Assignment#2 Q2 Screen Shot

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Sorry if the screen shot is not clear enough, next time I will try to submit latex/word file!

Q2 1. According to ~~Exercise~~ 1, the original square is cornered with $\{(1,0),(0,-1),(-1,0),(0,-1)\}$, and scale the square as $x'=2x$, $y'=2y$, $z'=2z$

.3.1 ercises

$\therefore$ The affine transformation is A (Scaling)

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\because$ the inverse transformation of Scaling A is A^{-1}

$\therefore$ the inverse transformation is

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. 1) A none affinie transformation is not invertible

* Here is the counter example:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$ is a non-affine transformation

Howereve its invertible

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$ as $A \cdot A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A^{-1}A$

2) Counter Example. Vector $\vec{p} = [x, y, 1]^T$

Translation $T(3,2) = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ $R_{45^\circ} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore T(3,2)(R_{45^\circ})\vec{p}$

$$= T(3,2) \cdot \begin{bmatrix} \frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y \\ \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y + 3 \\ \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y + 2 \\ 1 \end{bmatrix}$$

Then we compute $R_{45^\circ}(T_{(3,2)}\vec{p})$

$$= R_{45^\circ} \begin{bmatrix} x+3 \\ y+2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y + \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y + \frac{5}{2}\sqrt{2} \\ 1 \end{bmatrix}$$

So $T(3,2)(R_{45^\circ}\vec{p})$ doesn't equal to $R_{45^\circ}(T_{(3,2)}\vec{p})$ which proves the statement of commutative property is not correct

3. Define 2 parrallel lines L_1, L_2 as

$$L_1 = P_1 + t_1\vec{d}$$

$$L_2 = P_2 + t_2\vec{d} \quad (P_1 P_2 \text{ are points}, \vec{d} \text{ is unitary vector})$$

Apply Affine transformation T to both lines

$$TL_1 = (TP_1) + t_1(T\vec{d})$$

$$TL_2 = (TP_2) + t_2(T\vec{d})$$

$\because$ The multiplication of transformand and point remains to be points

the multiplication of transformation and vector remains to be vectors

~~define~~ (TP_1) as point P_1', (TP_2) as point P_2'

$T\vec{d}$ as vector $\vec{u}$

$$\therefore TL_1 = P_1' + t_1(\vec{u})$$

$$TL_2 = P_2' + t_2(\vec{u})$$

$\therefore$ TL_1, TL_2 are still parrallel lines, $(TL_1) \parallel (TL_2)$

$\therefore$ The statement is correct

4. <1> Rotation $R_{45^\circ} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

<2> Scaling $S(1,\frac{1}{2}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

<3> Rotation $R_{-30} = \begin{bmatrix} \cos(-30^\circ) & -\sin(-30^\circ) & 0 \\ \sin(-30^\circ) & \cos(-30^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore$ The Shear Transformation

$$Sh = R(-30^\circ) \cdot [S(1,\tfrac{1}{2})(R_{45^\circ})]$$

$$= R(-30^\circ) \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{8} & -\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{8} & 0 \\ -\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{8} & \frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{8} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$