

Assignment 8

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Computer Graphics 578

November 23, 2015

Exercise 1. (478 and 578) Express the point $(0,2,0)$ as a quaternion. Following the steps shown in class for computing the result of a rotation using quaternions, find the result of rotating this point 30 degrees around the line that passes through the origin and the point $(1,1,1)$.

Answer:

$$\because q_p = (0, \vec{p}), \quad q = [\cos(\phi/2); \sin(\phi/2)\vec{n}], \quad q'_p = qq_pq^{-1}$$

$$\text{Also, } \because \vec{p} = (0, 2, 0), \quad \vec{n} = \frac{(1, 1, 1)}{\sqrt{1^2 + 1^2 + 1^2}}, \quad \phi = 30^\circ$$

$$\therefore q = \left(\frac{\sqrt{6}+\sqrt{2}}{4}, \frac{\sqrt{6}-\sqrt{2}}{4}\vec{n}\right), \quad q_p = (0, (0, 2, 0)), \quad q^{-1} = \left(\frac{\sqrt{6}+\sqrt{2}}{4}, \left(\frac{\sqrt{6}-3\sqrt{2}}{12}, \frac{\sqrt{6}-3\sqrt{2}}{12}, \frac{\sqrt{6}-3\sqrt{2}}{12}\right)\right)$$

$$\therefore qq_p = \left(\frac{\sqrt{6}-3\sqrt{2}}{6}, \left(\frac{\sqrt{6}-3\sqrt{2}}{6}, \frac{\sqrt{6}+\sqrt{2}}{2}, \frac{3\sqrt{2}-\sqrt{6}}{6}\right)\right)$$

$$\therefore qq_pq^{-1} = \left(0, \left(\frac{2-2\sqrt{3}}{3}, \frac{2+2\sqrt{3}}{3}, \frac{2}{3}\right)\right)$$

$$\therefore q'_p = \left(\frac{2-2\sqrt{3}}{3}, \frac{2+2\sqrt{3}}{3}, \frac{2}{3}\right)$$

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