Formally verifying properties of a toy language

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CS550 - Formal Verification

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Outline

Introduction

- Vast majority of programming languages do not have a formally verified core
- Functional languages are studied a lot, but the real world is messy and dominated by imperative languages
- The goal is to define a toy language for which we can formally reason about some properties

Language

Description

- Imperative
- The only value type is a **boolean**
- All variables are "heap" allocated
- Lexical scoping
- Started ambitious (product types, references, deep mutability), *quickly* humbled

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Syntax

```
1  let myVar = true
2  let other = false
3
4  if other ↑ other {
5      myVar := myVar ↑ other
6      free other
7  }
8
9  while myVar {
10     let p = myVar ↑ true
11     myVar := p ↑ (p ↑ p)
12 }
```

$$
\begin{array}{lcll}
\langle expr \rangle & ::= & \mathsf{true} \mid \mathsf{false} & \texttt{Bool} \\
& \mid & \langle expr \rangle_1 \uparrow \langle expr \rangle_2 & \texttt{Nand} \\
& \mid & \langle name \rangle & \texttt{Ident} \\
& \mid & (\langle expr \rangle) & \texttt{Group} \\
\\
\langle stmt \rangle & ::= & \mathsf{let}\ \langle name \rangle = \langle expr \rangle & \texttt{Decl} \\
& \mid & \langle name \rangle := \langle expr \rangle & \texttt{Assign} \\
& \mid & \mathsf{if}\ \langle expr \rangle\ \{\ \langle stmt \rangle\ \} & \texttt{If} \\
& \mid & \mathsf{while}\ \langle expr \rangle\ \{\ \langle stmt \rangle\ \} & \texttt{While} \\
& \mid & \mathsf{free}\ \langle name \rangle & \texttt{Free} \\
& \mid & \langle stmt \rangle_1\ \langle stmt \rangle_2 & \texttt{Seq}
\end{array}
$$

Semantics

- `Free` conservatively forbids further usage of the variable

```
let var = true
if false {
    free var
}
var = false # illegal
```

- `Decl` defines a variable in its scope

```
if false {
    let var = true
}
var = false # not accessible
```

- `Decl` cannot shadow

```
let var = true
if false {
    let var = true # illegal
}
```

Approach

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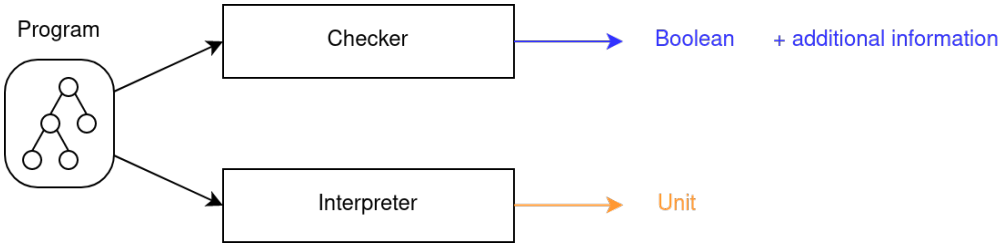
Implement an interpreter for our toy language. Given that a program is valid, prove that its execution by the interpreter enjoys some properties.



Goal: Program is valid $\Rightarrow$ Program executes successfully (does not throw)

Abstract state machine

Environment	$env : \text{Name} \to \text{Abstract location}$
Memory	$mem : \text{Abstract location} \to \text{Value}$
Allocator	$alloc : \_ \to \text{Abstract location}$

Abstract state: $(env, mem, alloc)$

Properties

Closedness

All variable accesses exist in the current environment.

Definition (Closedness)

A program is closed if whenever evaluating `Ident`⟨name⟩ or `Assign`⟨name, expr⟩, *env*(name) is defined.

```
1 var1 := true # error
2 if var2 { # error
3 }
```

No redeclarations

A declaration cannot declare an already declared name.

Definition (No redeclarations)

A program has no redeclarations if whenever evaluating $\texttt{Decl}\langle \text{name}, \text{expr} \rangle$, $\mathit{env}(\text{name})$ is not defined.

```
1 let var = true
2 let var = false # error
3 if true {
4     let var = false # error
5 }
```

Unique ownership

No two variables in the environment point to the same location.

Definition (Unique ownership)

A program exhibits unique ownership when *env* is injective at all times.

No use-after-free

All variable accesses point to existing memory.

Definition (No use-after-free)

A program has no uses-after-free if whenever evaluating $\texttt{Ident}\langle\text{name}\rangle$ or $\texttt{Assign}\langle\text{name}, \text{expr}\rangle$, $\mathit{mem}(\mathit{env}(\text{name}))$ is defined.

```
1 let var = true
2 free var
3 var := true # error
```

Implementation

Implementation

- Stainless interpreter (big-step flavour)
- Lean interpreter
- **Stainless tracer (small-step flavour)**

Implementation details:

- Avoid throwing in Stainless: interpretation functions return either a set of exceptions or the actual result.
- Limited interoperability between Maps and Sets in Stainless: introduce several axioms.

Stainless interpreter

Interpreter : *Prog* $\rightarrow$ *State*
Given a program *Prog*, the interpreter returns the final state *State*.

def evalStmt(stmt: Stmt, state: State): Either[Set[LangException], State]

- Pros: Most natural design, straightforward implementation, *closer* to the checker
- Cons: Symmetries with checker and proofs

Stainless tracer

Interpreter problem with whiles: non termination.

Given a program P the tracer returns a list of states.

We mainly focus on the part of the tracer that given a program P and a state S returns the program P' the state S' given by one step of execution.

$$T : \text{Prog} \to \text{State}^*$$

$$T_1 : \text{Prog} \times \text{State} \to \text{Prog} \times \text{State}$$

- Pros: More control over intermediate states, interesting properties about the trace.
- Cons: Many preconditions about the input state, prove preservation of properties.

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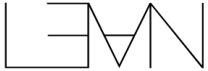
Lean interpreter



```
1  partial def evalStmt
2      (stmt : Stmt) (env : Env) (mem : Memory)
3      (h : isTypeCheckedStmt stmt (keySet env))
4      : Env × Memory := match stmt with
5    -- ...
6    | Stmt.conditional condition body =>
7      let cond := evalExpr condition env mem (typeCheck_conditionalCond h)
8      let (newEnv, newMem) := if cond
9          then evalStmt body env mem (typeCheckStmt_conditionalBody h)
10         else (env, mem)
11
12     -- we drop the new env, but keep the new mem
13     (env, newMem)
14   -- ...
```

Conclusions

Discussion

- Performance has to be traded for provable correctness
- Symmetricity between properties and implementation
- Requires intermediate lemmas proving correlation between properties and implementation
- Proving correctness is hard and very time consuming
- Despite the language being a subset of our original design, we are happy with the results

Future work

- Lack of memory leaks
- More language features

Merci!