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Problem 8

CW is the language for closed walk, it is represented as a string over the alphabet {N,S,E,W}.

We want to prove that CW is not context free

Assume, by way of contradiction, that CW is context-free.

Then it has a CFG in Chomsky Normal Form with k non-terminals.

Then, by the Pumping Lemma for CFLs, every w ∈ CW with |w| > 2^{k-1} can be written

w = $uvxyz$ where

- $vy \neq \varepsilon$,
- $|vxy| \leq 2^k$,
- and for all i ≥ 0 we have $uv^i xy^i z \in$ CW.

let M > 2^k-1 and let w be the following string:

$$w \ = \ N^M E^M S^M W^M$$

$w \ = \ N...NE...ES...SW...W$

This satisfies the rule for CW, so $w \in$ CW.

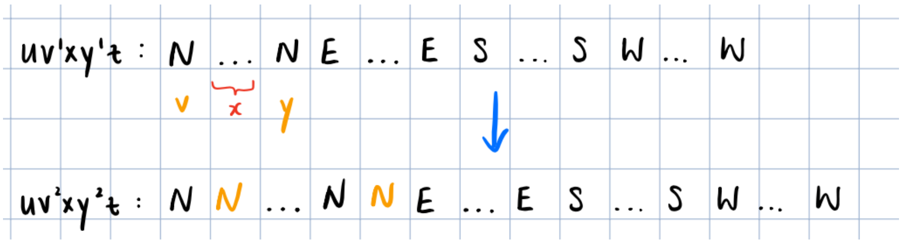
We can also see that our w has a length of $|w|$ > 4(2^k-1), so w satisfies the $|w|$ > 2^{k-1} condition in the Pumping Lemma for CFLs.

Since the window size must be lesser than or equal to 2^k, i.e $|vxy| \leq 2^k$. The window size must be lesser than or equal to M+1, i.e $|vxy| \leq$ M+1, where M > 2^k-1.

Now consider all possible divisions of w into five parts, $w \ = \ uvxyz$. Consider, in particular, the possible locations for v and y within w. We have several cases

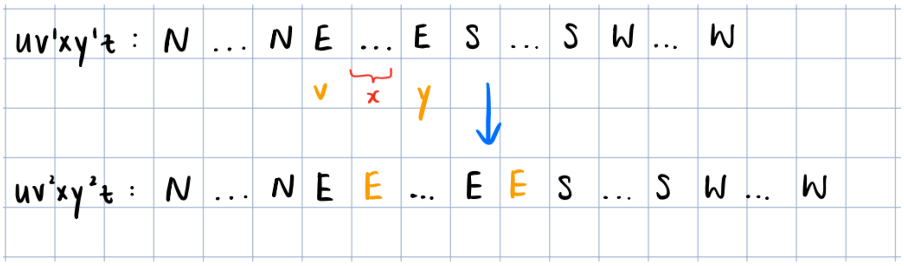
Case 1:

If vxy falls entirely within stretch of N, then uv^2xy^2z has number of S, W and E the same as w except that the stretch of N is now larger (since vy $\neq$ ε). As a result, it cannot be a closed walk since number of N is larger than number of S in w, i.e |N| ≠ |S|. So $uv^2xy^2z \notin$ CW.



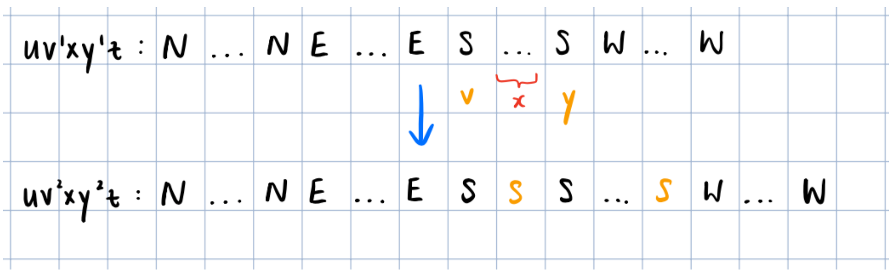
Case 2 (Similar to Case 1):

If vxy falls entirely within stretch of S, then uv^2xy^2z has number of N, W and E the same as w except that the stretch of S is now larger (since vy $\neq$ ε). As a result, it cannot be a closed walk since number of S is larger than number of N in the new string, i.e |S| ≠ |N|. So $uv^2xy^2z \notin$ CW.



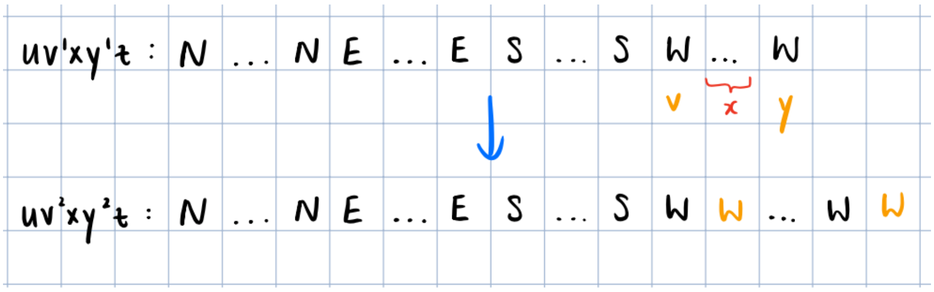
Case 3 (Similar to Case 1):

If vxy falls entirely within stretch of E, then uv^2xy^2z has number of N, S and W the same as w except that the stretch of E is now larger (since vy $\neq$ ε). As a result, it cannot be a closed walk since number of E is larger than number of W in the new string, i.e |E| ≠ |W|. So $uv^2xy^2z \notin$ CW.



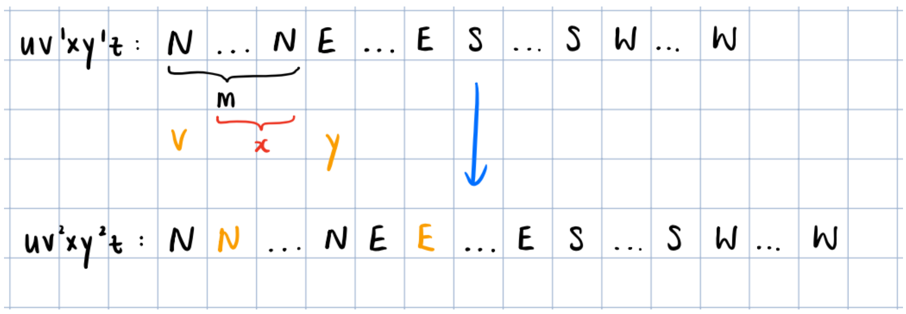
Case 4 (Similar to Case 1):

If vxy falls entirely within stretch of W, then uv^2xy^2z has number of N, S and E the same as w except that the stretch of W is now larger (since vy⊭ ε). As a result, it cannot be a closed walk since number of W is larger than number of E in the new string, i.e |W| ≠ |E|. So $uv^2xy^2z \notin$ CW.



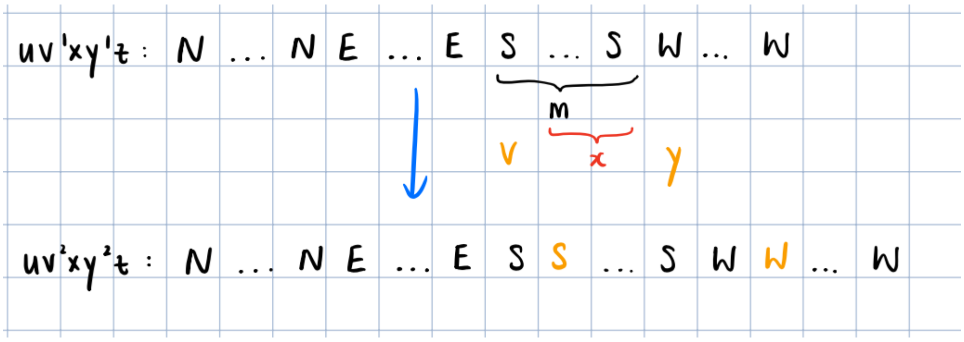
Case 5:

If v falls within N and y falls within E, then uv^2xy^2z has number of S and W the same as w except that the stretch of N or E or both N and E is now larger (since vy⊭ ε). As a result, it cannot be a closed walk since number of N is larger than number of S, as well as number of E is larger than number of W, i.e |N| ≠ |S| and |E| ≠ |W|. So $uv^2xy^2z \notin$ CW.



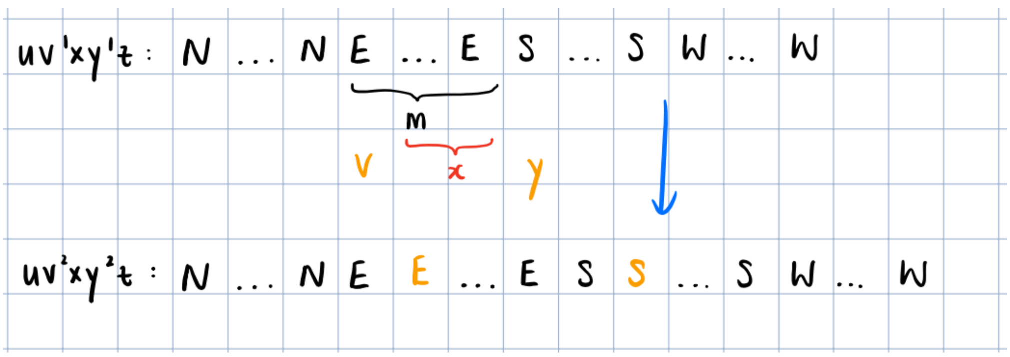
Case 6 (Similar to Case 5):

If v falls within S and y falls within W, then uv^2xy^2z has number of N and E the same as w except that the stretch of S or W or both S and W is now larger (since vy⊭ ε). As a result, it cannot be a closed walk since number of S is larger than number of N, as well as number of W is larger than number of E, i.e |S| ≠ |N| and |W| ≠ |E|. So $uv^2xy^2z \notin$ CW.



Case 7 (Similar to Case 5):

If v falls within E and y falls within S, then uv^2xy^2z has number of N and W the same as w except that the stretch of E or S or both E and S is now larger (since vy⊭ ε). As a result, it cannot be a closed walk since number of E is larger than number of W, as well as number of S is larger than number of N, i.e |E| ≠ |W| and |S| ≠ |N|. So $uv^2xy^2z \notin$ CW.



Conclusion

I have now covered all possibilities for the division of w into five parts, $w = uvxyz$ with vy ⊭ ε. In each case, I found a value of i such that $uv^i xy^i z$ ∉ CW. This contradicts the conclusion of the Pumping Lemma for CFLs. So the initial assumption, that CW is context-free, was wrong. Therefore CW is not context-free.

Note:

Since $|vxy|$ ≤ M+1, it is impossible for v to fall within stretch of N and at the same time y to fall within stretch of S. Hence there is no possibility for N and S to be pumped at the same time, resulting in string $uv^i xy^i z$ which has the same number of N and S. The same goes to E and W.