IE538: Non-Linear Optimization Algorithms and Models

Final Computing Project

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1) **Problem Number 21: (**"Test Examples for Nonlinear Programming Codes" by Klaus Schittkowski, (http://www.ai7.uni-bayreuth.de/test_problem_coll.pdf)

$$min.\ f(x_1, x_2) = 0.01{x_1}^2 + {x_2}^2 - 100$$
$$Subject\ to$$
$$10x_1 - x_2 - 10 \geq 0$$
$$2 \leq x_1 \leq 50$$
$$-50 \leq x_2 \leq 50$$

$$Initial\ Starting\ Point\ x_0 = (-1, -1)(Infeasible)$$
$$Optimal\ Solution\ x^* = (2,0)$$
$$Optimal\ Objective\ Function\ Value\ f^* = -99.96$$

2) The feasible region of the above problem is a **polyhedral set which is compact in nature**. Moreover the **objective function is continuous over the constraint set**. Hence the above problem has an optimal solution.

3) The objective function is convex since the Hessian is positive definite

$$H = \begin{bmatrix} 1/50 & 0 \\ 0 & 2 \end{bmatrix}$$

The constraint set is a polyhedral set which is also convex. Since the problem involves **minimization of a convex function over a convex set the problem is convex.**

4) **i)** The algorithm chosen is the Interior Point Method (IPM). The algorithm implemented here can handle infeasible starting points but is not suited for handling infeasible problems by itself. But Interior Point Method Algorithm is suited to handle infeasibility.

ii) The algorithm converges to a stationary point but in presence of LICQ it satisfies the first order optimality conditions converging to a local optimal point. In our case, we converge to the global optimum, since we have convexity and linear inequality constraints.

Condition for convergence in our case is μ tending towards 0 while converging and objective function is non - decreasing in feasible directions. Gradient of Lagrangian is close to zero.

iii) No, the convergence result stated above does not depend on starting point. Due to the implementation of merit function and second order correction we have ensured global convergence. This is also proved as the starting point for the problem is (-1,-1) which is infeasible by itself.

iv) Convergence rate is super linear

v) Termination condition used is max(norm(gradient of lagrangian),norm(z-inv(S)*11), norm(equality constraints), norm(inequality constraints)) < tolerance (0.041).