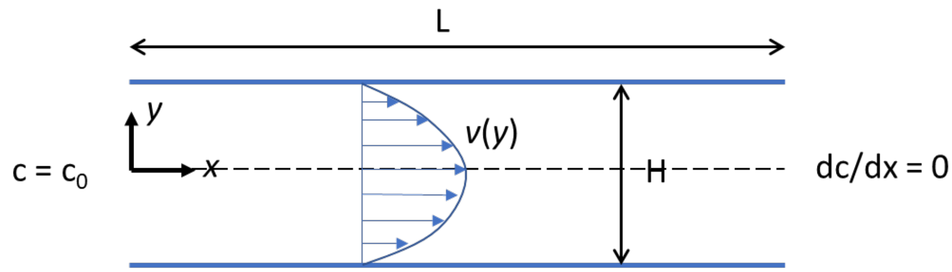
Homework Assignment 4
ENM 502 – Numerical Methods
(Due Friday, April 29)

Problem Statement:

You are to analyze numerically using the finite element method the steady-state concentration of a reactive solute in a rectangular channel of height H and length L in which a solvent is flowing according to a fully developed parabolic (laminar) flow; see the figure below. The walls of the channel are inert so that no flux conditions hold at the upper and lower boundaries. The inlet concentration of the solute is $c_0 = 1$. The channel can be assumed to be long enough so that at the outlet, the concentration gradient is zero in the direction of the flow.



The governing equation describing this situation is given by

$$v(y)\frac{\partial c}{\partial x} = D\nabla^2 c - kc$$

where $v(y) = \alpha\left(\frac{H^2}{4} - y^2\right)$ is the parabolic flow velocity profile, D is the solute diffusion coefficient and k is the (first order) reaction rate constant.

(a) Develop on paper the weak form that results from applying Galerkin's method. Make sure you explicitly write out all the expressions for components of the system of algebraic equations that result as we have done in class.

(b) Implement a finite element code in two dimensions in MATLAB to solve the reaction-convection-diffusion BVP that describes the solute concentration in the channel. Use a uniform grid (i.e., constant element size across the domain) consisting of N_x elements in the x-direction and N_y elements in the y-direction. Note that the elements do not have to be square. Use bilinear elements and express all integrals in terms of integrals over the unit element. The integrals are to be evaluated numerically using 3x3 point Gaussian quadrature (see class notes).

(c) Solve for the solute concentration for a channel with dimensions $H = 2$ and $L = 10$. The problem is controlled by three parameters, α (the flow velocity), D (the diffusivity) and k (the reaction rate). Explore how the solution changes as you change these parameters, starting from a base case of ($\alpha = 1$; $D = 1$; $k = 1$). In particular, what happens as you increase the flow velocity or the reaction rate relative to the diffusivity?

(d) For the base case parameters, explore how the solution accuracy changes as a function of the mesh resolution.