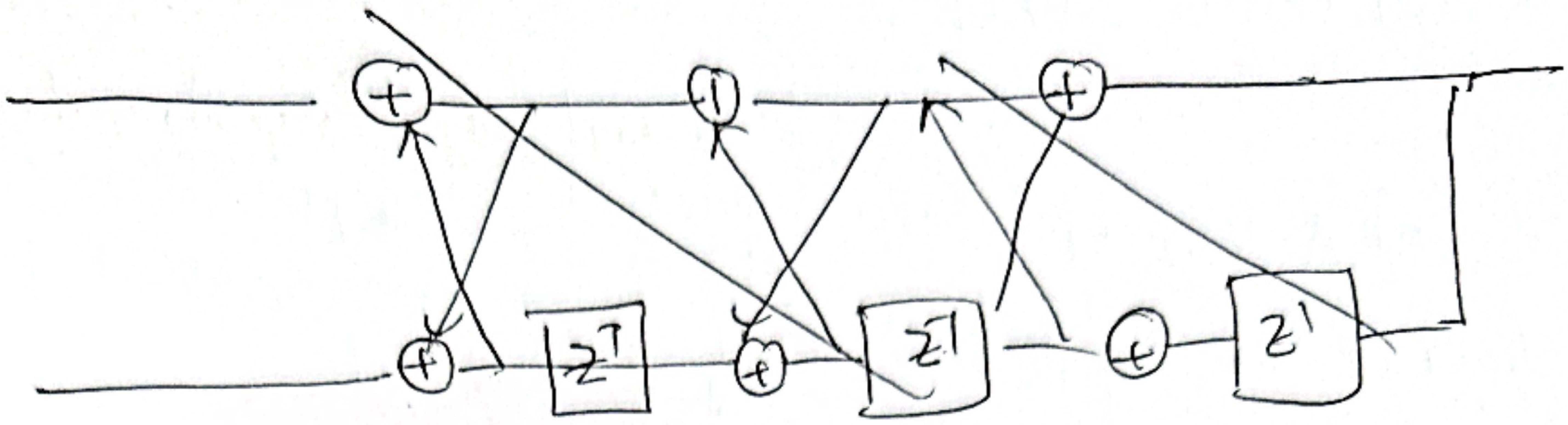
$m = 2$

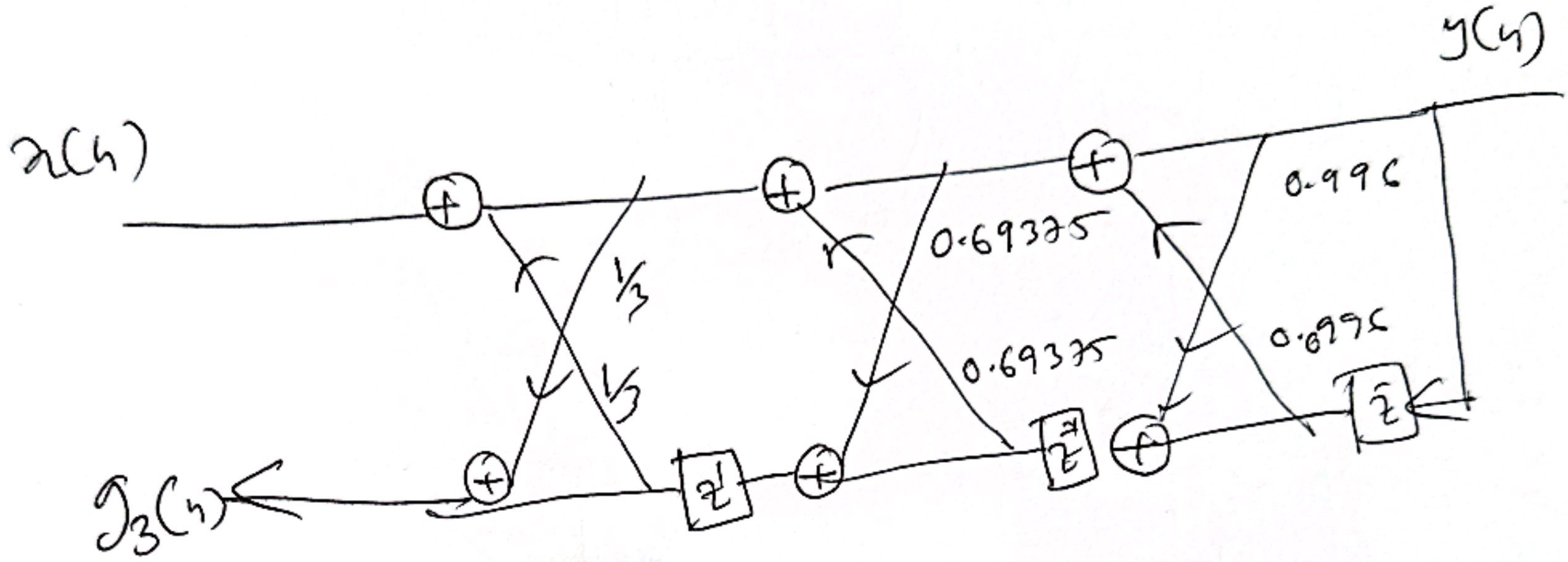
$i = 3$

$$a_1(1) = \frac{a_2(1) - k_2\, a_2(1)}{1 - k_2^2}$$

$$= \frac{0.16875 - (0.69875)(0.16875)}{1 - 0.69375}$$

$$= 0.0976 = k_1$$





② $H(z) = \dfrac{1+2z^{-1}}{1+\frac{3}{4}z^{-1}+\frac{1}{4}z^{-2}}$

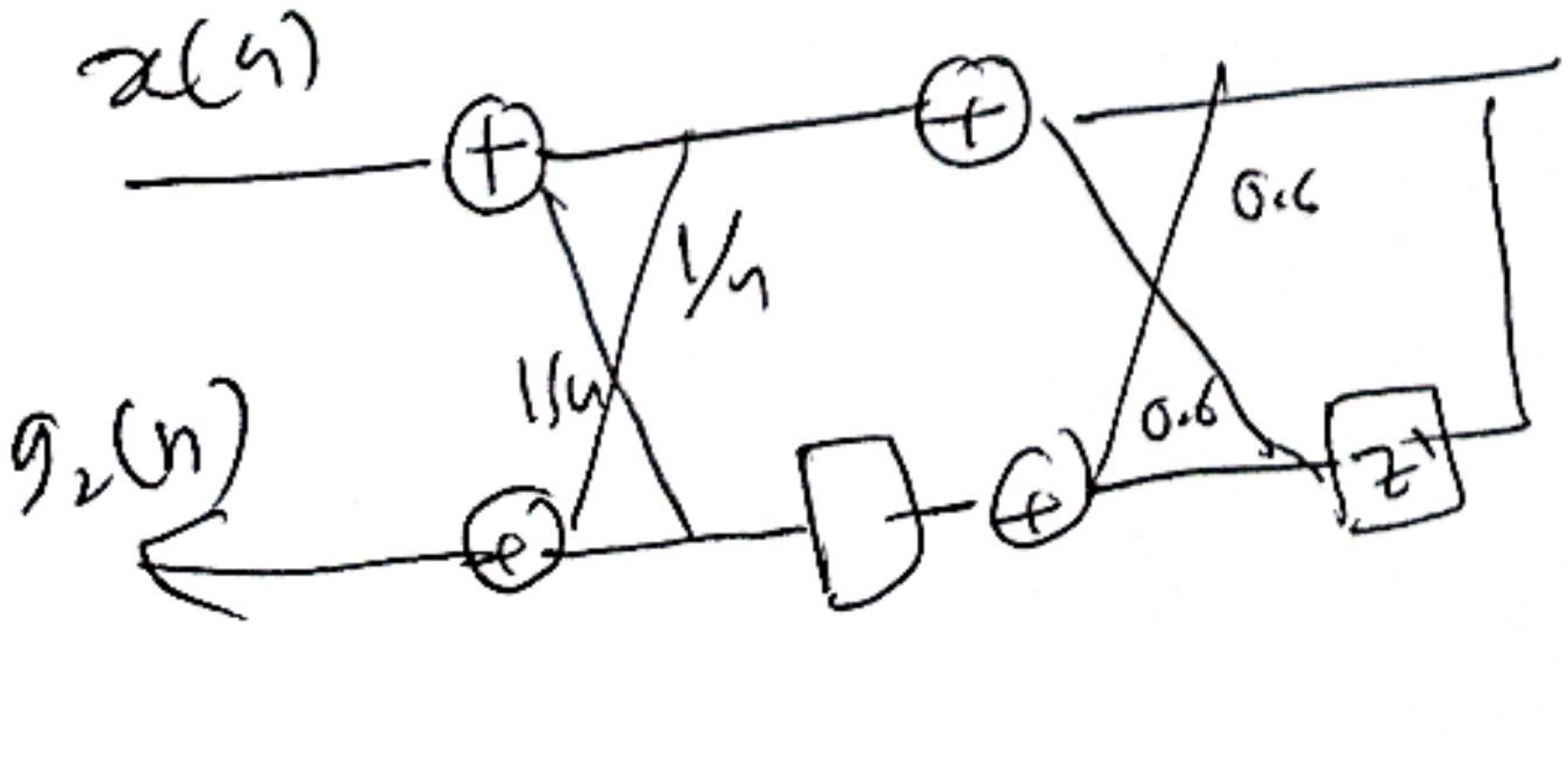
$a_2(0)=1$ $\qquad a_2(2) = \frac{1}{4} = K_3$

$a_2(1)=3/4$

$m=2$

$i=1 \quad a_1(1) = \dfrac{a_2(1) - K_2\, a_2(1)}{1-K_2^2}$

$= \dfrac{3/4 - 1/4 \cdot 3/4}{1-(1/4)^2}$



$K_1 = a_1(1) = 0.6$