Decimal counting

Digits → 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

000 → 0
001 → 1
002 → 2
003 → 3
004 → 4
005 → 5
006 → 6
007 → 7
008 → 8
009 → 9
010 → 10
011 → 11
012 → 12
013 → 13
014 → 14
015 → 15
016 → 16
017 → 17
018 → 18
019 → 19
020 → 20

Binary Counting

Digits → 0, 1

0000 → 0
0001 → 1
0010 → 10
0011 → 11
0100 → 100
0101 → 101
0110 → 110
0111 → 111
1000 → 1000
1001 → 1001
1010 → 1010
1011 → 1010
1100 → 1100
1101 → 1101
1110 → 1110
1111 → 1111

Convert Deci to Deci

$$
\begin{aligned}
273 \\
&= 200 + 70 + 3 \\
&= 2\times100 + 7\times10 + 3\times1 \\
&= 2\times10^2 + 7\times10^1 + 3\times10^0 \\
&= 2\times b^2 + 2\times b^1 + 3\times b^0
\end{aligned}
$$

For decimal: $b = 10$
For binary : $b = 2$
For Octal : $b = 8$
For Hexadecimal: $b = 16$

Convert binary to decimal

1101

$= 1 \times b^3 + 1 \times b^2 + 0 \times b^1 + 1 \times b^0$

$= 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 8 + 4 + 1 = 13$

Convert decimal to binary

$\frac{13}{2} = 6$; remainder = 1 (LSB → Least Significant bit) ↓ goes to the right

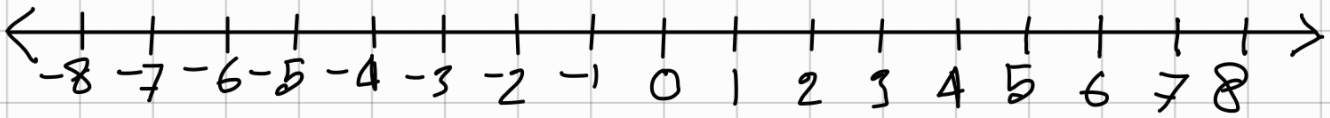
$\frac{6}{2} = 3$; remainder = 0

$\frac{3}{2} = 1$; remainder = 1

$\frac{1}{2} = 0$; remainder = 1 (MSB → Most Signi. Bit) ↳ goes to the left

1101

Types of Numbers



Real number: Any point on the number line

Positive number: Right to the zero

Negative number: Left to the zero

Non-negative number: Positive and zero

Natural number: $\{1, 2, 3, \ldots\}$

Whole number: $\{0, 1, 2, 3, \ldots\}$

Integers: $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Rational number: A rational number is any number that can be expressed as a fraction or ratio of two integers. (Note. $\frac{a}{b}, b \neq 0$)

$2.5 = \frac{5}{2}$, $0.5 = \frac{1}{2}$, $0.\underline{3}\underline{3}\underline{3}\ldots = \frac{1}{3}$,

$0.\underline{36}\underline{36}\underline{36}\ldots = \frac{4}{11}$

Irrational number: An irrational number is a number that cannot be expressed as a simple fraction or ratio of two integers and its decimal representation goes on forever <u>without repeation</u>.

$3.14159\ldots$, $\sqrt{2}$

Division

Divisor ↑ 5) Dividend ↑ 17 (3 → Quotient
15
2 → Remainder

Divisibility Check

There are two ways:

- Check if the remainder is zero.
  - 12 % 3 = 0 is divisible by 3.
  - 12 % 5 = 2 is not divisible by 5.
    ↳ mod operation
- what do we get if we just do the the division?
  - 12/3 = 4 → integer → divisible
  - 12/5 = 2.4 → not integer → not divisible

Prime and Composite Numbers

Factor: What does it mean when we say P is a "factor" of Q?

Answer: P divides Q evenly. (Remainder = 0)
Ex: 3 is a factor of 12.

Prime Numbers: Only two factors. 1 and itself.

Ex: 2, 3, 5, 7, 11, 13, ----

Composite Numbers: There exists at least one factor other than 1 and itself.

Ex: 4, 6, 8, 9, 10, 12, 14, ----

Note: 1 is neither prime nor composite rather it's a special number.

Even and Odd Numbers

Even: 0, 2, 4, 8, ---
Odd: 1, 3, 5, 7, ---

How to check?
- Divide the number with 2
- If remainder is 0, even! Otherwise, odd!

H.W: Convert the natural numbers to binary and see if you can find a pattern for even and odd numbers there.

Deci =	1	2	3	4	5	6	7	8	9	10
Bina =	1	10	11	100	101	110	111	1000	1001	1010

Pattern: Even: The last digit of every num is 0
Odd : " " " " " " " 1

PRACTICE DAY

1, 3, 5, 7
2, 4
8
16

- 1101011 is a binary number. Find its equivalent decimal number.
- Find the binary representation for 73 (a decimal number).
- Is 77 a prime number?
- What about 169? Prime or composite?
- Find out the 12^{th} prime number.
- Find out sum of first n odd numbers. Try $n = 1,2,3, \ldots$
  - Can you see any pattern?

Floor, Ceiling, Round

Floor → Nearest integer below
Ceiling → " " above
Round → Nearest integer

Ex: 12.65 → F=12, C=13, R=13
9.21 → F=9, C=10, R=9
24 → F=24, C=24, R=24

Divisor Counting

Problem statement: Given an integer n. How many divisors are there for n?

For prime number: 1 and the number itself.
For composite number: Take all the integers (1 to n) and test if remainder is 0.

Example: Let's take 12

naive method

1	12%1=0	√
2	12%2=0	√
3	12%3=0	√
4	12%4=0	√
5	12%5=2	x
6	12%6=0	√

7	12%7=5	x
8	12%8=4	x
9	12%9=3	x
10	12%10=2	x
11	12%11=1	x
12	12%12=0	√

The divisors: 1, 2, 3, 4, 6, 12.

Number of divisors: 6

Better Approach

→ We already know 1 and n will be there.
→ So, 2 to (n-1) need to be tested.
→ But, there won't be any divisor more than n/2. So we only need to check for 2 to n/2.

Even better Approach

If we just test from 2 to $\sqrt{n}$, then we'll get the all divisors of n.

Ex. 16

$1 \times 16 = 16$
$2 \times 8 = 16$
$4 \times 4 = 16$ → $\sqrt{16} = 4$
$8 \times 2 = 16$
$16 \times 1 = 16$

1, 2, 4, 8, 16

Note: We do not need to test any num greater than $\sqrt{n}$.

Ex. 12

$\sqrt{12} = 3.464 = 3$ (Floor num)

$1 \times 12 = 12$
$2 \times 6 = 12$
$3 \times 4 = 12$

1, 2, 3, 4, 6, 12

Primality Test

* Problem statement: Given an integer n.

Determine if n is a prime number or not?

Take numbers from 2 to $\sqrt{n}$
- If any of them divide n, Not prime.

Ex. is 77 a prime number?

$\sqrt{77} = 8.77 = 8$

77 % 2 = 1
77 % 3 = 2
77 % 4 = 1
77 % 5 = 2
77 % 6 = 5
77 % 7 = 0 ✓ → Factor: 7 and 77/7 = 11
77 % 8 = 5

So, it is not a prime number.

But there's even better and effective approach.

Sieve of Eratosthenes

This idea detects all the prime numbers in a given range very fast.

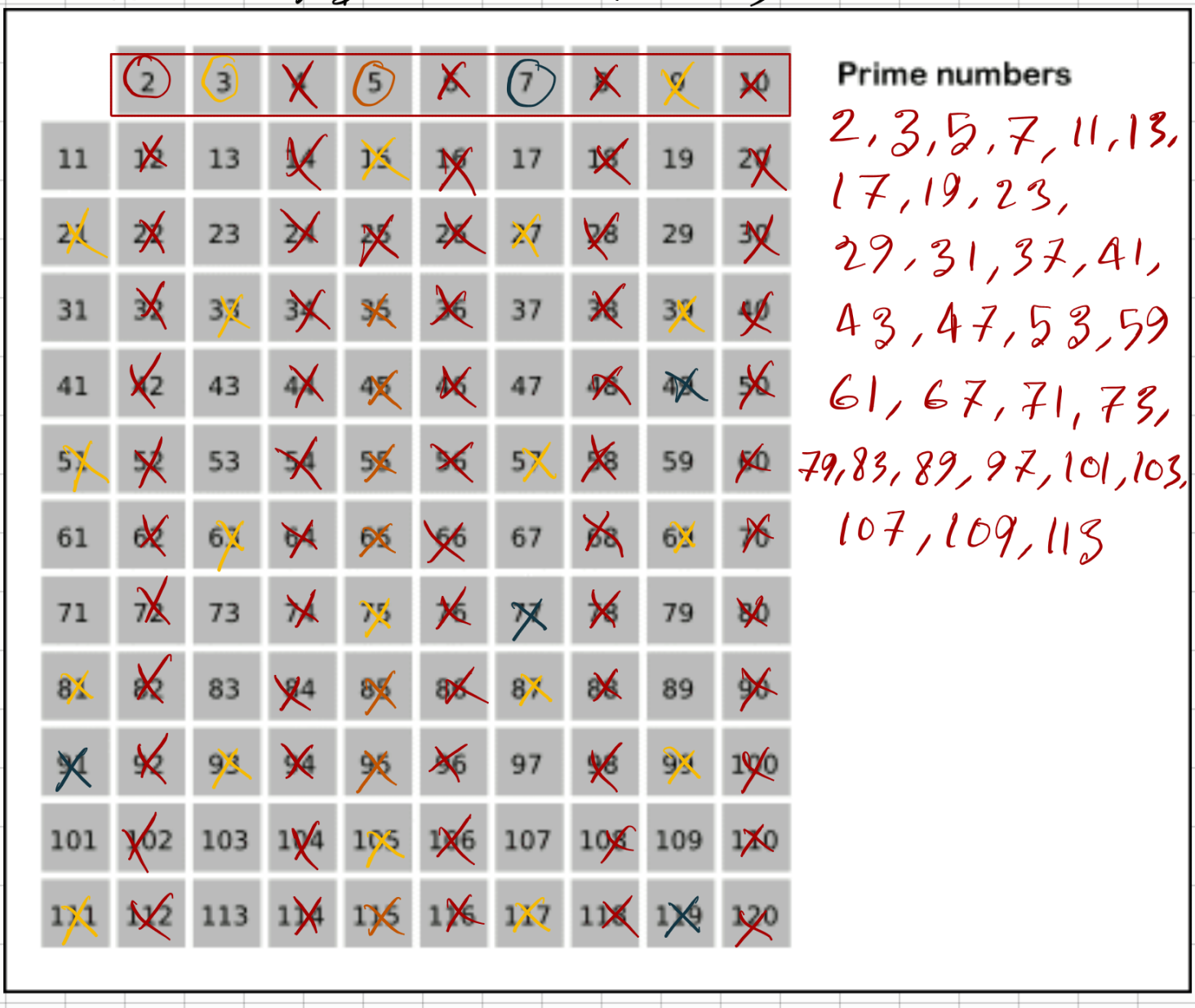
Steps: 1 → Take a number (start with 2) and find its multiplies in the range and cross them out.

2 → Proceed to the next number.
3 → If it's prime go to step-1, other-

else step - 2.

Ex: Find out all the prime num between 1 to 120. (We're taking sqrt coz if we just cross out $\sqrt{n}$, then all the divisors will be automatically crossed out.)

$\sqrt{120} = 10.95 = 10$ (Floor value)



Prime numbers

2, 3, 5, 7, 11, 13,
17, 19, 23,
29, 31, 37, 41,
43, 47, 53, 59
61, 67, 71, 73,
79, 83, 89, 97, 101, 103,
107, 109, 113

Extract Digits from an Integer

Find the i-th digit of a number (say n = 91408) from right. [i = 0, 1, 2, 3, 4]

$i = 0$, $n \% 10 = 8$

$i = 1$, $\text{Floor}(\frac{n}{10}) \% 10 = 0$

$i = 2$, $\text{Floor}(\frac{n}{100}) \% 10 = 4$

$i = 3$, $\text{Floor}(\frac{n}{1000}) \% 10 = 1$

$i = 4$, $\text{Floor}(\frac{n}{10000}) \% 10 = 9$

$i = 0$

$10 \overline{)91408} (9140$

90

14

10

40

40

08 ✓

$i = 4$, 91408

$(10000) \% 10 = 9$

$i = 1, \quad \frac{91408}{10} = 9140.8$

$= 9140$

10 | 9140 | 914

90

14

10

40

40

0 ✓

$i = 2, 3, \ldots$

- General Formula: $\text{Floor}\left(\frac{n}{10^i}\right) \% 10 = \left\lfloor \frac{n}{10^i} \right\rfloor \% 10$

Question 01:
Choose a prime number from the range of 50 to 100. We will call it P. Write in Details

Question 02:
Find the binary representation of P. You must show the process.

Question 03:
You and your friends in class might be choosing different values for P. Can anybody choose a P that is an odd number? Explain your answer.

Question 04:
Ternary numbers are formed with a number system with base 3. Given the ternary number 10212, find its decimal value. Write in Details

Question 05:
Build a sieve of Eratosthenes to determine if 19 is a prime number. Show the state of the table at each step.

Question 06:
Take two natural numbers X and Y. If X * Y = Z, is Z even or odd when X and Y -
(Case 1) both are even: even / odd
(Case 2) both are odd: even / odd
(Case 3) One of them is odd, the other one is even: even / odd
Note: (Answer in even or odd) Write in Details

GCD/HCF – The Euclidean Algorithm

Understanding GCD – Example 2

Naive approach

	25	150
Divisors	1, 5, 25	1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150
Common Divisors	1, 5, 25	
Greatest Common Divisor (GCD)	25	

∴ GCD(25, 150) = 25

Euclid's Algorithm for finding GCD

Find the GCD(12, 33).

Quotient	A	B	Remainder
2	33	12	9
1	12	9	3
3	9	3	0
X	3	0	X

33/12
12/9
9/3
3/0 → STOP

0 | 3

General formula: $gcd(a, b) = gcd(b, a\%b)$

$gcd(P, 0) = P \rightarrow gcd$

Ex: $gcd(15, 6) = gcd(6, 15\%6)$

$gcd(6, 3) = (3, 6\%3)$

$gcd(3, 0) = 3$

Note: Changing Order doesn't matter

$$\gcd(6,15) = \gcd(15, 6\%15) = (15,6)$$

Co-prime: If the GCD of two numbers is 1, then they're co-prime to each other.

Ex- 9 and 16 are co-prime to each other.

LCM/LCF

$$\gcd(a,b) \times \text{lcm}(a,b) = a \times b$$

Ex- $\text{lcm}(15,6) = \frac{15 \times 6}{3} = 30$

→ we got through Euclidean Algorithm

Factorial

We need factorial to find all kinds of possible combinations of multiple things.

$4! = 4 \times 3 \times 2 \times 1 = 24$

Note: $0! = 1$

Matrix/Matrices

→ Singular → plural

→ A collection of numbers which are arranged in rows and columns.

$$A = \begin{bmatrix} -2 & 5 & 6 \\ 5 & 2 & 7 \end{bmatrix}$$

← rows

↑ columns

$A_{2,3} = 7$

→ Rows and columns calculation: $r \times c = 2 \times 3$

→ Adding matrices:

$$\text{Midterm} = \begin{bmatrix} 40 & 36 \\ 28 & 32 \\ 30 & 27 \end{bmatrix}, \quad \text{Final} = \begin{bmatrix} 19 & 35 \\ 48 & 12 \\ 33 & 48 \end{bmatrix}$$

(Student-1, Student-2; → sub-1, → sub-2, → sub-3)

Note: column → students (3)
row → subjects (2)

$$M+F = \begin{bmatrix} 59 & 71 \\ 76 & 44 \\ 63 & 75 \end{bmatrix}$$

Note: Must be same shape.

Power and Roots

Power: b^x (x → exponent, b → base)

Root: $\sqrt[n]{a}$ (n^{th} root of a) ← opposite →

Assume, $\sqrt[n]{a} = x$

$a = x^n$

$\sqrt[2]{16} = 4$

$16 = 4^2$

$\sqrt[3]{8} = 2$

$8 = 2^3$

Introduction to Sets

→ A collection of objects/numbers.

↳ objects are called elements

Infinite set: Natural num set: {1, 2, 3, ...}

Finite Set : Favorite sports : {Cricket, Football, badminton}

Subset: A is a subset of B if all the elements of A is also in B.

Ex: $A = \{10, 12, 29\}$, $B = \{101, 10, 12, 29, 30\}$

Universal set: Depends on context.

Ex: When talking about numbers, universal set might be the set of Real Numbers.

Empty / Null set: Set with Zero element. way to write: $\{\}$ or $\emptyset$

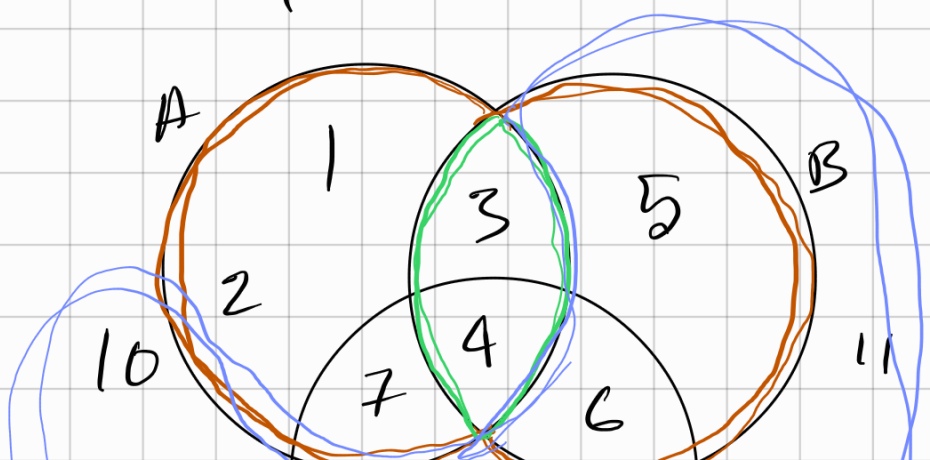
Three set operations: → Union ($\cup$)
→ Intersection ($\cap$)
→ Complement ($'$)

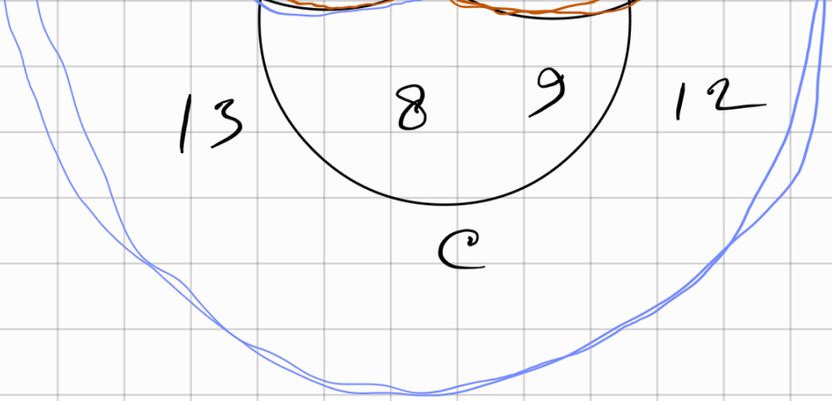
Ex - $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$

$A = \{1, 2, 3, 4, 7\}$

$B = \{3, 4, 5, 6\}$

$C = \{4, 6, 7, 8, 9\}$





Points and Lines

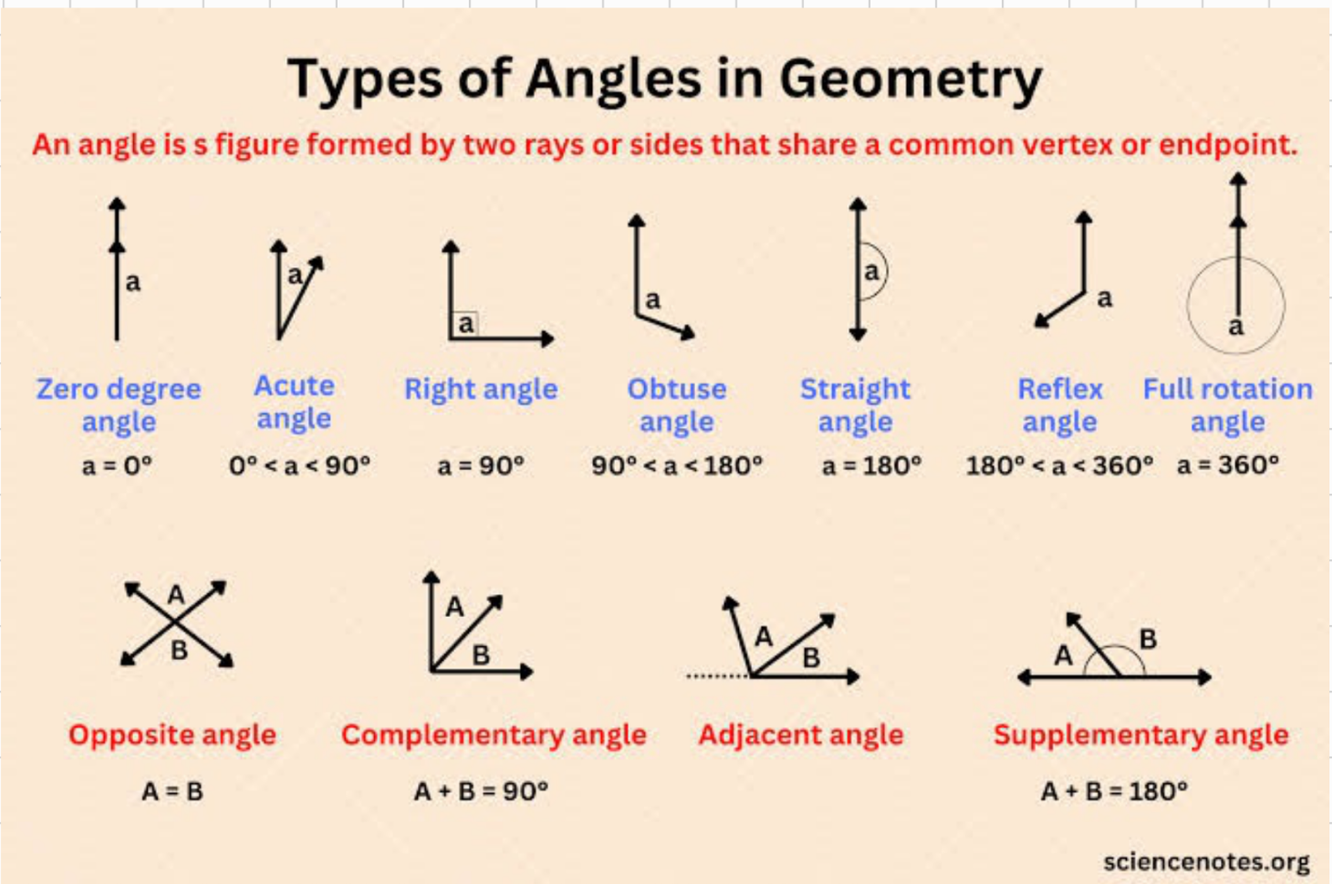
Point: A dot which has no shape, lenghth, width. It only gives a location.

Segment: If you connect two different points taking the shortest path (straight), you'll get a line segment.

Line: Line is extended in both ends of segment

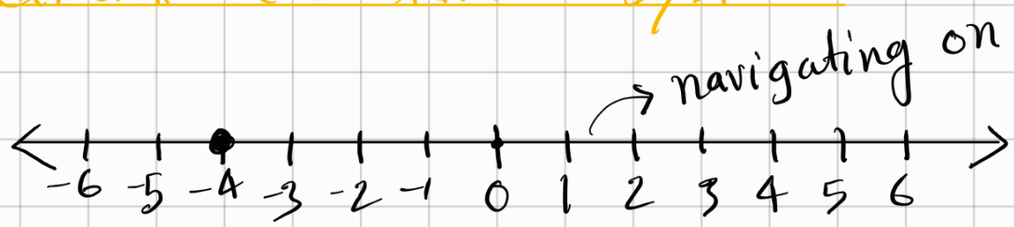
Ray: Ray " " " one " " "

Angles



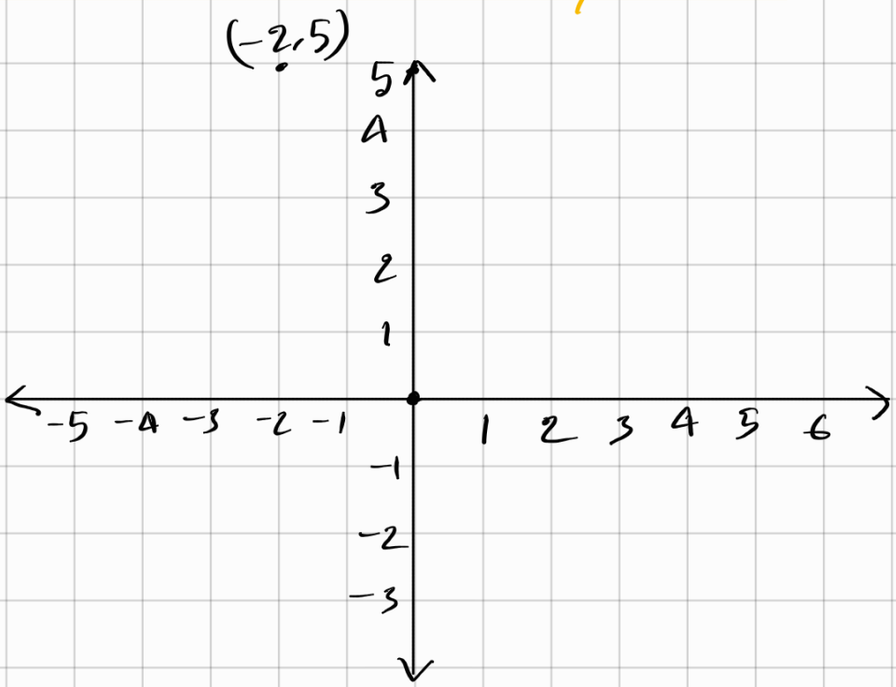
Coordinate System

One-dimensional coordinate system:

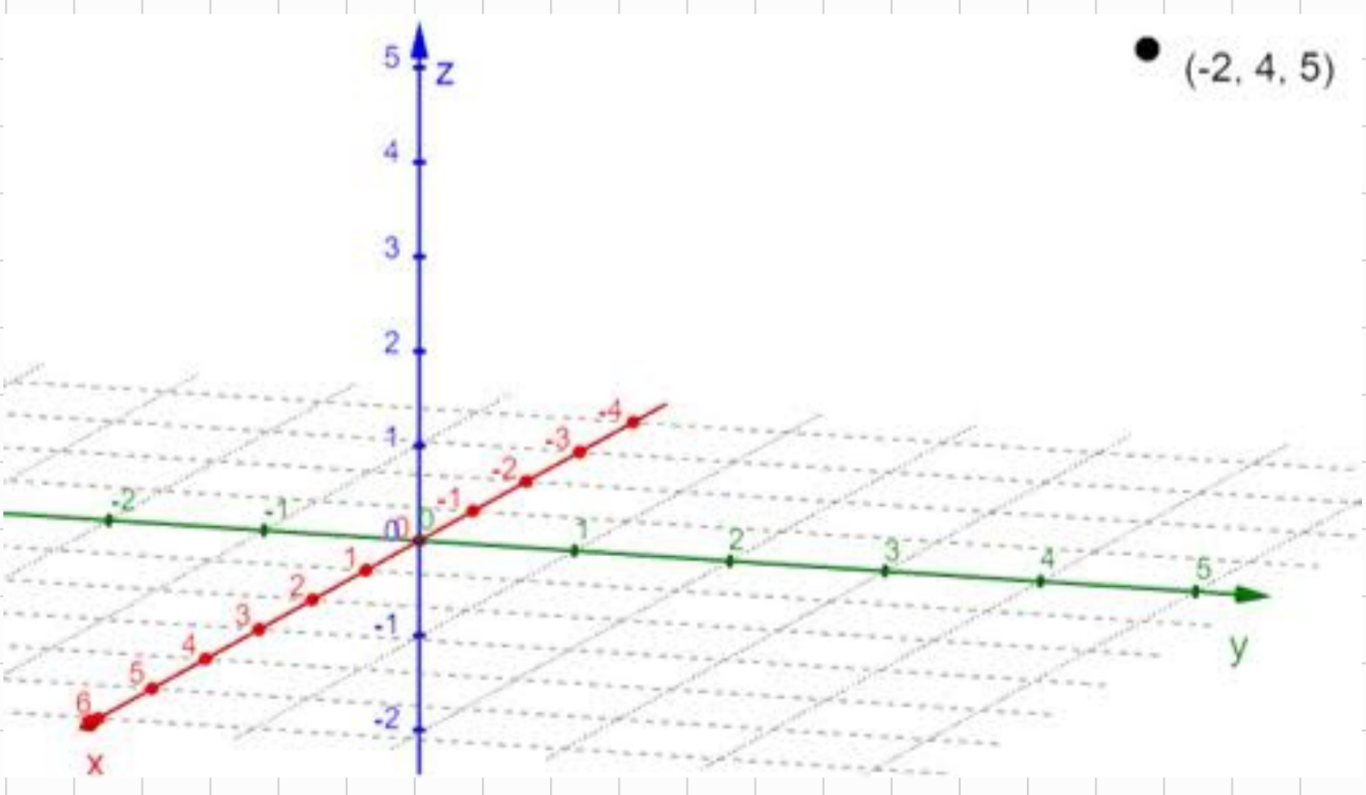


(-4) → taking one number to navigate

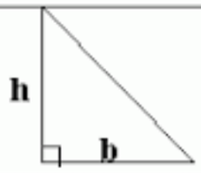
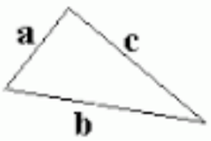
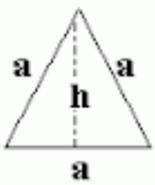
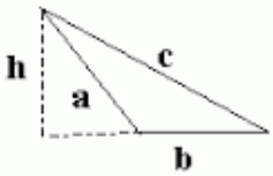
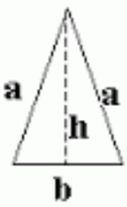
Two-dimensional coordinate system:



Three Dimensional coordinate system:



Triangles

Right Angle Triangle		h = height b = base	Area = 1/2 (b x h) = 1/2 (Product of the sides containing the right angle)
Scalene Triangle: with length a,b,c		a = side b = side c = side	Area = $\sqrt{s(s-a)(s-b)(s-c)}$ Where $s = \frac{(a+b+c)}{2}$ Perimeter = a + b + c
Equilateral Triangle		a = three equal sides	Area = $\sqrt{3}/4 \times a^2$ Perimeter = a + a + a
Obtuse Angle Triangle		h = height b = base	Area = 1/2 (b x h) Perimeter = a + b + c
Isosceles Triangle		a = two equal sides h = height b = base	Area = 1/2 (b x h) = 1/2 x a x b sin c Perimeter = a + a + c

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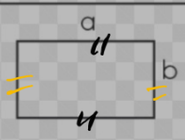
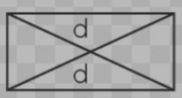
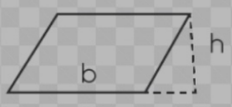
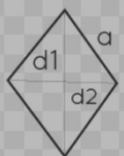
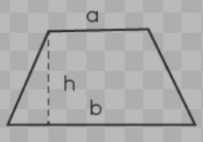
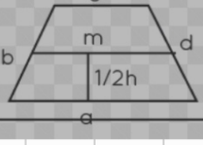
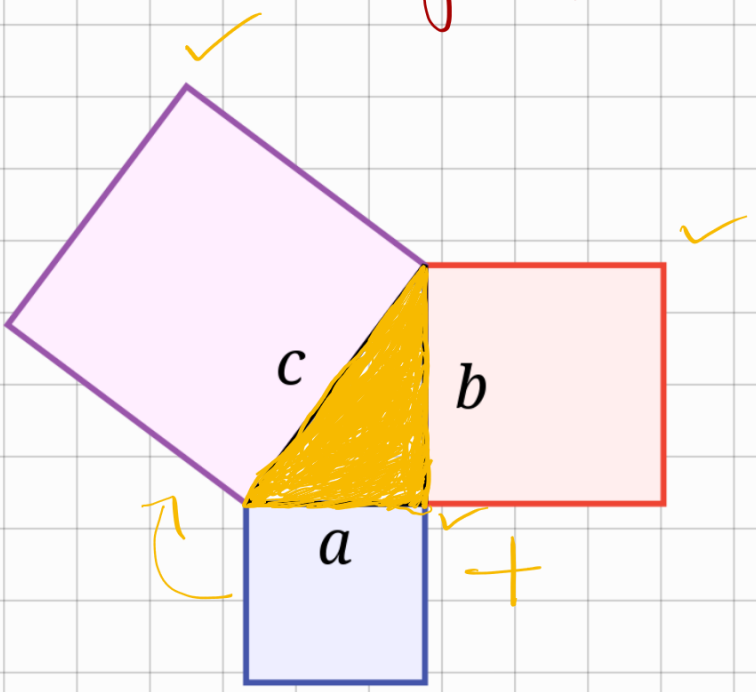
Quadrilaterals

Square		a = side Area = a x a = a^2 Perimeter = 4a	 d = diagonal = $\sqrt{2}a$ Area = (1/2) x d^2
Rectangle		a = length, b = width Area = a x b Perimeter = 2a+2b =2(a+b)	 d = diagonal Area = (1/2) x d^2
Parallelogram		h=height b=base	Area=bxh Perimeter = a+b+a+b = 2a+2b
Rhombus		d1=diagonal d2=diagonal	Area=1/2 x d1 x d2 Perimeter = a+a+a+a = 4a
Trapezoid or Trapezium		h=height or distance between the parallel sides a and b = lengths of parallel sides	Area=1/2 x h(axb) Perimeter = a+b+c+d
			Perimeter = a+b+c+d = 2m+c+d

Pythagorean Theorem

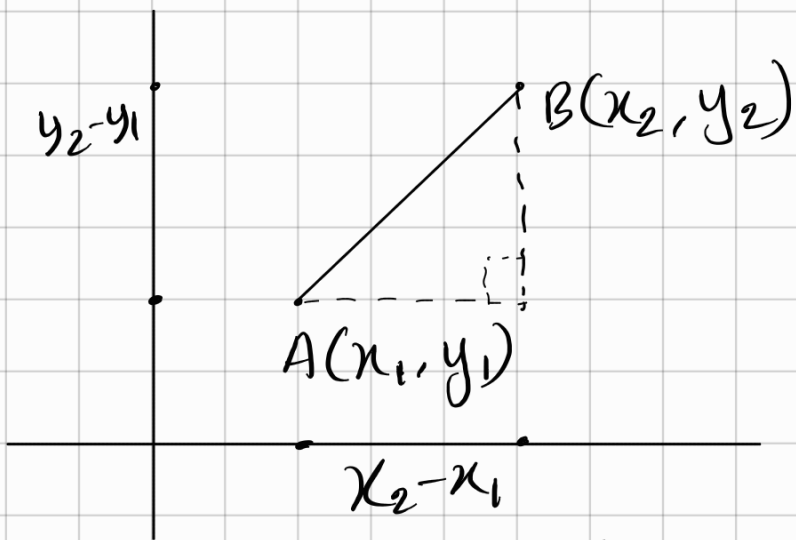


$c^2 = a^2 + b^2$

Ex- $5^2 = 3^2 + 4^2$

$13^2 = 5^2 + 12^2$

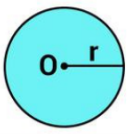
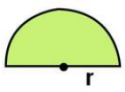
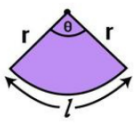
Distance between two points



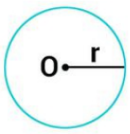
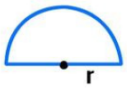
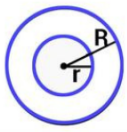
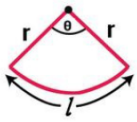
$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Circle

Figure	Area
Circle 	πr^2 or $\frac{\pi d^2}{4}$ Diameter (d) = 2r
Semicircle 	$\frac{\pi r^2}{2}$
Quadrant 	$\frac{\pi r^2}{4}$
Ring 	$\pi(R^2 - r^2)$ R = Outer Radius r = Inner Radius
Sector 	(i) $\frac{\theta}{360} \times \pi r^2$ (ii) $\frac{1}{2} lr$ Where $l = \frac{\theta}{360} \times 2\pi r$ l = Arc Length
Segment 	$\frac{\theta}{360} \pi r^2 - \frac{1}{2} r^2 \sin\theta$

Note: θ is in degrees

Figure	Perimeter
Circle 	$2\pi r$ *or* πd
Semicircle 	$\pi r + 2r$
Quadrant 	$\frac{\pi r}{2} + 2r$
Ring 	$2\pi R$ *(Outer Circumference)* + $2\pi r$ *(Inner Circumference)*
Sector 	$\frac{\theta}{360} \times 2\pi r + 2r$
Segment 	$\frac{\pi r\theta}{180} + 2r\sin\frac{\theta}{2}$

Note: θ is in degrees

Series

Sequence: List of numbers with some order or pattern.

Ex- ① 1, 3, 5, 7, 9 → pattern = Odd number List

② 1, 1, 2, 3, 5, 8, 13 → sum of two previous terms → called fibonacchi Series.

Series: Sum of elements in a sequence.

Ex - ① $S_1 = 1+3+5+7+8$ → finite

② $S_2 = 1+1+2+3+5+8+13+\cdots$
↳ infinite

Arithmetic series ↓ vs Geometric series ↓

$1+6+11+16+21+\cdots$
↳ 5 extra added in each previous term
↓
operations $(+,-)$

$2+6+18+54+\cdots$
3 multiplied to previous term
↓
operations $(\times,\div)$

Sum of natural numbers

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

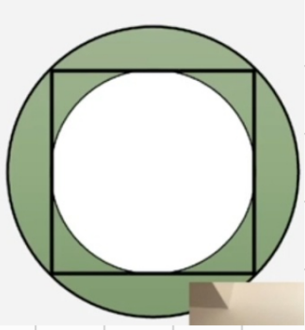
Sum of Arithmetic Series

n^{th} term in arithmetic series: $a+(n-1)d$
Sum of first n terms: $\frac{n}{2}\{2a+(n-1)d\}$

PRACTICE DAY

- Find the straight line (shortest) distance between two points in a 3-dimensional space.
  - Points: $A(-10,23,5)$ & $B(31,-11,76)$
- Find the area of the green region if the side length of the square is 10
- Find the sum of first n odd numbers. Calculate for $n = 10,100,1000$

$1, 3, 7, \cdots, 19$



Unitary Method → per unit

Unit = একক, Unitary = একক

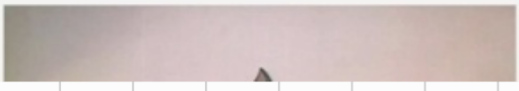
* Problem solving method: Find value for single unit.

Example: ↓

- **Problem: You and your 6 friends went to a restaurant. All ordered the same meal, and the total bill was 1218 Taka. Now if you go there with your 2 best friends and have the same meal, what would be the bill this time?**
  - Given: Cost for 7 persons
  - Goal: Finding cost for 3 persons
  - Unitary method: Find cost for 1 person first

Cost for 7 persons = 1218
Cost for 1 person = 1218/7 = 174
Cost for 3 persons = 174 × 3 = 522

- **Tom finishes his homework within 15 hours while Jerry takes 10 hours. How many hours will the same homework take to be done if they work together?**
- Pause the video and try



Percentage → per 100

Per hundred quantity
Scaling reference to 100.
* You may need to use unitary method as well.

Ex-① You gave a test on 40 marks. Your score was 32. What would be the score if the test was taken with 100 marks?

40 → 32
100 → 32 × 100/40 = 80 marks

(100) 40 $\frac{32}{40} \times 100 = 80$ marks

This is per hundred quantity, therefore percentage

15%.10 → denotes modulus
15% → denotes percentage

(2) A clothing store is selling one of their most popular products at 870 Tk after 40% discount. What was the original price?

Let the original price be x.
discounted price $= x(100-40)\%$
$= x \times \frac{60}{100}$

Now, $\frac{6x}{10} = 870 \Rightarrow x = \frac{870 \times 10}{6} = 1450$
Ans

(3) The price of oil increased by 25% and then decreased by 15%. What is the net percentage of increase or decrease in oil price?

Let's assume the initial price = 100
increased by 25 %: updated price = 125
Decreased by 15%, price gets reduced to 85%:
updated price $= 125 \times 85\%$
$= 106.25$

Final price is more than 100
Net increase $(106.25 - 100) = 6.25$

Interest

Key words: Capital, interest, interest rate
↓
per 100
↓
time → 1 year, 6 months etc.

Two types of interest: ① simple interest
② compound interest

Simple interest: $I = Pnr$ → interest rate
↳ time
↳ capital
↓
interest

Ex- ① Your friend deposits 7000 tk in Sonali bank for 3 years which earn him an interest of 8%. What is the amount he gets after 3 years?

$$I = 7000 \times 3 \times \frac{8}{100} = 70 \times 24 = 1680$$ Ans

② You deposit 5400 tk and got back an amount of 6000 tk after 2 years. Find the simple interest rate of the bank.

$I = 6000 - 5400 = 600$

$600 = 5400 \times 2 \times r$ $\therefore r = 5.56\%$ Ans

Mean and Median

Mean:

$$\bar{x} = A = \frac{1}{n}\sum_{i=1}^{n} a_i$$

A = arithmetic mean $= \bar{x}$

n = number of values

a_i = data set values

- Example: You and your friends went to a restaurant. Everyone puts their money on the table and orders the same meal for all within budget. How much is the maximum budget for each person?
  - Money collected from 6 friends: 103, 210, 57, 85, 500, 180

$$\bar{x} = \frac{103+210+57+85+500+180}{6} = 189.167$$

Avg

Median:

[redacted]

if n is odd,

$$\text{median} = \left(\frac{n+1}{2}\right)^{th}$$

if n is even,

$$\text{median} = \frac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2}$$

n = number of terms
th = n(th) number

Why Median?

- Because sometimes mean can be misleading.
  Example: You walk everyday. Here's the history, 3.5 km, 7 km, 5 km, 4 km, 4.5 km, 24 km, 5 km, 5.5 km, 5 km, 6.5 km

Mean: 7 km

Leave out the outlier 24. Now, mean: 5.11 km

But median: $\underrightarrow{3.5, 4, 4.5, 5, 5, 5,} \; \underleftarrow{5.6, 6.5, 7, 24}$ (sorted)

$$= \frac{\left(\frac{16}{2} + \left(\frac{10}{2} + 1\right)\right)}{2} = \frac{5 + 5 + 1}{2} = 5.5$$

Even if we leave out 24, then media: 5

* Median cannot be deviated as much as mean by some outliers.