Vulnerability Analysis of Public Transport Networks

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Modelling Real World Problems

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Introduction

Public transport networks play a crucial role in ensuring efficient mobility for millions of people in major European cities. However, due to maintenance disruptions, overpopulation, epidemics or changing urban dynamics, these networks often face challenges related to efficiency, robustness and vulnerability. In order to improve the overall performance of public transport systems, one must address these challenges and come up with innovative solutions. In this project, we investigate the robustness of the public transport network of two major European cities. Our research is focused on analyzing the disruption vulnerability of different public transit networks. In order to do so, we address two different research questions; First, we determine how to define robustness and vulnerability in the context of public transit networks, and second, we investigate which of the covered cities is more/less vulnerable to disruptions.

Dataset

We make use of an online dataset with network graphs of the public transport networks of a few major European cities. The network data of each city contains network graphs for each mode of transportation (metro, bus, railway, etc.) and a combined graph, all as CSV files. Within the dataset for an individual mode of transportation, we can access the connection between a pair of nodes (i.e. stops), the distance and average duration between the two, the number of vehicles that operate through the connection in a day, and which routes the vehicles have operated for (Bieze, 2020).

Context

For the purposes of this research we decided to focus on the cities of Berlin and Helsinki. Both these cities have efficient public transport systems, but there are some notable differences between them. First of all, Berlin is a much larger city compared to Helsinki, both in terms of area and population. It has a population of over 3.6 million people, spread across a vast metropolitan area. Helsinki only has a population of just over 1.3 million people. In terms of transport modes, both Berlin and Helsinki offer buses, railways, subways and trams. However, due to its larger size and higher population, Berlin's public transport system generally offers more frequent services, especially during peak hours. Helsinki's public transport system maintains a good level of frequency, although it may not be as high as Berlin's due to the smaller size of the city.

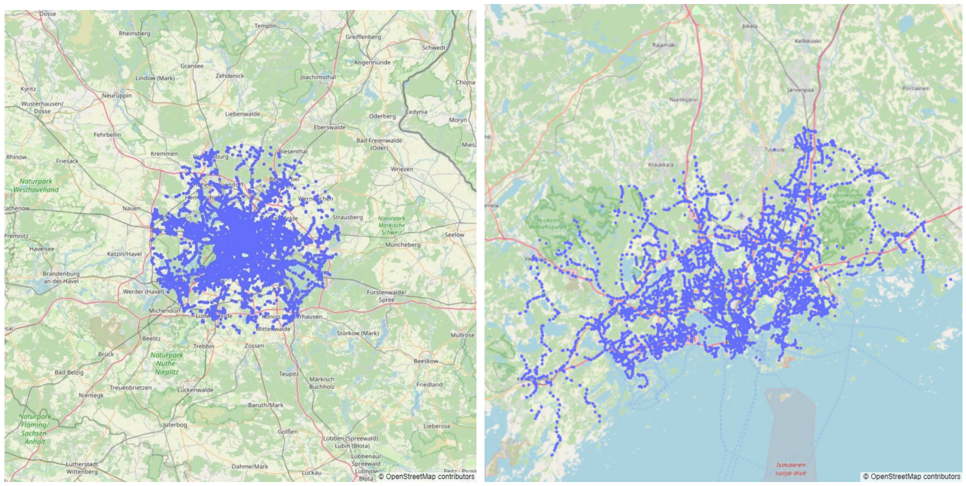

Figure 1. The public transport network of Berlin and Helsinki

Car ownership in Berlin is relatively high, but it is lower compared to many other major cities in Germany. The availability and efficiency of public transport options contribute to a lower dependence on private cars. Berlin has implemented various initiatives to promote sustainable modes of transportation and reduce car use, such as car-sharing services and dedicated cycling infrastructure. Car ownership in Helsinki is relatively lower compared to many other European cities. The city has actively promoted sustainable transportation options. Helsinki's compact city layout makes it convenient for residents to rely on alternative modes of transportation instead of cars. The city has also introduced car-sharing schemes and implemented policies to reduce traffic congestion and emissions. Both Berlin and Helsinki have a well-developed cycling infrastructure, with dedicated bicycle lanes, bike-sharing programs, and parking facilities. The cities encourage cycling as a sustainable means of transportation and provide amenities such as bike-sharing services.

Both these cities prioritize the development of sustainable and efficient public transport systems. While Berlin's system is more extensive and serves a larger population, Helsinki's public transport network is still well-designed and provides convenient connectivity within the city. Because they are similar in terms of the type of transportation they use, and both are major European capitals, they make for an interesting analysis.

Network Analysis

We began by performing some basic analysis on the graphs of the combined transportation networks of Berlin and Helsinki, which includes trains, buses, trams and subways. It is required for the graph to be connected to analyze the connectivity of a network based on measures of distance, reachability, and redundancy of paths between nodes. Since these were disconnected graphs, we split these into their connected components using a self-defined function which outputs the number of stops (nodes) and connections (edges) in each component. Berlin has 4 components, while Helsinki has 9 components. The three biggest components of each city can be summarized as follows:

Berlin:

Component	Stops	Connections
1	4593	12070
2	4	6
3	2	2

Helsinki:

Component	Stops	Connections
1	6879	8911
2	75	119
3	19	32

Figure 2: Connected components of cities

For both Berlin and Helsinki, the overwhelming majority of stops and connections fell into one of their connected components (highlighted in the tables) and therefore we picked those for the purposes of our analysis. It is interesting to note that Berlin has fewer disconnected components, and, overall, a fewer number of stops. However, the 4593 stops in the largest connected component have 12070 connections, resulting in an average of 2.63 connections per stop. In comparison, Helsinki has more disconnected components, but a larger amount of stops. The largest component of Helsinki with 6879 stops, however, has 8911 connections, resulting in an average of 1.30 nodes. This may indicate that Berlin has a higher resistance to disruptions through the possibility of alternative pathways, which is shown to be a resistance mechanism towards disruptions (Freitas et al., 2022; Snelder et al., 2018).

Distance Measures

1) **Average Distance:** This is the average distance between all pairs of nodes in the graph i.e. the number of stops needed to pass to reach from one node to another. For Berlin, this was 13.61, whereas for Helsinki it was 25.30.
2) **Diameter:** This is the maximum distance between any pair of nodes in the network. For Berlin, this was 46, whereas, for Helsinki, it was 74.
3) **Radius:** This is the minimum eccentricity of the network. Eccentricity is the largest distance between a node and all the other nodes. For Berlin, this was 23, whereas, for Helsinki, it was 39.
4) **Periphery:** The set of nodes that have eccentricity equal to diameter i.e. nodes that are on the outskirts of the graphs. For Berlin these were Tegelort (node 1714) and Niederlehme, Dahmestr. (node 7482), whereas for Helsinki these were Lankela (node 4584), and Hirvikalliontie (nodes 6625, 6658).
5) **Center:** The set of nodes that have eccentricity equal to the radius i.e. nodes that are in the center of the graph. For Berlin, this was node S Shöneweide Bhf (node 2933) whereas for Helsinki these were Talontie (nodes 963, 1011)

City	Distance Measure				
	Average Distance	**Diameter**	**Radius**	**Periphery (nodes)**	**Center**
Berlin	13.61	46	23	Tegelkort (1714), Niederlehme, Dahmestr. (7482)	S Shöneweide Bhf (2933),
Helsinki	25.30	74	39	Lankela (4584), Hirvikalliontie (6625, 6658)	Talontie (963, 1011)

Figure 3. Summary of distance measures in the transport networks of Berlin and Helsinki

Figure 3 summarizes the distance measures in Berlin and Helsinki. Overall, Berlin has a smaller average distance, diameter, and radius compared to Helsinki. This is in contrast to the cities' respective land surface areas, 891 km² and 184.47 km² (Britannica). Combining the fact that Berlin's largest connected component has 4593 nodes and 12070 connections, while Helsinki's has 6879 nodes with 8911 connections, Berlin's transport nodes seem to be more tightly connected than those of Helsinki.

Defining robustness

Robustness is the ability of a network to maintain its general structural properties when it faces disruptions or attacks (loses nodes or edges). For the purposes of this analysis, we define robustness as the property of maintaining connectivity. In this study, we consider the average travel time of all pairs of nodes in the network to measure the resistance of the networks towards edge removal. In this section, we consider vertex connectivity and edge connectivity measures. Node connectivity is defined as the "minimal number of [nodes] that need to be removed to disconnect a graph" and edge connectivity is the "minimal number of edges that need to be removed to disconnect a graph" (Freitas et al., 2022).

We found that removing just one node would disconnect the largest connected components of both Berlin and Helsinki.

- For Berlin, this was node 7334, which is the Schönefeld, Wehrmathen station
- For Helsinki, this was node 7523, which is the Talman Koulu station.

Schönefeld, Wehrmathen station operates 3 bus lines, while Talman Koulu operates one bus line. In either case, these nodes were located on the outskirts of the city. Therefore, these disruptions will likely not affect the majority of nodes located in the urban areas of the cities.

Similarly, we found that removing just one edge would disconnect the largest connected components of both Berlin and Helsinki.

- For Berlin, this was the connection between nodes 8147 and 8151 which are stations Erkner, Friedhof and Erkner, Jägerstr.
- For Helsinki, this was the connection between nodes 7441 and 7523 which are stations Mehuasema and Sommarnäsintie.

Again, these edges connect nodes located on the outskirts of the city, with one operating bus line. Therefore, the disconnected edges have no significant impact on the bigger network.

Generally, robust networks have large minimum node and edge connectivity. However, in the context of these transportation networks, these measures do not offer valuable insight into the robustness of the whole network, as the relevant nodes are located in the periphery of the network. Furthermore, the minimum_edge_cut and minimum_node_cut functions in the NetworkX package only give the minimum number of nodes and edges to disconnect a graph. Therefore, there may be more possible nodes and edges that are essential to the connection of the network.

Robustness Analysis

To test the robustness of the public transport networks in Helsinki and Berlin, we will perform bond percolation to simulate the disruption in connections between set(s) of nodes. We will utilize two attack strategies on the combined transportation network graphs to test the robustness against random failures and targeted attacks. To measure the average travel time of each network after percolation, the network must be connected. Therefore, the combined network graph will be supplemented by the walking distance dataset between sets of nodes. In theory and practice, there will always exist a walking path between two nodes, at an event of public transport disruption. When all transport edges are removed, the average travel time will equal the average duration of walking paths in the network. Furthermore, we assume the difference in average travel time between the two cities to be irrelevant in the analysis, considering the difference in size of the respective cities.

Random bond percolation

First, we perform uniform random bond percolation, where a fraction of edges are removed randomly from the network graph. The percolation process is parameterized by the occupation probability Φ, which is the probability that an edge is “present or functioning” in the network (Newman, 2010). To explore the relationship between percolation and robustness, we plot the percolation probability $1\text{-}\Phi$ against the observed average travel time of the network after percolation (Figure 3).

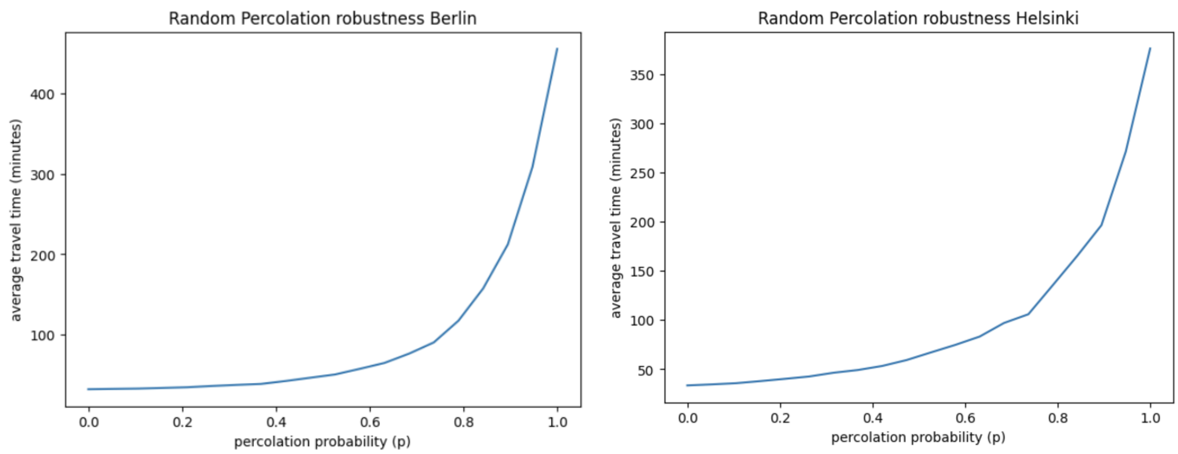

Figure 4. Average travel time as a function of occupation probability in the public transport networks of Berlin(left) and Helsinki(right)

In both cities' public transport networks, the average travel time increases exponentially at a remarkably similar rate. The non-linear growth in average travel time, or the percolation transition, roughly begins at $p \approx 0.7$ in both cities. Therefore, we can estimate that the percolation threshold of the networks to random percolation ϕ_c is approximately 0.65-0.75.

Degree-targeted bond percolation

Secondly, we perform bond percolation on the networks with targeted attacks on high-degree nodes. The procedure is performed such that edges are removed from nodes, in order of degree centrality measure (highest-degree nodes first), until a fraction p edges are removed. We plot the average travel time as a function of the fraction of nodes removed (Figure 3).

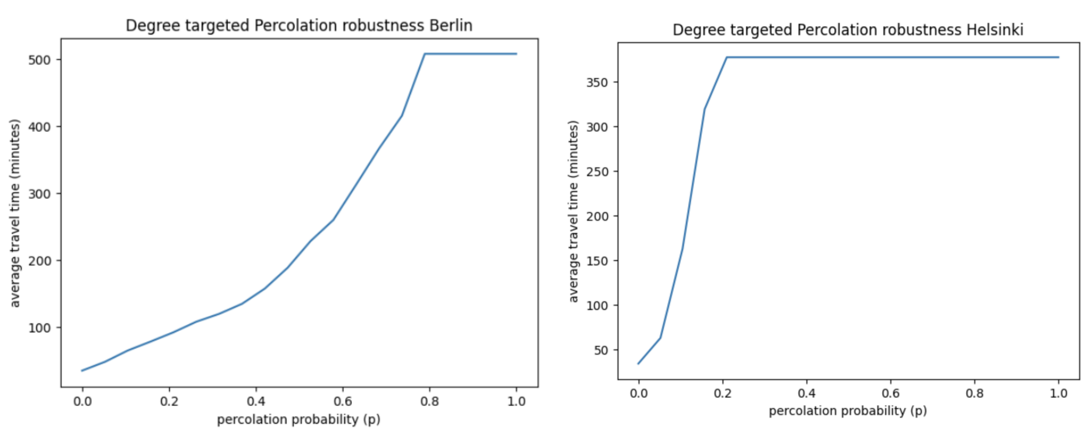

Figure 5. Average travel time as a function of occupation probability for targeted attacks on transport networks of Berlin (left) and Helsinki (right)

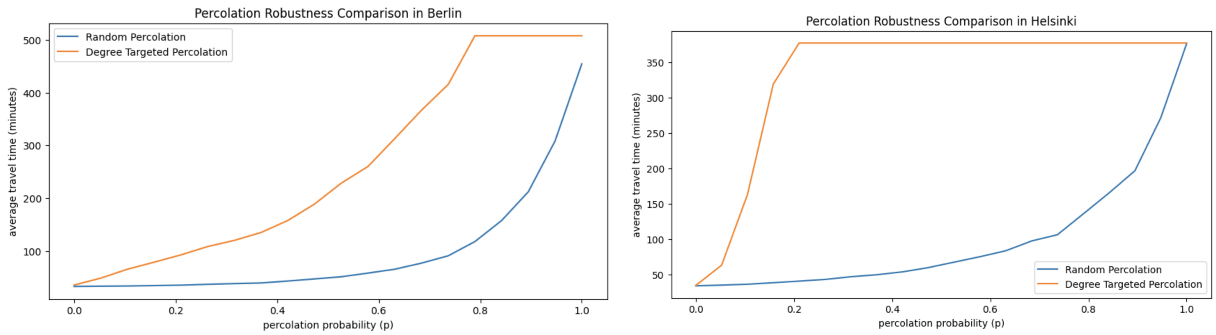

Figure 6. Average travel time as a function of occupation probability for random and degree targeted percolation on transport networks of Berlin (left) and Helsinki (right)

We see the results of bond percolation with targeted attacks on high-degree ‘hubs’. The robustness of the networks significantly decreases in comparison to random attacks, as we have seen in Figure 2. The average travel time increases roughly linearly with an increase in percolation probability. In both networks, the function growth plateaus at a critical point: at this percolation probability (p) value, the average travel time in by the network under degree-targeted attack becomes (approximately*) equivalent to a network under random percolation at $p = 1$ (all transport edges removed). In Berlin, this critical threshold value is observed at approximately $p = 0.8$; in Helsinki, this is observed at approximately $p = 0.2$. Therefore, Berlin’s public transport network is more resistant to degree-targeted percolation compared to Helsinki’s.

* The inconsistency of the average travel time of the Berlin transportation network at $p = 1$, could be due to stochasticity in our method. To reduce the computational burden, we calculate the average travel time in a random sample of 100 nodes (the sample size can be varied by changing a variable input in line 350 of auxiliaries.py).

Discussion

Through random and degree-based percolation, it can be concluded that the public transport networks of Berlin and Helsinki are relatively robust against random attacks ($\phi_c \approx 0.65\text{-}0.75$), but in general, are less resistant to degree-targeted attacks. Helsinki’s network is especially more vulnerable against such targeted attacks, considering that the average travel time converges to the walking travel time when 20% of the edges connected to highest-degree nodes have been removed. In comparison, at 80%, Berlin relatively has a high resistance to degree-targeted attacks.

Our findings reveal that Helsinki’s public transport network has a higher dependency on high-degree nodes or ‘hubs’ compared to Berlin. This may possibly be a result of the topology of the network or the lack of alternative pathways during the removal of an edge, which are further research directions. In a real-life application, simultaneous disruptions in major ‘hubs’ are unlikely to occur. Therefore, a high robustness of the networks against random attacks indicates a high level of resistance towards disruptions in general.

Limitations and Future Research

Although the current project provided valuable insights into public transport network analysis, the project is limited for a number of reasons, and future research is needed to expand on the findings. Firstly, the scope of the project is only on two major European cities, and the findings may not be generalizable to other cities with distinctly different characteristics. Future research should include a broader range of cities which could lead to more generalizable results. Secondly, the model used in the project is a severely simplified representation of real-world data, and models in future research could be made more sophisticated. The model could include a more realistic travel-time metric, real-time passenger flow, and historical data on where disruptions occur or are already occurring.

Furthermore, a relevant follow-up research question that emerged from the current research would be to investigate the effect of adding edges (connections) on the disruption vulnerability of the network. It could be determined which added connections or stops contribute the most to the network robustness, exposing real-life vulnerabilities and possible policy suggestions for urban development plans. Future research should also explore other aspects of public transport network robustness, such as the impact of social and economic factors on system performance, the integration of emerging technologies (e.g., autonomous vehicles), and the design of effective strategies for managing disruptions in real time.

Appendix

The dataset used in this project can be found at Zenodo.org
https://zenodo.org/record/1186215#.ZEfKMXZBy5c

The code and implementation details of this research are available on GitHub at:
https://github.com/shantanu-555/Networks_Project

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