CS4248 Assignment 1

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1.

$$p(k) = \frac{\binom{n}{k}}{\binom{N}{n}} = \frac{n!}{k!(n-k)!} \frac{N!(N-n)!}{n!}$$
$$= \frac{N!(N-n)!}{k!(n-k)!}$$

2. Some Witten-Bell smoothing shit.

	$P_{WB}(w c_1)$	$P_{WB}(w c_2)$	
body			
fun			
is			
jogging			
John			
loves			
Mary			
our			
strengthens			
swimming			

3. Table for edit distance.

р	5	4	3	4	3	4
а	4	3	2	3	4	5
е	3	2	1	2	3	4
h	2	1	2	3	4	5
С	1	2	3	4	5	6
	0	1	2	3	4	5
		h	е		р	s

$$\begin{split} H(X,Y) &= -\sum_{x \in X} \sum_{y \in Y} \, p(x,y) \log \, p(x,y) \\ &= -\sum_{x \in X} \sum_{y \in Y} \, p(x,y) \left(\log \, p(y \, | \, x) \, p(x) \right) \\ &= -\sum_{x \in X} \sum_{y \in Y} \, p(x,y) \left(\log \, p(y \, | \, x) + \log \, p(x) \right) \\ &= -\left(\sum_{x \in X} \sum_{y \in Y} \, p(x,y) \log \, p(y \, | \, x) + \sum_{x \in X} \sum_{y \in Y} \, p(x,y) \log \, p(x) \right) \\ &= -\left(\sum_{x \in X} \sum_{y \in Y} \, p(x,y) \log \, p(y \, | \, x) + \sum_{x \in X} \log \, p(x) \sum_{y \in Y} \, p(x,y) \right) \end{split}$$

Marginalising over y,

$$\begin{split} &= -\left(\sum_{x \in X} \sum_{y \in Y} \, p(x,y) \log \, p(y \,|\, x) + \sum_{x \in X} \, p(x) \log \, p(x)\right) \\ &= \left(-\sum_{x \in X} \sum_{y \in Y} \, p(x,y) \log \, p(y \,|\, x)\right) + \left(-\sum_{x \in X} \, p(x) \log \, p(x)\right) \\ &= \, H(Y \,|\, X) + \, H(X) \end{split}$$