The Half-Cycle Correction: Banish Rather Than Explain It

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The half-cycle correction is often used in discrete Markov models to estimate state memberslip. This article shows that the correction, in addition to being unintuitive, actually produces the wrong results in many circumstances. These include quality-adjusted life year (QALY) weights and unit costs that differ by cycle. The half-cycle correction is also incompatible with discounting of the obtained stream of state membership. It is furthermore shown that the life table method of estimating state membership obtains correct results under these circumstances and is also much more transparent. The article concludes that the half-cycle correction should be dropped in favor of the life table method. Key words: Markov Models; mathematical models and decision analysis; statistical methods. (Med Decis Making 2009;29:500-502)

Naimark and others' are correct in their observation that the half-cycle correction leaves many students stumped. This is not surprising because the half-cycle correction is a typical example of what in computer programming circles is referred to as a *.kludge.*

What is a kludge? From Wikipedia: "a kludge is a 'solution' to a problem ... that is inefficient, inelegant, or even unfathomable, but which nevertheless (more or less) works." This sums it up nicely: • the half-cycle correction more or less works, but it is inelegant, understandably baffles students (and not just students), and in many common circumstances actually breaks down.

Nevertheless, the half-cycle correction seems to have attained the status of "gold standard" in Markov modeling, which is regrettable because a perfectly good, transparent, real solution to the problem it tries to address has been used by demographers for decades.

In this note, I will briefly describe what the problem is that the half-cycle correction aims to solve, point out how demographers solve it, and show that the demographic solution is superior to the half-cycle correction by giving some examples where the latter fails but the former performs without any problem.

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THE PROBLEM

The problem the half-cycle correction aims to solve is introduced by the use of discrete time in the Markey model. This means that state membership is known at the beginning and at the end of each cycle (or equivalently the beginning of the next cycle) but not in between.

The usual assumption is that, on average, people will transit to another state halfway through the cycle, This means that when state membership is measured at the beginning of the cycle, it is systematically overestimated, and when measured at the end, systematically underestimated. As Naimark and others explain, when cumulative state membership is calculated by summing end-of-cycle membership over all cycles, adding a half cycle's worth of the membership at the start of the first cycle provides a very good estimate of the surface under a continuous-time membership curve.[1]

THE LIFE TABLE SOLUTION

Demographers face a similar problem when calculating life expectancy using a life table (which is not astonishing, given that a discrete life table is also a Markov model), Life expectancy is the average number of years a person of a given age (often 0) will live, given currently observed age-specific mortality probabilities. Those mortality probabilities are applied to a life table cohort of arbitrary initial size (often 100,000), which is subsequently whittled away until extinct.[2]

To calculate the life expectancy correctly, the demographer needs the total number of life years lived by the initial life table cohort. Available are the numbers of people alive at each exact age (analog to "start of cycle"), by convention denoted by, i_x, where x is the index for age. In a so-called complete (as opposed to abridged) life table, calculations are done by single year of age, so we need to assess for each age the number of life years lived in'the interval x . x 1, which is denoted by L_x. There exist several methods, but usually L_x is obtained by the following equation:

$$L_x \text{—} \frac{-/x+ix+1}{2}$$

In other words, the years lived in the interval are estimated by taking the average of the number of people at the start and end of the age interval .(in Markov terms: the average of membership at start and end of the cycle).

An equivalent equation that is particularly useful as the last age because it does not refer to the next age is

$$= i_x — 0.5\, cl, ,$$

where cl_x is the number of deaths in the age interval $x\ x+1$. This can be explained as the number alive at the start of the age interval minus half the number of deaths, half because of the assumption that people transit on average halfway through the age interval. The 2 *bxpressions* are equivalent because $ix + \ dx$ •

Table 1 illustrates the life table method (to limit the number of rows, an unlikely high annual mortality probability has been used, which is also improbable for being constant with age). The equivalence of the life table with a Markov model is obvious from this table: just replace "age" with "cycle."

Table 1 also displays a column where the number of life years lived is calculated using the half-cycle correction. Although the entries at each age/cycle between the L_x and the half-cycle correction columns are different, the sum over all ages/cycles is the same, thus demonstrating the equivalence of both methods with respect to cumulative state membership.

Table I. The Life Table Method

Age	qx	lx	dx	L_x	Half-Cycle
0	0.52	1000	520	740	980
1	0.52	480	250	355	230
2	0.52	230	120	170	111
a	0.52	111	58	82	53
4	• 0.52	53	28	39	25
5	0.52	25	13	19	12
6	0.52	12	6	9	6
7	0.52	6	3	4	3
8	0.52	3	1	2	1
9	0.52	1	1	1	1
10	0.52	1	0	0	0
			Sum	1423	1423

q,: annual mortality probability. life table cohort; for $x > 0$, — $d_{x\_}$, *d,:* number of deaths in age interval $x. . x+1; ri_x=1$ L„: life years lived in the interval $x \ldots x+1$; 0.5(4 + 4_{+1}l; for the last entry, the equivalent calculation is —0.5d„ Half-cycle: life years calculated using the half-cycle correction; for $x\ 0$, this column is calculated as 0.54 +1,4.1; for $x > 0$, it equals 4_+, , which is equal to end-of-cycle

PROBLEMS WITH THE HALF-CYCLE CORRECTION

However, the half-cycle correction is equivalent to the life table method only in a very limited way. Naimark and others[1] already mention that the half-cycle correction needs adjustment when the Markov chain terminates with people still in the state. This is a well-known problem with a well-known solution (which is pointed out by Naimark and others). The life table method needs no special correction in that case.

Other problems with the half-cycle correction exist without such easy solutions. They originate from the fact that in the case of the half-cycle correction, the distribution of state membership over the cycles is not correct (only the sum over all cycles is). This plays havoc with calculations that depend on this distribution.

I illustrate this point by 3 examples that all commonly occur:

- .p unit costs differ by cycle
- o Quality-adjusted life year (QALY) weights differ by cycle
- Discounting

It will be clear that cost and QALY calculations using state membership will be correct only when the unit costs and QALY weights are constant across all cycles. With, for example, an intervention in cycle 0, this will clearly not be the case.

Discounting will always be thrown off by the incorrect state membership distribution over the cycles: with a large additional membership in the first cycle and too low membership in all the others, the discounted stream of state membership will always be too high. For example, the 5% discounted cumulative state membership of Table 1 is 1363 for the life table method and 1384 for the half-cycle correction. The discounting problem of course affects costs and QALYs as well, even when unit costs and QALY weights are constant over the cycles. Because the life table method yields a correct distribution of state membership over the cycles, it is not affected by any of these problems.

The life table method can also handle without problems cycles of different lengths. Abridged life tables often have 5-year age intervals, but to accommodate the high infant mortality, the youngest age group is usually split into 0-1 and 14. It is not clear how such a situation could be resolved adequately using a half-cycle correction.

DISCUSSION

I have shown that the problems with the half-cycle correction are not limited to explaining it to students; in many circumstances, it will produce the wrong result. These circumstances include discounting, as well as unit costs and QALY weights that differ by cycle. I know of very few relevant Markov models in medical decision making where QALY weights and unit costs are constant across all cycles, and no discounting is done.

There seems to be very little awareness of the problems with the half-cycle correction. They are not mentioned by Naimark and others,' who only discuss the termination problem. In their discussion of Markov models, Hunink and others[3] mention both discounting and the half-cycle correction but not that these are incompatible. Similarly, no mention of this incompatibility is made in the Australian guidelines for preparing submissions to the Pharmaceutical Benefits Advisory Committee.[4]

The life table method of calculating state membership not only is superior in the sense that it does not suffer from the problems of the half-cycle correction but also is much easier to explain. Few students will fail to see that the average of start and end of cycle membership will provide a good estimate of state membership in that cycle.

CONCLUSION

It is time that we recognize the half-cycle correction for what it is: an ugly kludge with many problems. We can avoid those problems by adopting the life table method for calculating state membership, which has the additional advantage of being easy to explain.

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