ML ALGO

Aúc / Curve

Logistic Regression :- (0 to 1) values

pred. Prob b/w 0 & 1

Binary classification.

→ sigmoid f^n → take real values & map into 0 & 1

$$\boxed{y} = \frac{1}{1+e^{-x}} \qquad e = 2.71828$$

i.e. dependant var = categorical

linear regression is unbounded & so we call logistic regression.

eg: Say to check whether person is malignant / not?

so say a class is malignant with pred. cont value = 0.4,

threshold = 0.5 (all >= 0.5 is malignant)

so $\boxed{0.4 < 0.5}$ it will ~~state~~ that not malignant thus leading to huge consequences.

So linear regression ✗

O/P = 0 or 1

hypothesis :- $Z = wx + B$

$Z \to (\infty)$, $Y(\text{pred}) = 1$

$$\left(\frac{1}{1+e^{-\infty}}\right) = \frac{1}{1+0} = 1$$

$$x \to (-\infty) = \frac{1}{1+e^{-(-\infty)}} = \frac{1}{1+e^{\infty}}$$
$$= \frac{1}{\infty} = 0$$

Analysis of hypothesis :-

O/P of the hypothesis is the estimated probability. This is need to infer how confident can predicted value be actual value when given i/p X.

→ **Binary LR** :- $\boxed{2}$ possible outcomes
eg: spam / Not spam

→ **Multinomial LR** :- ($\boxed{\geq 3}$ categories)
eg: Iris (setosa, virginica, versicolour)

→ **Ordinal LR** : ($\boxed{\geq 3}$ but with $\boxed{\text{ordering}}$
Movie rating 1 to 5.

Decision Boundary : To predict which class a data belongs, a threshold can be set.

$$\boxed{h_\theta(x) : \text{Sigmoid}(Z)}$$
theta

Cost f^n (Not as of linear reg) :-

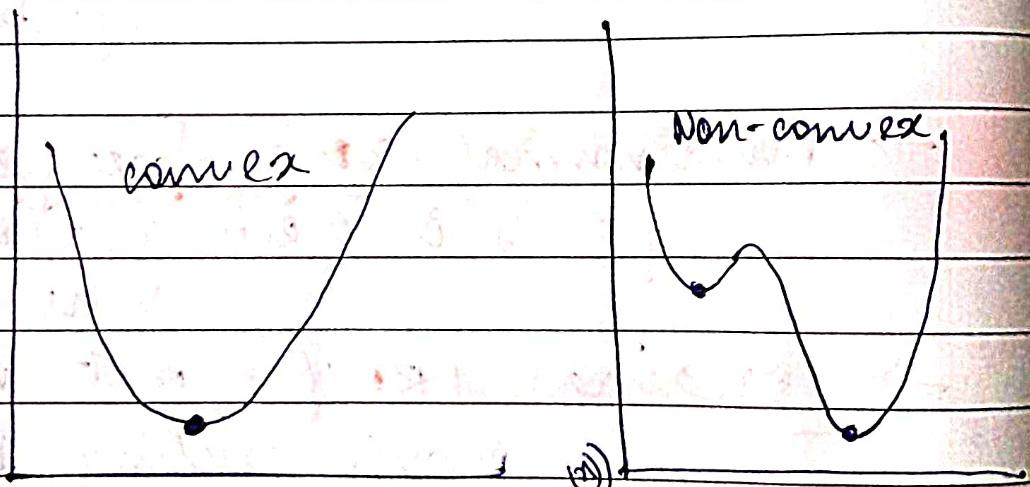
$$\text{Cost}(h_\theta(x), Y(\text{actual})) = -\log(h_\theta(x))$$ $\boxed{\text{if } y = 1}$

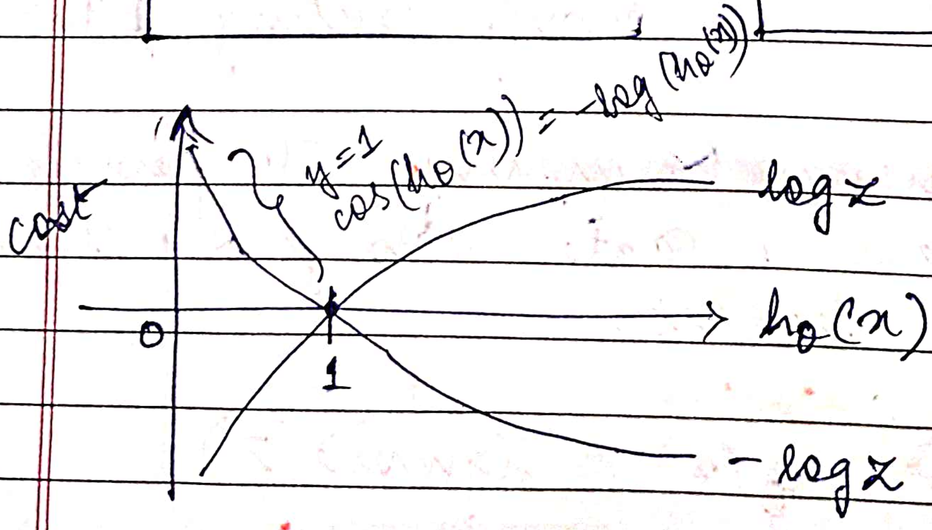
$$= -\log(1 - h_\theta(x)) \text{ if } \boxed{y = 0}$$

Q.) cost f^n of linear regress (is NOT) the cost " " Logistic " —:

linear regression uses MSE as its cost f^n. If this is used for logistic regression, then it will be a non-convex f^n of parameters (θ).

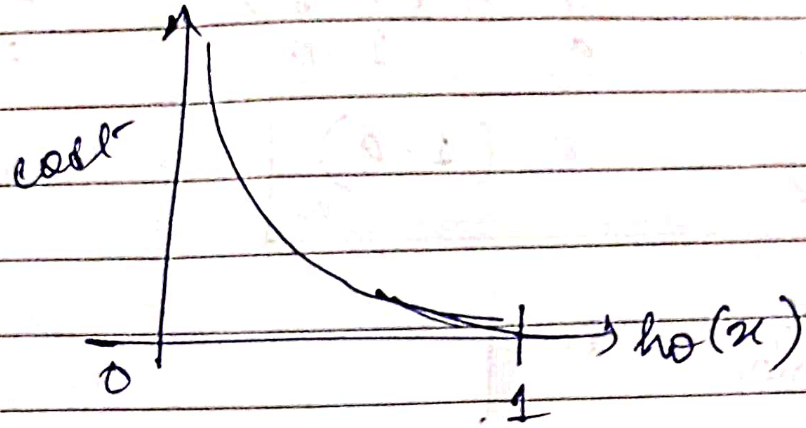
Gradient descent (GD) will converge into global minimum only if the function is convex.





$$\text{cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) \rightarrow y = 1 \\ -\log(1 - h_\theta(x)) \rightarrow y = 0 \end{cases}$$

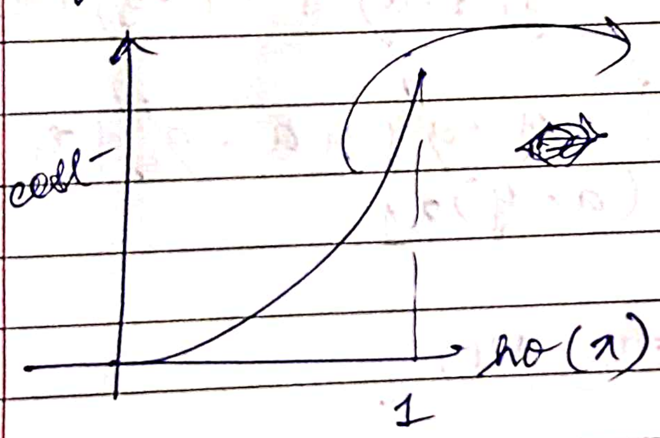
if $y = 1$,
$$cost(h_\theta(x), y) = -\log(h_\theta(x))$$



$cost = 0, \; h_\theta(x) = 1$
$cost = \infty, \; h_\theta(x) = 0$

if $h_\theta(x) = 0 \longrightarrow P(y=1|x;\theta) = 0$

$-\log(1-z)$ $\boxed{y=0}$



$cost = 0, \; h_\theta(x) = 0$
$cost = \infty, \; h_\theta(x) = 1$
$\downarrow$
$P(0|x;\theta) = 0$

GD:

$$z = w_1 x_1 + w_2 x_2 + b.$$
$$\hat{y} = a = \underbrace{\sigma}_{\text{sigma}}(z) \longrightarrow L(\hat{y}, y)$$
$a = \hat{y}$

$$\Big[\underbrace{L(\hat{y}, y) = -y\log(h_\theta(x)) - (1-y)\log(1-h_\theta(x))}_{\text{cost fn simplified } (y = 0 \text{ or } 1)}\Big]$$

$$\boxed{w_1} = \frac{\partial(L)}{\partial w_1} = \frac{\partial L}{\partial a} \cdot \frac{\partial a}{\partial z} \cdot \frac{\partial z}{\partial w_1}$$

$$\therefore \frac{\partial L}{\partial a} = \frac{\partial}{\partial a}\left(-y\log a - (1-y)\log(1-a)\right)$$

$$= -y\left(\frac{1}{a}\right) + \frac{(1-y)}{(1-a)}$$

$$\boxed{\frac{\partial L}{\partial a} = \frac{-y}{a} + \frac{(1-y)}{1-a}}$$

$\frac{\sigma(z)}{\partial z}$

$$\boxed{\frac{\partial a}{\partial z} = a(1-a)}$$

$a = h_\theta(x)$
$= \sigma(z)$

$$\boxed{\frac{\partial z}{\partial w_1} = x_1}$$

$$\therefore \frac{\partial L}{\partial w_1} = \left(\left(-\frac{y}{a} + \frac{(1-y)}{1-a}\right) \cdot (a)(1-a)\right) x_1$$

$$= \left[-y(1-a) + a(1-y)\right] x_1$$

$$= \left[-y + \cancel{ay} + a - \cancel{ay}\right] x_1$$

$$= (a-y)x_1$$

$\therefore$ <u>update for w_1;</u>

$$\frac{\partial L}{\partial w_1} = (a-y)x_1$$

Here $a - y = \frac{\partial L}{\partial z}$

$$w_1 = w_1 - \alpha \frac{\partial L}{\partial w_1}$$

$$\therefore w_i = w_i - \alpha \frac{\partial L}{\partial w_i}$$

$\frac{\partial L}{\partial b} = (a-y)$ $\qquad b = b - \alpha \frac{\partial L}{\partial b}$