Exercise 28.1 Apply $C(\alpha, C_1, C_2)$

1. In RGB Model
$$\begin{cases} C_1 = r_1 R + g_1 G + b_1 B \\ C_2 = r_2 R + g_2 G + b_2 B \end{cases}$$

Then interpolate using RGB Model to get $C(\alpha \cdot C_1 \cdot C_2) = (1-\alpha) C_1 + \alpha C_2$ <1>

$$C_{interpolate} = [(1-\alpha) r_1 + \alpha r_2] R + [(1-\alpha) g_1 + \alpha g_2] G + [(1-\alpha) b_1 + \alpha b_2] B \quad \text{<2>}$$

2. In YIQ Model, from RGB to YIQ:

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = M \begin{bmatrix} R \\ G \\ B \end{bmatrix}, \quad M = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & -0.322 \\ 0.211 & -0.523 & 0.312 \end{bmatrix}$$

∴ YIQ
$$\begin{cases} C_1' = M \cdot C_1 \\ C_2' = M C_2 \end{cases}$$
So $C'_{interpolate} = (1-\alpha) C_1' + \alpha C_2'$
$= (1-\alpha) M C_1 + \alpha C_2$ <3>

When transforming from YIQ to RGB

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = M^{-1} \begin{bmatrix} Y \\ I \\ Q \end{bmatrix} \quad \text{<4>}$$

$\therefore C_{interpolate} = M^{-1} C'_{interpolate}$

$= (1-\alpha) M^{-1} M \, C_1 + \alpha \cdot M^{-1} M C_2$

$= (1-\alpha) C_1 + \alpha C_2$ <5>

According to <1> and <5>, using different model can get exactly same result.

system: CIE XYZ, YIQ, sRGB

Because they all can transform using

$$\text{System} = M \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

which is invertible.

Exercise of.2

① In RGB model, the mixing color is C

$$C = (0.7+\frac{\varepsilon}{2},\ 0.4-\varepsilon,\ 0.3-\frac{\varepsilon}{2})$$

② In $L^*u^*v^*$

$$L^* = \begin{cases} 116\,(Y/Y_n)^{\frac{1}{3}}-16 & \frac{Y}{Y_n} < 0.008856 \\ 903.3\,Y & \frac{Y}{Y_n} \geq 0.008856 \end{cases}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \to M \begin{bmatrix} G \\ R \\ B \end{bmatrix}$$

$\therefore C_{xyz} = M C_{RGB}$, (X_n, Y_n, Z_n) is White $=(1,1,1)$

$$M = \begin{bmatrix} 3.2410 & -1.5374 & -0.4986 \\ -0.9692 & 1.8760 & 0.0416 \\ 0.0556 & -0.2040, & 1.057 \end{bmatrix}$$

$\therefore C_{L^*u^*v^*}$, $u' = \frac{4x}{X+15Y+3Z}$ $\quad u_n = \frac{4}{19}$ $\quad v' = \frac{9}{19}$

$v' = \frac{9Y}{X+15Y+3Z}$

$u^* = 13L^*(u'-u_n')$

$v^* = 13L^*(v'-v_n')$

According to ① and ②

In RGB $C_0 = (0.705, 0.39, 0.295)$ when $\varepsilon = 0.01$

$C_1 = (0.725, 0.35, 0.275)$ $\quad \varepsilon = 0.05$

$C_2 = (0.825, 0.15, 0.175)$ $\quad \varepsilon = 0.25$

In $L^*u^*v^*$ $Y/Y_n \geq 0.008856$ when $\varepsilon = 0.01$ or $\varepsilon = 0.05$ or $\varepsilon = 0.25$

$$\therefore \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = M \begin{bmatrix} R \\ G \\ B \end{bmatrix} \quad \therefore$$

For $(r,g,b) = (0.7, 0.4, 0.3) \to XYZ = (24.68, 19.62, 9.51) \to L^*u^*v^*$ $(51.41, 57.63, 26.60)$

$\varepsilon = 0.01$ $rgb = (0.71, 0.38, 0.29) \to L^*u^*v^*$ $(50.54, 63.23, 26.57)$

$\varepsilon = 0.05$ $rgb = (0.75, 0.3, 0.25) \to L^*u^*v^*$ $(47.95, 87.34, 24.89)$

$\varepsilon = 0.25$ $rgb = (0.95, 0, 0.05) \to L^*u^*v^*$ $(50.58, 165.40, 34.65)$

$\therefore$ After Interpolate $C_0 = L^*u^*v^* = (50.975, 60.455, 26.435) \to rgb\ (0.705, 0.39, 0.296)$

$C_1 = L^*u^*v^* = (49.680, 72.485, 25.745) \to rgb\ (0.727, 0.35, 0.28)$

$C_2 = L^*u^*v^* = (50.995, 111.525, 30.625) \to rgb\ (0.839, 0.18, 0.22)$

When ε is small, the result is nearly same; when ε increase, the variance will increase

Exercise 28.7

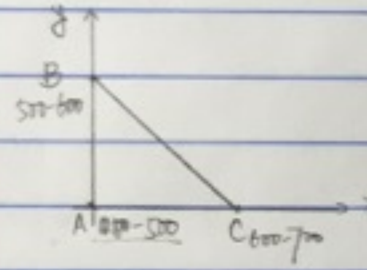
(a)

rgb(1,0,0) → XYZ(0,

∴ 400~500 x,y,z=(0,0,1) A

500~600 xyz = (0,1,0) B

600~700 xyz= (1,0,0) C



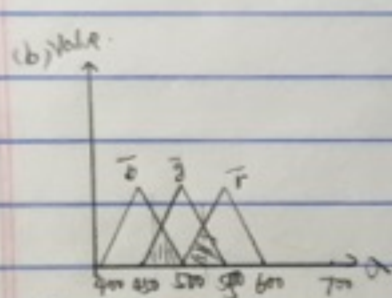
CIE Diagrams is a Triangle of △ABC

Need 3 primary (A,B,C) to reproduce color

A=(450~500) C(600~700) B(500,600)

Inside △ABC is the combination of rgb color

(b)

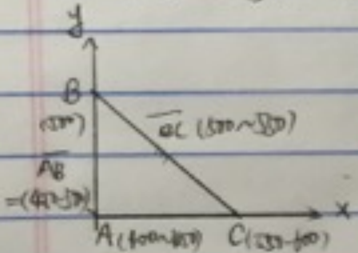


400~450 A(0,0,1)

$\overline{AB}$ 450~500 (0,y,z) $\overline{BC}$ 500~550 (X,y,0) $\overline{BC}$

500 B(0,1,0)

550~600 C(1,0,0)



Need 3 primaries to represent a triangle.

CIE is a triangle.

A is (400~450)

B is (500

C is (550~600)

Line AB is (450~500)

Line BC is (500~550)