

# Formally verifying properties of a toy language

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- No use-after-free

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# Introduction

- Vast majority of programming languages do not have a formally verified core
- Functional languages are studied a lot, but the real world is messy and dominated by imperative languages
- The goal is to define a toy language for which we can formally reason about some properties

# Language

# Description

- Imperative
- The only value type is a **boolean**
- All variables are "heap" allocated
- Lexical scoping
- Started ambitious (product types, references, deep mutability), *quickly* humbled

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# Syntax

```

1  let myVar = true
2  let other = false
3
4  if other ↑ other {
5      myVar := myVar ↑ other
6      free other
7  }
8
9  while myVar {
10     let p = myVar ↑ true
11     myVar := p ↑ (p ↑ p)
12 }
    
```

$\langle expr \rangle$	$::=$ true   false	Bool
	$\langle expr \rangle_1 \uparrow \langle expr \rangle_2$	Nand
	$\langle name \rangle$	Ident
	$(\langle expr \rangle)$	Group
$\langle stmt \rangle$	$::=$ let $\langle name \rangle = \langle expr \rangle$	Decl
	$\langle name \rangle := \langle expr \rangle$	Assign
	if $\langle expr \rangle$ { $\langle stmt \rangle$ }	If
	while $\langle expr \rangle$ { $\langle stmt \rangle$ }	While
	free $\langle name \rangle$	Free
	$\langle stmt \rangle_1 \langle stmt \rangle_2$	Seq

# Semantics

- Free conservatively forbids further usage of the variable

```
let var = true
if false {
  free var
}
var = false # illegal
```

- Decl defines a variable in its scope

```
if false {
  let var = true
}
var = false # not accessible
```

- Decl cannot shadow

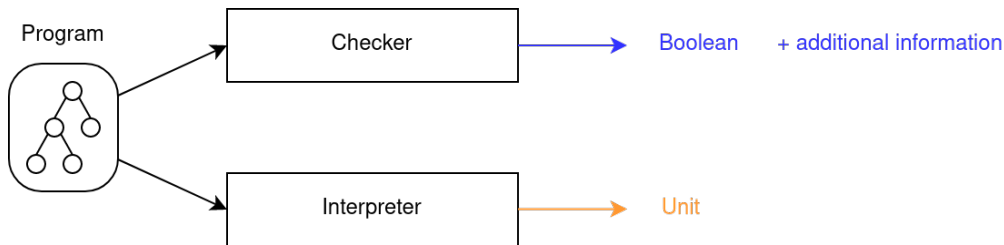
```
let var = true
if false {
  let var = true # illegal
}
```



# Approach

## Approach

Implement an interpreter for our toy language. Given that a program is valid, prove that its execution by the interpreter enjoys some properties.



**Goal:** Program is valid  $\Rightarrow$  Program executes successfully (does not throw)

# Abstract state machine

Environment     $env : \text{Name} \rightarrow \text{Abstract location}$   
 Memory         $mem : \text{Abstract location} \rightarrow \text{Value}$   
 Allocator       $alloc : \_ \rightarrow \text{Abstract location}$

**Abstract state:**  $(env, mem, alloc)$

# Properties

# Closedness

All variable accesses exist in the current environment.

## Definition (Closedness)

A program is closed if whenever evaluating  $\text{Ident}\langle\text{name}\rangle$  or  $\text{Assign}\langle\text{name}, \text{expr}\rangle$ ,  $\text{env}(\text{name})$  is defined.

```
1 var1 := true # error
2 if var2 { # error
3 }
```

# No redeclarations

A declaration cannot declare an already declared name.

## Definition (No redeclarations)

A program has no redeclarations if whenever evaluating  $\text{Decl}\langle \text{name}, \text{expr} \rangle$ ,  $\text{env}(\text{name})$  is not defined.

```
1 let var = true
2 let var = false # error
3 if true {
4     let var = false # error
5 }
```

# Unique ownership

No two variables in the environment point to the same location.

## Definition (Unique ownership)

A program exhibits unique ownership when *env* is injective at all times.

# No use-after-free

All variable accesses point to existing memory.

## Definition (No use-after-free)

A program has no uses-after-free if whenever evaluating  $\text{Ident}\langle\text{name}\rangle$  or  $\text{Assign}\langle\text{name}, \text{expr}\rangle$ ,  $\text{mem}(\text{env}(\text{name}))$  is defined.

```
1 let var = true
2 free var
3 var := true # error
```



# Implementation

# Implementation

- Stainless interpreter (big-step flavour)
- Lean interpreter
- Stainless tracer (small-step flavour)

---

## Implementation details:

- Avoid throwing in Stainless: interpretation functions return either a set of exceptions or the actual result.
- Limited interoperability between Maps and Sets in Stainless: introduce several axioms.

## Stainless interpreter

*Interpreter* :  $Prog \rightarrow State$

Given a program *Prog*, the interpreter returns the final state *State*.

`def evalStmt(stmt: Stmt, state: State): Either[Set[LangException], State]`

- Pros: Most natural design, straightforward implementation, *closer* to the checker
- Cons: Symmetries with checker and proofs

## Stainless tracer

Interpreter problem with whiles: non termination.

Given a program  $P$  the tracer returns a list of states.

We mainly focus on the part of the tracer that given a program  $P$  and a state  $S$  returns the program  $P'$  the state  $S'$  given by one step of execution.

$$T : \text{Prog} \rightarrow \text{State}^*$$

$$T_1 : \text{Prog} \times \text{State} \rightarrow \text{Prog} \times \text{State}$$

- Pros: More control over intermediate states, interesting properties about the trace.
- Cons: Many preconditions about the input state, prove preservation of properties.

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# Lean interpreter

```
1 partial def evalStmt
2   (stmt : Stmt) (env : Env) (mem : Memory)
3   (h : isTypeCheckedStmt stmt (keySet env))
4   : Env × Memory := match stmt with
5   -- ...
6   | Stmt.conditional condition body =>
7     let cond := evalExpr condition env mem (typeCheck_conditionalCond h)
8     let (newEnv, newMem) := if cond
9       then evalStmt body env mem (typeCheckStmt_conditionalBody h)
10      else (env, mem)
11
12     -- we drop the new env, but keep the new mem
13     (env, newMem)
14   -- ...
```

# Conclusions



# Discussion

- Performance has to be traded for provable correctness
- Symmetricity between properties and implementation
- Requires intermediate lemmas proving correlation between properties and implementation
- Proving correctness is hard and very time consuming
- Despite the language being a subset of our original design, we are happy with the results

## Future work

- Lack of memory leaks
- More language features

# Merci!