Morphological Image Processing

ICT4201: DIP

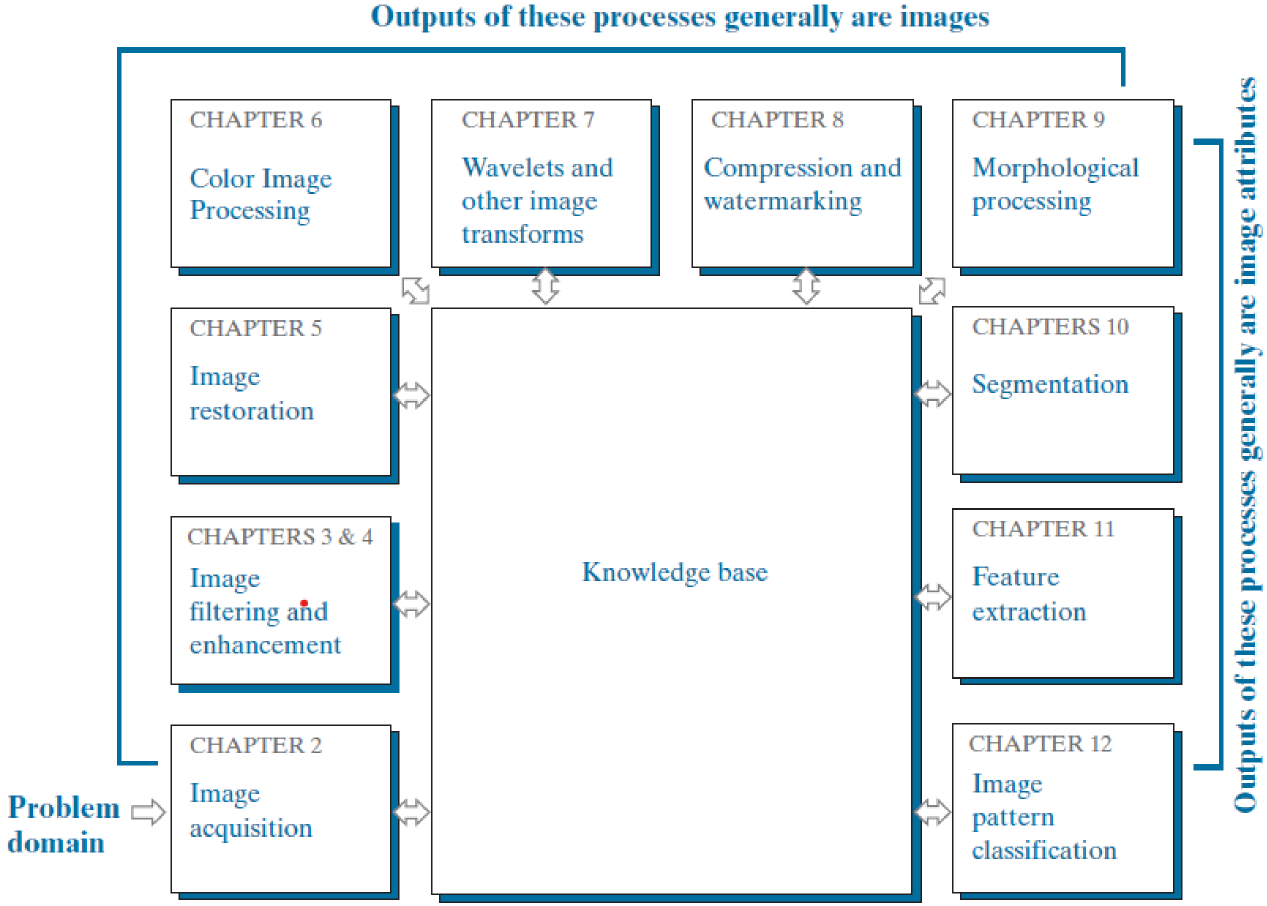
Introduction

- **Morphology**: a branch of biology that deals with the form and structure of animals and plants

- Morphological image processing or mathematical morphology is used to extract image components for representation and description of region shape, such as boundaries, skeletons, and the convex hull

Fundamental Steps of DIP

FIGURE 1.23 Fundamental steps in digital image processing. The chapter(s) indicated in the boxes is where the material described in the box is discussed.



Preliminaries (1)

- **Reflection**

The reflection of a set B, denoted $\hat{B}$, is defined as

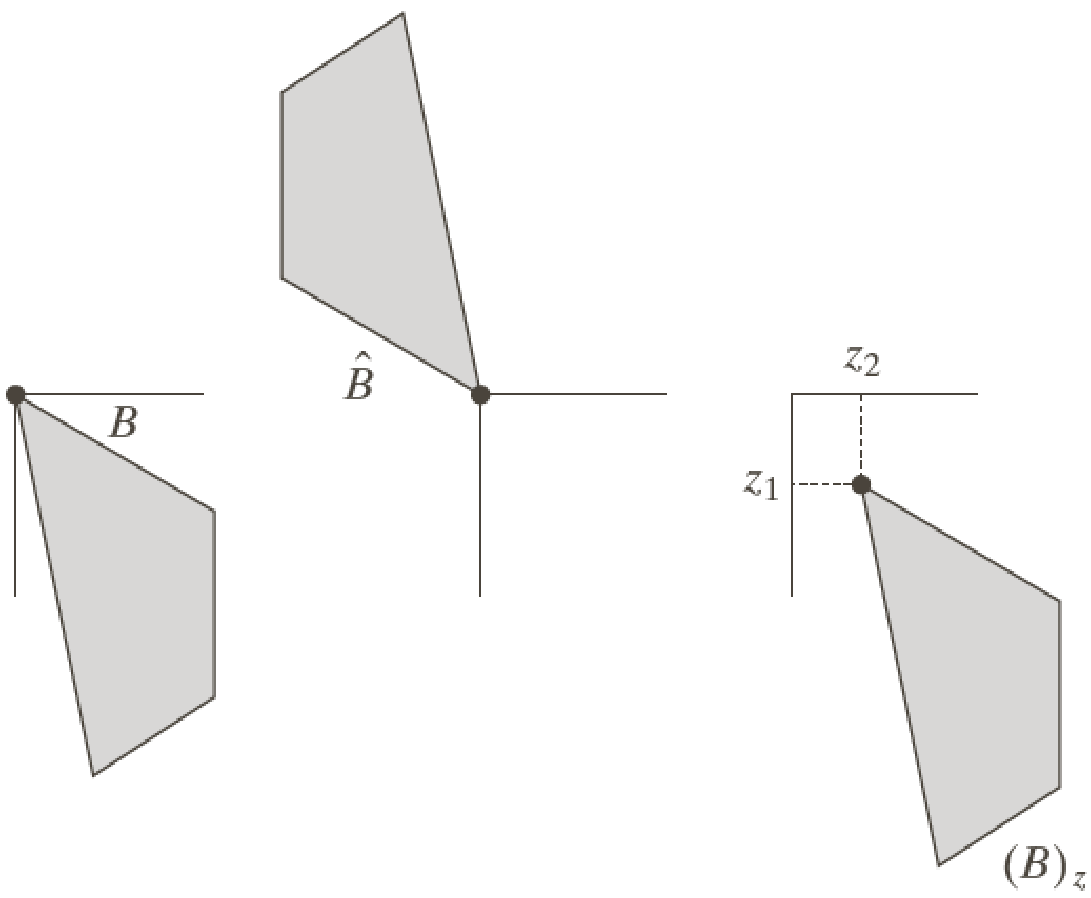
$$\hat{B} = \{w \mid w = -b, \text{for } b \in B\}$$

- **Translation**

The translation of a set B by point $z = (z_1, z_2)$, denoted $(B)_Z$, is defined as

$$(B)_Z = \{c \mid c = b + z, \text{for } b \in B\}$$

Example: Reflection and Translation



a b c

FIGURE 9.1
(a) A set, (b) its reflection, and (c) its translation by z.

Preliminaries (2)

- **Structure elements (SE)**

Small sets or sub-images used to probe an image under study for properties of interest

Examples: Structuring Elements (1)

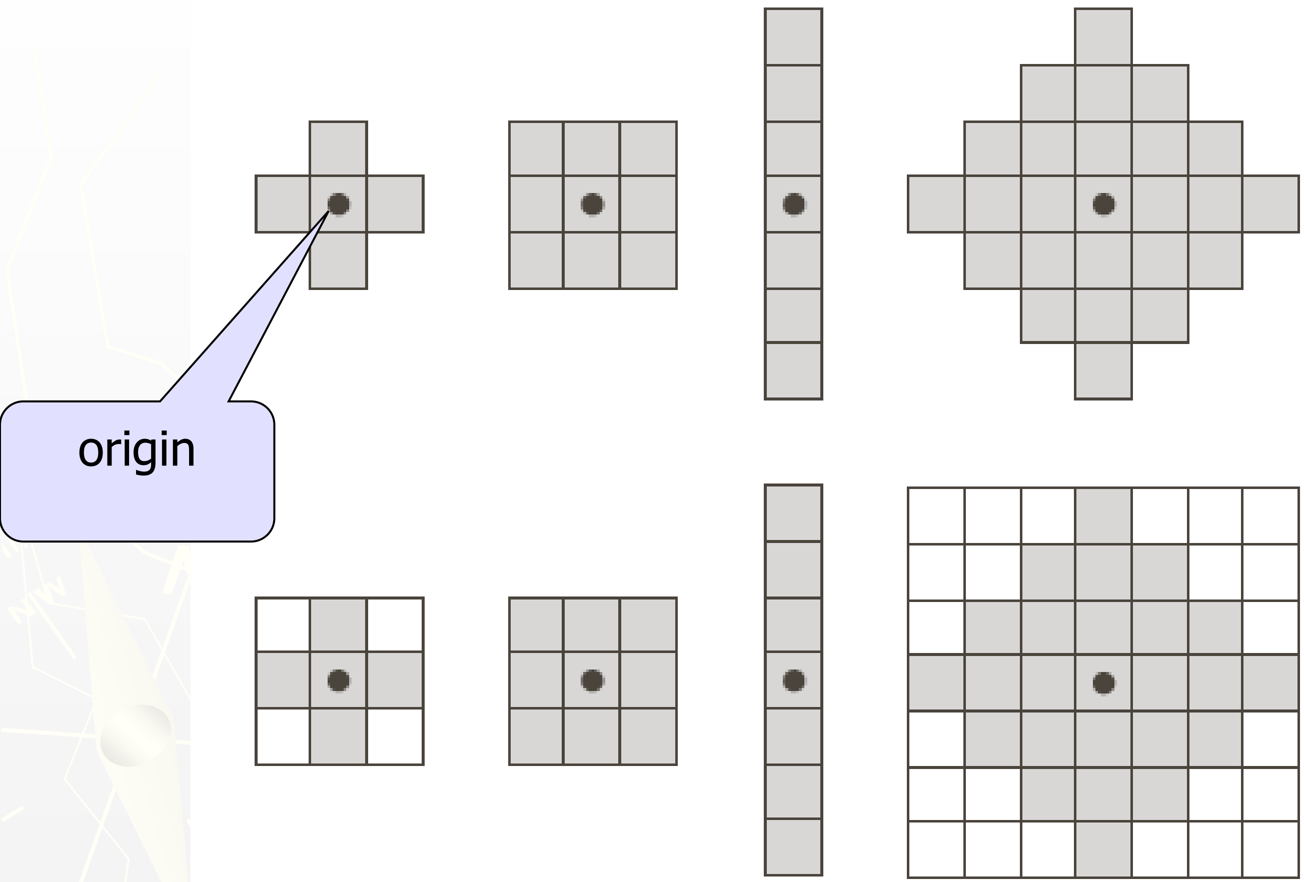
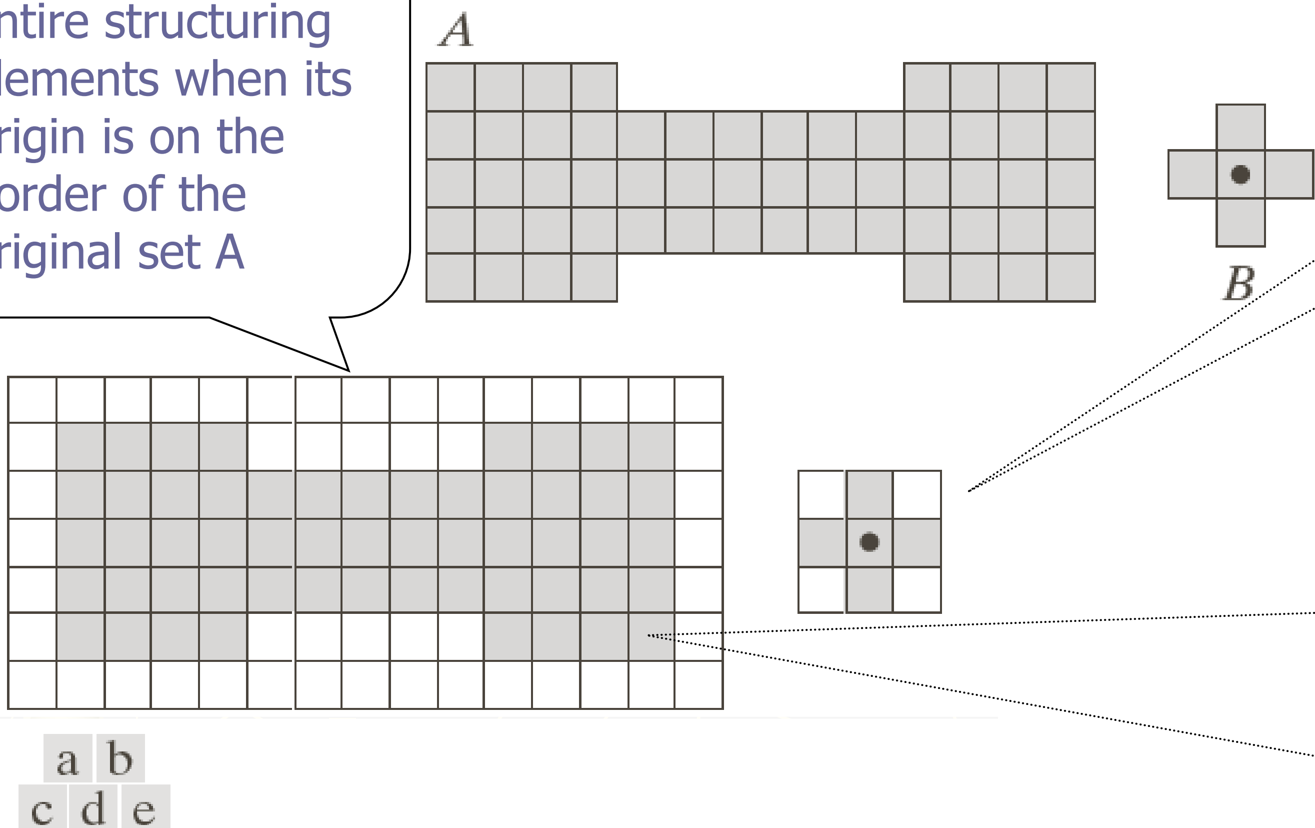


FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

Examples: Structuring Elements (2)

Accommodate the entire structuring elements when its origin is on the border of the original set A

Origin of B visits every element of A



At each location of the origin of B, if B is completely contained in A, then the location is a member of the new set, otherwise it is not a member of the new set.

FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

Erosion

With A and B as sets in Z^2, the erosion of A by B, denoted $A \ominus B$, defined

$$A \ominus B = \{ z \mid (B)_Z \subseteq A \}$$

The set of all points z such that B, translated by z, is contained by A.

$$A \ominus B = \{ z \mid (B)_Z \cap A^c = \varnothing \}$$

Example of Erosion (1)

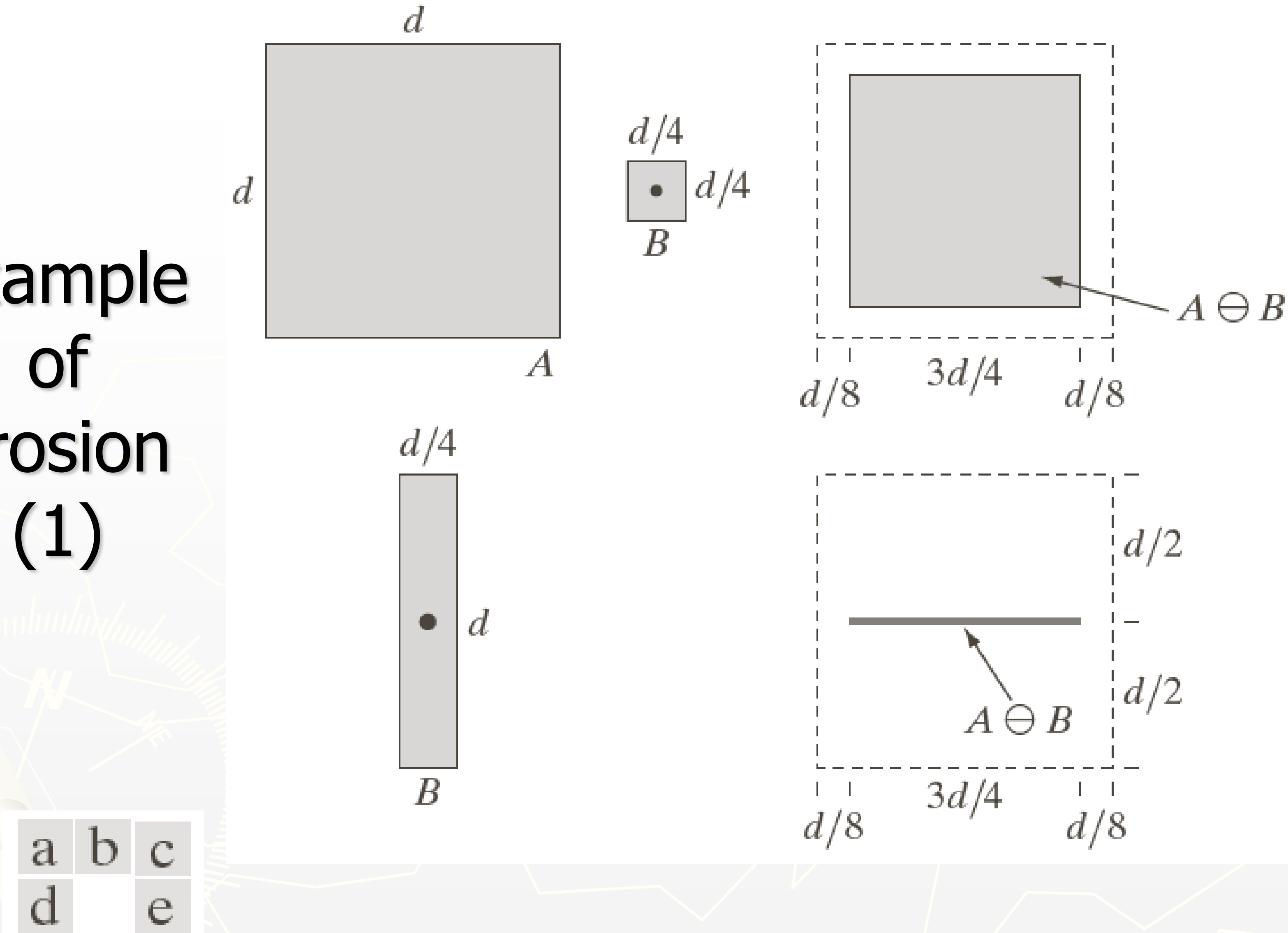
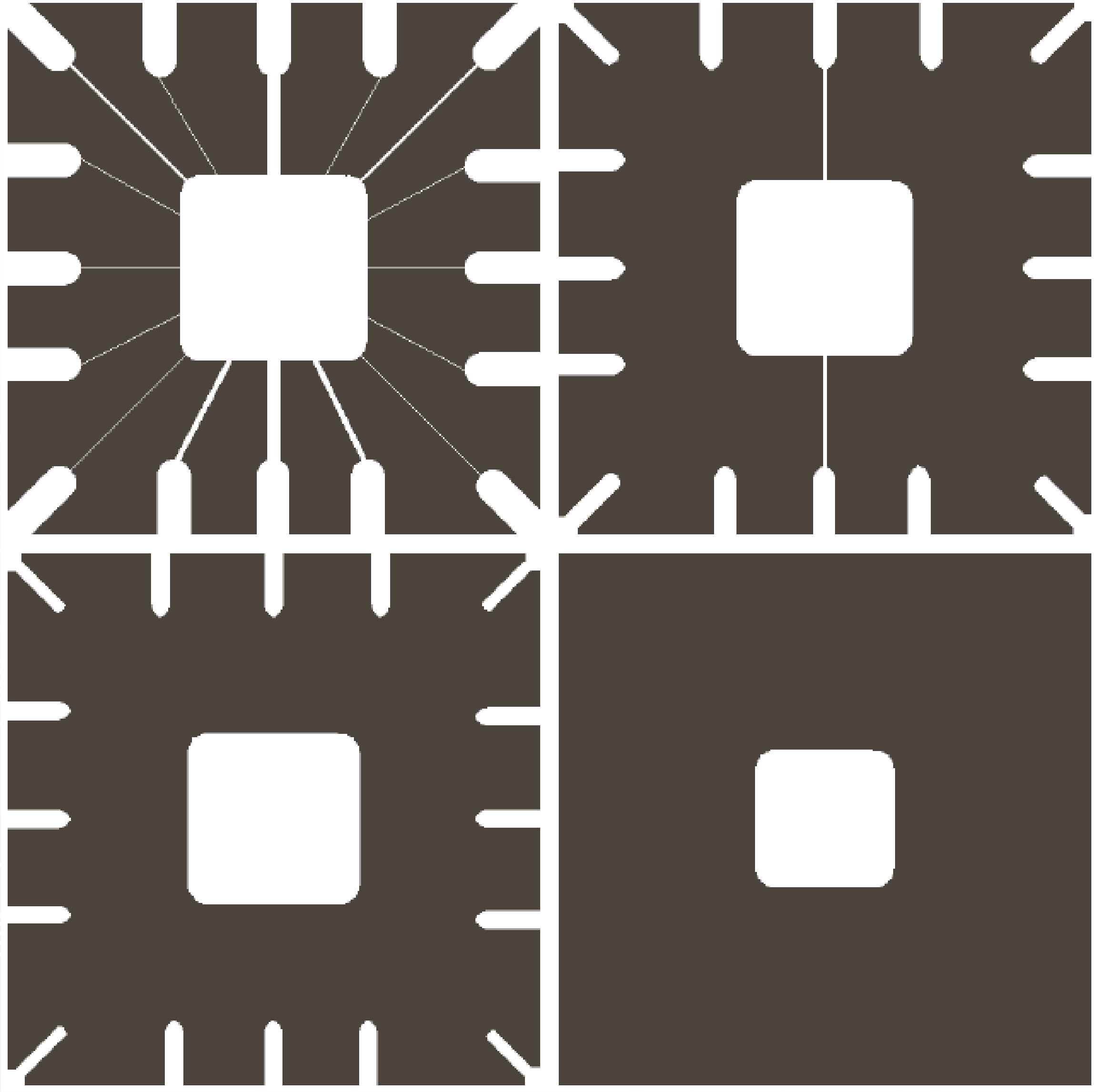


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

Example of Erosion (2)



a b
c d

FIGURE 9.5 Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45, respectively. The elements of the SEs were all 1s.

Dilation

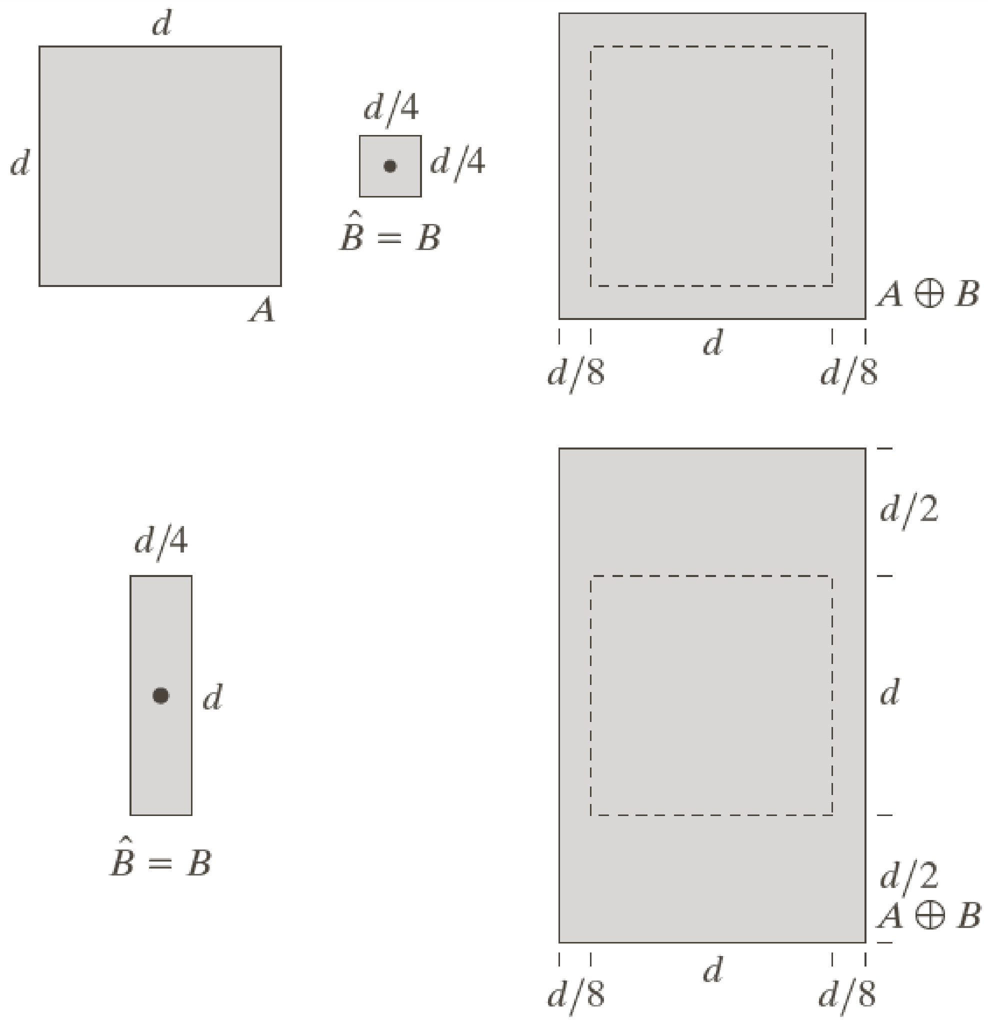
With A and B as sets in Z^2, the dilation of A by B, denoted $A \oplus B$, is defined as

$$\mathrm{A} \oplus \mathrm{B} = \left\{ z \middle| \left(\hat{B}\right)_z \cap A \neq \emptyset \right\}$$

The set of all displacements z, the translated B and A overlap by at least one element.

$$A \oplus B = \left\{ z \middle| \left[\left(\hat{B}\right)_z \cap A \right] \subseteq A \right\}$$

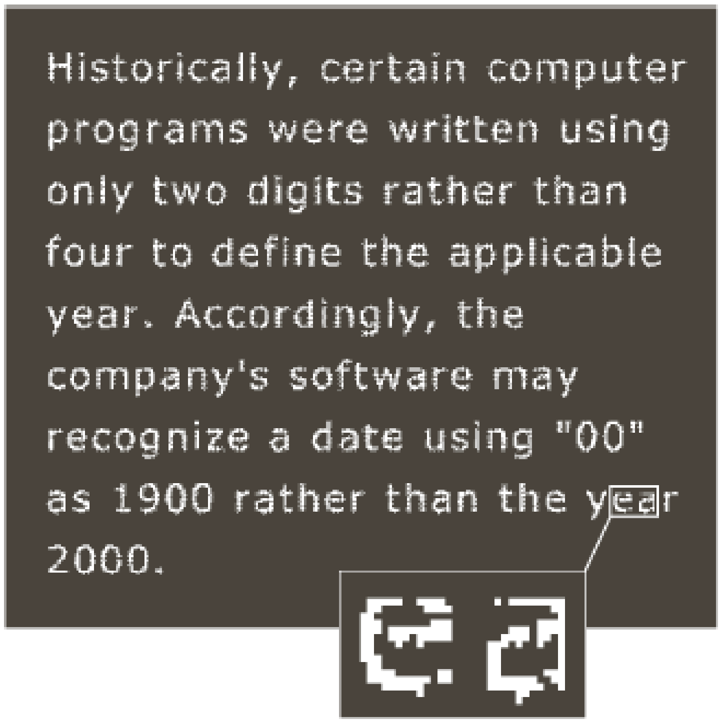
Examples of Dilation (1)

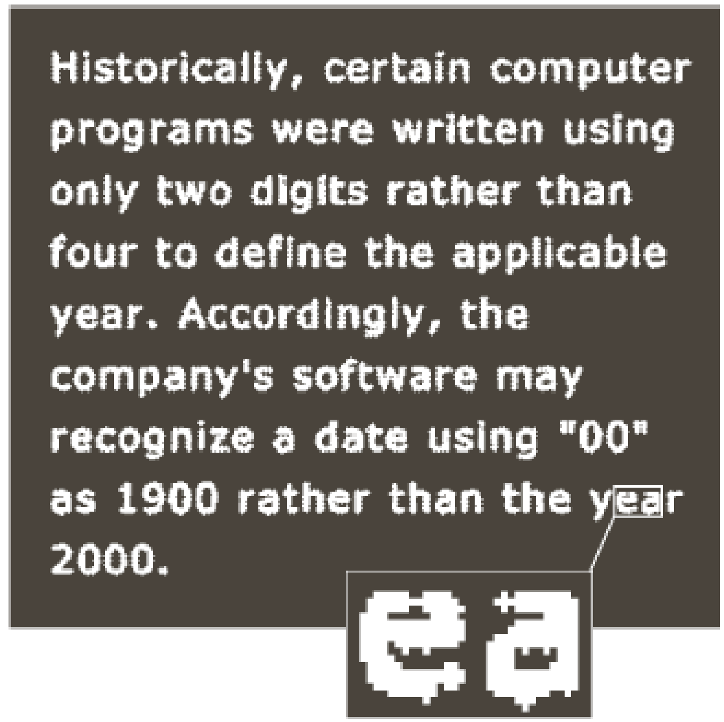


a	b	c
d		e

FIGURE 9.6
(a) Set A.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

Examples of Dilation (2)





0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Duality

- Erosion and dilation are duals of each other with respect to set complementation and reflection

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

and

$$(A \oplus B)^c = A^c - \hat{B}$$

Duality

- Erosion and dilation are duals of each other with respect to set complementation and reflection

$$\left(A \ominus B\right)^c = \left\{z \mid \left(B\right)_Z \subseteq A\right\}^c$$

Duality

- Erosion and dilation are duals of each other with respect to set complementation and reflection

$$
\begin{aligned}
(A \oplus B)^c &= \left\{z \mid \left(\hat{B}\right)_Z \cap A \neq \emptyset\right\}^c \\
&= \left\{z \mid \left(\hat{B}\right)_Z \cap A^c = \emptyset\right\} \\
&= A^c - \hat{B}
\end{aligned}
$$

Opening and Closing

- Opening generally smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions

- Closing tends to smooth sections of contours but it generates fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour

Opening and Closing

- The opening of set A by structuring element B, denoted $A \circ B$, is defined as

$$A \circ B = (A \ominus B) \oplus B$$

- The closing of set A by structuring element B, denoted $A \bullet B$, is defined as

$$A \bullet B = (A \oplus B) \ominus B$$

Opening and Closing

- The opening of set A by structuring element B, denoted $A \circ B$, is defined as

$$A \circ B = \cup \{(B)_Z | (B)_Z \subseteq A\}$$

The opening of A by B is the union of all the translations of B so that B fits entirely in A.

- Similarly, we can write the closing of A by B as

$$A \bullet B = \left[\cup \left\{(B)_z \,\middle|\, (B)_z \cap A = \phi\right\}\right]^c$$

The closing is then the complement of the union of all translations of B that do not overlap A

Example: Opening

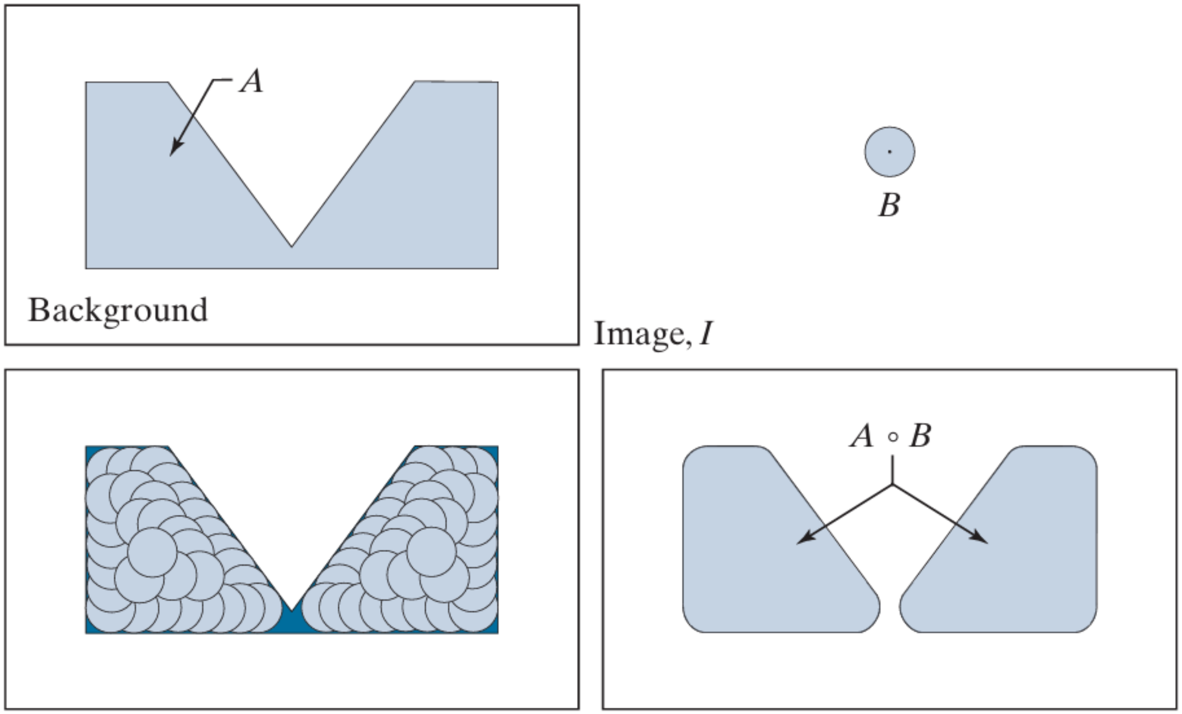
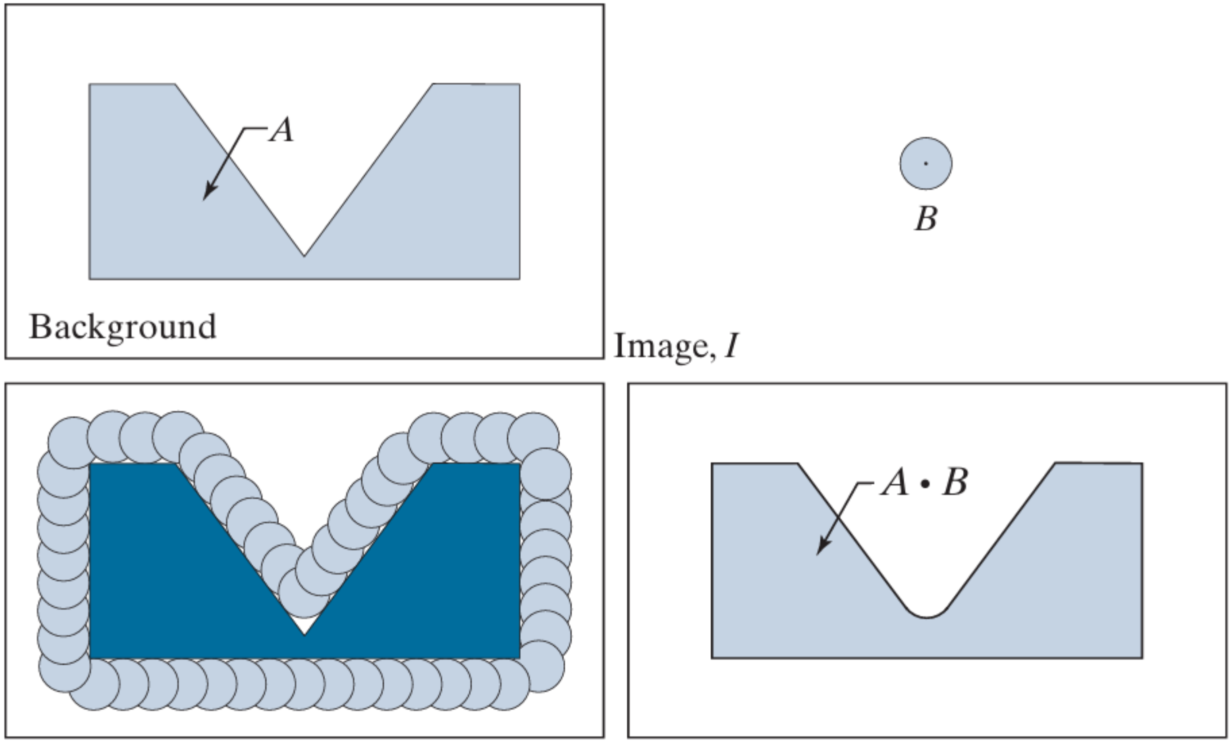


FIGURE 9.8
(a) Image I, composed of set (object) A and background.
(b) Structuring element, B.
(c) Translations of B while being contained in A. (A is shown dark for clarity.)
(d) Opening of A by B.

Example: Closing

a	b
c	d

FIGURE 9.9
(a) Image I, composed of set (object) A, and background.
(b) Structuring element B.
(c) Translations of B such that B does not overlap any part of A. (A is shown dark for clarity.)
(d) Closing of A by B.



Duality of Opening and Closing

- Opening and closing are duals of each other with respect to set complementation and reflection

$$(A \circ B)^c = \left(A^c \bullet \hat{B}\right)$$

$$(A \bullet B)^c = \left(A^c \circ \hat{B}\right)$$

The Properties of Opening and Closing

Morphological opening has the following properties:

(a) $A \circ B$ is a subset of A.

(b) If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$.

(c) $(A \circ B) \circ B = A \circ B$.

Similarly, closing satisfies the following properties:

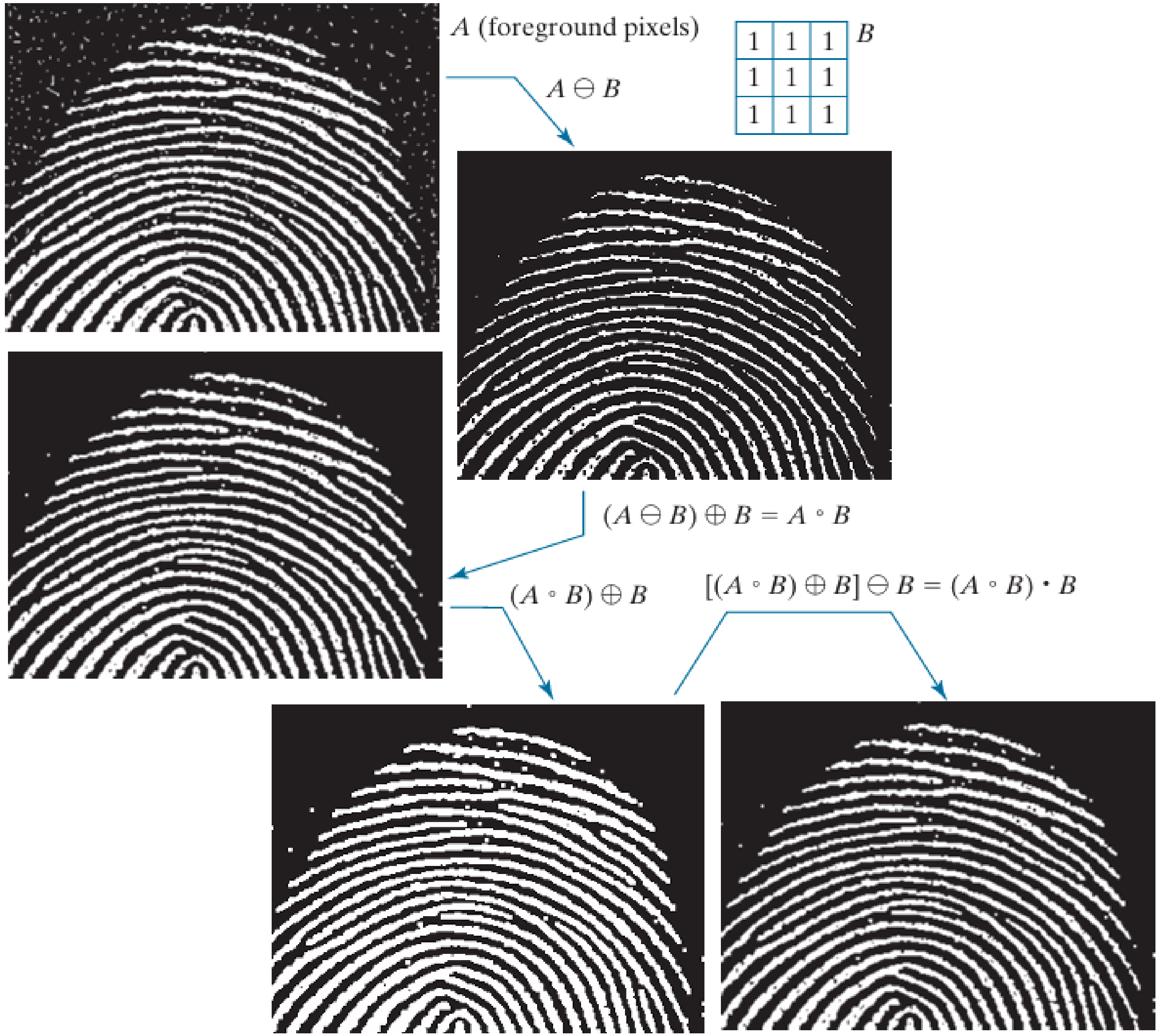
(a) A is a subset of $A \bullet B$.

(b) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$.

(c) $(A \bullet B) \bullet B = A \bullet B$.

a b c d e f

FIGURE 9.11
(a) Noisy image. (b) Structuring element. (c) Eroded image. (d) Dilation of the erosion (opening of A). (e) Dilation of the opening. (f) Closing of the opening. (Original image courtesy of the National Institute of Standards and Technology.)



The Hit-or-Miss Transformation

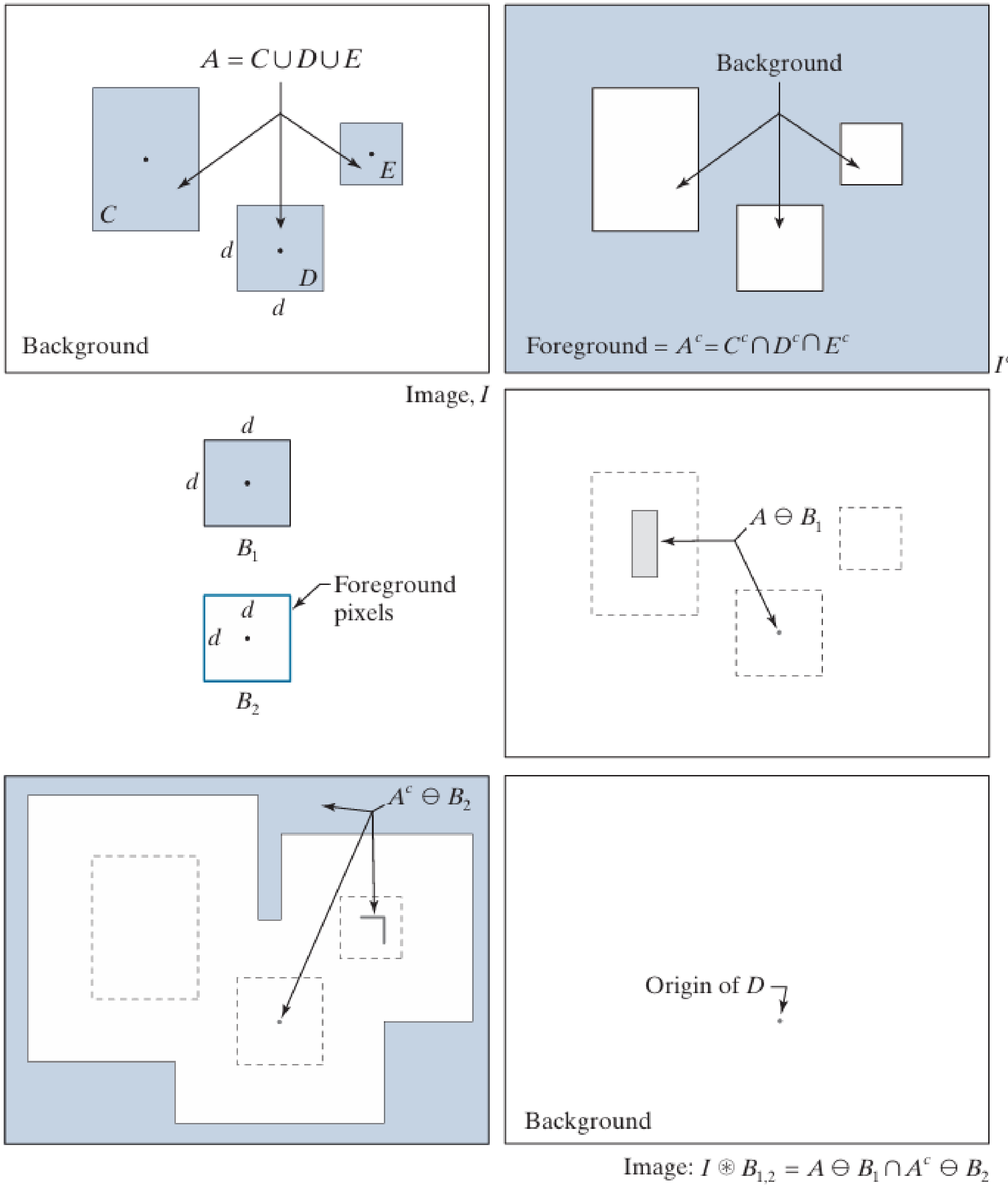
- The morphological hit-or-miss transform (HMT) is a basic tool for shape detection.
- Let I be a binary image composed of foreground (A) and background pixels, respectively. Unlike the morphological methods discussed thus far, the HMT utilizes two structuring elements: B_1 for detecting shapes in the foreground, and B_2, for detecting shapes in the background.
- The HMT of image I is defined as

$$I \circledast B_{1,2} = \left\{ z \middle| (B_1)_z \subseteq A \text{ and } (B_2)_z \subseteq A^c \right\}$$
$$= (A \ominus B_1) \cap (A^c \ominus B_2)$$

- HMT is the set of translations, z, of structuring elements B_1 and B_2 such that, simultaneously, B_1 found a match in the fore ground (i.e., B_1 is contained in A) and B_2 found a match in the background (i.e., B_2 is contained in A^c). The word "simultaneous" implies that z is the same translation of both structuring elements.
- The word "miss" in the HMT arises from the fact that B_2 finding a match in A^c is the same as B_2 not finding (missing) a match in A.

a	b
c	d
e	f

FIGURE 9.12
(a) Image consisting of a foreground (1's) equal to the union, A, of set of objects, and a background of 0's.
(b) Image with its foreground defined as A^c.
(c) Structuring elements designed to detect object D.
(d) Erosion of A by B_1.
(e) Erosion of A^c by B_2.
(f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origin of their respective components. Each dot is a single pixel.



Some Basic Morphological Algorithms (1)

- **Boundary Extraction**

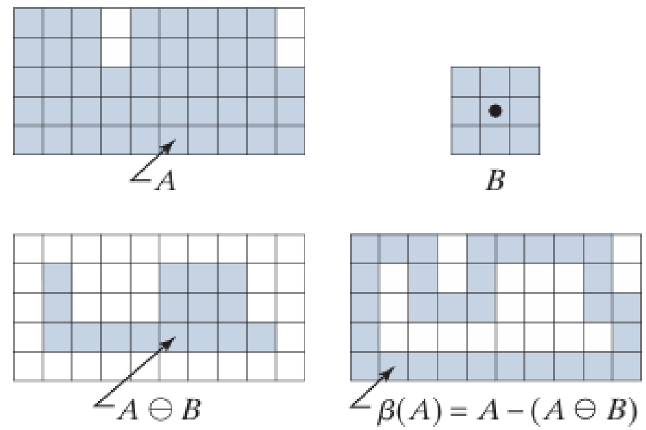
The boundary of a set A, can be obtained by first eroding A by B and then performing the set difference between A and its erosion.

$$\beta(A) = A - (A \ominus B)$$

Example 1

a b
c d

FIGURE 9.15
(a) Set, A, of foreground pixels.
(b) Structuring element.
(c) A eroded by B.
(d) Boundary of A.



1	1	1	0	1	1	1	1	1	0
1	1	1	0	1	1	1	1	1	0
1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1

1	1	1
1	1	1
1	1	1

0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	1	1	1	0	0
0	1	0	0	0	1	1	1	0	0
0	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0

1	1	1	0	1	1	1	1	1	0
1	0	1	0	1	0	0	0	1	0
1	0	1	1	1	0	0	0	1	1
1	0	0	0	0	0	0	0	0	1
1	1	1	1	1	1	1	1	1	1

1	1	1	0	1	1	1	1	1	0
1	0	1	0	1	0	0	0	1	0
1	0	1	1	1	0	0	0	1	1
1	0	0	0	0	0	0	0	0	1
1	1	1	1	1	1	1	1	1	1

Example 2

FIGURE 9.16 (a) A binary image. (b) Result of using Eq. (9-18) with the structuring element in Fig. 9.15(b).

Some Basic Morphological Algorithms (2)

- **Hole Filling**

  A hole may be defined as a background region surrounded by a connected border of foreground pixels.

  Let A denote a set whose elements are 8-connected boundaries, each boundary enclosing a background region (i.e., a hole). Given a point in each hole, the objective is to fill all the holes with 1s.

Some Basic Morphological Algorithms (2)

- **Hole Filling**

  1. Forming an array X_0 of 0s (the same size as the array containing A), except the locations in X_0 corresponding to the given point in each hole, which we set to 1.

  2. $X_k = (X_{k-1} \oplus B) \cap A^c \quad k=1,2,3,\ldots$

  Stop the iteration if $X_k = X_{k-1}$

Example

a	b	c
d	e	f
g	h	i

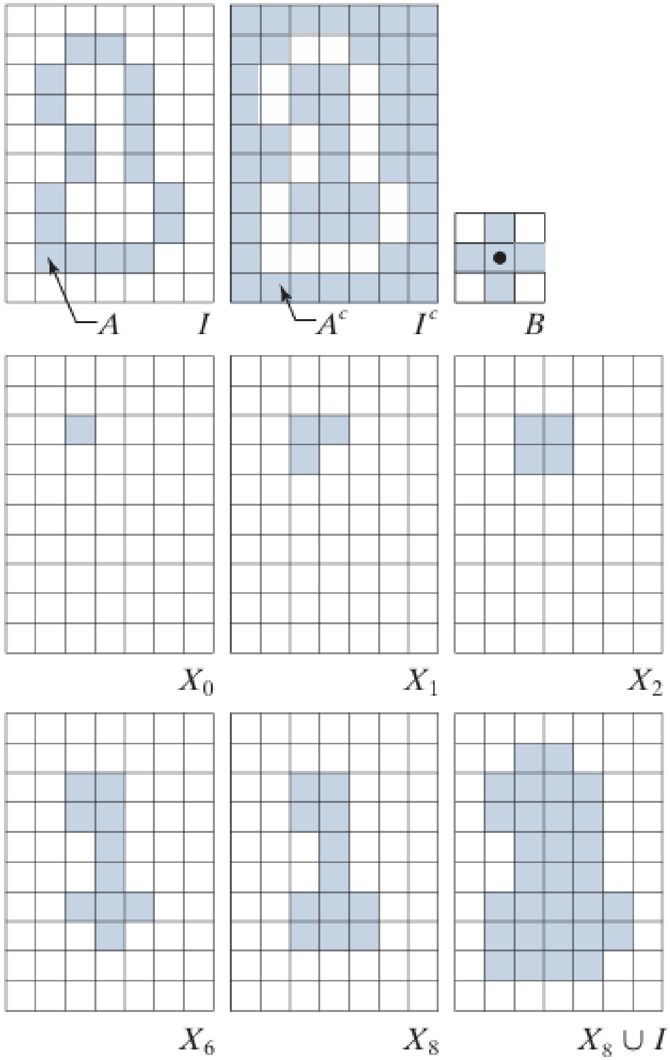


FIGURE 9.17
Hole filling.
(a) Set A (shown shaded) contained in image I.
(b) Complement of I.
(c) Structuring element B. *Only* the foreground elements are used in computations
(d) Initial point inside hole, set to 1.
(e)–(h) Various steps of Eq. (9-19).
(i) Final result [union of (a) and (h)].

Some Basic Morphological Algorithms (3)

- **Extraction of Connected Components**

  Central to many automated image analysis applications.

  Let A be a set containing one or more connected components, and form an array X_0 (of the same size as the array containing A) whose elements are 0s, except at each location known to correspond to a point in each connected component in A, which is set to 1.

Some Basic Morphological Algorithms (3)

- **Extraction of Connected Components**

  Central to many automated image analysis applications.

$$X_k = (X_{k-1} \oplus B) \cap A$$

$$B : \text{structuring element}$$

$$\text{until } X_k = X_{k\text{-}1}$$

Example

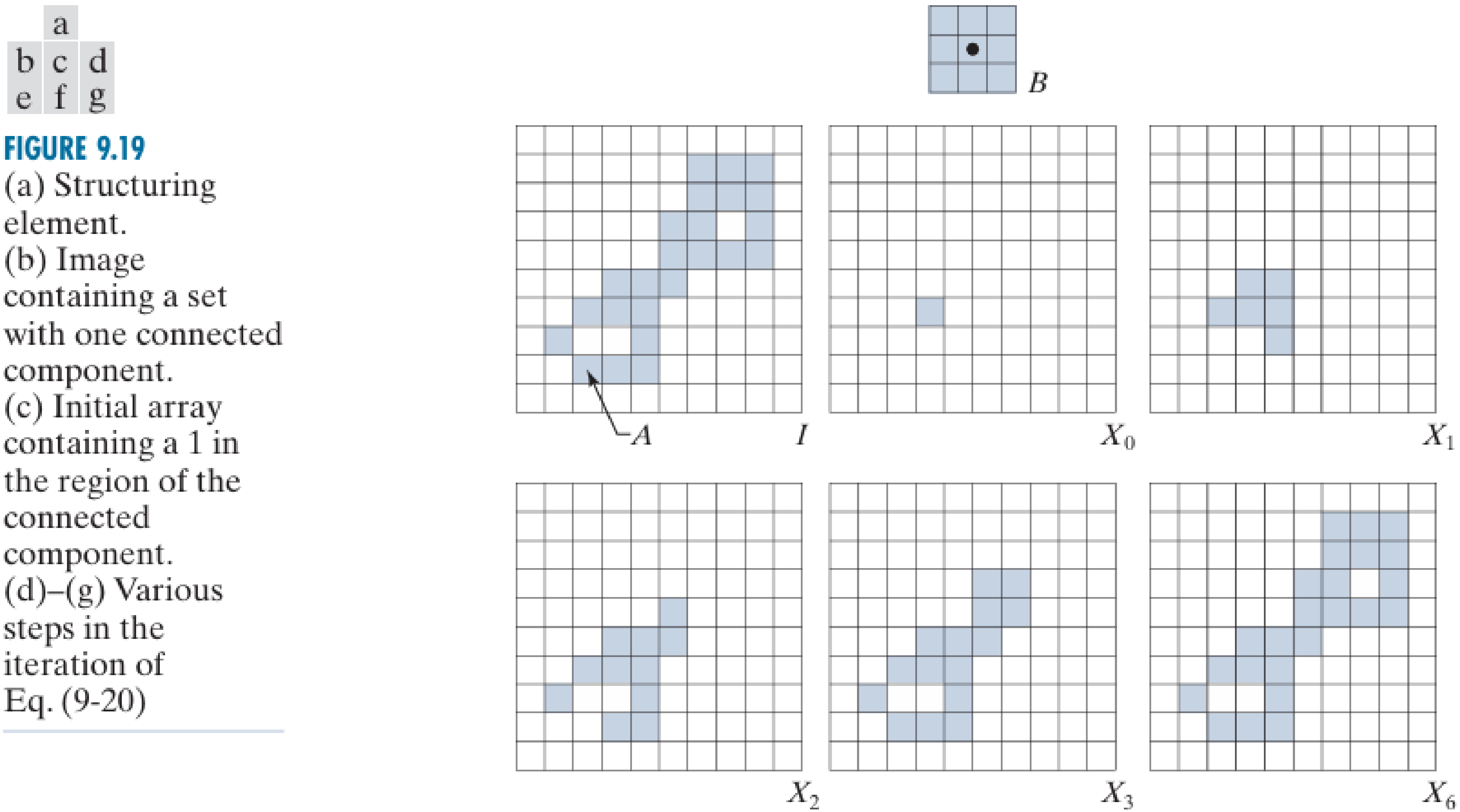
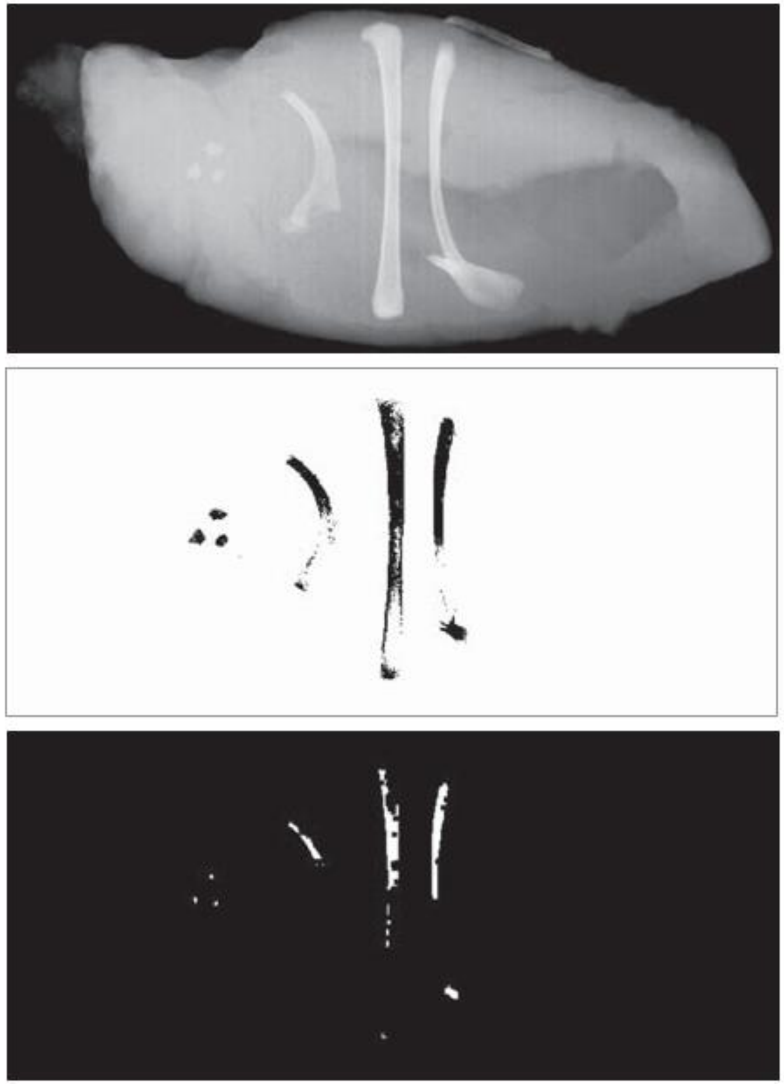


FIGURE 9.19
(a) Structuring element.
(b) Image containing a set with one connected component.
(c) Initial array containing a 1 in the region of the connected component.
(d)–(g) Various steps in the iteration of Eq. (9-20)

a
b
c d

FIGURE 9.20
(a) X-ray image of a chicken filet with bone fragments.
(b) Thresholded image (shown as the negative for clarity).
(c) Image eroded with a 5×5 SE of 1's.
(d) Number of pixels in the connected components of (c).
(Image (a) courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Some Basic Morphological Algorithms (4)

- **Convex Hull**

  A set *A* is said to be ***convex*** if the straight line segment joining any two points in *A* lies entirely within *A*.

  The ***convex hull*** *H* or of an arbitrary set *S* is the smallest convex set containing *S*.

Some Basic Morphological Algorithms (4)

- **Convex Hull**

Let B^i, $i = 1,\ 2,\ 3,\ 4$, represent the four structuring elements.

The procedure consists of implementing the equation:

$$X_k^i = (X_{k-1} \circledast B^i) \cup A$$
$$i = 1, 2, 3, 4 \ \text{ and } \ k = 1, 2, 3, \ldots$$

with $X_0^i = A$.

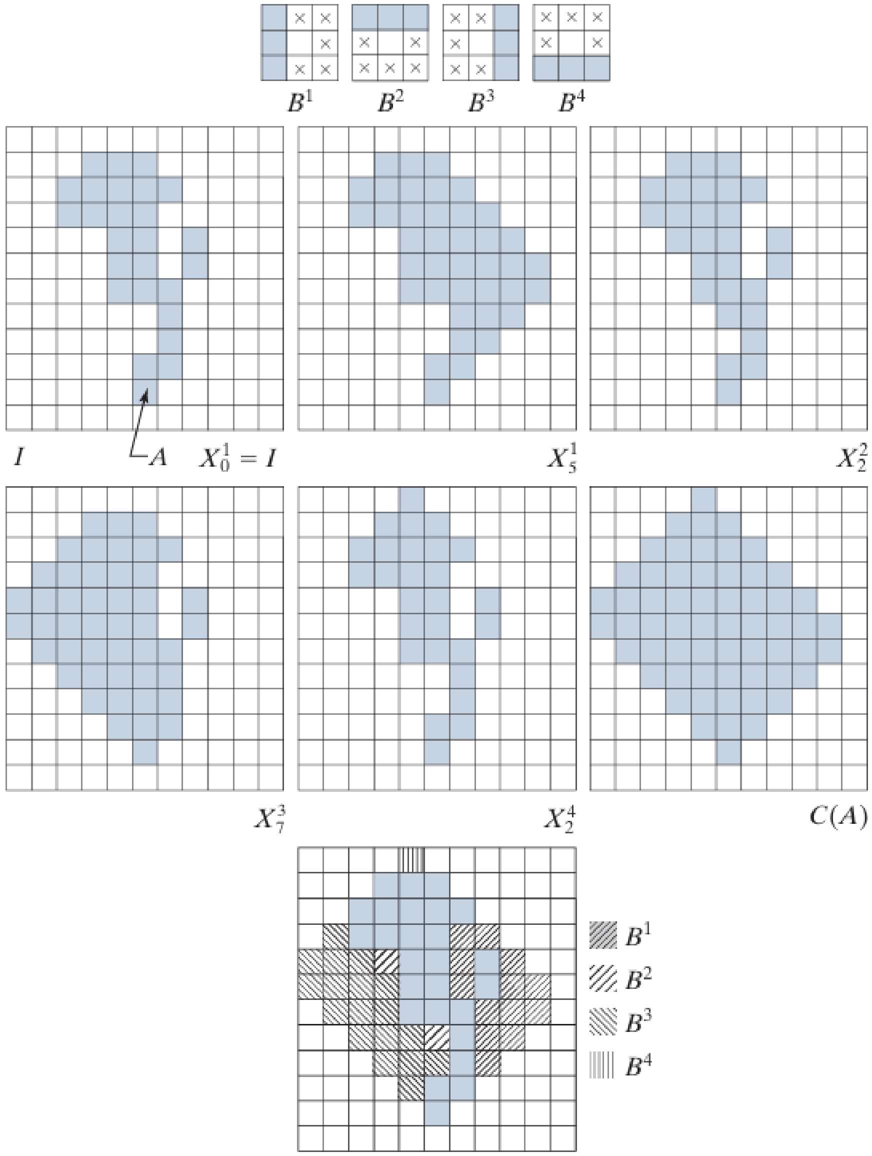
When the procedure converges, or $X_k^i = X_{k-1}^i$, let $D^i = X_k^i$,

the convex hull of A is

$$C(A) = \bigcup_{i=1}^{4} D^i$$

a
b c d
e f g
h

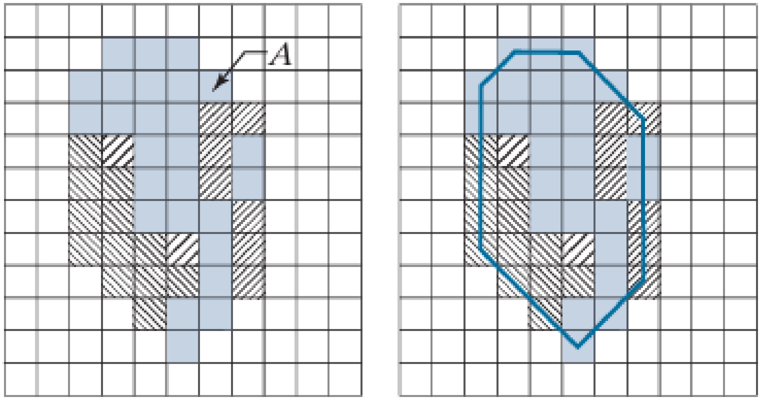
FIGURE 9.21
(a) Structuring elements.
(b) Set A.
(c)–(f) Results of convergence with the structuring elements shown in (a).
(g) Convex hull.
(h) Convex hull showing the contribution of each structuring element.



Example

a b

FIGURE 9.22
(a) Result of limiting growth of the convex hull algorithm.
(b) Straight lines connecting the boundary points show that the new set is convex also.



Some Basic Morphological Algorithms (5)

- Thinning

  The thinning of a set A by a structuring element B, defined

$$A \otimes B = A - (A \circledast B)$$

$$= A \cap (A \circledast B)^c$$

Some Basic Morphological Algorithms (5)

- A more useful expression for thinning A symmetrically is based on a sequence of structuring elements:

$$\{B\} = \left\{B^1, B^2, B^3, \ldots, B^n\right\}$$

where B^i is a rotated version of $B^{i\text{-}1}$

The thinning of A by a sequence of structuring element $\{B\}$

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$

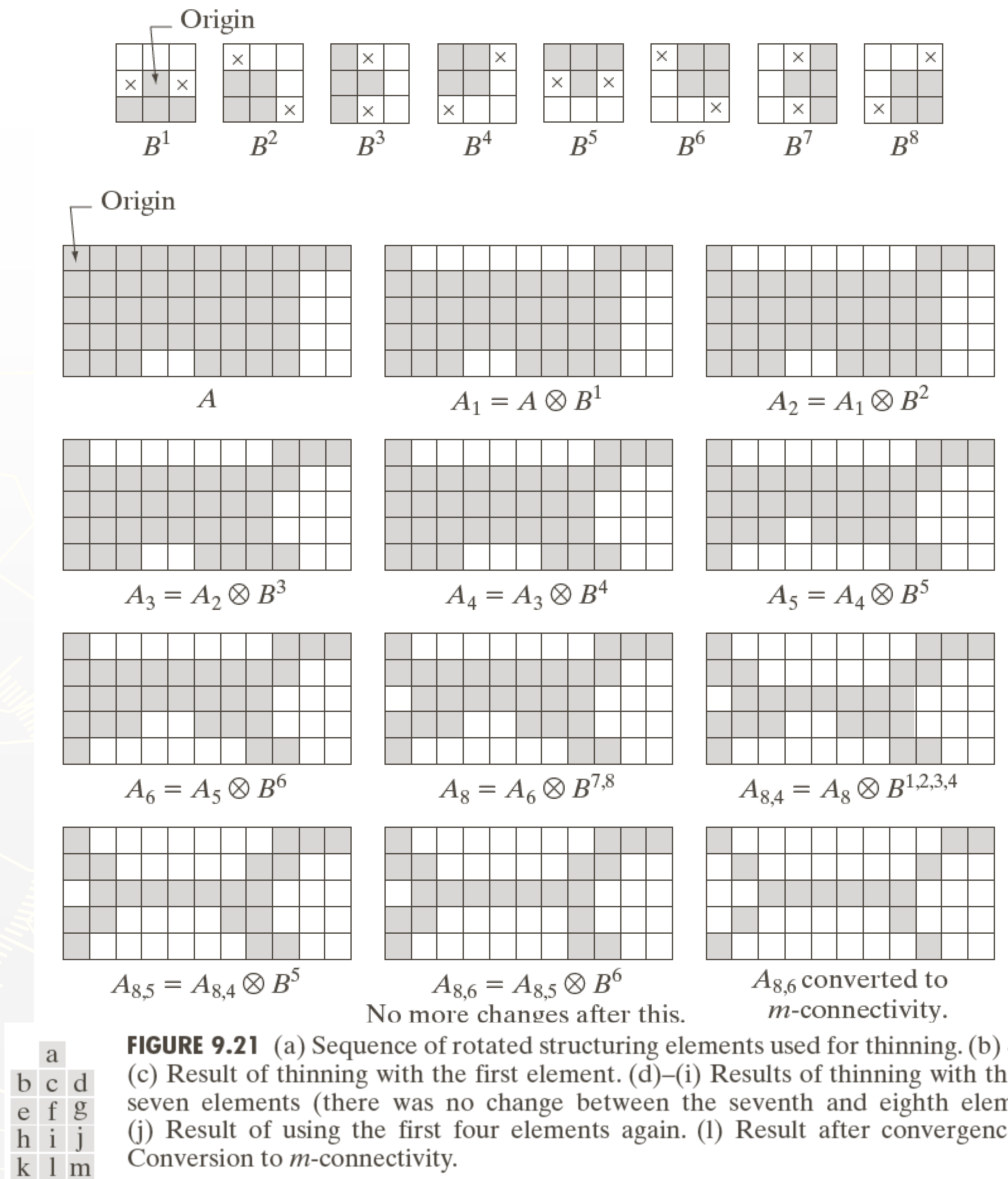


FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to m-connectivity.

Some Basic Morphological Algorithms (6)

- Thickening:

The thickening is defined by the expression

$$A \odot B = A \cup (A * B)$$

The thickening of A by a sequence of structuring element $\{B\}$

$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$$

In practice, the usual procedure is to thin the background of the set and then complement the result.

Some Basic Morphological Algorithms (6)

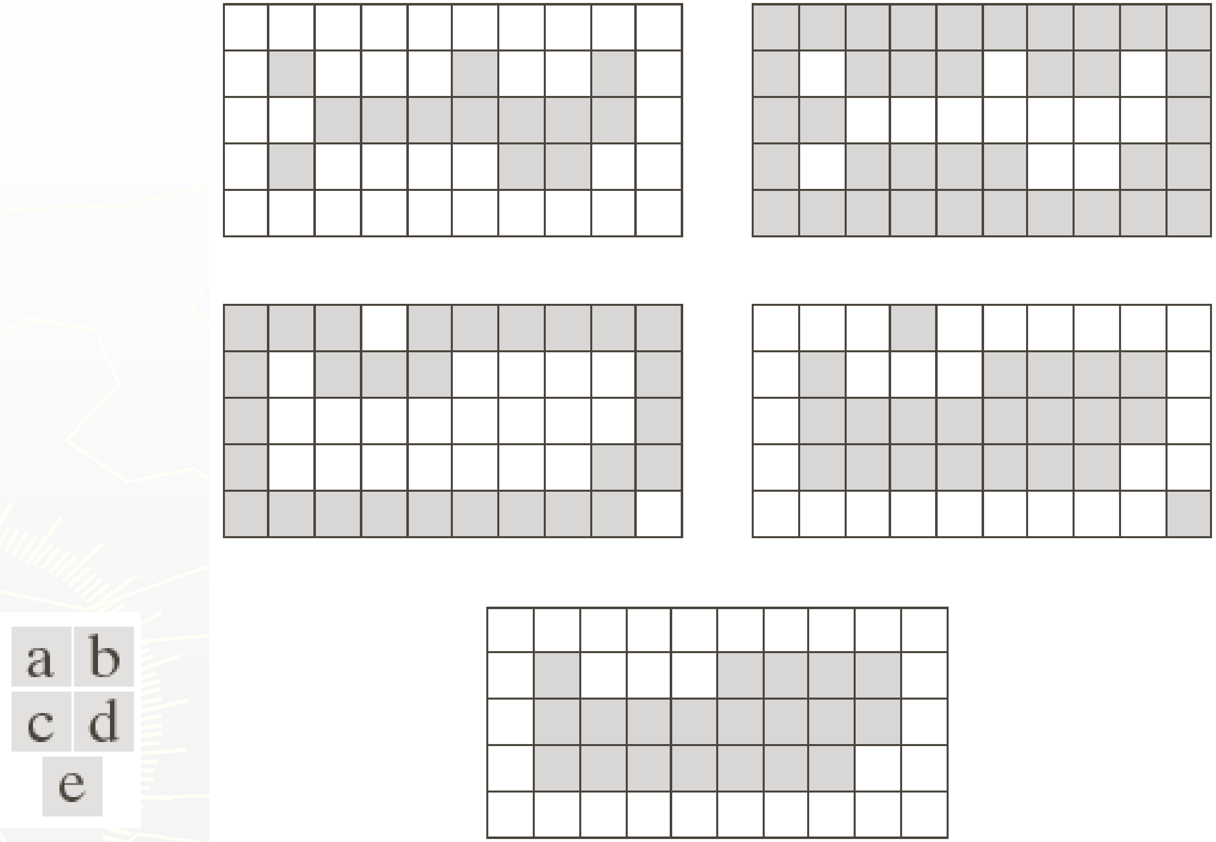


FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Some Basic Morphological Algorithms (7)

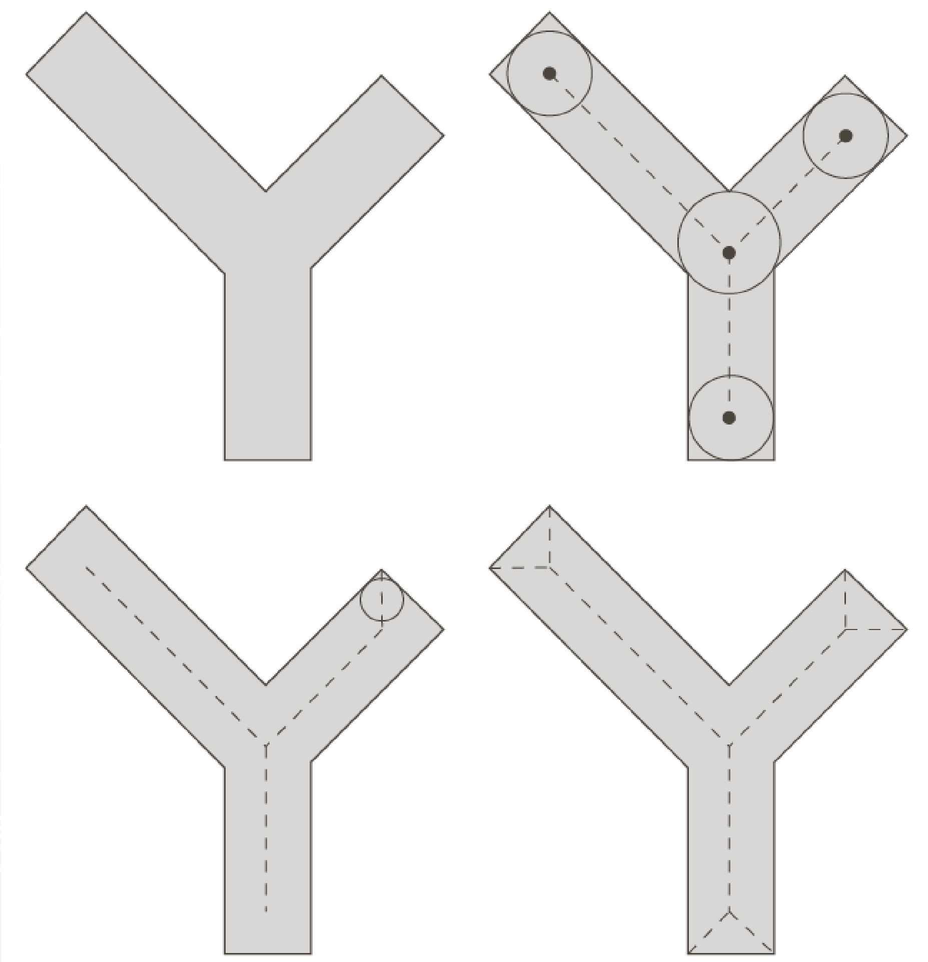
- Skeletons

A skeleton, $S(A)$ of a set A has the following properties

a. if z is a point of $S(A)$ and $(D)_z$ is the largest disk centered at z and contained in A, one cannot find a larger disk containing $(D)_z$ and included in A. The disk $(D)_z$ is called a maximum disk.

b. The disk $(D)_z$ touches the boundary of A at two or more different places.

Some Basic Morphological Algorithms (7)



a	b
c	d

FIGURE 9.23
(a) Set A.
(b) Various positions of maximum disks with centers on the skeleton of A.
(c) Another maximum disk on a different segment of the skeleton of A.
(d) Complete skeleton.

Some Basic Morphological Algorithms (7)

The skeleton of A can be expressed in terms of erosion and openings.

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

with $K = \max\{k \mid A \ominus kB \neq \phi\};$

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

where B is a structuring element, and

$$(A \ominus kB) = ((..((A \ominus B) \ominus B) \ominus ...) \ominus B)$$

k successive erosions of A.

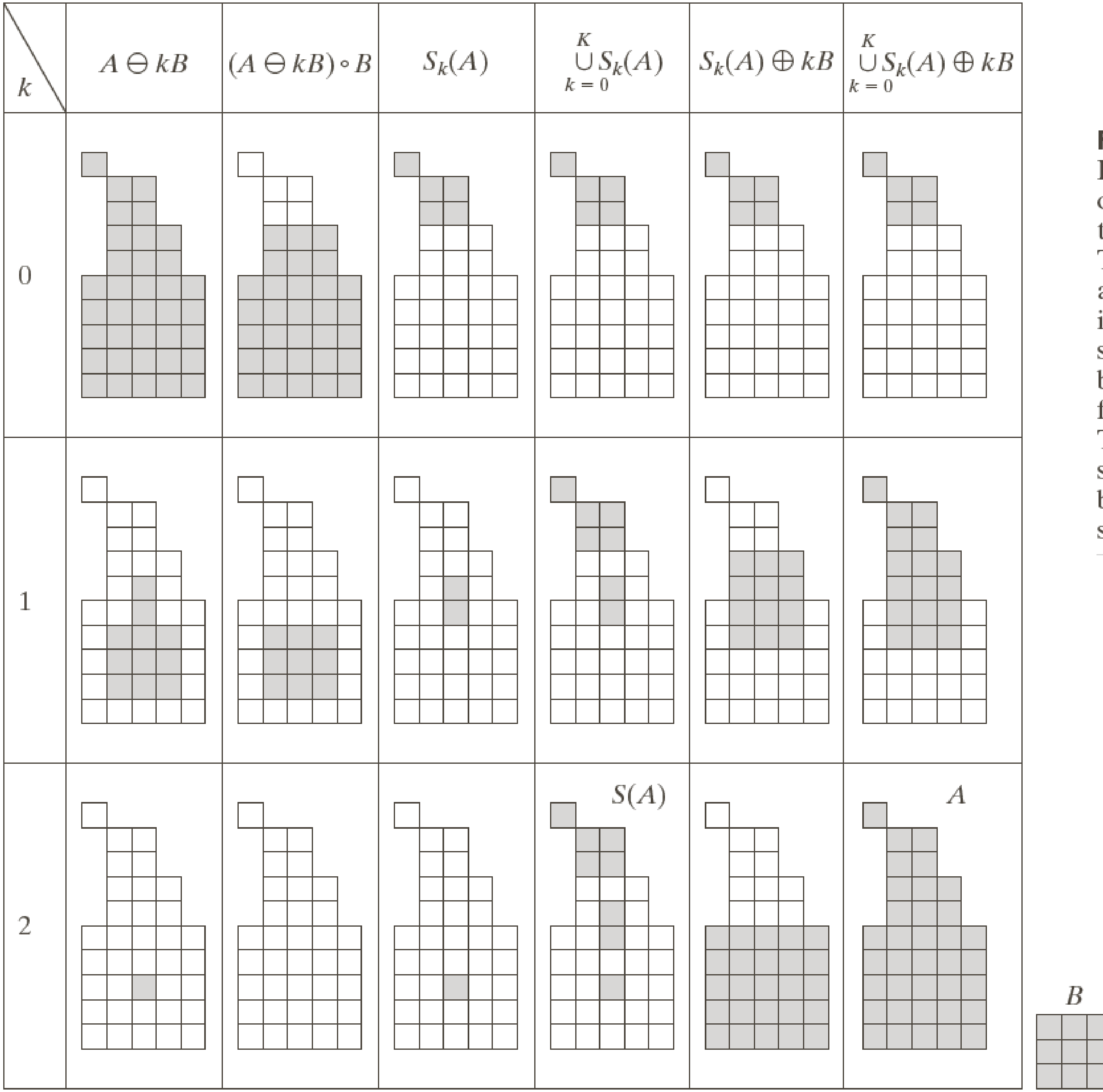


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

Some Basic Morphological Algorithms (7)

A can be reconstructed from the subsets by using

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

where $S_k(A) \oplus kB$ denotes k successive dilations of A.

$$(S_k(A) \oplus kB) = ((...((S_k(A) \oplus B) \oplus B)... \oplus B)$$

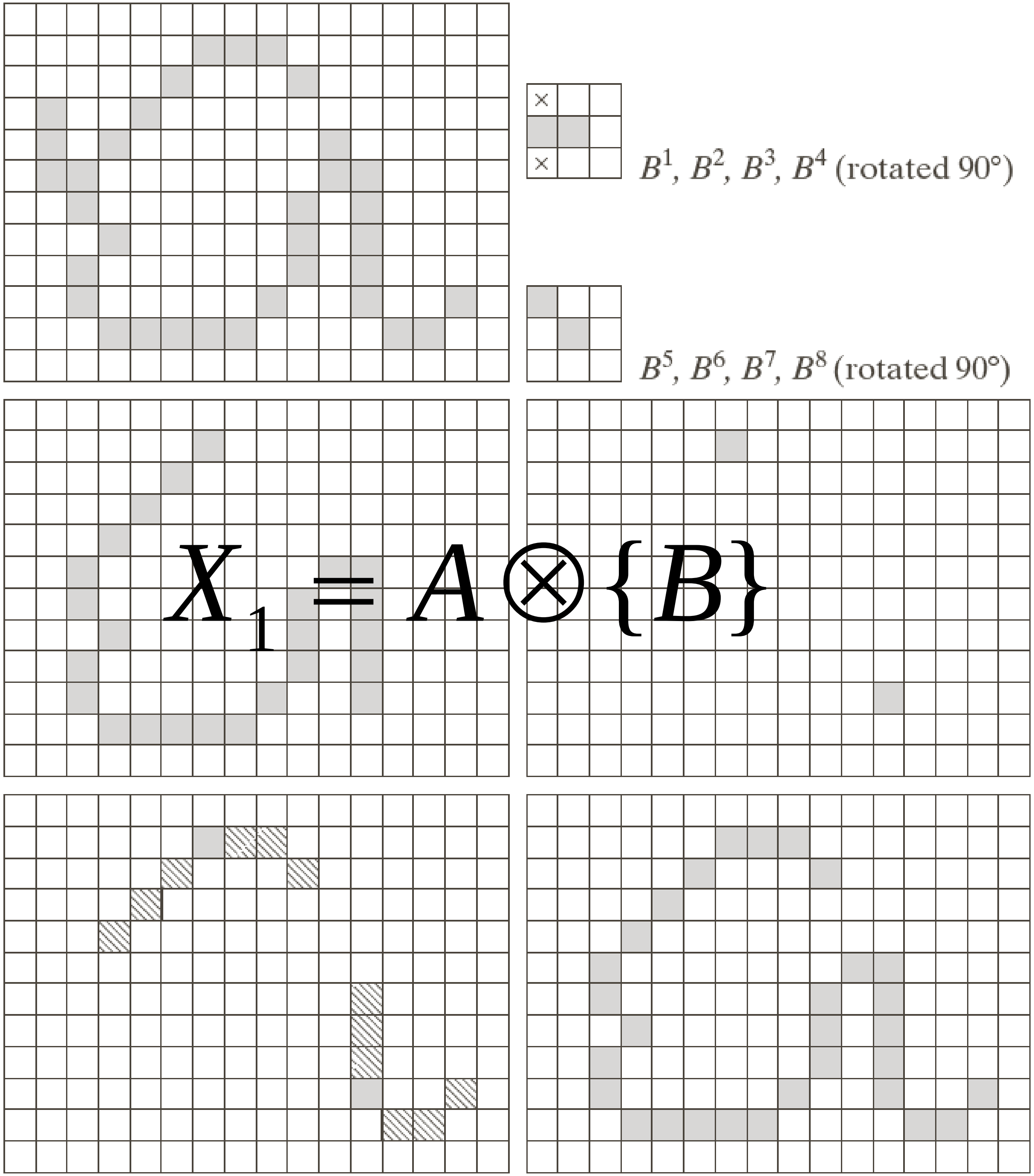
Some Basic Morphological Algorithms (8)

- Pruning

a. Thinning and skeletonizing tend to leave parasitic components

b. Pruning methods are essential complement to thinning and skeletonizing procedures

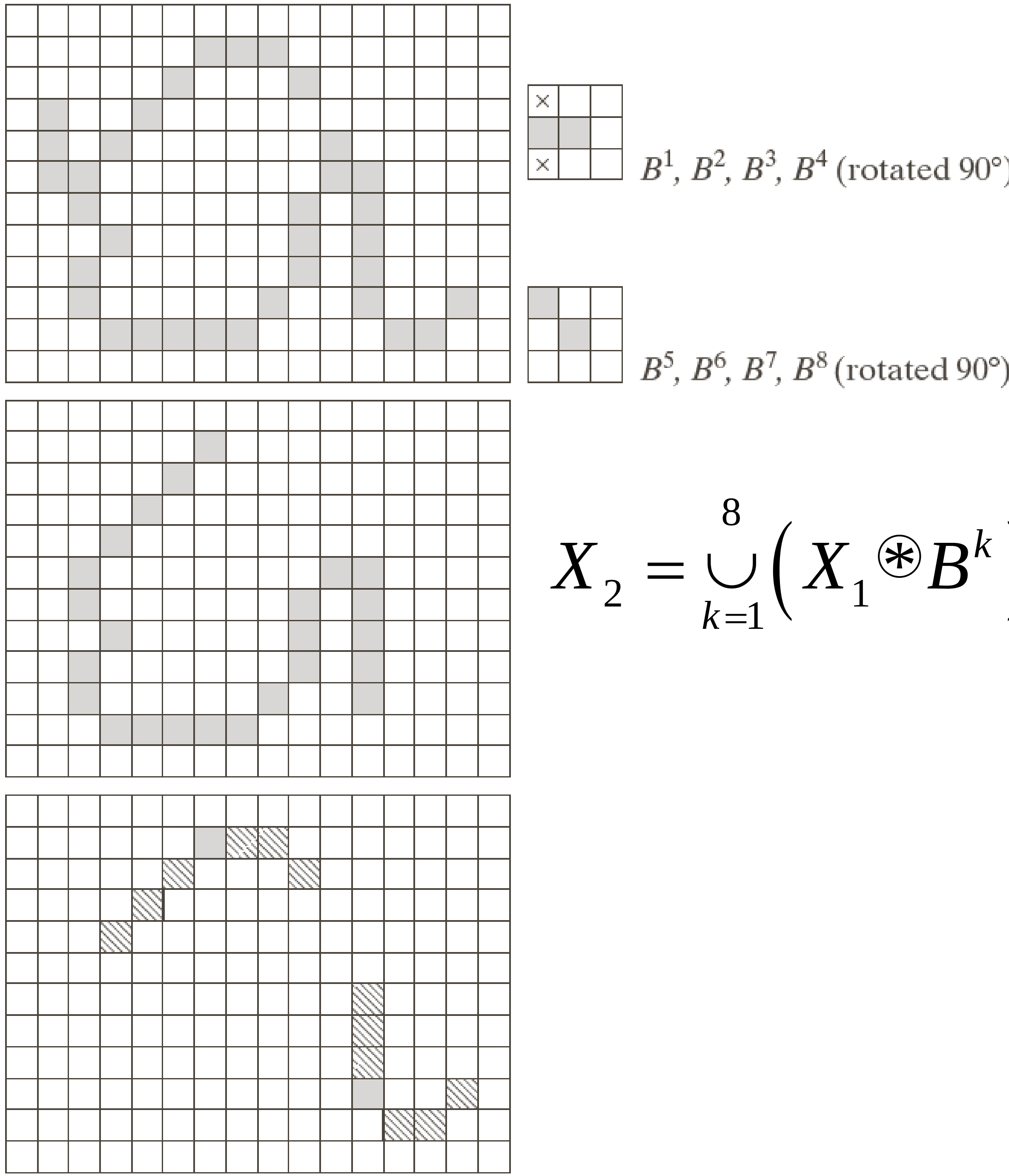
Pruning: Example



$$X_1 = A \otimes \{B\}$$

FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

Pruning: Example

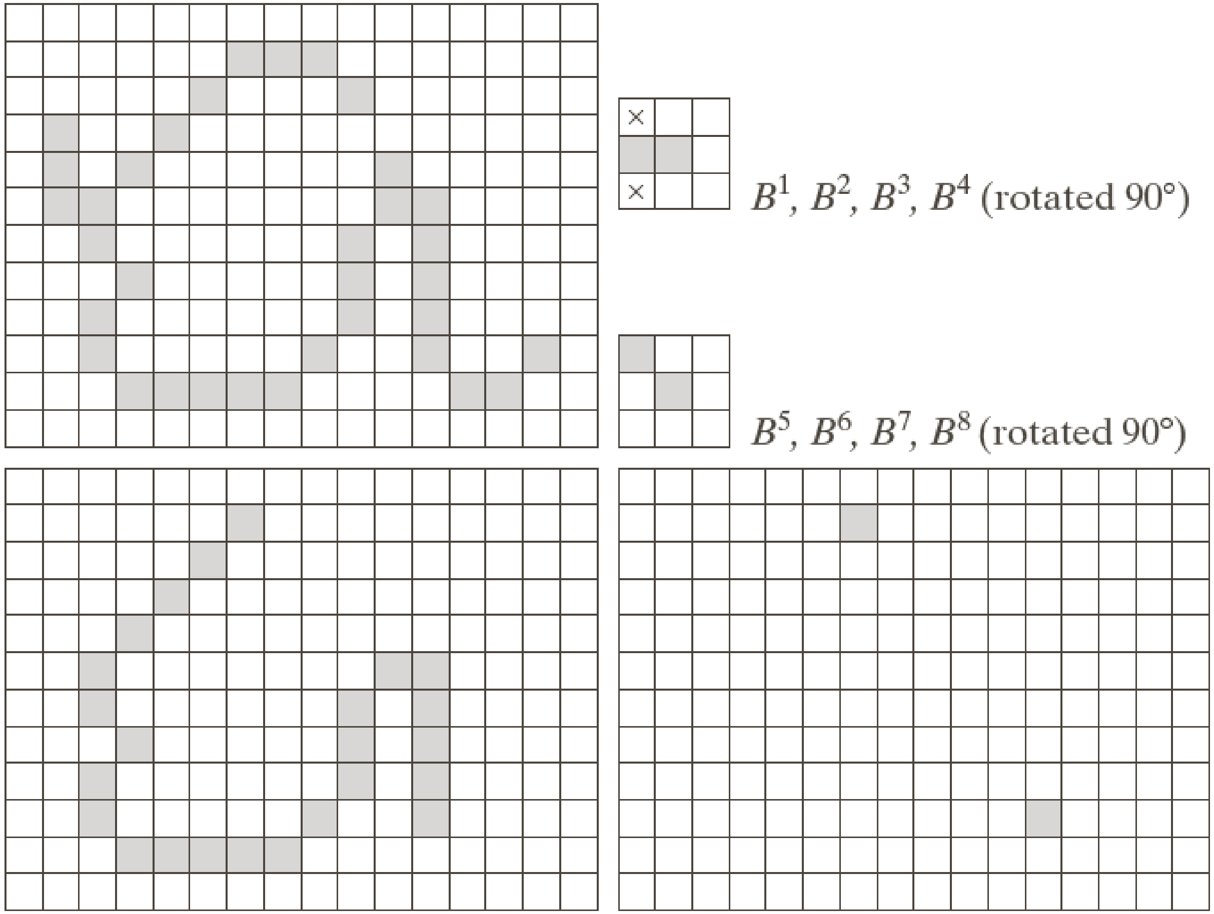


$$X_2 = \bigcup_{k=1}^{8} \left(X_1 \circledast B^k \right)$$

a b
c
d e
f g

FIGURE 9.25
(a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

Pruning: Example



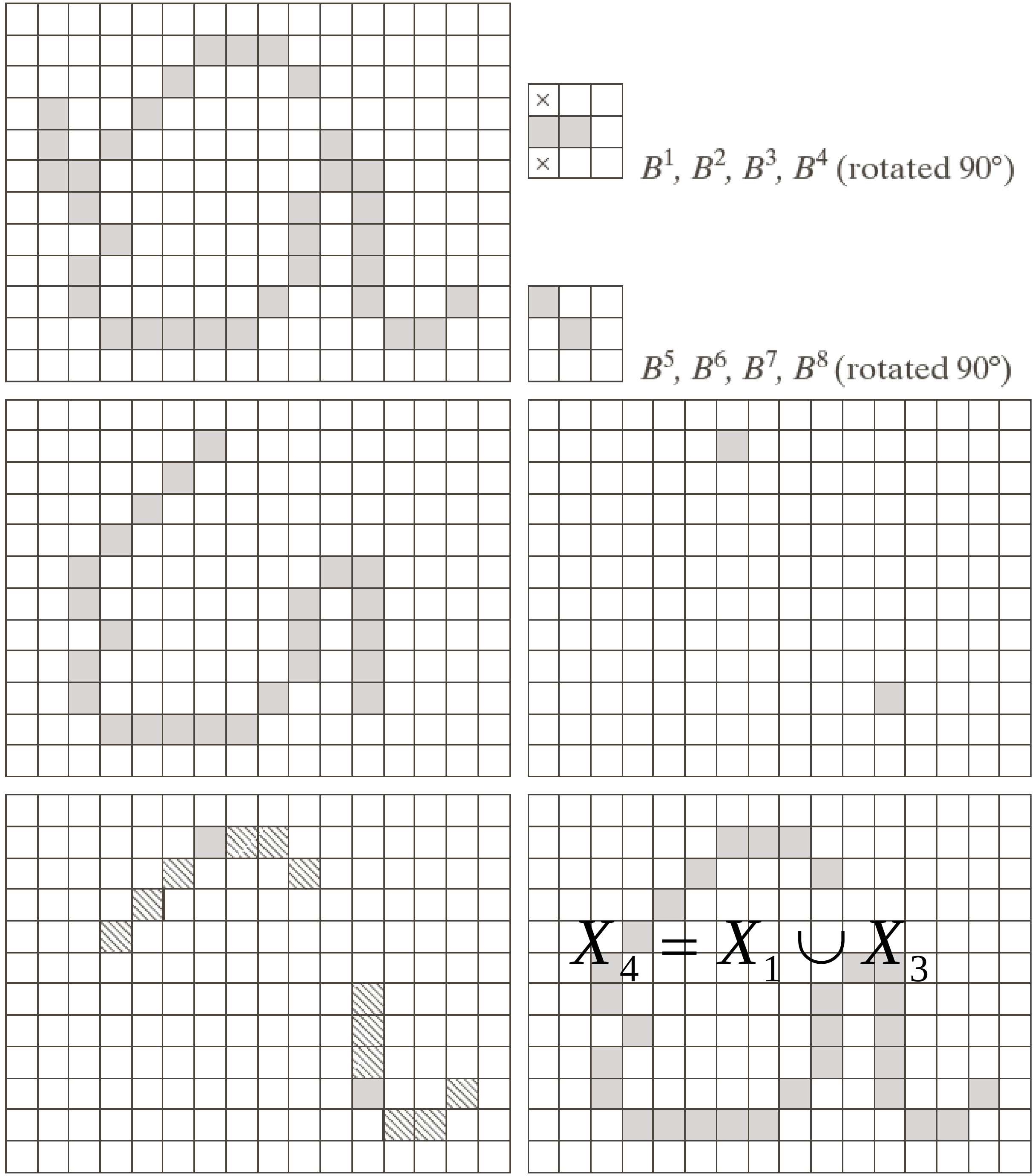
a b
c
d e
f g

FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

$$X_3 = \left(X_2 \oplus H \right) \cap A$$

$H : 3 \times 3$ structuring element

Pruning: Example

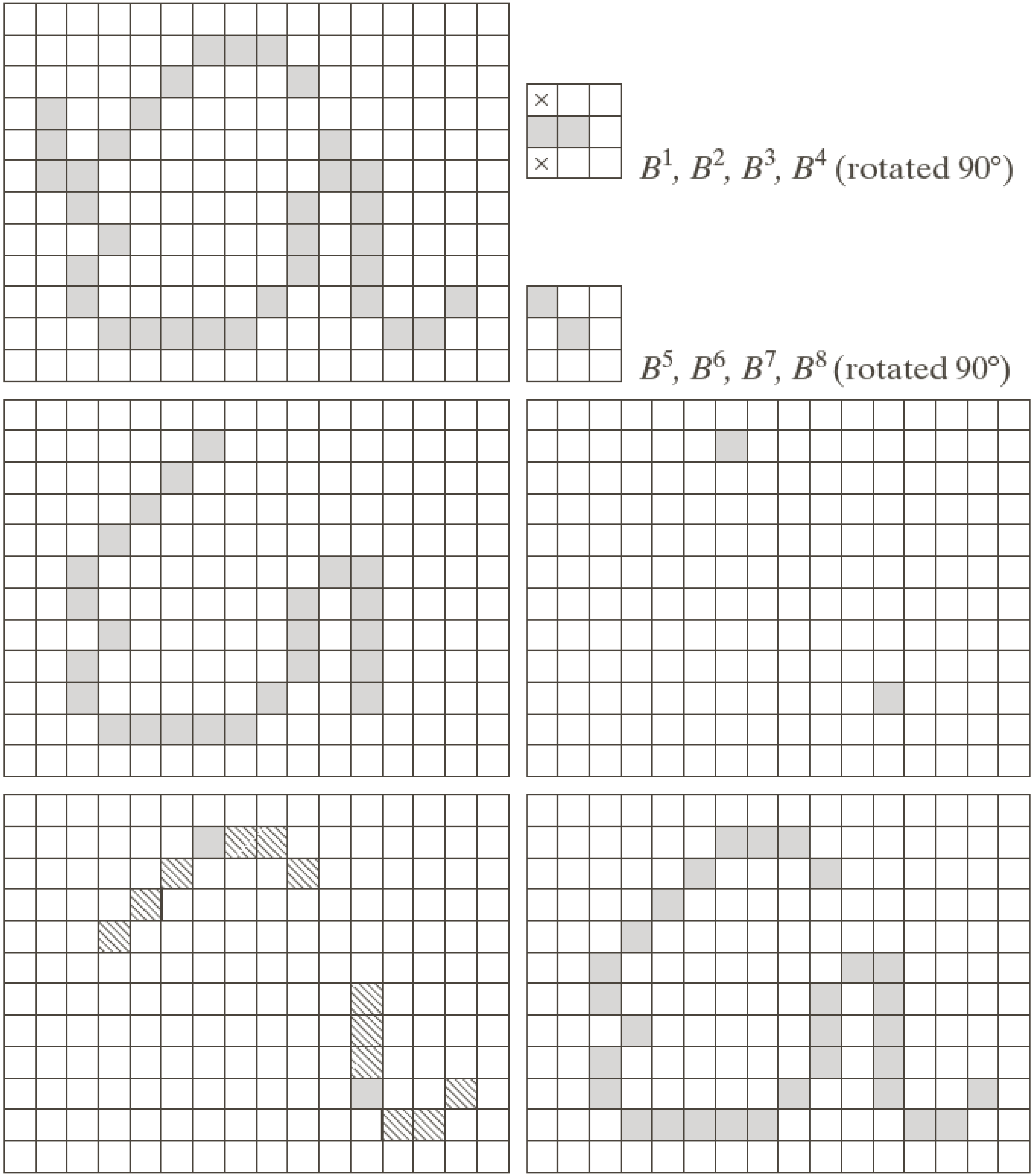


$$X_4 = X_1 \cup X_3$$

a b
c
d e
f g

FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

Pruning: Example



a b
c
d e
f g

FIGURE 9.25
(a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

Some Basic Morphological Algorithms (9)

- Morphological Reconstruction

  It involves two images and a structuring element

  a. One image contains the starting points for the transformation (The image is called marker)
  b. Another image (mask) constrains the transformation
  c. The structuring element is used to define connectivity

Morphological Reconstruction: Geodesic Dilation

Let F denote the marker image and G the mask image, $F \subseteq G$. The geodesic dilation of size 1 of the marker image with respect to the mask, denoted by $D_G^{(1)}(F)$, is defined as

$$D_G^{(1)}(F) = (F \oplus B) \cap \mathrm{G}$$

The geodesic dilation of size n of the marker image F with respect to G, denoted by $D_G^{(n)}(F)$, is defined as

$$D_G^{(n)}(F) = D_G^{(1)}(F)\left[D_G^{(n-1)}(F)\right]$$

with $D_G^{(0)}(F) = F$.

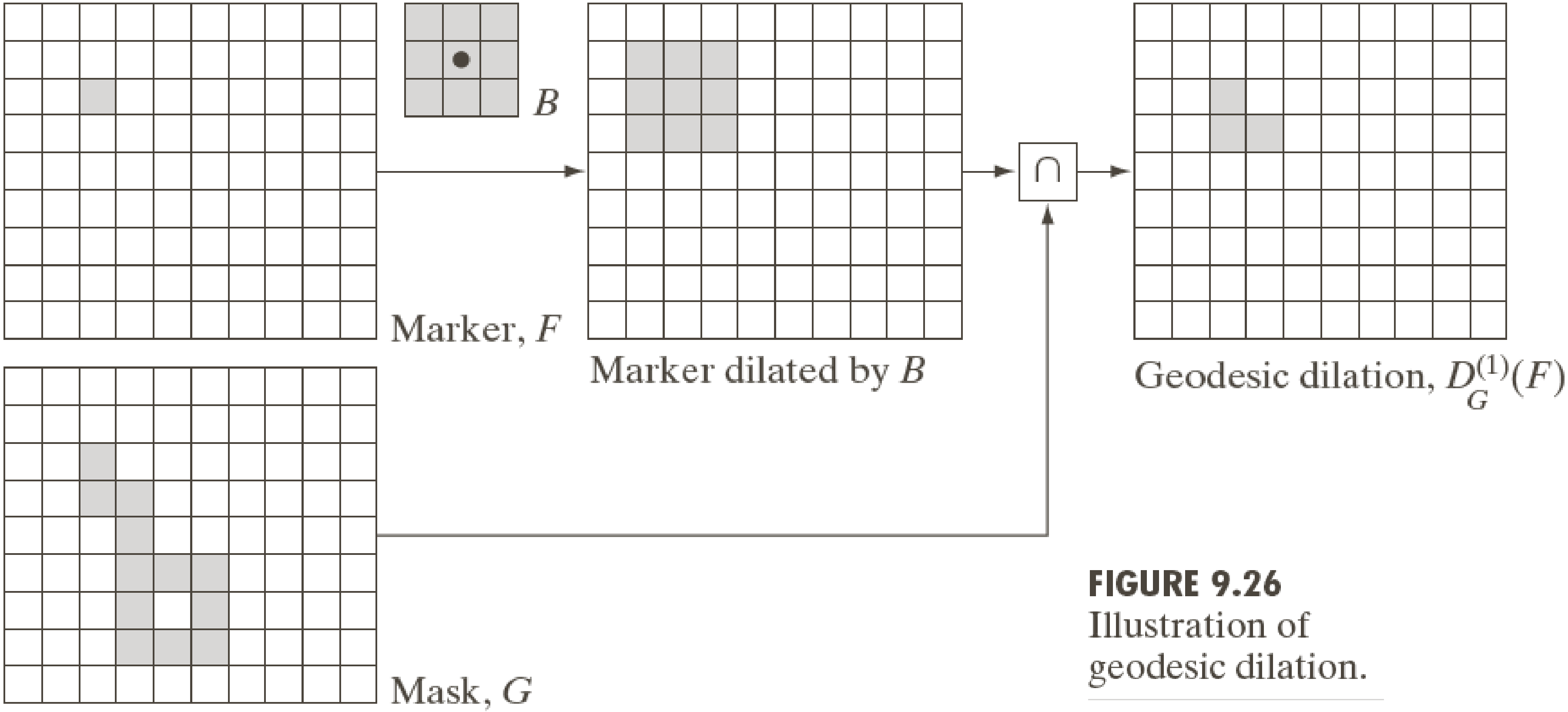


FIGURE 9.26 Illustration of geodesic dilation.

Morphological Reconstruction: Geodesic Erosion

Let F denote the marker image and G the mask image.

The geodesic erosion of size 1 of the marker image with respect to the mask, denoted by $E_G^{(1)}(F)$, is defined as

$$E_G^{(1)}(F) = \left(F \ominus B\right) \cup \mathrm{G}$$

The geodesic erosion of size n of the marker image F with respect to G, denoted by $E_G^{(n)}(F)$, is defined as

$$E_G^{(n)}(F) = E_G^{(1)}(F)\left[E_G^{(n-1)}(F)\right]$$

with $E_G^{(0)}(F) = F$.

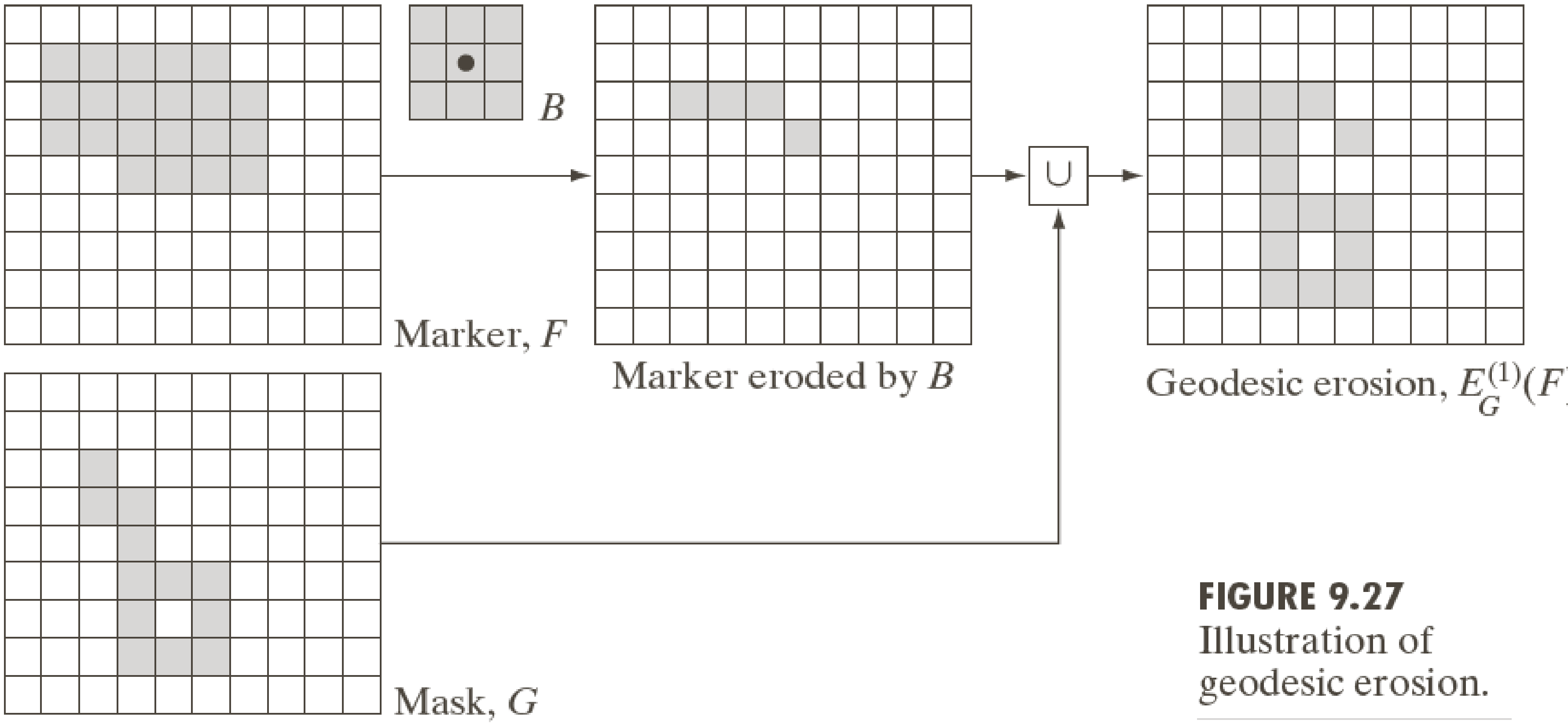


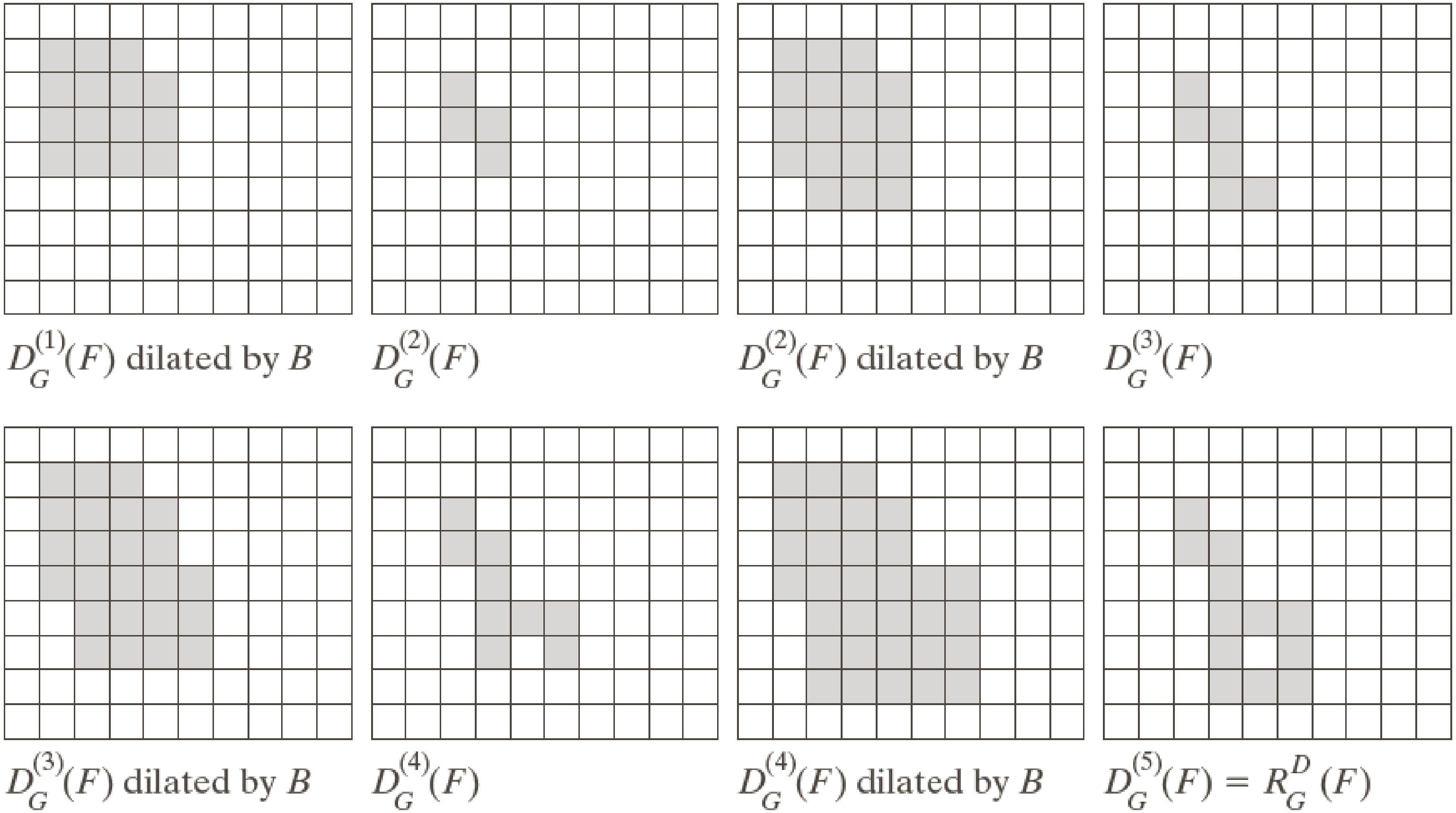
FIGURE 9.27
Illustration of geodesic erosion.

Morphological Reconstruction by Dilation

Morphological reconstruction by dialtion of a mask image G from a marker image F, denoted $R_G^D(F)$, is defined as the geodesic dilation of F with respect to G, iterated until stability is achieved; that is,

$$R_G^D(F) = D_G^{(k)}(F)$$

with k such that $D_G^{(k)}(F) = D_G^{(k-1)}(F)$.



a b c d
e f g h

FIGURE 9.28 Illustration of morphological reconstruction by dilation. F, G, B and $D_G^{(1)}(F)$ are from Fig. 9.26.

Morphological Reconstruction by Erosion

Morphological reconstruction by erosion of a mask image G from a marker image F, denoted $R_G^E(F)$, is defined as the geodesic erosion of F with respect to G, iterated until stability is achieved; that is,

$$R_G^E(F) = E_G^{(k)}(F)$$

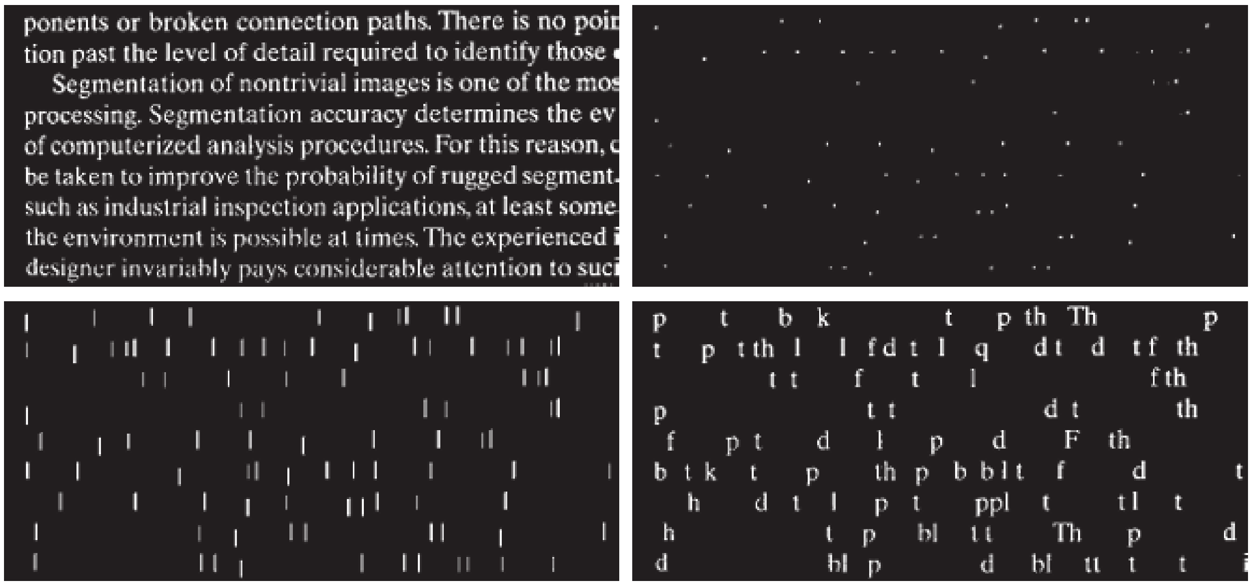
with k such that $E_G^{(k)}(F) = E_G^{(k-1)}(F)$.

Opening by Reconstruction

The opening by reconstruction of size n of an image F is defined as the reconstruction by dilation of F from the erosion of size n of F; that is

$$O_R^{(n)}(F) = R_F^D\left[\left(F \ominus nB\right)\right]$$

where $\left(F \ominus nB\right)$ indicates n erosions of F by B.



a b
c d

FIGURE 9.31 (a) Text image of size 918×2018 pixels. The approximate average height of the tall characters is 51 pixels. (b) Erosion of (a) with a structuring element of size 51×1 elements (all 1's). (c) Opening of (a) with the same structuring element, shown for comparison. (d) Result of opening by reconstruction.

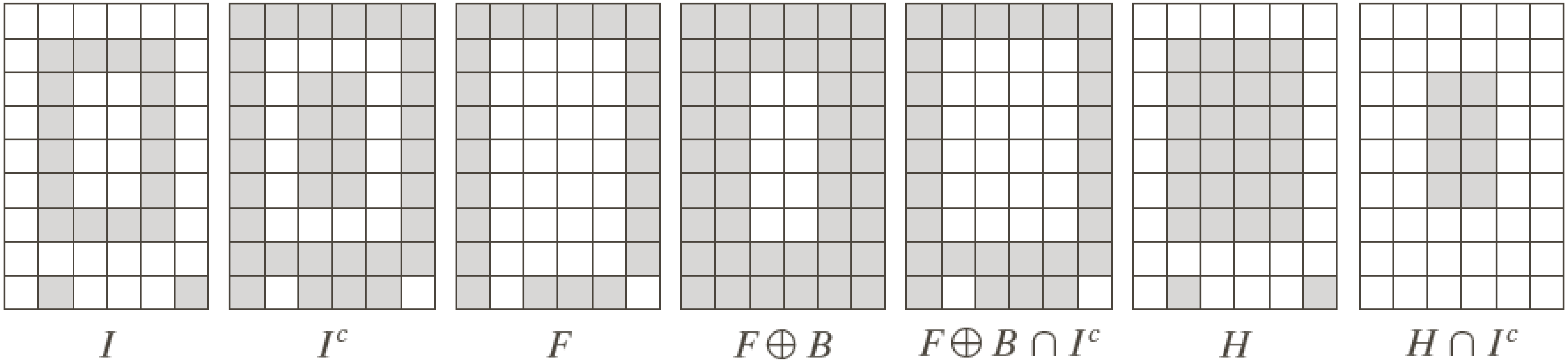
Filling Holes

Let $I(x,y)$ denote a binary image and suppose that we form a marker image F that is 0 everywhere, except at the image border, where it is set to $1-I$; that is

$$F(x,y)=\begin{cases}1-I(x,y) & \text{if } (x,y) \text{ is on the border of } I\\ 0 & \text{otherwise}\end{cases}$$

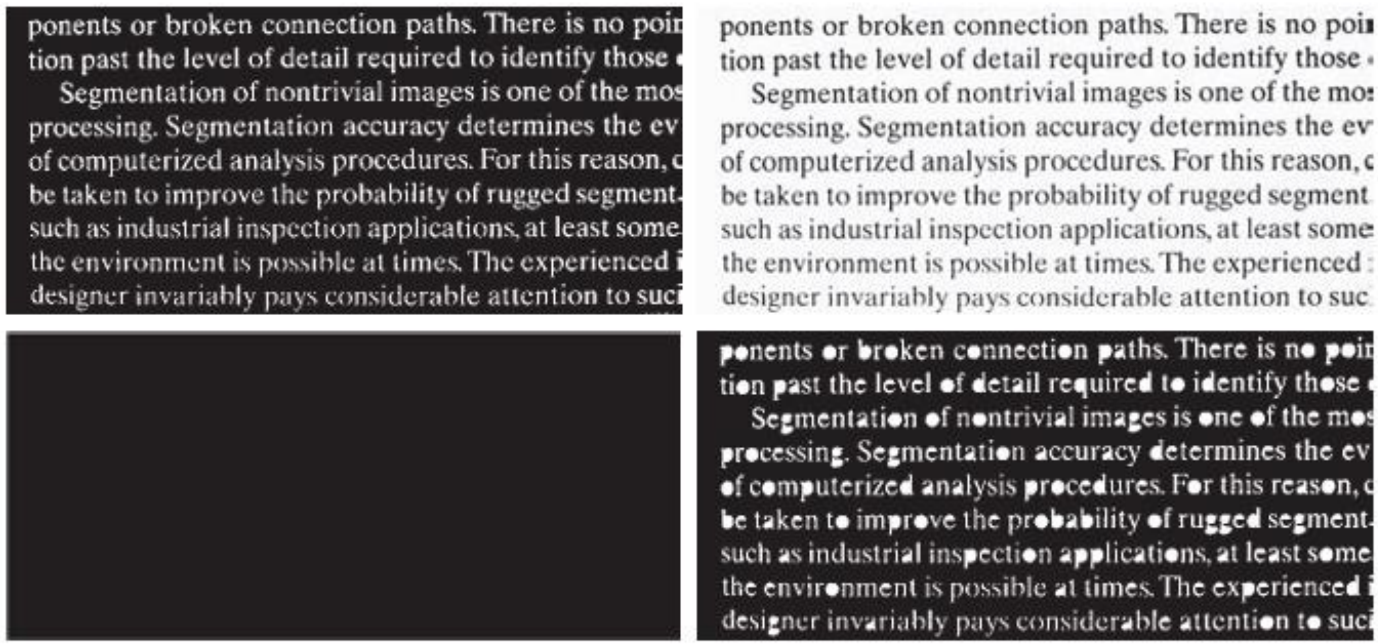
then

$$H=\left[R_{I^c}^{D}(F)\right]^c$$



SE : 3×3 1s.

FIGURE 9.30 Illustration of hole filling on a simple image.



a b
c d

FIGURE 9.33
(a) Text image of size 918 × 2018 pixels.
(b) Complement of (a) for use as a mask image.
(c) Marker image.
(d) Result of hole-filling using Eqs. (9-45) and (9-46).

$$H = \left[R_{I^c}^{D}(F) \right]^c$$

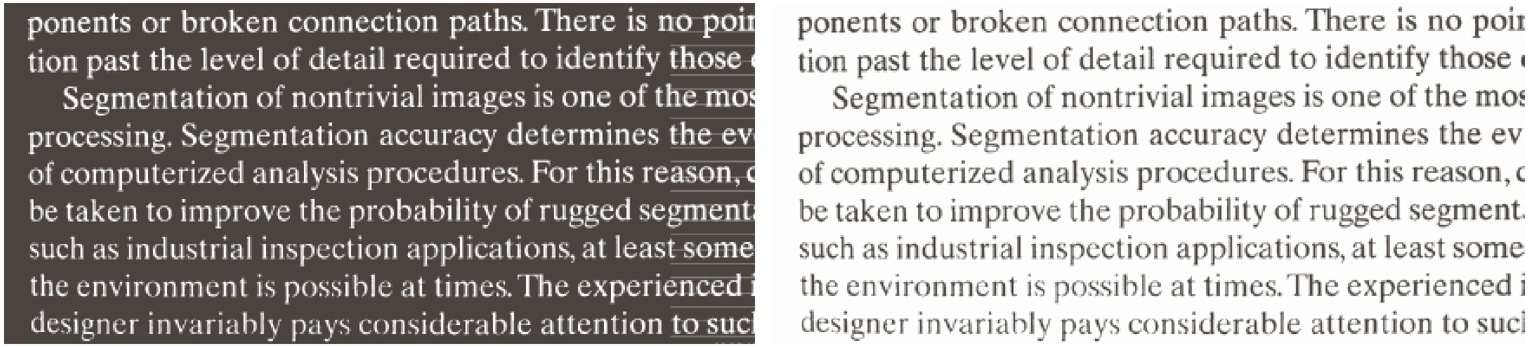
Border Clearing

It can be used to screen images so that only complete objects remain for further processing; it can be used as a singal that partial objects are present in the field of view.

The original image is used as the mask and the following marker image:

$$F(x,y) = \begin{cases} I(x,y) & \text{if } (x,y) \text{ is on the border of } I \\ 0 & \text{otherwise} \end{cases}$$

$$X = I - R_I^D(F)$$



ponents or broken connection paths. There is no poi
tion past the level of detail required to identify those
Segmentation of nontrivial images is one of the mos
processing. Segmentation accuracy determines the ev
of computerized analysis procedures. For this reason,
be taken to improve the probability of rugged segment
such as industrial inspection applications, at least some
the environment is possible at times. The experienced
designer invariably pays considerable attention to suc

ponents or broken connection paths. There is no poi
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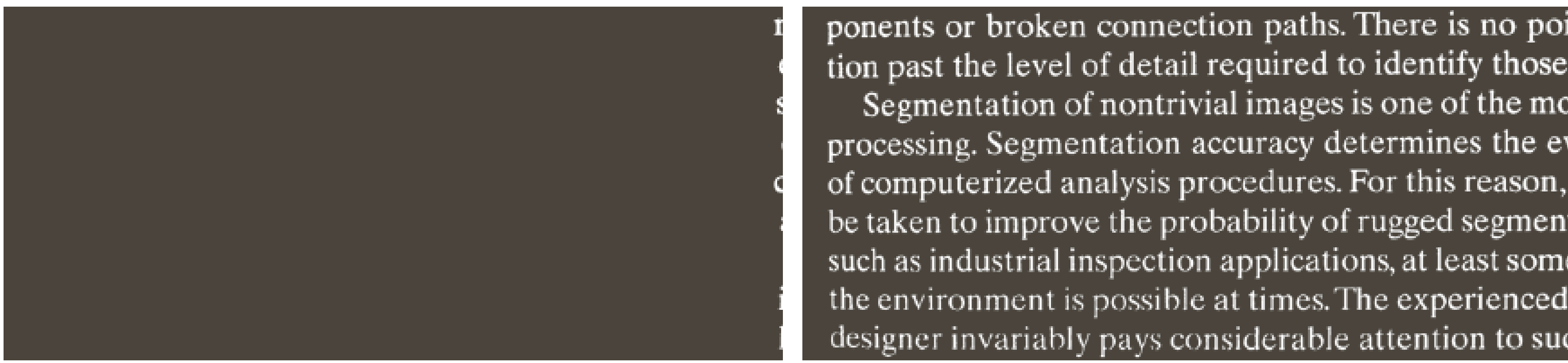

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be taken to improve the probability of rugged segment
such as industrial inspection applications, at least some
the environment is possible at times. The experienced
designer invariably pays considerable attention to suc

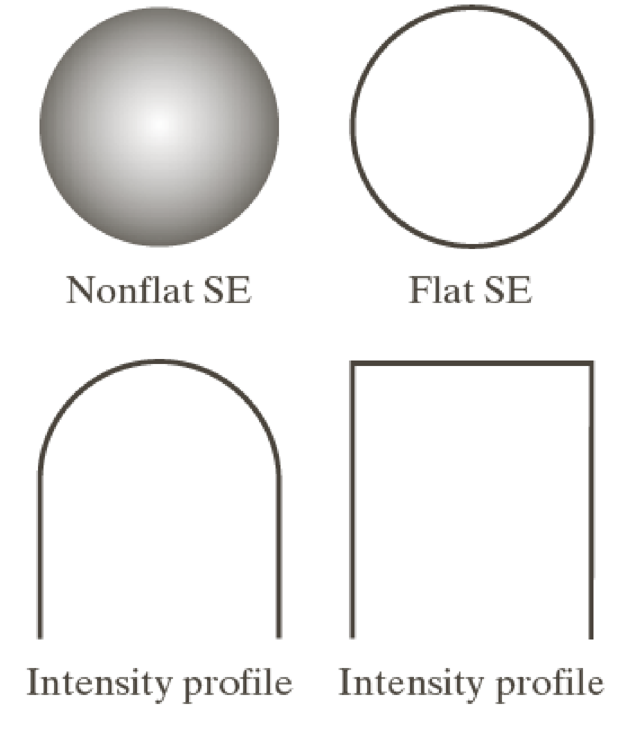
a b

FIGURE 9.32 Border clearing. (a) Marker image. (b) Image with no objects touching the border. The original image is Fig. 9.29(a).

Gray-Scale Morphology

$f(x, y)$: gray-scale image

$b(x, y)$: structuring element



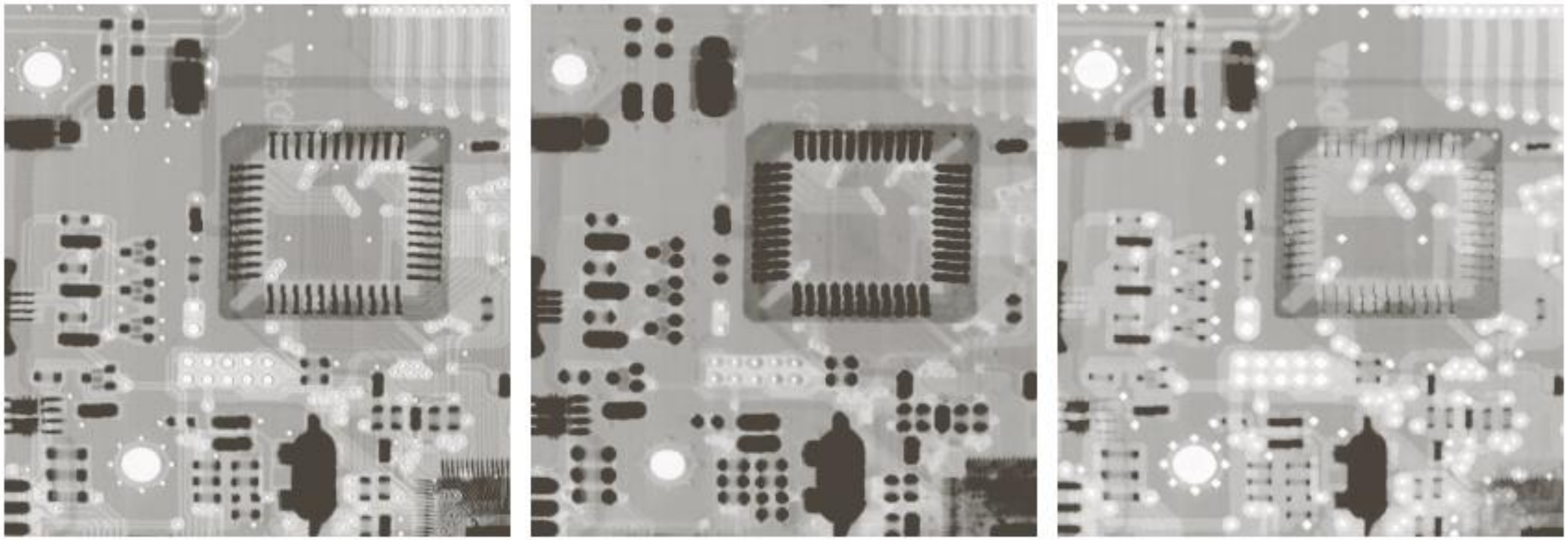
a b
c d

FIGURE 9.34 Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their center. All examples in this section are based on flat SEs.

Gray-Scale Morphology: Erosion and Dilation by Flat Structuring

$$[f \ominus b](x, y) = \min_{(s,t)\in b} \{ f(x+s, y+t) \}$$

$$[f \oplus b](x, y) = \max_{(s,t)\in b} \{ f(x-s, y-t) \}$$



a b c

FIGURE 9.35 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

Properties of Gray-scale Opening

$$(a) \quad f \circ b \subseteq f$$

$$(b) \quad \text{if } f_1 \subseteq f_2, \text{ then } \left(f_1 \circ b \right) \subseteq \left(f_2 \circ b \right)$$

$$(c) \quad \left(f \circ b \right) \circ b = f \circ b$$

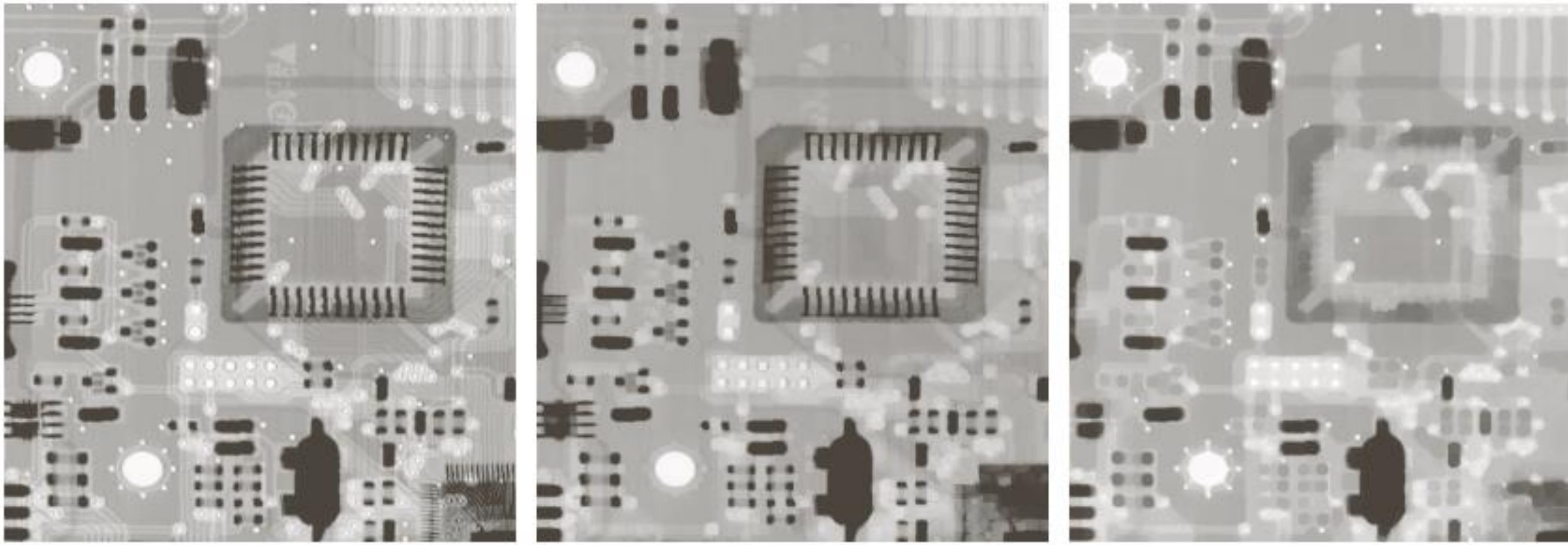
where $e \subseteq r$ denotes e is a subset of r and also $e(x, y) \le r(x, y)$.

Properties of Gray-scale Closing

$$(a) \quad f \dashv f \bullet b$$

$$(b) \quad \text{if } f_1 \dashv f_2, \text{ then } \left(f_1 \bullet b \right) \dashv \left(f_2 \bullet b \right)$$

$$(c) \quad \left(f \bullet b \right) \bullet b = f \bullet b$$



a b c

FIGURE 9.37 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.

Morphological Smoothing

- Opening suppresses bright details smaller than the specified SE, and closing suppresses dark details.

- Opening and closing are used often in combination as morphological filters for image smoothing and noise removal.

Morphological Smoothing

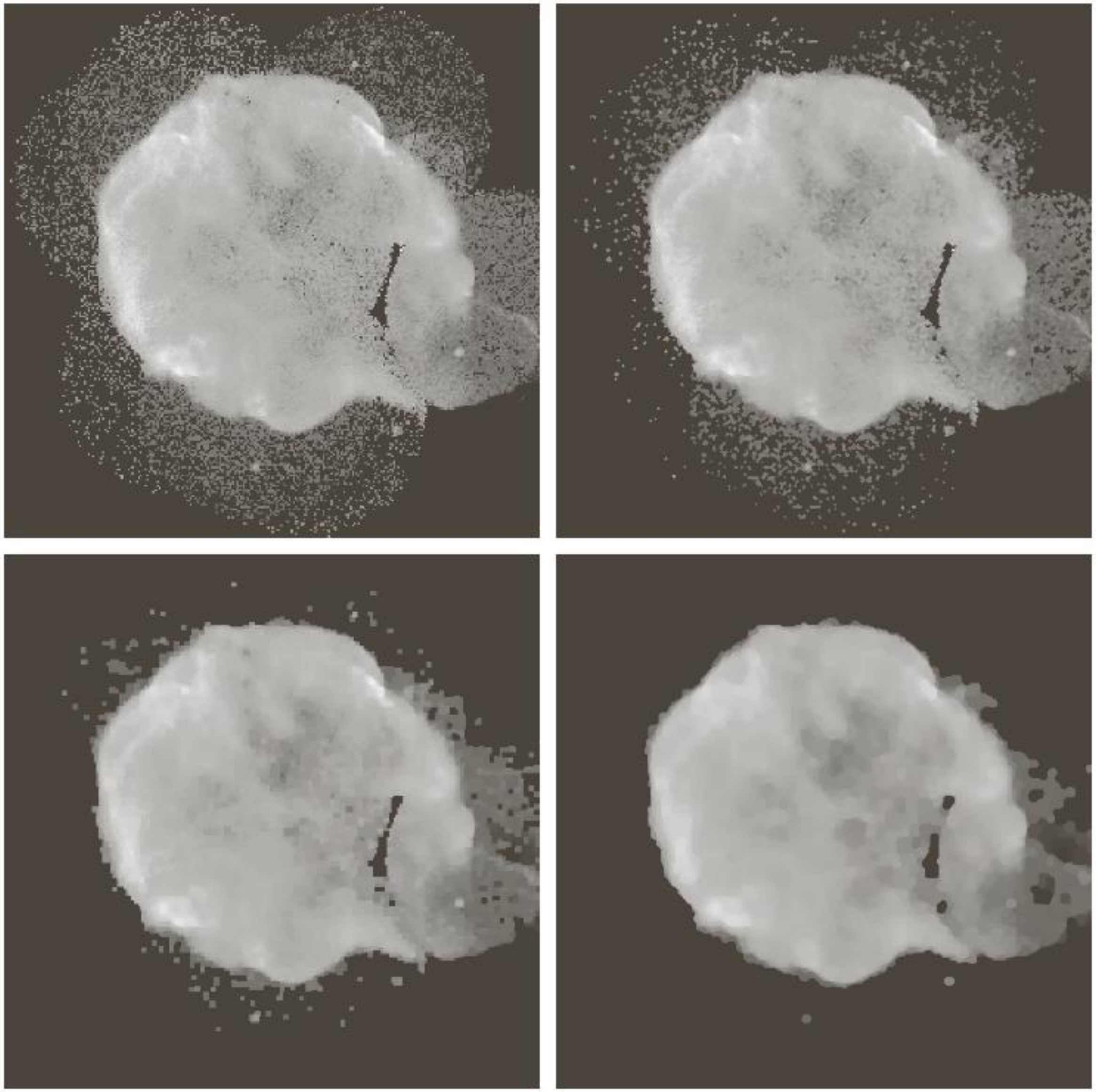


FIGURE 9.38 (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

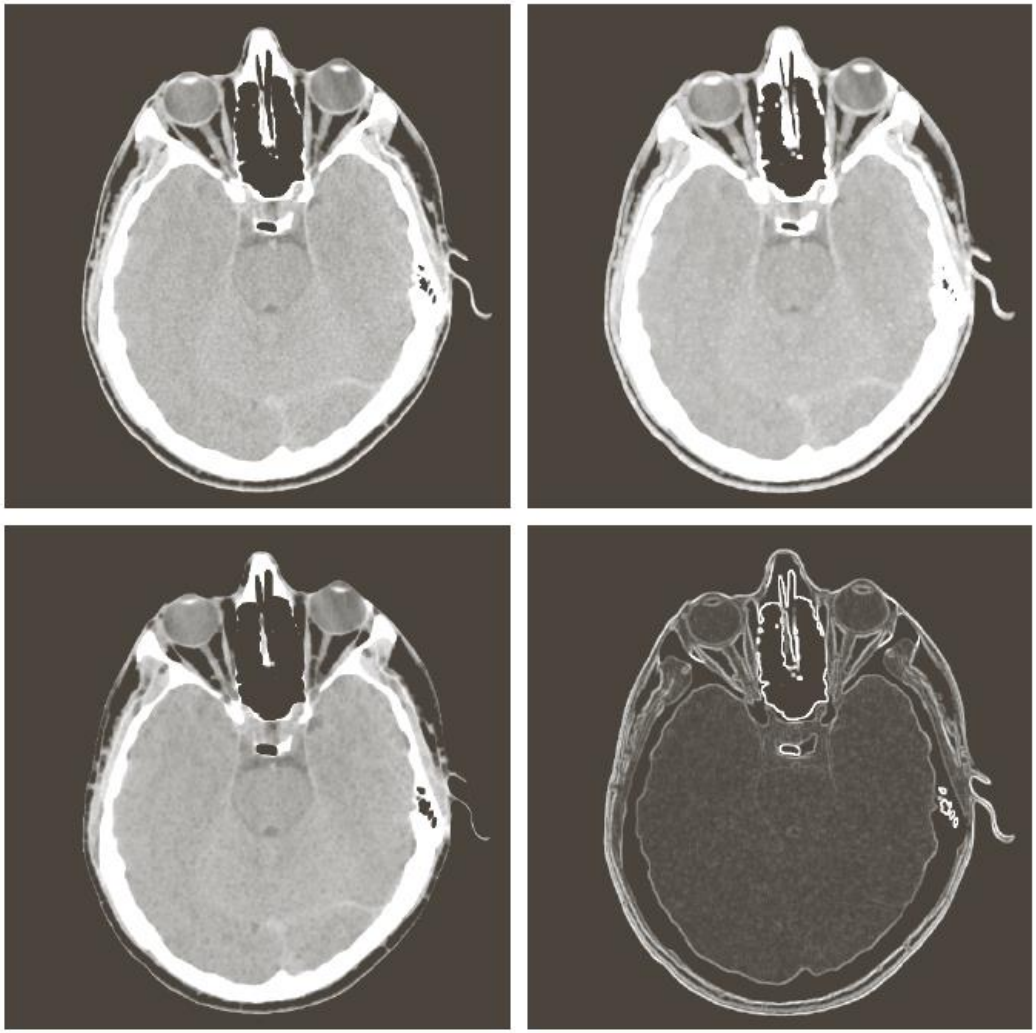
Morphological Gradient

- Dilation and erosion can be used in combination with image subtraction to obtain the morphological gradient of an image, denoted by g,

$$g = (f \oplus b) - (f \ominus b)$$

- The edges are enhanced and the contribution of the homogeneous areas are suppressed, thus producing a “derivative-like” (gradient) effect.

Morphological Gradient



a b
c d

FIGURE 9.39 (a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

Top-hat and Bottom-hat Transformations

- The top-hat transformation of a grayscale image f is defined as f minus its opening:

$$T_{hat}(f) = f - (f \circ b)$$

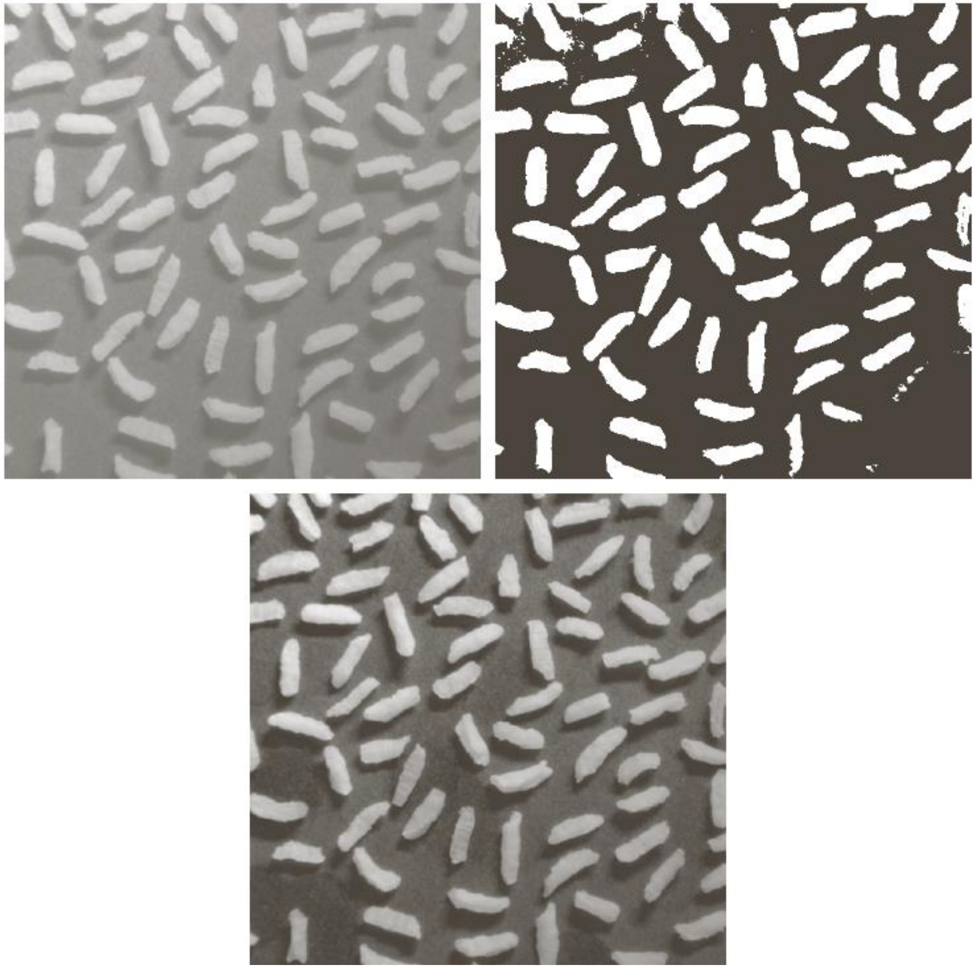
- The bottom-hat transformation of a grayscale image f is defined as its closing minus f:

$$B_{hat}(f) = (f \bullet b) - f$$

Top-hat and Bottom-hat Transformations

- One of the principal applications of these transformations is in removing objects from an image by using structuring element in the opening or closing operation

Example of Using Top-hat Transformation in Segmentation



a b
c d e

FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size 600 × 600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

Granulometry

- Granulometry deals with determining the size of distribution of particles in an image
- Opening operations of a particular size should have the most effect on regions of the input image that contain particles of similar size
- For each opening, the sum (**surface area**) of the pixel values in the opening is computed

Example

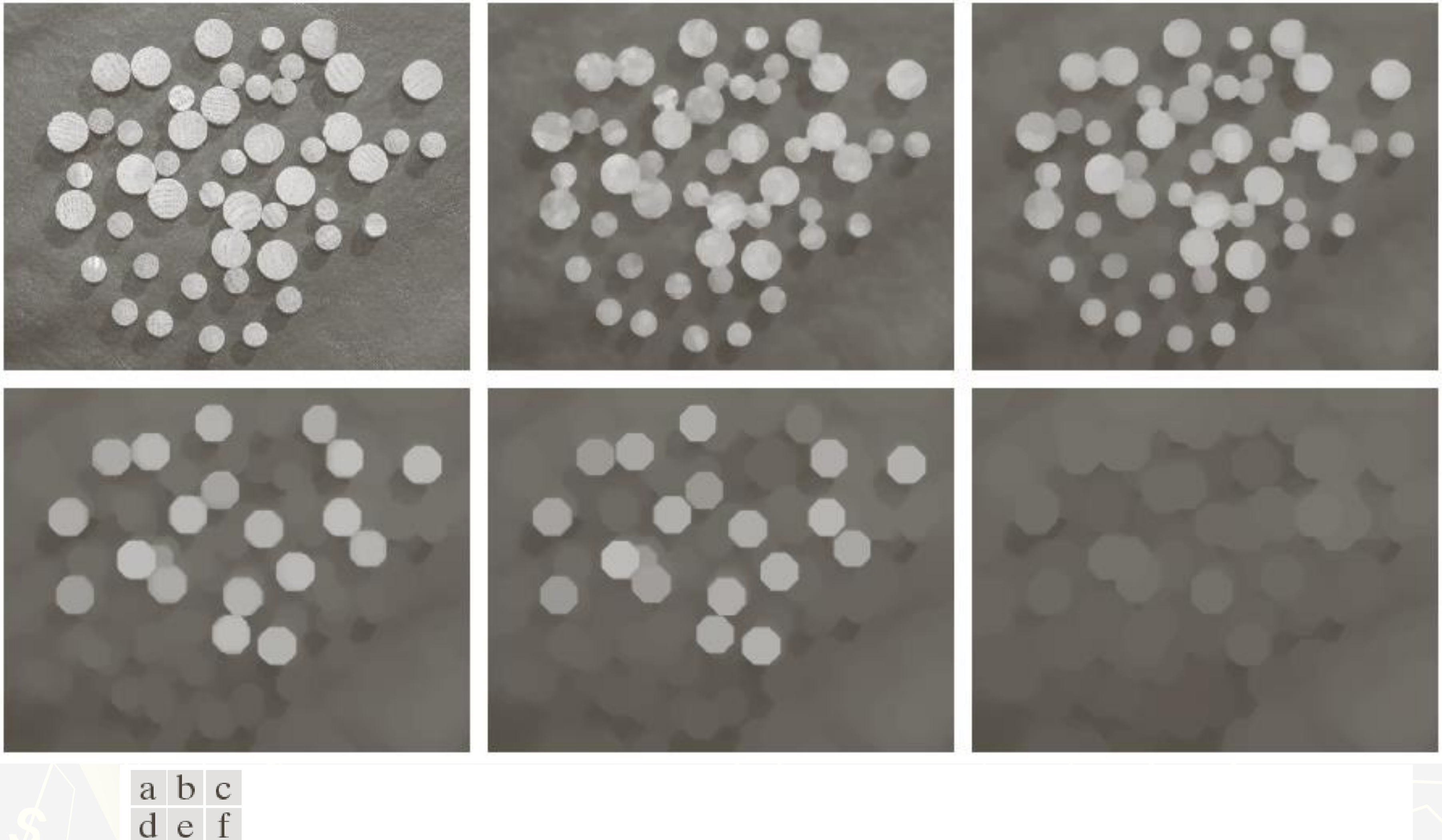


FIGURE 9.41 (a) 531 × 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

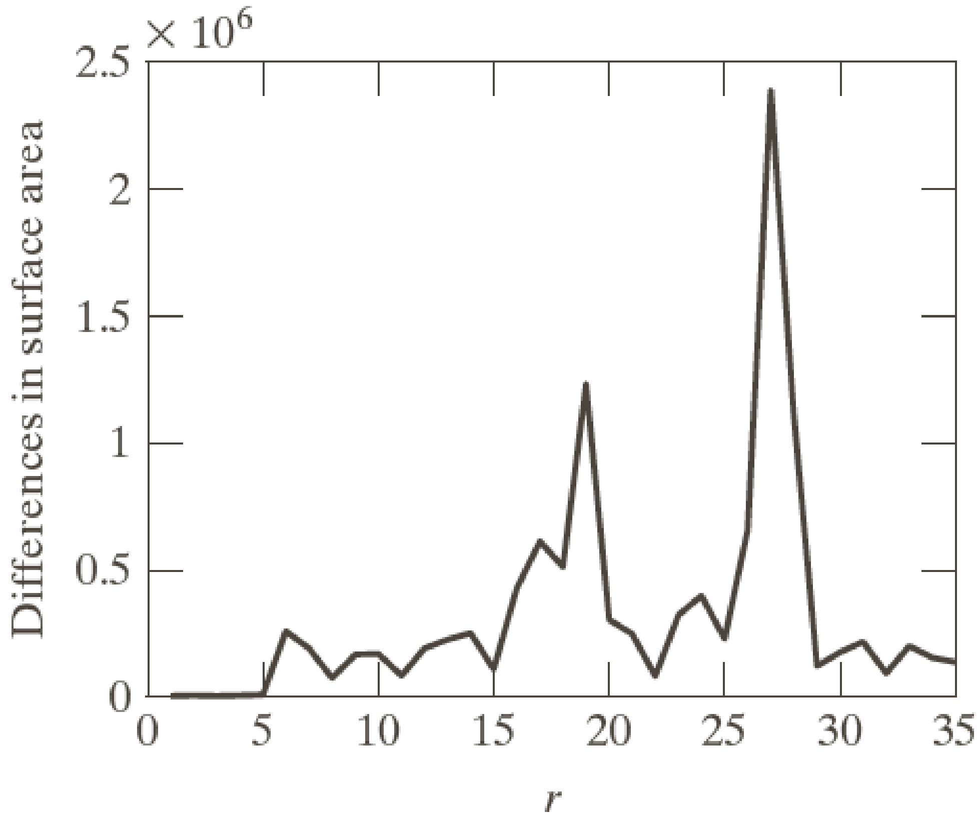


FIGURE 9.42 Differences in surface area as a function of SE disk radius, *r*. The two peaks are indicative of two dominant particle sizes in the image.

Textual Segmentation

- Segmentation: the process of subdividing an image into regions.

Textual Segmentation

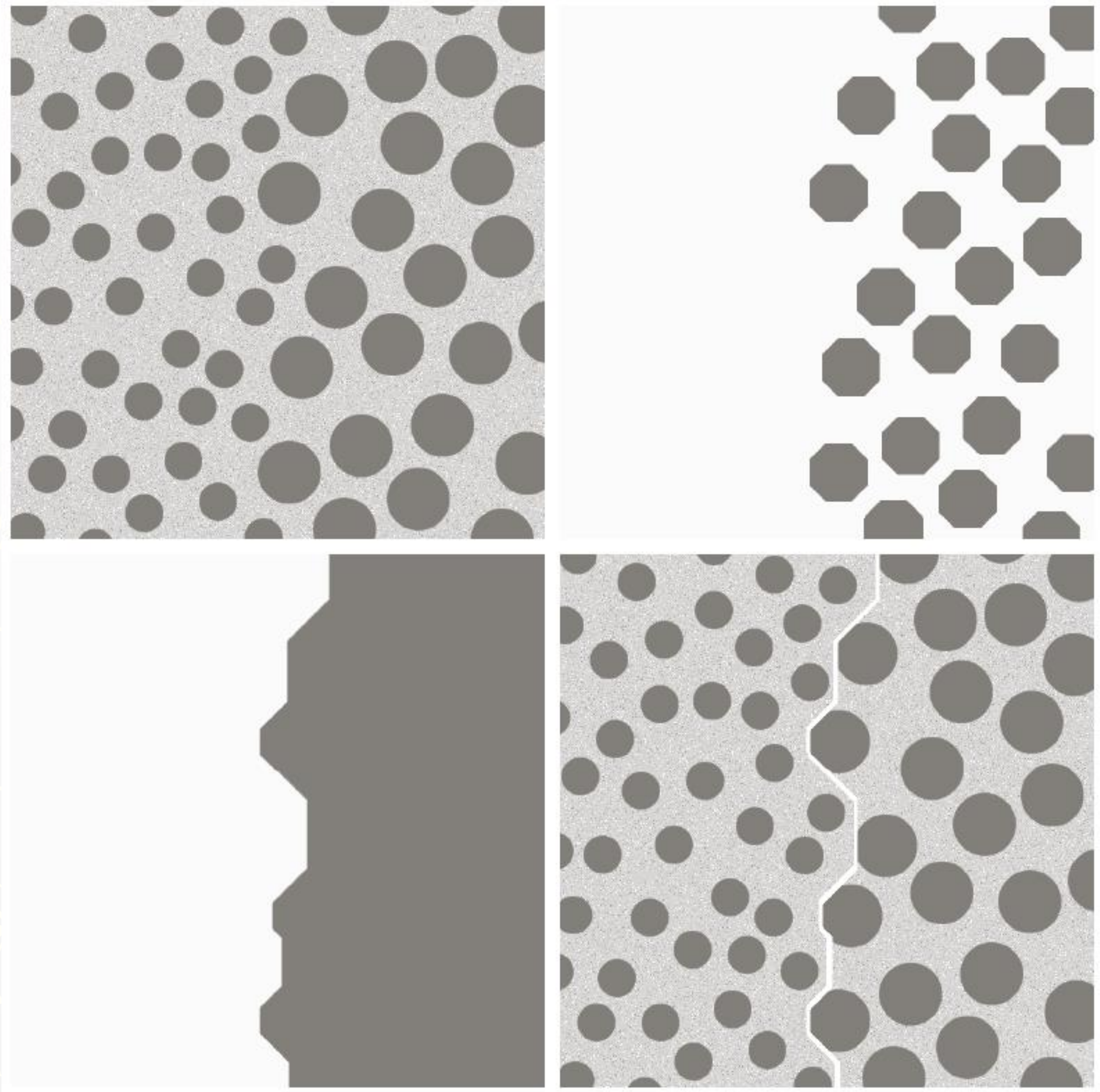


FIGURE 9.43 Textural segmentation. (a) A 600 × 600 image consisting of two types of blobs. (b) Image with small blobs removed by closing (a). (c) Image with light patches between large blobs removed by opening (b). (d) Original image with boundary between the two regions in (c) superimposed. The boundary was obtained using a morphological gradient operation.

Gray-Scale Morphological Reconstruction (1)

- Let f and g denote the marker and mask image with the same size, respectively and f ≤ g.

The geodesic dilation of size 1 of f with respect to g is defined as

$$D_g^{(1)}(f) = \left(f \oplus g\right) \wedge g$$

where $\wedge$ denotes the point-wise minimum operator.

The geodesic dilation of size n of f with respect to g is defined as

$$D_g^{(n)}(f) = D_g^{(1)}\left[D_g^{(n-1)}(f)\right] \text{ with } D_g^{(0)}(f) = f$$

Gray-Scale Morphological Reconstruction (2)

- The geodesic erosion of size 1 of f with respect to g is defined as

$$E_g^{(1)}(f) = \left(f \ominus g \right) \vee g$$

where $\vee$ denotes the point-wise maximum operator.

The geodesic erosion of size n of f with respect to g is defined as

$$E_g^{(n)}(f) = E_g^{(1)}\left[E_g^{(n-1)}(f) \right] \text{ with } E_g^{(0)}(f) = f$$

Gray-Scale Morphological Reconstruction (3)

- The morphological reconstruction by dilation of a gray-scale mask image g by a gray-scale marker image f, is defined as the geodesic dilation of f with respect to g, iterated until stability is reached, that is,

$$R_g^D(f) = D_g^{(k)}(f)$$

$$\text{with k such that } D_g^{(k)}(f) = D_g^{(k+1)}(f)$$

The morphological reconstruction by erosion of g by f is defined as

$$R_g^E(f) = E_g^{(k)}(f)$$

$$\text{with k such that } E_g^{(k)}(f) = E_g^{(k+1)}(f)$$

Gray-Scale Morphological Reconstruction (4)

- The opening by reconstruction of size n of an image f is defined as the reconstruction by dilation of f from the erosion of size n of f; that is,

$$O_R^{(n)}(f) = R_f^D\left[f \ominus nb\right]$$

The closing by reconstruction of size n of an image f is defined as the reconstruction by erosion of f from the dilation of size n of f; that is,

$$C_R^{(n)}(f) = R_f^E\left[f \oplus nb\right]$$

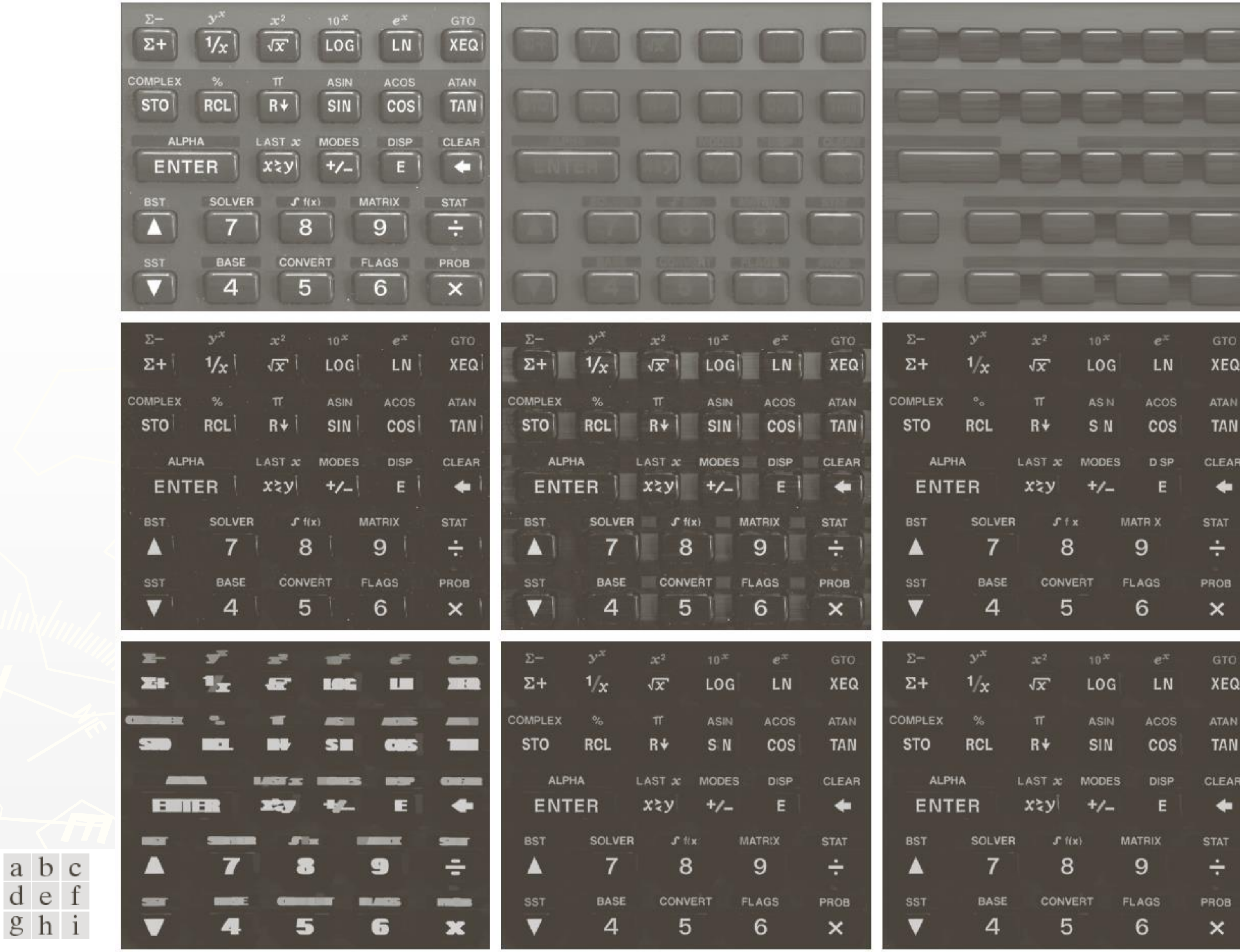


FIGURE 9.44 (a) Original image of size 1134 × 1360 pixels. (b) Opening by reconstruction of (a) using a horizontal line 71 pixels long in the erosion. (c) Opening of (a) using the same line. (d) Top-hat by reconstruction. (e) Top-hat. (f) Opening by reconstruction of (d) using a horizontal line 11 pixels long. (g) Dilation of (f) using a horizontal line 21 pixels long. (h) Minimum of (d) and (g). (i) Final reconstruction result. (Images courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

Steps in the Example

1. Opening by reconstruction of the original image using a horizontal line of size 1x71 pixels in the erosion operation

$$O_R^{(n)}(f) = R_f^D\left[f \ominus nb\right]$$

2. Subtract the opening by reconstruction from original image

$$f' = f - O_R^{(n)}(f)$$

3. Opening by reconstruction of the f′ using a vertical line of size 11x1 pixels

$$f1 = O_R^{(n)}(f') = R_f^D\left[f' \ominus nb'\right]$$

4. Dilate f1 with a line SE of size 1x21, get f2.

Steps in the Example

5. Calculate the minimum between the dilated image f2 and and f’, get f3.

6. By using f3 as a marker and the dilated image f2 as the mask,

$$R_{f2}^{D}(f3) = D_{f2}^{(k)}\left(f3\right)$$

with k such that $D_{f2}^{(k)}\left(f3\right) = D_{f2}^{(k+1)}\left(f3\right)$