Exercise 28.1 Apply $C(\alpha, C_1, C_2)$

i. In RGB Model $\begin{cases} C_1 = r_1R + g_1G + b_1B \\ C_2 = r_2R + g_2G + b_2B \end{cases}$

Then interpolate using RGB Model to get $C(\alpha, C_1, C_2) = (1-\alpha)C_1 + \alpha C_2$ <1>

$$C_{interpolate} = [(1-\alpha)r_1 + \alpha r_2]R + [(1-\alpha)g_1 + \alpha g_2]G + [(1-\alpha)b_1 + \alpha b_2]B \quad <2>$$

ii. In YIQ Model, from RGB to YIQ:

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = M \begin{bmatrix} R \\ G \\ B \end{bmatrix}, \quad M = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & -0.322 \\ 0.211 & -0.523 & 0.312 \end{bmatrix}$$

∴ In YIQ $\begin{cases} C_1' = M \cdot C_1 \\ C_2' = MC_2 \end{cases}$ So $C'_{interpolate} = (1-\alpha)C_1' + \alpha C_2'$

$= (1-\alpha)MC_1 + \alpha C_2$ <3>

When tranforming from YIQ to RGB

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = M^{-1} \begin{bmatrix} Y \\ I \\ Q \end{bmatrix} \quad <4>$$

$\therefore C_{interpolate} = M^{-1}C'_{interpolate}$

$= (1-\alpha)M^{-1}MC_1 + \alpha \cdot M^{-1}MC_2$

$= (1-\alpha)C_1 + \alpha C_2$ <5>

According to <1> and <5>, using different model can get exactly same result.

System: CIE XYZ, YIQ Because they all can tranform using sRGB System $= M \begin{bmatrix} R \\ G \\ B \end{bmatrix}$

which is invertible.

Exercise 2.2

① In RGB model, the mixing color C

$$C = (0.7 + \frac{\varepsilon}{2},\ 0.4 - \varepsilon,\ 0.3 - \frac{\varepsilon}{2})$$

② In $L^*u^*v^*$

$$L^* = \begin{cases} 116(Y/Y_n)^{\frac{1}{3}} - 16 & \frac{Y}{Y_n} < 0.008856 \\ 903.3\,Y & \frac{Y}{Y_n} \geq 0.008856 \end{cases}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow M \begin{bmatrix} G \\ R \\ B \end{bmatrix} \qquad \therefore C_{xyz} = M C_{RGB}, \ (X_n, Y_n, Z_n) \text{ is White} = (1,1,1)$$

$$M = \begin{bmatrix} 3.2410 & -1.5374 & -0.4986 \\ -0.9692 & 1.8760 & 0.0416 \\ 0.0556 & -0.2040 & 1.0570 \end{bmatrix}$$

$$\therefore C_{L^*u^*v^*},\ u' = \frac{4X}{X+15Y+3Z} \qquad u_n = \frac{4}{19} \qquad v' = \frac{9}{19}$$

$$v' = \frac{9Y}{X+15Y+3Z}$$

$$u^* = 13L^*(u - u_n')$$

$$v^* = 13L^*(v - v_n')$$

According to ① and ②

In RGB $C_0 = (0.705, 0.39, 0.295)$ when $\varepsilon = 0.01$

$C_1 = (0.725, 0.35, 0.275)$ $\varepsilon = 0.05$

$C_2 = (0.825, 0.15, 0.175)$ $\varepsilon = 0.25$

In $L^*u^*v^*$ $Y/Y_n \geq 0.008856$ when $\varepsilon = 0.01$ or $\varepsilon = 0.05$ or $\varepsilon = 0.25$

$$\therefore \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = M \begin{bmatrix} R \\ G \\ B \end{bmatrix} \qquad \therefore$$

For $(r,g,b) = (0.7, 0.4, 0.3) \rightarrow XYZ = (24.68, 19.62, 9.51) \rightarrow L^*u^*v^*$ (51.41, 57.63, 26.60)

$\varepsilon = 0.01$ $rgb = (0.71, 0.38, 0.29) \rightarrow L^*u^*v^*$ (50.54, 63.28, 26.27)

$\varepsilon = 0.05$ $rgb = (0.75, 0.3, 0.25) \rightarrow L^*u^*v^*$ (47.95, 87.34, 24.89)

$\varepsilon = 0.25$ $rgb = (0.95, 0, 0.05) \rightarrow L^*u^*v^*$ (50.58, 165.40, 34.65)

∴ After Interpolate $C_0 = L^*u^*v^* = (50.975, 60.455, 26.485) \rightarrow$ rgb (0.705, 0.39, 0.296)

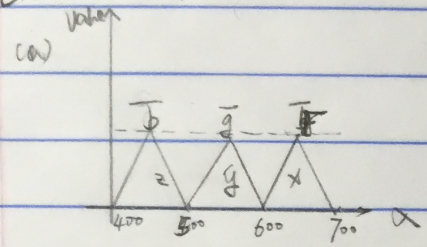
$C_1 = L^*u^*v^* = (49.680, 72.485, 25.745) \rightarrow$ rgb (0.727, 0.35, 0.29)

$C_2 = L^*u^*v^* = (50.995, 111.515, 30.625) \rightarrow$ rgb (0.83, 0.8, 0.22)

When ε is small, the result is nearly same; when ε increase, the variance will increase.

Exercise 28.7

(a)



rgb(1,0,0) → XYZ(0,

∴ 400~500 x,y,z=(0,0,1) A

500~600 xyz = (1,1,0) B

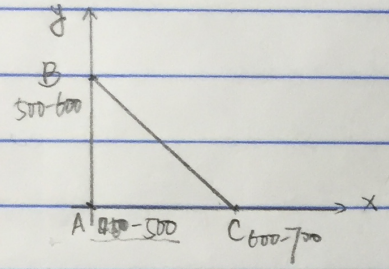
600~700 xyz = (1,0,0) C

CIE Diagrams is a Triangle of △ABC

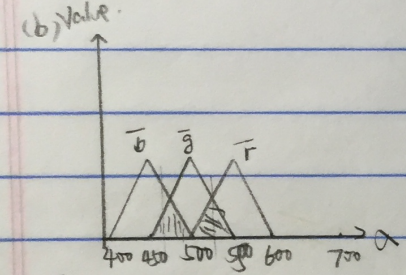
Need 3 primary (A,B,C) to reproduce color

A=(400~500) C(600~700) B(500,600)

Inside △ABC is the combination of rgb color



(b)

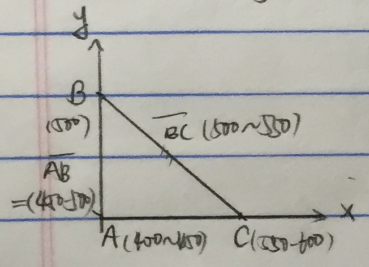


400~450 A(0,0,1)

$\overline{AB}$ 450~500 (0,y,z) $\overline{BC}$ 500~550 (x,y,0) $\overline{BC}$

500 B(0,1,0)

550~600 C(1,0,0)



Need 3 primaries to represent a triangle.

CIE is a triangle.

A is (400~450)

B is (500

C is (550~600)

Line AB is (450-500)

Line BC is (500-550)