

Assignment 8

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Computer Graphics 578

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Exercise 2. (478 and 578) Consider a 2D velocity field defined by $(v_x, v_y) = ((y-5) + (5-x)/2, ((5-y)/2 + (5-x)))$ where the velocity is in m/s. Starting an object at the point $(x, y) = 4, 6$, plot (using whatever code you want, please include your source) the path of the object moving for 5 seconds using:

- a) a time step of 1 second using Euler's method
- b) a time step of 0.1 second using Euler's method
- c) a time step of 1 second using the Mid-Point method
- d) a time step of 0.1 second using the Mid-point method

Answer:

$$\because q_p = (0, \vec{p}), \quad q = [\cos(\phi/2); \sin(\phi/2)\vec{n}], \quad q'_p = qq_pq^{-1}$$

$$\text{Also, } \because \vec{p} = (0, 2, 0), \quad \vec{n} = \frac{(1, 1, 1)}{\sqrt{1^2 + 1^2 + 1^2}}, \quad \phi = 30^\circ$$

$$\therefore q = \left(\frac{\sqrt{6} + \sqrt{2}}{4}, \frac{\sqrt{6} - \sqrt{2}}{4}\vec{n} \right), \quad q_p = (0, (0, 2, 0)), \quad q^{-1} = \left(\frac{\sqrt{6} + \sqrt{2}}{4}, \left(\frac{\sqrt{6} - 3\sqrt{2}}{12}, \frac{\sqrt{6} - 3\sqrt{2}}{12}, \frac{\sqrt{6} - 3\sqrt{2}}{12} \right) \right)$$

$$\therefore qq_p = \left(\frac{\sqrt{6} - 3\sqrt{2}}{6}, \left(\frac{\sqrt{6} - 3\sqrt{2}}{6}, \frac{\sqrt{6} + \sqrt{2}}{2}, \frac{3\sqrt{2} - \sqrt{6}}{6} \right) \right)$$

$$\therefore qq_pq^{-1} = \left(0, \left(\frac{2 - 2\sqrt{3}}{3}, \frac{2 + 2\sqrt{3}}{3}, \frac{2}{3} \right) \right)$$

$$\therefore q'_p = \left(\frac{2 - 2\sqrt{3}}{3}, \frac{2 + 2\sqrt{3}}{3}, \frac{2}{3} \right)$$

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