Labor Market Power

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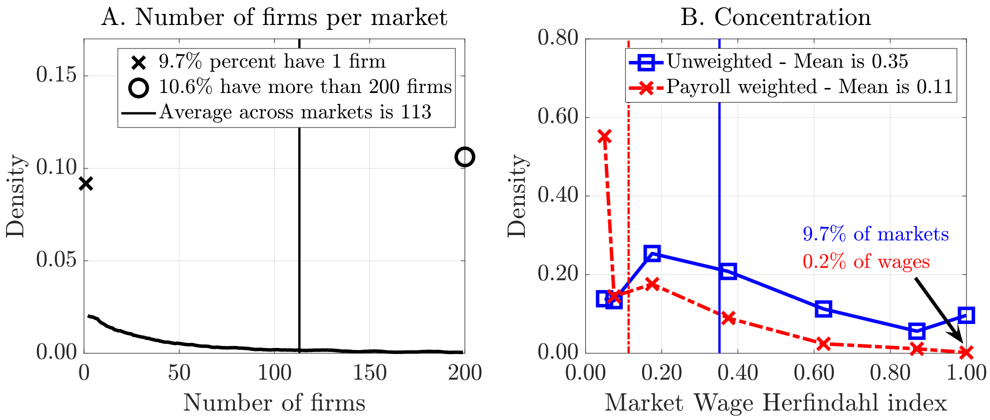
The views expressed herein are those of the authors and not those of the Census or the Federal Reserve System.

Measuring labor market power in the U.S. labor market

- Fact — Labor markets are concentrated. Many employers, concentrated employment.
- Model — Tractable general equilibrium oligopsony model with strategic interaction
- Estimate — Match reduced form responses to changes in tax policy in Census data
- Validate — $\underbrace{\text{(i) Pass-through,}}_{\text{Kline et al, 2018}}$ $\underbrace{\text{(ii) Strategic response}}_{\text{Staiger et al, 2010}}$, $\underbrace{\text{(iii) Mergers}}_{\text{Arnold, 2020}}$
- **Micro** — Measure labor market power in terms of $\underbrace{\text{wage markdowns on MRPL}}_{w_i=\mu_i \times MRPL_i\ ,\ \mathbb{E}[\mu_i]=0.92\ ,\ \mu^*=0.71}$
- **Macro** — Measure labor market power in terms of (i) $\underbrace{\text{welfare,}}_{\text{4 - 7\%}}$ (ii) $\underbrace{\text{Wages vs. Misalloc.}}_{\approx \text{ 1/2 each}}$
- Apply — Link concentration $\leftrightarrow$ labor share.

Motivation



- Market - NAICS3 $\times$ Commuting Zone

Literature

1. Theory

   Oligopsony Robinson (1933), Hotelling (1929), Salop (1979), Bhaskar, Manning & To (2002)

   Oligopoly Atkeson Burstein (2008), Amiti, Itskhoki, Konings (2019), Edmond Midrigan Xu (2015, 2019)

   Frictional Burdett Mortensen (1998), Flinn (2010), Manning (2003, 2006)

   Competitive Card Cardoso Heining Kline (2018), Lamadon Mogstad Setzler (2019)

   New - GE model of strategic interaction in local labor mkts taken to firm-level Census data

2. Empirics

   Concentration Benmelech et al (2018), Azar et al (2018), Rinz (2018)
   Hershbein, Macaluso, Yeh (2019), Rossi-Hansberg et al (2018)

   Corporate Taxes Giroud, Rauh (2019), Suarez Serrato, Zidar (2016)

   Wage pass-through Kline, Petkova, Williams, Zidar (2018), Card, Cardoso, Heining, Kline (2016)

   Wage responses Staiger et al (2010), Derenoncourt et al (2021)

   New - Quantitatively interpret empirical evidence

Model

Environment

Representative family

- Continuum of labor markets $j \in [0, 1]$
- Labor market j has a fixed number of firms $i \in \{1, 2, \ldots, M_j\}$
- Disutility of supplying workers $\{n_{ijt}\}$ across firms

Firms

- Firm i has idiosyncratic productivity z_{ijt}, DRS production
- Hire workers n_{ijt}, rent capital k_{ijt} to produce identical final good

Markets

- Local, Cournot competition for labor
- National, Walrasian markets for output and capital

Household

Preferences

$$\mathcal{U}_0 = \max_{\{n_{ijt}, c_{ijt}, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U\Big(\mathbf{C}_t, \mathbf{N}_t\Big) \quad , \quad \beta \in (0,1) \quad , \quad \varphi > 0$$

Disutility of labor supply

$$\mathbf{N}_t := \left[\int_0^1 \mathbf{N}_{jt}^{\frac{\theta+1}{\theta}} \, dj\right]^{\frac{\theta}{\theta+1}} \quad , \quad \theta > \varphi$$

$$\mathbf{N}_{jt} := \left[n_{1jt}^{\frac{\eta+1}{\eta}} + \cdots + n_{M_j jt}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}} \quad , \quad \eta > \theta$$

Budget constraint

$$\mathbf{C}_t + \Big[K_{t+1} - (1-\delta)K_t\Big] = \int_0^1 \Big[w_{1jt}n_{1jt} + \cdots + w_{M_j jt}n_{M_j jt}\Big] \, dj + R_t K_t + \Pi_t,$$

$$\mathbf{C}_t := \int_0^1 \Big[c_{1jt} + \cdots + c_{M_j jt}\Big] \, dj.$$

Discussion of preferences

1. Across markets

$$\mathbf{N}_t := \left[\int_0^1 \mathbf{N}_{jt}^{\frac{\theta+1}{\theta}}\, dj\right]^{\frac{\theta}{\theta+1}}$$

$\theta \to 0$: Fixed labor supply to each market

$\ldots\ \theta$ proxies *inter*-market mobility costs (e.g. moving)

2. Within markets

$$\mathbf{N}_{jt} := \left[n_{1jt}^{\frac{\eta+1}{\eta}} + \cdots + n_{M_j jt}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}$$

$\eta \to \infty$: All workers to firm with highest wage

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Equivalence result - Nested logit individual choice model

Anderson, De Palma, Thisse (EL 1987) , Verboven (EL 1996)

Firms - Cournot competition

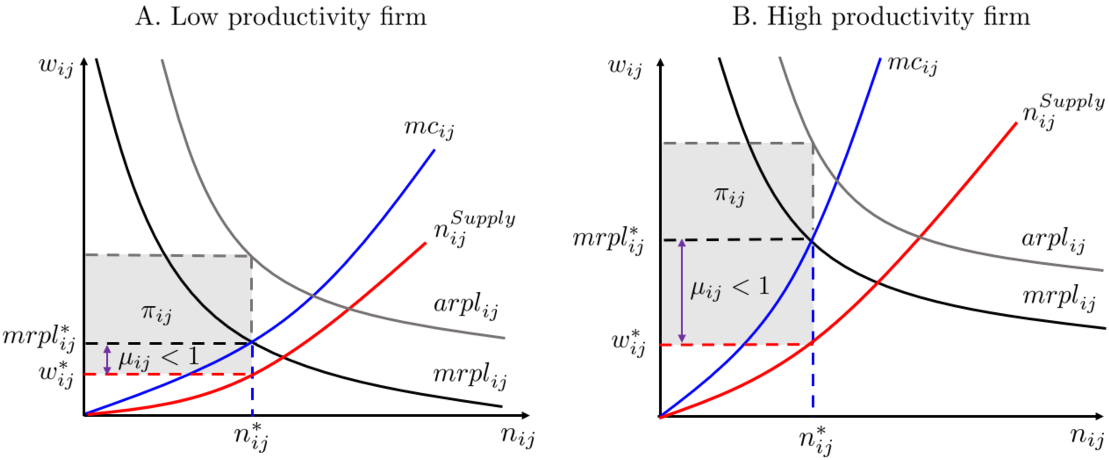
$$\max_{k_{ijt},n_{ijt}} \pi_{ijt}\left(k_{ijt}, n_{ijt}, n^*_{-ijt}\right) = \underbrace{\overline{Z} z_{ijt}\left(k_{ijt}^{1-\gamma} {n_{ijt}}^{\gamma}\right)^{\alpha} - R_t k_{ijt}}_{\widetilde{Z}\widetilde{z}_{ijt} {n_{ijt}}^{\widetilde{\alpha}}} - w\left(n_{ijt}, n^*_{-ijt}, \mathbf{N}_t\right) n_{ijt}$$

subject to

$$w\left(n_{ijt}, n^*_{-ijt}, \mathbf{N}_t\right) = \left(\frac{n_{ijt}}{\mathbf{N}_{jt}}\right)^{\frac{1}{\eta}} \left(\frac{\mathbf{N}_{jt}}{\mathbf{N}_t}\right)^{\frac{1}{\theta}} \mathbf{W}_t \quad , \quad \mathbf{W}_t = -\frac{U_N\left(\mathbf{C}_t, \mathbf{N}_t\right)}{U_C\left(\mathbf{C}_t, \mathbf{N}_t\right)}$$

$$\mathbf{N}_{jt} = \left[{n^*_{1jt}}^{\frac{\eta+1}{\eta}} + \ldots {n_{ijt}}^{\frac{\eta+1}{\eta}} + \ldots {n^*_{M_j jt}}^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}}$$

Partial equilibrium



- **Result** - Endogenous negative $cov\left(\mu_{ij}, z_{ij}\right) < 0$

▸ Example - Shares, Markdowns, Wages, Employment

Market equilibrium - Markdowns

Wages

$$w_{ijt} \quad = \quad \mu_{ijt} \quad \underbrace{mrpl_{ijt}}_{\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ijt} n_{ijt}^{\widetilde{\alpha}-1}} \quad = \quad \mu_{ijt} \; \widetilde{\alpha} \quad \left(\frac{va_{ijt}}{n_{ijt}}\right)$$

Market equilibrium - Markdowns

Wages

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Markdown

$$\underbrace{\mu_{ijt} = \frac{\varepsilon_{ijt}}{\varepsilon_{ijt}+1}}_{\text{Markdown}} \quad , \quad \underbrace{\varepsilon_{ijt} := \left. \frac{\partial \log n_{ijt}}{\partial \log w_{ijt}} \right|_{n^*_{-ijt}} = \left[s_{ijt}^{wn} \frac{1}{\theta} + \left(1 - s_{ijt}^{wn}\right) \frac{1}{\eta} \right]^{-1}}_{\textit{Equilibrium labor supply elasticity}} \quad , \quad \underbrace{s_{ijt}^{wn} = \frac{w_{ijt} n_{ijt}}{\sum_{k \in j} w_{kjt} n_{kjt}}}_{\text{Wage bill share}}$$

Market equilibrium - Markdowns

Wages

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Result 1 - Market equilibrium $\left\{ s_{ijt}, \mu_{ijt} \right\}_{i \in j}$ is independent of aggregates.

▸ Tilde variables   ▸ Product market competition

Market equilibrium - Markdowns

Wages

$$\Uparrow w_{ijt} = \mu_{ijt} \underbrace{mrpl_{ijt}}_{\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ijt} n_{ijt}^{\widetilde{\alpha}-1}} = \Downarrow \mu_{ijt}\, \widetilde{\alpha} \Uparrow \left(\frac{va_{ijt}}{n_{ijt}}\right)$$

Markdown

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Result 2 - Pass-through is less than one

Tilde variables   Product market competition

General equilibrium - Two wedges

1. General equilibrium

- Aggregates $\{\mathbf{W}, \mathbf{N}, \mathbf{C}, \mathbf{Y}\}$ depend on misallocation ω and markdown μ

$$\text{Output:} \quad \mathbf{Y} = \omega \times \widetilde{Z}\mathbf{N}^{\widetilde{\alpha}} \quad , \quad \text{Labor demand:} \quad \mathbf{W} = \mu \times \widetilde{\alpha}\widetilde{Z}\mathbf{N}^{\widetilde{\alpha}-1}$$

$$\text{Goods market clearing:} \quad \mathbf{C} = \text{Const.} \times \mathbf{Y} \quad , \quad \text{Labor supply:} \quad \mathbf{W} = -\frac{U_N(\mathbf{C},\mathbf{N})}{U_C(\mathbf{C},\mathbf{N})}$$

- Negative cov. b/w productivity ($z_{ij} \uparrow$) and markdowns ($\mu_{ij} \downarrow$) induces misallocation:

$$\omega = \int \left(\frac{\widetilde{z}_j}{\widetilde{z}}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \left(\frac{\mu_j}{\mu}\right)^{\frac{\alpha\theta}{1+\theta(1-\widetilde{\alpha})}} \omega_j \, dj \quad , \quad \omega_j = \sum_{i \in j} \left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}} \left(\frac{\mu_{ij}}{\mu_j}\right)^{\frac{\eta\widetilde{\alpha}}{1+\eta(1-\widetilde{\alpha})}}$$

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2. Two *"Monopsony limits"*

- If (i) $\theta \to \eta$, or (ii) $s_{ij} \to 0$, then markdowns are identical, $\mu = \dfrac{\eta}{\eta+1}$ and $\omega = 1$.

General equilibrium - Labor share

Concentration

$$HHI^{wn} := \int_0^1 s_j^{wn} HHI_j^{wn} dj \in [0,1] \quad , \quad HHI_j^{wn} := \sum_{i \in j} \left(s_{ij}^{wn}\right)^2$$

General equilibrium - Labor share

Concentration

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Labor share

$$LS = \tilde{\alpha}\frac{\boldsymbol{\mu}}{\boldsymbol{\omega}} = \underbrace{\alpha\gamma}_{\text{Comp. } LS} \times \underbrace{\left[HHI^{wn}\left(\frac{\theta}{\theta+1}\right)^{-1} + \left(1 - HHI^{wn}\right)\left(\frac{\eta}{\eta+1}\right)^{-1}\right]^{-1}}_{\text{Labor market power adjustment}}$$

▸ *HHI* measures over time ▸ Labor share algebra ▸ Bias of employment HHI relative to wage bill HHI

Concentration, 1977 to 2013

Data: LBD, **Market:** NAICS3 $\times$ Commuting Zone, (e.g. Mnpls X furniture mfg.)

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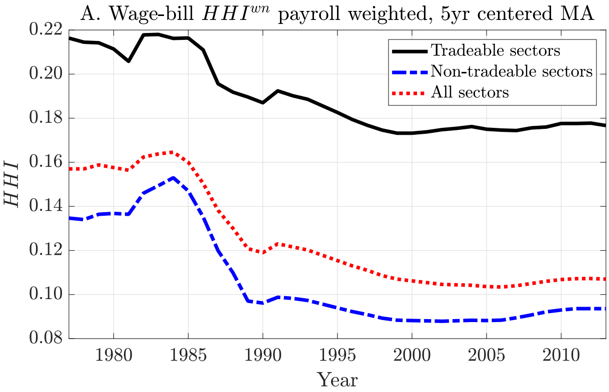

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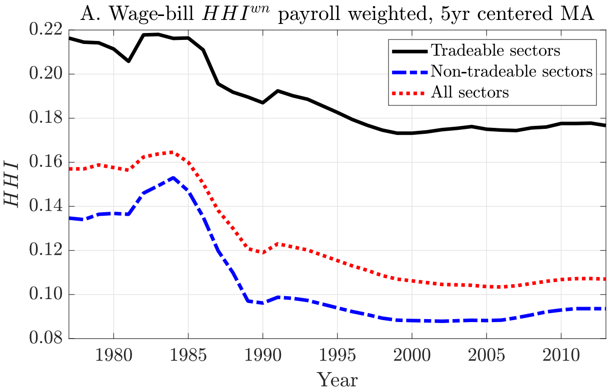

$(HHI^{wn})^{-1}$ increased from 6.25 in 1977 to 9.43 in 2013

Result - At estimated $\{\theta, \eta, \alpha, \gamma\}$ increasing competition added 4.32 ppt to Labor Share

Calibration

Indirect inference

- *Equilibrium labor supply elasticities*, if known, would identify (θ, η)

$$\varepsilon\left(s_{ij}, \theta, \eta\right) := \left.\frac{\partial \log n_{ij}}{\partial \log w_{ij}}\right|_{n^*_{-ij}} = \left[s_{ij}\frac{1}{\theta} + (1 - s_{ij})\,\frac{1}{\eta}\right]^{-1}$$

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- Even a perfect quasi-experiment is only going to deliver *reduced form elasticities*

$$\epsilon\left(s_{ij}, \theta, \eta, \dots\right) := \frac{\Delta \log n_{ijt}}{\Delta \log w_{ijt}} \approx \frac{\varepsilon\left(s_{ijt}, \theta, \eta\right)}{1 + \varepsilon\left(s_{ijt}, \theta, \eta\right)\left(\frac{\eta-\theta}{\theta\eta}\right)\left(\frac{\sum_{k\neq i} s_{kjt}\Delta \log n_{kjt}}{\Delta \log n_{ijt}}\right)}$$

Monopsony limits

- Under either limit $\varepsilon = \epsilon$

Indirect inference

- *Equilibrium labor supply elasticities*, if known, would identify (θ, η)

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Indirect inference

1. Data Quasi-experiment to estimate average relationship $\widehat{\epsilon}^{Data}(s)$
2. Model Replicate in the model and $\min_{\theta,\eta} \left|\widehat{\epsilon}^{Data}(s) - \widehat{\epsilon}^{Model}(s, \theta, \eta)\right|$

1. Empirical estimates of reduced form elasticities - $\widehat{\epsilon}^{Data}(s)$

State corporate tax changes

- Policy Large changes in state corporate taxes (Giroud Rauh, JPE 2019)
- Variation Within market-state j, s, across firm payroll share $i \in j$
- Sample Tradeable C-corps operating in 2 markets in state s, 2002-2012

Specification

$$\log n_{ijt} = \alpha_{ij} + \phi_t + \psi s^{wn}_{ijt-1} + \beta_n \tau_{s(j)t} + \gamma_n \left(s^{wn}_{ijt-1} \times \tau_{s(j)t} \right) + \Gamma X_{s(j)t} + e_{ijt}$$

$$\widehat{\epsilon}^{Data}(s_{ijt}) = \frac{\widehat{d\log n_{ijt}}}{\widehat{d\log w_{ijt}}} = \frac{\widehat{\beta}_n + \widehat{\gamma}_n s^{wn}_{ijt}}{\widehat{\beta}_w + \widehat{\gamma}_w s^{wn}_{ijt}}$$

Table - Sample summary statistics

Table - Regression results

2. Model simulation of reduced form elasticities - $\widehat{\epsilon}^{Model}(s,\theta,\eta)$

Firms maximize post-tax profits

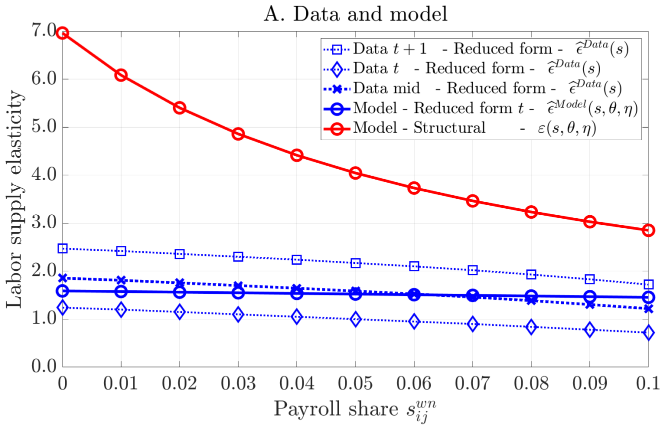
$$\pi_{ijt} = \left(1-\tau_C\right)\lambda_C\, z_{ijt}\left(n_{ijt}^{\gamma}k_{ijt}^{1-\gamma}\right)^{\alpha} - \left(1-\tau_C\right)w_{ijt}n_{ijt} - \left(1-\tau_C\lambda_K\right)R_t k_{ijt}$$

1. Tax on profits — $\tau_C = 7.15\%$ (Giroud Rauh, 2019)
2. Distorts after tax return on fraction of capital — $\lambda_K = 0.31$ (Graham et. al., 2014)
3. Only affects C-Corps — 43% of firms (CBP)
4. C-Corps are $\lambda_C > 1$ times more productive: — 66% of emp. (CBP)
5. Simulate a tax cut — $\Delta\tau_C = -1$ ppt (Giroud Rauh, 2019)

- SMM: Data: $\left\{\tau_C, \lambda_K, \Delta\tau_C, G(M_j)\right\}$. Estimate: $\left\{\lambda_C, \theta, \eta, \widetilde{\alpha}, F(z), \overline{Z}, \overline{\varphi}\right\}$

▸ Details - How corporate taxes map to $MRPL_{ijt}$ through Accounting vs. Economic profits

Reduced form and *Structural* elasticities $(\theta = 0.45, \eta = 6.96)$



- Implies 13% markdown at atomistic firms ($s = 0$), 70% markdown at large firm ($s = 1$)
- Strategic interactions imply $\varepsilon > \epsilon$ – large firms shift labor supply to small firms

Calib Table
Figure - Distribution of ε_{ij} and markdowns μ_{ij}
Figure - Bias of idiosyncratic shocks: $\hat{\epsilon} > \varepsilon$

Parameters and moments

Parameter	Description	Value	Moment	Model	Data
A. Common					
Parameter	**Description**	**Value**	**Moment**	**Model**	**Data**
r	Risk free rate	0.04			
δ	Depreciation rate	0.10			
φ	Aggregate Frisch elasticity	0.50			
J	Number of markets	5,000			
B. Tradeable					
$G(M_j)$	Mix two paretos		Mean, Std. Dev., Skewness of distribution		
			15 percent of markets have only 1 firm		
ω_C	Share of firms that are C-corps	0.42	Share of estabs. that are C-corps (CBP, 2014)		
τ_C	State corporate tax rate	0.069	Mean of state corp. tax rate $\tau_{C,st}$		
Δ_τ	State corporate tax rate increase	0.010	Std. dev. of annual $\tau_{C,st}$		
λ_K	Fraction of capital debt financed	0.213	Tradeable industries (Compustat, 2014)		
Estimated					
η	Within market substitutability	6.96	Average $\widehat{\epsilon}^{Data}(s^{wn})$ for $s^{wn} \in [0, 0.05]$	1.55	1.70
θ	Across market substitutability	0.45	Average $\widehat{\epsilon}^{Data}(s^{wn})$ for $s^{wn} \in [0.05, 0.10]$	1.48	1.38
Δ_C	Relative productivity of C-corps	1.41	Emp. share of C-corps	0.66	0.66
$\sigma_{\tilde{z}}$	Productivity dispersion	0.248	Payroll weighted $\mathbb{E}[HHI^{wn}]$	0.17	0.17
α	DRS parameter	1.000	Labor share	0.53	0.54
γ	Labor exponent	0.799	Capital share	0.19	0.19
$\widetilde{Z}$	Productivity shifter	1.53e+04	Ave. firm size	34.6	34.6
$\overline{\varphi}$	Labor disutility shifter	2.261	Ave. payroll per worker (\$000)	58.3	58.3

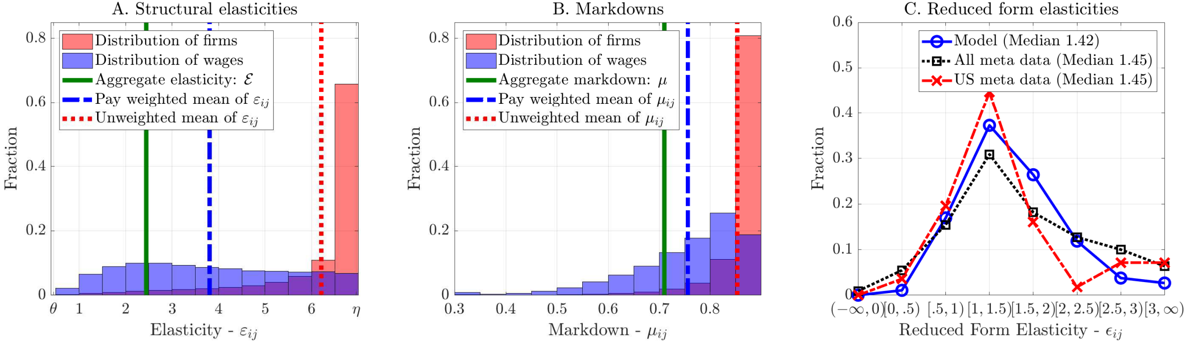
▸ Figure - Distribution of M_j. Details - Calibration of $G(M_j)$
▸ Figure - Distribution of elasticities ε_{ij} and markdowns μ_{ij}

Parameters and moments

C. Economy-wide					
Parameter	**Description**	**Value**	**Moment**	**Model**	**Data**
$G(M_j)$	Mix two paretos		Mean, Std. Dev., Skewness of distribution		
			9 percent of markets have only 1 firm		
η	Within market substitutability	6.96	Held fixed at estimated tradeable value		
θ	Across market substitutability	0.45	Held fixed at estimated tradeable value		
Estimated					
σ_z	Productivity dispersion	0.327	Payroll weighted $\mathbb{E}[HHI^{wn}]$	0.11	0.11
α	DRS parameter	0.957	Labor share	0.57	0.57
γ	Labor exponent	0.812	Capital share	0.18	0.18
$\overline{Z}$	Productivity shifter	1.59e+04	Ave. firm size	22.8	22.8
$\overline{\varphi}$	Labor disutility shifter	3.081	Ave. payroll per worker ($000)	43.8	43.8

- Fewer markets with 1 firm
- Lower concentration
- Smaller firms with lower pay

Distribution of labor supply elasticities and markdowns



▸ Back - Share dependent labor supply elasticity

▸ Back - Calibration table

Next

1. Validation
   (i) Pass-through (Kline, Petkova, Williams, Zidar, QJE 2019)
   (ii) Strategic interactions (Staiger, Spetz, Phibs, JOLE 2010)
   (iii) Mergers (Arnold, JMP 2020)
2. Measurement
   (i) Welfare gains associated with efficient allocation
   (ii) Decomposition into μ and ω
3. Applications
   - Concentration and the labor share

VALIDATION

1. Pass-through

- Replicate Kline et al (2018) patent quasi-experiment
- Shock to $\uparrow z_{ij}$ to match average increase in $\uparrow (va_{ij} / n_{ij})$ of 13 percent
- Compute pass-through in logs

$$\Delta \log w_{it} = \gamma_i + \beta \Delta \log vapw_{it} + e_{it} \quad , \quad \widehat{\beta} = 0.47$$

- Monopsony limits vs. Oligopsony
    - In either of the monopsony limits, then μ_{ijt} is constant, and $\widehat{\beta} = 1$

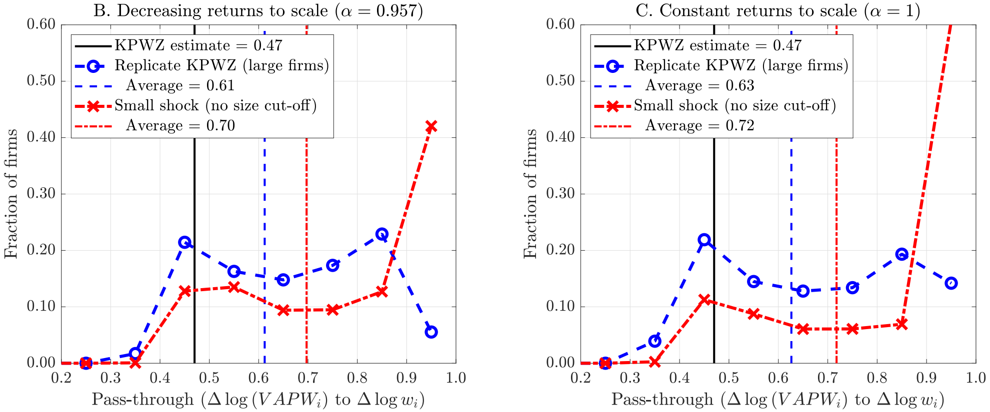
$$\Delta \log w_{ijt} = \Delta \log \mu_{ijt} + \Delta \log vapw_{ijt}$$

    - In our model, if $\eta > \theta$, then $\Delta \log \mu_{ijt} < 0$ and $\widehat{\beta} < 1$

Details - Implementation of experiment in the model

Details - Data and model summary statistics. Increase in value added per worker etc.

1. Pass-through



▸ Details - Implementation of experiment in the model

▸ Details - Data and model summary statistics. Increase in value added per worker etc.

2. Competitor responses

- Replicate Staiger et al (2010) VA hospital quasi-experiment
- Firm VA, j increases wages from 2-3 percent below market average to market average
- Compute competitor wage responses

$$\Delta \log w_{ij} = \alpha_0 + \alpha_1 \Delta \log w_{VA,j} + e_{ij} \quad , \quad \widehat{\alpha}_1 = 0.13$$

- Monopsony limits vs. Oligopsony
  - In either of the monopsony limits, competitors do not respond and $\widehat{\alpha}_1 = 0$

$$\Delta \log w_{ij} = \Omega(s_{ij}) \Delta \log vapw_{ij} + \Big(1 - \Omega(s_{ij})\Big) \sum_{k \neq i} \left(\frac{s_{kj}}{1 - s_{ij}}\right) \Delta \log w_{kj}$$

  - In our model, if $\eta > \theta$, then $\Omega(s_{ij}) < 1$ and $\widehat{\alpha}_1 > 0$

▸ Details - Implementation of experiment in the model

▸ Details - Data and model summary statistics. Increase in value added per worker etc.

2. Competitor responses

- Replicate Staiger et al (2010) VA hospital quasi-experiment
- Firm VA, j increases wages from 2-3 percent below market average to market average
- Compute competitor wage responses

$$\Delta \log w_{ij} = \alpha_0 + \alpha_1 \Delta \log w_{VA,j} + e_{ij} \quad , \quad \widehat{\alpha}_1 = 0.13$$

	Model	Data Staiger et al (2010)
Log avg. market wage minus focal firm log wage ex-ante ($\log \bar{w}_{-ij} - \log w_{ij}$)	0.02	0.02
Number of firms in market	10.89	10.90
Elasticity of competitor wages WRT focal wage ($\frac{d \log \bar{w}_{-ij}}{d \log w_{ij}}$)	0.07	0.13

▸ Details - Implementation of experiment in the model

▸ Details - Data and model summary statistics. Increase in value added per worker etc.

3. Mergers

- Replicate Arnold et al (2020) mergers in local labor markets
- Merge firms 1 and 2: $\mu(s_{1j}) \to \mu(s'_{1j} + s'_{2j})$
- Compute merging firm and market changes in employment and wages
- Monopsony limits vs. Oligopsony
  - In either of the monopsony limits, competitors do not respond
  - In our economy: (i) merging firms' wages and employment fall, (ii) merging firms' combined shares fall and competitors' combined shares increase, (iii) market wage and market employment fall

▸ Details - Implementation of experiment in the model
▸ Details - Data and model summary statistics. Increase in value added per worker etc.

3. Mergers

Calibrate size of merging entities to match descriptive statistics in Arnold (2020)

Moment	A. Arnold (2020) Reference	A. Arnold (2020) Value	B. Replicate Value
Part I. Outcomes at merging firms			
Target: Median employment pre-merger	Table 1	116.0	116.3
Change in log employment (weighted)	Table 3(1)	-0.144	-0.070
Change in log payroll (weighted)	Table 3(4)	-0.121	-0.083
Change in log worker earnings	Table 5(2)	-0.008	-0.013
... high concentration market	Table 6(1)	-0.031	-0.041
... medium concentration market	Table 6(2)	-0.008	-0.012
$\Delta HHI_j = \alpha + \beta \Delta \widehat{HHI}_j$	Table 8(1)	0.834	0.904
Part II. Market outcomes in markets with large predicted changes in hhi_j			
Target: Average change in log HHI_j	Figure 8A	0.170	0.171
Elasticity of market wage to *HHI*	Table 10(3)	-0.219	-0.475
... above median *HHI*	Table 10(6)	-0.259	-0.505

WELFARE

Counterfactual

How much would consumption have to increase to be indifferent between U.S. labor market and a competitive labor market?

Counterfactual

How much would consumption have to increase to be indifferent between U.S. labor market and a competitive labor market?

Efficient allocation

- Corresponds to a competitive equilibrium in which $\mu_{ij}^* = 1$
- Implies aggregates $\boldsymbol{\mu}^* = 1$, and $\boldsymbol{\omega}^* = 1$
- Compare this to our benchmark economy with $\boldsymbol{\mu}_o = 0.72$, and $\boldsymbol{\omega}_o = 0.97$
- Identical system of aggregate conditions

$$\mathbf{Y} = \boldsymbol{\omega} \times \widetilde{\mathbf{Z}}\mathbf{N}^{\widetilde{\alpha}} \quad , \quad \mathbf{W} = \boldsymbol{\mu} \times \widetilde{\alpha} Z \mathbf{N}^{\widetilde{\alpha}-1}$$

Results

Define - Welfare gain associated with competitive labor market, λ_{SS}

$$U\Big((1+\lambda_{SS})\mathbf{C}_o,\mathbf{N}_o\Big)=U\Big(\mathbf{C}^*,\mathbf{N}^*\Big)\quad,\quad U\Big(\mathbf{C},\mathbf{N}\Big)=\frac{\mathbf{C}^{1-\sigma}}{1-\sigma}-\frac{1}{\overline{\varphi}^{\frac{1}{\varphi}}}\frac{\mathbf{N}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$$

Results

Define - Welfare gain associated with competitive labor market, λ_{SS}

$$U\Big((1+\lambda_{SS})\mathbf{C}_o, \mathbf{N}_o\Big) = U\Big(\mathbf{C}^*, \mathbf{N}^*\Big) \quad , \quad U\Big(\mathbf{C}, \mathbf{N}\Big) = \frac{\mathbf{C}^{1-\sigma}}{1-\sigma} - \frac{1}{\overline{\varphi}^{\frac{1}{\varphi}}} \frac{\mathbf{N}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$$

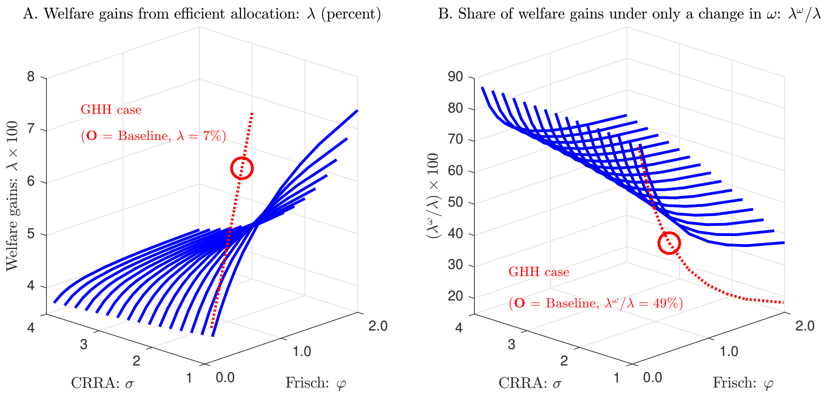
Frisch elasticity	A. Welfare				B. Labor market		C. Concentration	
	GHH		$\sigma = 1.0$	$\sigma = 2.0$				
	Steady state	Transition	Steady state	Steady state	Ave. wage	Agg. emp.	Unweighted	Weighted
φ	$\lambda_{SS} \times 100$	$\lambda_{Trans} \times 100$	$\lambda_{SS} \times 100$	$\lambda_{SS} \times 100$	$\mathbf{E}[w_{it}]$	$\sum_i n_{it}$	ΔHHI^{wn}	ΔHHI^{wn}
0.2	4.8	4.2	4.0	3.6	48.5	1.1	0.19	0.15
0.5	7.0	5.7	5.3	4.5	48.1	11.3	0.19	0.15
0.8	9.2	7.2	6.0	4.8	47.5	22.5	0.19	0.15

Results

- Labor market power leads to between 4% to 9% welfare losses
- Increase in competition but increase in concentration

Figure - Transition Dynamics
Back - Measurement 1 - Welfare

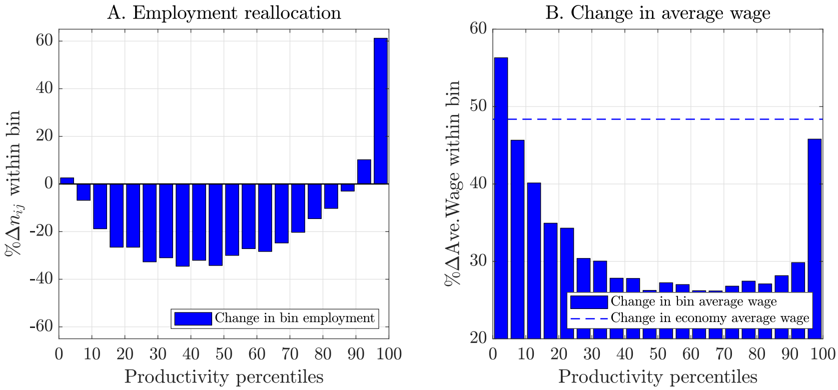
Decomposition and robustness



Interpretation

- ↑ Frisch – Larger gains (working more less painful), less gains from reallocation
- ↑ Wealth effects – Higher labor supply in distorted oligopoly economy

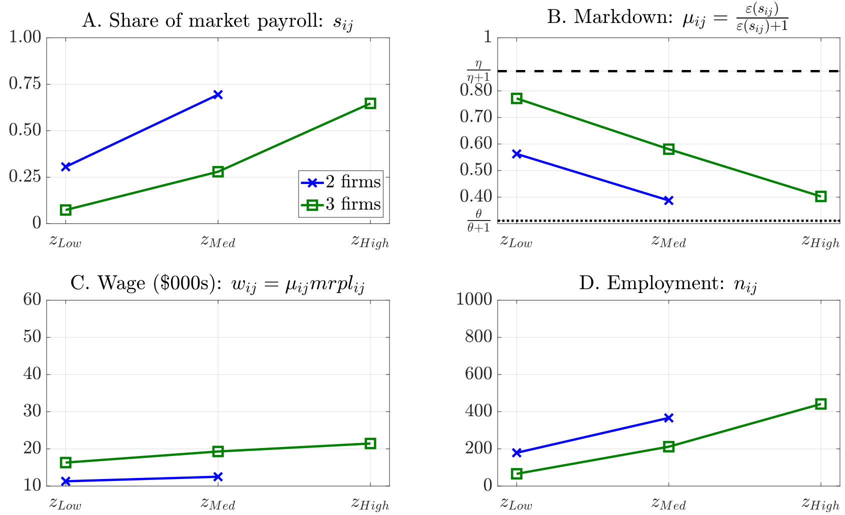
Reallocation



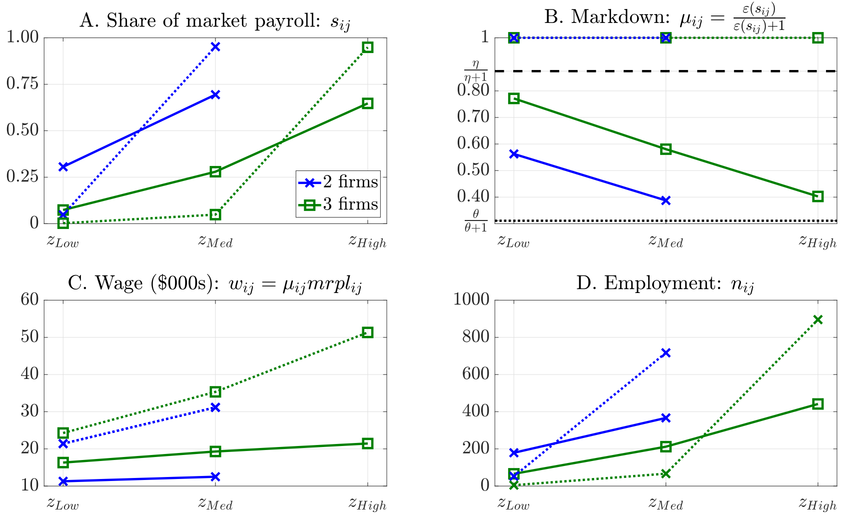
Interpretation

- Significant reallocation of employment toward higher productivity firms
- Achieved through higher (shadow) wages

Reallocation



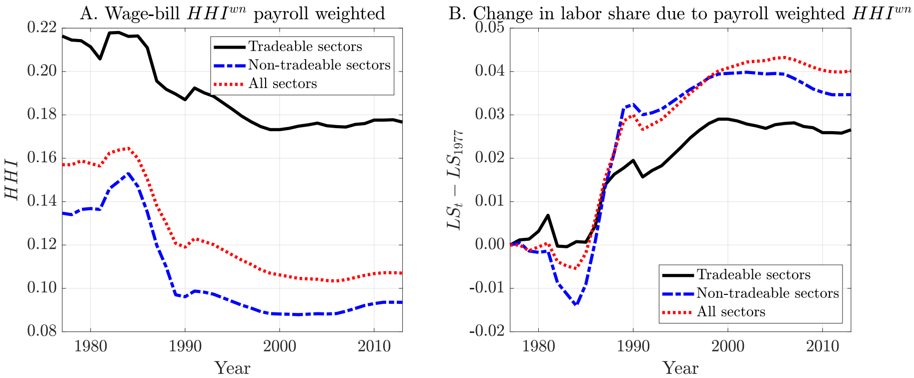
Reallocation



APPLICATION

What are implications for labor share?

$$LS = \alpha\gamma \left[\left(1 - HHI^{wn}\right) \left(\frac{\eta}{\eta+1}\right)^{-1} + HHI^{wn} \left(\frac{\theta}{\theta+1}\right)^{-1} \right]^{-1}$$



Result - Labor market concentration does not explain declining labor share

Contributions

1. Develop and validate general equilibrium oligopsony model
   - Provides a model relevant concentration measure
   - Kick the tires hard: Replicate pass-through, strategic interactions, and merger responses
2. New evidence on size dependent corporate tax response, used in estimation
   - Quantitatively important to model strategic interaction if inferring labor market power via employment and wage responses to identified shocks
3. Welfare losses from labor market power are large: 4% to 9%
4. Welfare and concentration both increase in efficient allocation
5. Declining payroll concentration from 1976-2014 implies a +4.3 ppt labor share rise

THANK YOU!

APPENDIX

Representation - Logit model

- Workers $m \in [0,1]$ with committed income $y_m \sim F(y)$
- Minimize total labor disutility of attaining y_m

$$\min_{ij} \ \log h_m - \xi_{ij} \qquad \text{s.t.} \qquad w_{ij} h_m = y_m$$

- Random labor disutility

$$F\left(\xi_{11}, \ldots, \xi_{ij}, \ldots \xi_{NJ}\right) = \exp\left[- \sum_{j=1}^{J} \left(\sum_{i=1}^{M_j} e^{-(1+\eta)\xi_{ij}} \right)^{\frac{1+\theta}{1+\eta}} \right]$$

- Labor supply

$$n_{ij} = \frac{w_{ij}^{\eta}}{\sum_{i=1}^{M_j} w_{ij}^{1+\eta}} \frac{\left[\sum_{i=1}^{M_j} w_{ij}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{l=1}^{J}\left[\sum_{k=1}^{M_l} w_{kl}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}} Y. \tag{1}$$

- **Result** Delivers same supply system as rep. agent CES Back

Firms - Notation

- Optimizing out capital

$$\pi_{ijt} = \max_{n_{ijt}} \; \widetilde{Z}\widetilde{z}_{ijt} n_{ijt}^{\widetilde{\alpha}} - w_{ijt} n_{ijt}$$

- The '*widetilde*' variables are defined as follows:

$$\begin{aligned}
\widetilde{\alpha} &:= \frac{\alpha\gamma}{1-(1-\gamma)\alpha} \\
\widetilde{z}_{ijt} &:= [1-(1-\gamma)\alpha]\left(\frac{(1-\gamma)\alpha}{R_t}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} z_{ijt}^{\frac{1}{1-(1-\gamma)\alpha}} \\
\widetilde{Z} &:= \overline{Z}^{\frac{1}{1-(1-\gamma)\alpha}}
\end{aligned}$$

- Note that $(1-\gamma)\alpha$ is capital's share of income

▸ Back - Nash equilibrium markdowns

Computation

A firm's wage-bill share is defined by their relative wage:

$$s_{ij}^{wn} = \left(\frac{w_{ij}}{\mathbf{w}_j}\right)^{1+\eta}$$

Within a market, an equilibrium can be solved by iterating through the following conditions given a guess of $\mathbf{s}_j^{wn} = \left(s_{1j}^{wn}, \ldots, s_{M_j j}^{wn}\right)$

$$\varepsilon_{ij} = \begin{cases} s_{ij}^{wn}\theta + \left(1 - s_{ij}^{wn}\right)\eta & \text{Bertrand} \\ \left[s_{ij}^{wn}\frac{1}{\theta} + \left(1 - s_{ij}^{wn}\right)\frac{1}{\eta}\right]^{-1} & \text{Cournot} \end{cases}$$

$$\mu_{ij} = \frac{\varepsilon_{ij}}{\varepsilon_{ij} + 1}$$

$$w_{ij} = \mu_{ij} MRPL_{ij}$$

$$\mathbf{w}_j = \left[\int_0^1 w_{ij}^{1+\eta} dj\right]^{\frac{1}{1+\eta}}$$

$$s_{ij}^{wn(NEW)} = \left(\frac{w_{ij}}{\mathbf{w}_j}\right)^{1+\eta}$$

We guess equal shares, and then iterate until $\mathbf{s}_j^{wn(NEW)} = \mathbf{s}_j^{wn}$. Back

DRS Computation

Sub in inverse supply curve for n_{ij}:

$$MRPL_{ij} = \omega \mathbf{W}^{(1-\widetilde{\alpha})(\theta-\varphi)} \widehat{z}_{ij} \left\{ w_{ij}^{-\eta} \mathbf{w}_j^{\eta-\theta} \right\}^{1-\widetilde{\alpha}}$$

Write the wage in terms of the marginal revenue product of labor:

$$\begin{aligned} w_{ij} &= \mu_{ij} MRPL_{ij} \\ &= \mu_{ij} \omega \mathbf{W}^{(1-\widetilde{\alpha})(\theta-\varphi)} \widehat{z}_{ij} \left\{ w_{ij}^{-\eta} \mathbf{w}_j^{\eta-\theta} \right\}^{1-\widetilde{\alpha}} \end{aligned}$$

Use $\mathbf{w}_j = w_{ij} s_{ij}^{-\frac{1}{\eta+1}}$: $w_{ij} = \omega^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \mathbf{W}^{\frac{(1-\widetilde{\alpha})(\theta-\varphi)}{1+(1-\widetilde{\alpha})\theta}} \mu_{ij}^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \widehat{z}_{ij}^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} s_{ij}^{-\frac{(1-\widetilde{\alpha})(\eta-\theta)}{\eta+1} \frac{1}{1+(1-\widetilde{\alpha})\theta}}$

We will solve for an equilibrium in 'hatted' variables, and then rescale:

$$\widehat{w}_{ij} := \mu_{ij}^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \widehat{z}_{ij}^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} s_{ij}^{-\frac{(1-\widetilde{\alpha})(\eta-\theta)}{\eta+1} \frac{1}{1+(1-\widetilde{\alpha})\theta}}$$

$$\widehat{\mathbf{w}}_j := \left[\sum_{i \in j} \widehat{w}_{ij}^{\eta+1} \right]^{\frac{1}{\eta+1}}$$

$$\widehat{\mathbf{W}} := \left[\int \widehat{\mathbf{w}}_j^{\theta+1} dj \right]^{\frac{1}{\theta+1}}$$

$$\widehat{n}_{ij} := \left(\frac{\widehat{w}_{ij}}{\widehat{\mathbf{w}}_j} \right)^{\eta} \left(\frac{\widehat{\mathbf{w}}_j}{\widehat{\mathbf{W}}} \right)^{\theta} \left(\frac{\widehat{\mathbf{W}}}{1} \right)^{\varphi}$$

▸ Back

DRS Computation

These definitions imply that

$$w_{ij} = \omega^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \mathbf{W}^{\frac{(1-\widetilde{\alpha})(\theta-\varphi)}{1+(1-\widetilde{\alpha})\theta}} \widehat{w}_{ij}$$

$$\mathbf{w}_j = \omega^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \mathbf{W}^{\frac{(1-\widetilde{\alpha})(\theta-\varphi)}{1+(1-\widetilde{\alpha})\theta}} \widehat{\mathbf{w}}_j$$

$$\mathbf{W} = \omega^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \mathbf{W}^{\frac{(1-\widetilde{\alpha})(\theta-\varphi)}{1+(1-\widetilde{\alpha})\theta}} \widehat{\mathbf{W}}$$

These definitions allow us to compute the equilibrium market shares in terms of ‘hatted’ variables:

$$s_j^{wn} = \left(\frac{w_{ij}}{\mathbf{w}_j}\right)^{\eta+1} = \left(\frac{\widehat{w}_{ij}}{\widehat{\mathbf{w}}_j}\right)^{\eta+1} \tag{2}$$

▸ Back

DRS Computation

For a given set of values for parameters $\{\overline{\varphi}, \widetilde{Z}, \widetilde{\alpha}, \beta, \delta\}$, we can solve for the non-constant returns to scale equilibrium as follows:

1. Guess $\mathbf{s}_j^{wn} = (s_{1j}^{wn}, \ldots, s_{M_j j}^{wn})$
2. Compute $\{\epsilon_{ij}\}$ and $\{\mu_{ij}\}$ using the industry eq formulas.
3. Construct the 'hatted' equilibrium values as follows:

$$\widehat{w}_{ij} = \mu_{ij}^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \widehat{z}_{ij}^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} s_{ij}^{-\frac{(1-\widetilde{\alpha})(\eta-\theta)}{\eta+1}\frac{1}{1+(1-\widetilde{\alpha})\theta}}$$

$$\widehat{\mathbf{w}}_j = \left[\sum_{i\in j} \widehat{w}_{ij}^{\eta+1}\right]^{\frac{1}{\eta+1}}$$

$$\widehat{\mathbf{W}} = \left[\int \widehat{\mathbf{w}}_j^{\theta+1} dj\right]^{\frac{1}{\theta+1}}$$

$$\widehat{n}_{ij} = \left(\frac{\widehat{w}_{ij}}{\widehat{\mathbf{w}}_j}\right)^{\eta} \left(\frac{\widehat{\mathbf{w}}_j}{\widehat{\mathbf{W}}}\right)^{\theta} \left(\frac{\widehat{\mathbf{W}}}{1}\right)^{\varphi}$$

4. Update the wage-bill share vector using previous expression (prior slide).
5. Iterate until convergence of wage-bill shares. Back

DRS Computation

Recovering true equilibrium values from 'hatted' equilibrium: Once the 'hatted' equilibrium is solved, we can construct the true equilibrium values by rescaling as follows:

$$\omega = \frac{\widetilde{Z}}{\overline{\varphi}^{1-\widetilde{\alpha}}} \tag{3a}$$

$$\mathbf{W} = \omega^{\frac{1}{1+(1-\widetilde{\alpha})\varphi}} \widehat{\mathbf{W}}^{\frac{1+(1-\widetilde{\alpha})\theta}{1+(1-\widetilde{\alpha})\varphi}} \tag{3b}$$

$$w_{ij} = \omega^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \mathbf{W}^{\frac{(1-\widetilde{\alpha})(\theta-\varphi)}{1+(1-\widetilde{\alpha})\theta}} \widehat{w}_{ij} \tag{3c}$$

$$\mathbf{w}_j = \omega^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \mathbf{W}^{\frac{(1-\widetilde{\alpha})(\theta-\varphi)}{1+(1-\widetilde{\alpha})\theta}} \widehat{\mathbf{w}}_j \tag{3d}$$

$$n_{ij} = \overline{\varphi} \left(\frac{w_{ij}}{\mathbf{w}_j}\right)^{\eta} \left(\frac{\mathbf{w}_j}{\mathbf{W}}\right)^{\theta} \left(\frac{\mathbf{W}}{1}\right)^{\varphi} \tag{3e}$$

Back

DRS Computation

We set the scale parameters $\overline{\varphi}$ and $\widetilde{Z}$ in order to match average firm size observed in the data ($AveFirmSize^{Data} = 27.96$ from Table 5), and average earnings per worker in the data ($AveEarnings^{Data} = \$65,773$ from Table 5):

$$\widehat{AveFirmSize}^{Data} = \frac{\int \left\{ \sum_{i \in j} n_{ij} \right\} dj}{\int \left\{ M_j \right\} dj} \tag{4a}$$

$$\widehat{AveEarnings}^{Data} = \frac{\int \left\{ \sum_{i \in j} w_{ij} n_{ij} \right\} dj}{\int \left\{ \sum_{i \in j} n_{ij} \right\} dj} \tag{4b}$$

Back

DRS Computation

To compute the values of $\overline{\varphi}$ and $\widetilde{Z}$ that allow us to match $AveFirmSize^{Data}$ and $AveEarnings^{Data}$, we substitute the model's values for n_{ij}, w_{ij}, and M_j into $AveFirmSize^{Data}$ and $AveEarnings^{Data}$. We repetitively substitute equations (3a) through (3e) into (4a) and (4b). We then solve for $\overline{\varphi}$ and $\widetilde{Z}$:

$$\overline{\varphi} = \frac{\frac{AveFirmSize^{Data}}{\widehat{AveFirmSize}^{Model}}}{\left(\frac{AveEarnings^{Data}}{\widehat{AveEarnings}^{Model}}\right)^{\varphi}} \tag{5}$$

$$\widetilde{Z} = \overline{\varphi}^{1-\widetilde{\alpha}} \left(\frac{AveEarnings^{Data}}{\widehat{AveEarnings}^{Model}}\right)^{1+(1-\widetilde{\alpha})\varphi} \times \widehat{\boldsymbol{W}}^{-(1-\widetilde{\alpha})(\theta-\varphi)} \tag{6}$$

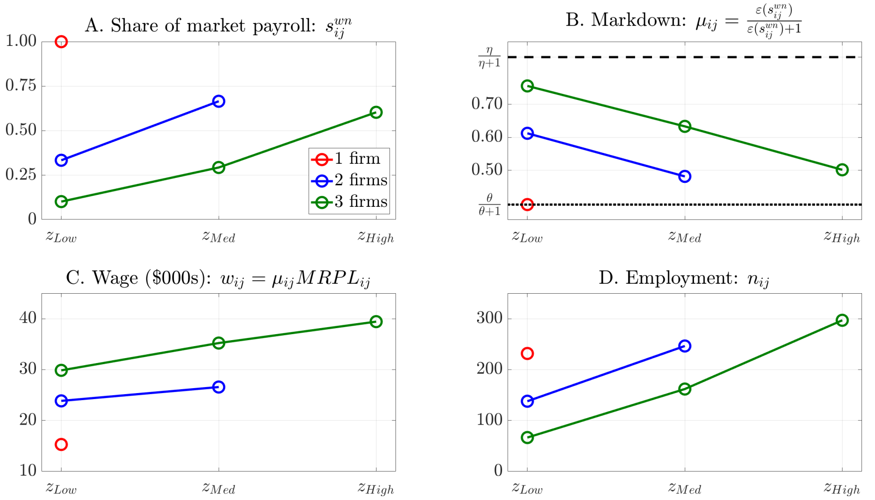
where

$$\widehat{AveFirmSize}^{Model} = \frac{\int \left\{\sum_{i\in j} \widehat{n}_{ij}\right\} dj}{\int \left\{M_j\right\} dj}$$

$$\widehat{AveEarnings}^{Model} = \frac{\int \left\{\sum_{i\in j} \widehat{w}_{ij}\widehat{n}_{ij}\right\} dj}{\int \left\{\sum_{i\in j} \widehat{n}_{ij}\right\} dj}$$

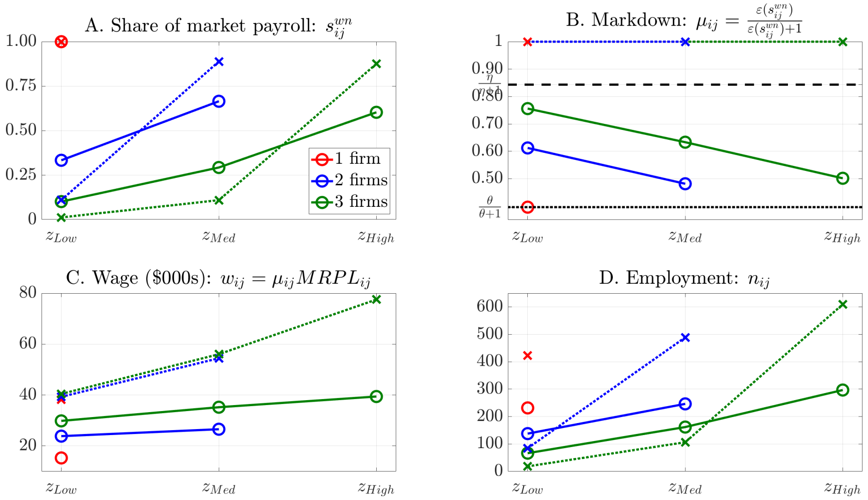
▸ Back

Firms - Local labor market equilibrium



Back to simple graph

Firms - Local labor market equilibrium - Competitive



Back to simple graph

Aggregation – Labor share and concentration

$$ls_{ij} = \frac{w_{ij}n_{ij}}{\widetilde{z}_{ij}\widetilde{Z}n_{ij}^{\widetilde{\alpha}}}$$

$$ls_{ij} = \widetilde{\alpha}\frac{w_{ij}}{\widetilde{\alpha}\widetilde{z}_{ij}\widetilde{Z}n_{ij}^{\widetilde{\alpha}-1}}$$

$$ls_{ij} = \widetilde{\alpha}\frac{w_{ij}}{MRPL_{ij}}$$

$$ls_{ij} = \widetilde{\alpha}\mu_{ij}$$

Let $y_{ij} = \widetilde{z}_{ij}\widetilde{Z}n_{ij}^{\widetilde{\alpha}}$. At the market level, the labor share in market j, LS_j, is given by the following expression:

$$\begin{aligned} LS_j &= \left[\frac{\sum_i y_{ij}}{\sum_i w_{ij}n_{ij}}\right]^{-1} \\ &= \left[\sum_i \left(\frac{w_{ij}n_{ij}}{\sum_i w_{ij}n_{ij}}\right)\frac{y_{ij}}{w_{ij}n_{ij}}\right]^{-1} \end{aligned}$$

Back - Labor share and aggregation

Aggregation – Labor share and concentration

Using the definition of the wage-bill share,

$$LS_j^{-1} = \sum_i s_{ij}^{wn} \widetilde{\alpha}^{-1} \mu_{ij}^{-1}$$

$$LS_j^{-1} = \widetilde{\alpha}^{-1} \sum_i s_{ij}^{wn} \left[\frac{\eta+1}{\eta} + s_{ij}^{wn} \left(\frac{\theta+1}{\theta} - \frac{\eta+1}{\eta} \right) \right]$$

$$LS_j^{-1} = \widetilde{\alpha}^{-1} \frac{\eta+1}{\eta} + \widetilde{\alpha}^{-1} \left(\frac{\theta+1}{\theta} - \frac{\eta+1}{\eta} \right) HHI_j^{wn}$$

Define the inverse Herfindahl at the market level as $IHI_j^{wn} = (HHI_j^{wn})^{-1}$. Aggregating across markets yields the economy-wide labor share:

$$LS^{-1} = \frac{\int \sum y_{ij}}{\int \sum w_{ij} n_{ij}} = \int \frac{\sum w_{ij} n_{ij}}{\int \sum w_{ij} n_{ij}} \frac{\sum y_{ij}}{\sum w_{ij} n_{ij}} = \int s_j^{wn} LS_j^{-1}$$

$$LS^{-1} = \frac{1}{\widetilde{\alpha}} \left(\frac{\eta+1}{\eta} + \left(\frac{\theta+1}{\theta} - \frac{\eta+1}{\eta} \right) \int s_j^{wn} \left(IHI_j^{wn} \right)^{-1} dj \right)$$

Back - Labor share and aggregation

Aggregation – Labor share and concentration

$$\textbf{Wage Bill Herfindahl:} \quad HHI_j^{wn} \equiv \sum_i (s_{ij}^{wn})^2 \quad , \quad s_{ij}^{wn} = \frac{w_{ij} n_{ij}}{\sum_i w_{ij} n_{ij}}$$

$$\textbf{Employment Herfindahl:} \quad HHI_j^{n} \equiv \sum_i (s_{ij}^{n})^2 \quad , \quad s_{ij}^{n} = \frac{n_{ij}}{\sum_i n_{ij}}$$

Note:

$$HHI_j^{wn} = \sum_i \left(\frac{w_{ij}}{\sum_i s_{ij}^n w_{ij}} \right) \left(s_{ij}^n\right)^2$$

1. Employment Herfindahl yields less concentration:

   Since $cov\left(s_{ij}^n, w_{ij}\right) > 0$, then $HHI_j^{wn} > HHI_j^n$

2. $cov\left(s_{ij}^n, w_{ij}\right)$ is endogenous and depends on concentration

Back - Labor share and aggregation

Table: Summary Statistics, Longitudinal Employer Database 1976 and 2014

	(A) Firm-market-level averages	
	1976	**2014**
Total firm pay (000s)	470.90	1839.00
Total firm employment	37.09	27.96
Pay per employee	$ 12,696	$ 65,773
Firm-level observations	660,000	810,000
	(B) Market-level averages	
	1976	**2014**
Wage-bill Herfindahl (Unweighted)	0.45	0.45
Employment Herfindahl (Unweighted)	0.43	0.42
Wage-bill Herfindahl (Weighted by market's share of total employment)	0.19	0.14
Employment Herfindahl (Weighted by market's share of total employment)	0.18	0.12
Firms per market	42.56	51.60
Percent of markets with 1 firm	14.6%	14.7%
National employment share of markets with 1 firm	0.63%	0.36%
Market-level observations	15,000	16,000
	(C) Market-level correlations	
	1976	**2014**
Correlation of Wage-bill Herfindahl and number of firms	-0.22	-0.21
Correlation of Wage-bill Herfindahl and Std. Dev. Of Relative Wages	-0.49	-0.51
Correlation of Wage-bill Herfindahl and Employment Herfindahl	0.98	0.98
Correlation of Wage-bill Herfindahl and Market Employment	-0.20	-0.21
Market-level observations	15,000	16,000

Notes: Tradeable NAICS2 codes (11,21,31,32,33,55).

▸ Back to overline chart ▸ Back to calib

	(A) Firm-market-level averages 1976	2014
Total firm pay (000s)	209.40	1102.00
Total firm employment	19.43	23.21
Pay per employee	$ 10,777	$ 47,480
Firm-Market level observations	3,746,000	5,854,000

	(B) Market-level averages 1976	2014
Wage-bill Herfindahl (Unweighted)	0.36	0.34
Employment Herfindahl (Unweighted)	0.33	0.32
Wage-bill Herfindahl (Weighted by market's share of total wage-bill)	0.17	0.11
Employment Herfindahl (Weighted by market's share of total wage-bill)	0.15	0.09
Firms per market	75.70	113.20
Percent of markets with 1 firm	10.4%	9.4%
Market level observations	49,000	52,000

	(C) Market-level correlations 1976	2014
Correlation of Wage-bill Herfindahl and number of firms	-0.20	-0.17
Correlation of Wage-bill Herfindahl and Employment Herfindahl	0.97	0.97
Correlation of Wage-bill Herfindahl and Market Employment	-0.15	-0.16
Market-level observations	49,000	52,000

Notes: **All NAICS.** Back to overline chart Back to calib

Corporate taxes, labor and wages

	$\log n_{ijt}$ (1)	$\log n_{ijt}$ (2)	$\log w_{ijt}$ (3)	$\log w_{ijt}$ (4)
$\tau_{s(j)t}$	-0.00357***	-0.00368***	-0.00181***	-0.00187***
	(0.000644)	(0.000757)	(0.000584)	(0.000588)
s^{wn}_{ijt}		2.085***		0.214***
		(0.0467)		(0.00724)
$\tau_{s(j)t} \times s^{wn}_{ijt}$		0.0158***		0.00310***
		(0.00495)		(0.000749)
Year FE	Y	Y	Y	Y
Commuting Zone/Industry FE	Y	Y	Y	Y
Firm×State FE	Y	Y	Y	Y
R-squared	0.872	0.877	0.819	0.821
Round N	4,425,000	4,425,000	4,425,000	4,425,000

Notes: *** p<0.01, ** p<0.05, * p<0.1 Standard errors clustered at State×Year level. Tradeable C-Corps from 2002 to 2014.

- 1 ppt increase in $\tau_{s(j)t}$ causes a -0.36% change in employment
- Elast. is -0.32% at mean $s^{wn}_{ij} = 0.03$ and -0.15% for 1-std dev larger $s^{wn}_{ij} = 0.14$

Back - Regression specification

Data Appendix

Data:

- Isolate all plants (lbdnums) with non missing firmids, with strictly positive pay, strictly positive employment, non-missing county codes for the continental US (we exclude Alaska, Hawaii, and Puerto Rico)
- isolate all lbdnums with non-missing 2 digit NAICS codes equal to 11,21,31,32,33, or 55.
- We use the consistent 2007 NAICS codes provided by *Fort & Klimek* throughout the paper.
- Define a firm to be the sum of all establishments in a commuting zone with a common firmid and NAICS3 classification.

1. **Summary Statistics Sample:** Our summary statistics include all observations that satisfy the above criteria in 1976 and 2014.
2. **Corporate Tax Sample:** The corporate tax analysis includes all observations that satisfy the above criteria between 2002 and 2014 with an LFO of ‘C’. Firms must operate in at least two markets within a state.

Data Appendix

Table: Sample NAICS3 Codes.

NAICS3	Description	NAICS3	Description
111	Crop Production	322	Paper Manufacturing
112	Animal Production and Aquaculture	323	Printing and Related Support Activities
113	Forestry and Logging	324	Petroleum and Coal Products Manufacturing
114	Fishing, Hunting and Trapping	325	Chemical Manufacturing
115	Support Activities for Agriculture and Forestry	326	Plastics and Rubber Products Manufacturing
211	Oil and Gas Extraction	327	Nonmetallic Mineral Product Manufacturing
212	Mining (except Oil and Gas)	331	Primary Metal Manufacturing
213	Support Activities for Mining	332	Fabricated Metal Product Manufacturing
311	Food Manufacturing	333	Machinery Manufacturing
312	Beverage and Tobacco Product Manufacturing	334	Computer and Electronic Product Manuf.
313	Textile Mills	335	Electrical Equipment, Appliance, and Component Manuf.
314	Textile Product Mills	336	Transportation Equipment Manufacturing
315	Apparel Manufacturing	337	Furniture and Related Product Manufacturing
316	Leather and Allied Product Manufacturing	339	Miscellaneous Manufacturing
321	Wood Product Manufacturing	551	Management of Companies and Enterprises

Data Appendix

Table: Commuting Zone Examples

CZ ID, 2000	County Name	Metropolitan Area, 2003	County Pop. 2000	CZ Pop. 2000
58	Cook County	Chicago-Naperville-Joliet, IL Metropolitan Division	5,376,741	8,704,935
58	DeKalb County	Chicago-Naperville-Joliet, IL Metropolitan Division	88,969	8,704,935
58	DuPage County	Chicago-Naperville-Joliet, IL Metropolitan Division	904,161	8,704,935
58	Grundy County	Chicago-Naperville-Joliet, IL Metropolitan Division	37,535	8,704,935
58	Kane County	Chicago-Naperville-Joliet, IL Metropolitan Division	404,119	8,704,935
58	Kendall County	Chicago-Naperville-Joliet, IL Metropolitan Division	54,544	8,704,935
58	Lake County	Lake County-Kenosha County, IL-WI Metropolitan Division	644,356	8,704,935
58	McHenry County	Chicago-Naperville-Joliet, IL Metropolitan Division	260,077	8,704,935
58	Will County	Chicago-Naperville-Joliet, IL Metropolitan Division	502,266	8,704,935
58	Kenosha County	Lake County-Kenosha County, IL-WI Metropolitan Division	149,577	8,704,935
58	Racine County	Racine, WI Metropolitan Statistical Area	188,831	8,704,935
58	Walworth County	Whitewater, WI Micropolitan Statistical Area	93,759	8,704,935
47	Anoka County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	298,084	2,904,389
47	Carver County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	70,205	2,904,389
47	Chisago County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	41,101	2,904,389
47	Dakota County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	355,904	2,904,389
47	Hennepin County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	1,116,200	2,904,389
47	Isanti County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	31,287	2,904,389
47	Ramsey County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	511,035	2,904,389
47	Scott County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	89,498	2,904,389
47	Washington County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	201,130	2,904,389
47	Wright County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	89,986	2,904,389
47	Pierce County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	36,804	2,904,389
47	St. Croix County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	63,155	2,904,389

Summary Statistics

Table: Summary Statistics, C-Corp Sample

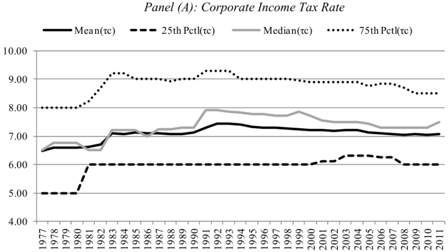
Variable	Mean	Std. Dev.
Corporate Tax Rate in Percent ($\tau_{s(j)t}$)	7.14	3.19
Change in Corporate Tax Rate	0.05	0.78
Total Pay At Firm (Thousands)	2148	19010
Total Employment At Firm	37.99	215.2
Wage Bill Share (s_{ijt}^{wn})	0.03	0.12
HHI - Wage Bill	0.10	0.16
Log Number of Firms per Market [exp(5.56)=259.8]	5.56	2.01
Log Total Employment ($\log n_{ijt}$) [exp(2.39)=10.9]	2.39	1.32
Log Wage ($\log w_{ijt}$) [exp(3.58)=$35k]	3.58	0.71
Observations		4,425,000

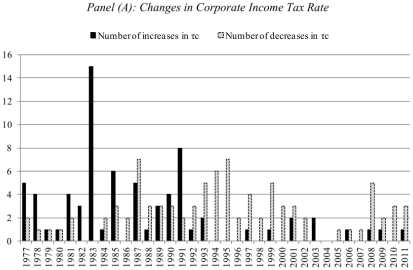
Notes: Tradeable C-Corps from 2002 to 2012.

Back - Calibration strategy

Summary Statistics

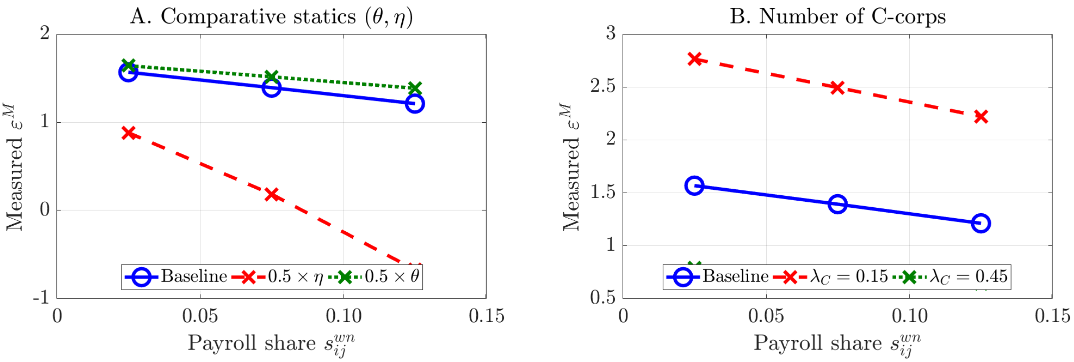
Reproduced from Giroud and Rauh (2011):



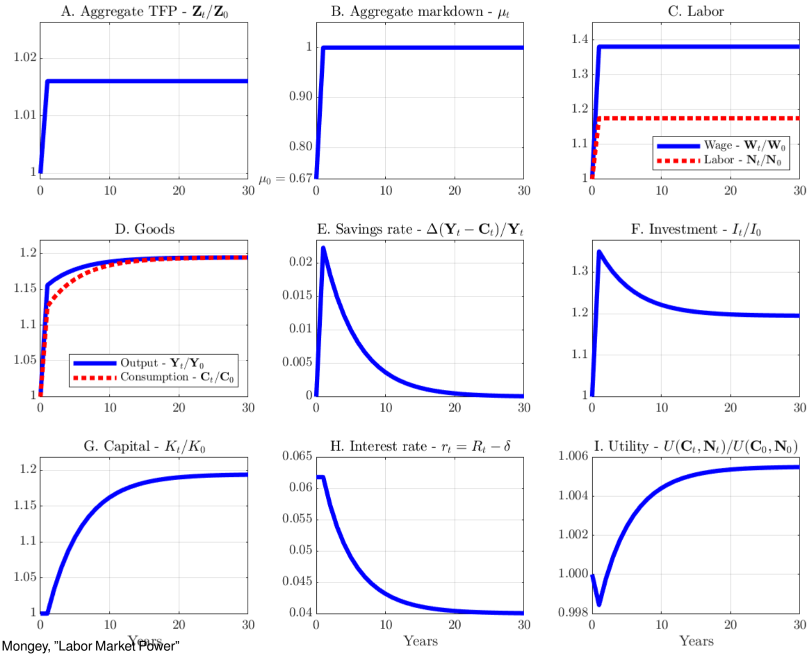


▸ Back - Calibration strategy

Bias in idiosyncratic shock case

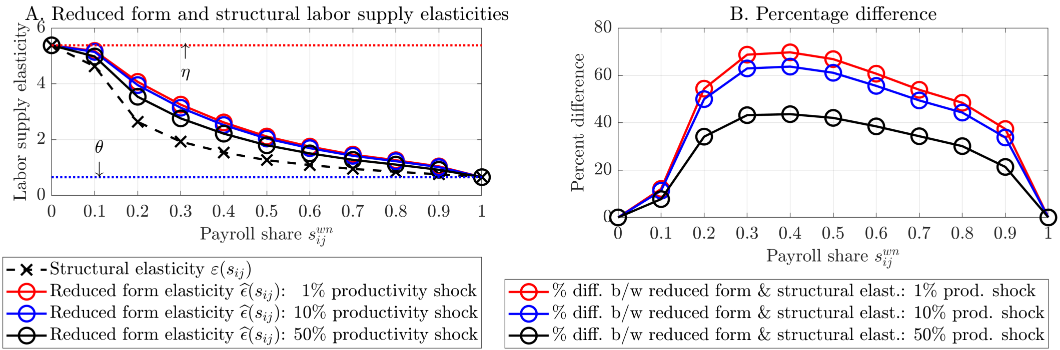


Back - Measured and perceived elasticities


A. Aggregate TFP - $\mathbf{Z}_t/\mathbf{Z}_0$
B. Aggregate markdown - μ_t
$\mu_0 = 0.67$
C. Labor
Wage - $\mathbf{W}_t/\mathbf{W}_0$
Labor - $\mathbf{N}_t/\mathbf{N}_0$
D. Goods
Output - $\mathbf{Y}_t/\mathbf{Y}_0$
Consumption - $\mathbf{C}_t/\mathbf{C}_0$
E. Savings rate - $\Delta(\mathbf{Y}_t - \mathbf{C}_t)/\mathbf{Y}_t$
F. Investment - I_t/I_0
G. Capital - K_t/K_0
H. Interest rate - $r_t = R_t - \delta$
I. Utility - $U(\mathbf{C}_t, \mathbf{N}_t)/U(\mathbf{C}_0, \mathbf{N}_0)$
Years
Years
Years

Back

Measured and perceived labor supply elasticities



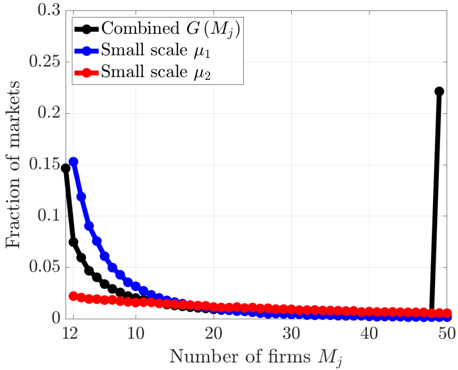
- **Method** Simulate purely idiosyncratic shock to a single firm
- **Result** Estimated *Reduced form* $\widehat{\epsilon}(s)$ is always larger than *Structural* $\varepsilon(s)$
- **Implication** Treating reduced form as structural understates labor market power

Back - Bias when using idiosyncratic shocks: $\widehat{\epsilon} > \varepsilon$

Calibration - Number of firms M_j

- 15% of markets have one firm $(M_j = 1)$
- Rest drawn from two Paretos, same shape γ, different scales μ_1, μ_2

Distribution of number of firms M_j	**Mean**	**Std.Dev.**	**Skewness**
Data (LBD, 2014)	51.6	264.9	29.9
Model	51.6	264.9	28.7



Back - Calibration table

Calibration

Table: Estimated parameters

Par.	Description	Value	Targeted Moment	Model	Data
$\widetilde{\alpha}$	DRS parameter	0.984	Labor share	0.57	0.57
$\sigma_{\widetilde{z}}$	Log Normal Standard Deviation	0.391	$E(HHI_j^{wn})$ Payroll wtd.	0.14	0.14
$\widetilde{Z}$	Productivity shifter	23,570	Avg. wage per worker	\$ 65,773	\$ 65,773
$\overline{\varphi}$	Aggregate labor disutility shifter	6.904	Avg firm size	27.96	27.96

Labor share:

- To recover labor-share, we must take a stance on capital's share of income
- Assume $KS = .18$ as in Barkai (2018)

$$\widetilde{\alpha} = \frac{\alpha\gamma}{1-(1-\gamma)\,\alpha}$$

$$\widetilde{\alpha} = \frac{\alpha\gamma}{1-KS}$$

Back

1. Non-targeted concentration measures

Moment	Model	Data
A. Unweighted		
Wage-bill Herfindahl (unweighted)	0.35	0.45
Std. Dev. of Wage-bill Herfindahl (unweighted)	0.33	0.33
Skewness of Wage-bill Herfindahl (unweighted)	1.07	0.48
B. Weighted		
Wage-bill Herfindahl (weighted by market's share of total payroll)	0.14	0.14
Std. Dev. of Wage-bill Herfindahl (weighted by market's share of total payroll)	0.03	0.20
Skewness of Wage-bill Herfindahl (weighted by market's share of total payroll)	3.01	2.20
C. Correlations of Wage-bill Herfindahl		
Number of firms	-0.52	-0.21
Std. Dev. Of Relative Wages	-0.31	-0.51
Employment Herfindahl	1.00	0.98
Market Employment	-0.75	-0.21

- Model generates $2x$ difference between wtd. and unwtd. HHI^{wn}

Back

Pass-through - Details

1. Replicate experiment in KPWZ (QJE 2019)

- Same sample properties: (i) median firm size, (ii) average VAPW increase
- Randomly sample 1% of firms with size greater than $\overline{n}$ employees
  - With $\overline{n} = 2$, match median size of 25 in KPWZ
- Then increase productivity by Δ
  - With $\Delta = 14\%$ match increase in $vapw_{ij}$ of 13 percent in KPWZ
- Repeat exercise 10 times, report average point estimates

2. Compare to following statistic from KPWZ

- Table 2, Panel A. Top dosage quintile - Mean $VAPW = \$120,160$, Median $n = 25.26$
- Table 5 col(4) - Event study. $\uparrow VAPW = \$15,740 \implies \uparrow 13\%$
- Table 8B col(1b) - IV regression $w_{ijt} = \alpha_{ij} + 0.23 vapw_{ijt}$. Elast. = $0.23 \left(\frac{\overline{vapw}}{\overline{w}}\right) = 0.47$

Back - Pass-through

2. Pass-through

Size cutoff	2.00
Fraction of Firms	0.01
N firms	906
Log change in VAPW (VAPW=$\widetilde{Z}\tilde{z}_{ij}n_{ij}^{\tilde{\alpha}-1}$)	0.13
Data Percent change in VAPW (Panel A Table 2 and Table 5 =15.74/120.16)	.13
Shock size	0.14
Nsims	1.00
Median firm size	25.07
Data median firm size	25.26
Mean firm size	56.37
Data mean firm size	61.49
Median VAPW (dollars)	90565
Data median VAPW (dollars)	86870
Mean VAPW (dollars)	92314
Data mean VAPW (dollars)	120160

▸ Back - Pass-through results

3. Size wage premium

- Size-wage premium regressions in Bloom et al (2018)

$$\log w_{ij} = \beta_0 + \beta_1 \log n_{ij} + \epsilon_{ij}$$

	Model (1)	**Bloom et al (2018), 1980** (2)	**Bloom et al (2018), 2013** (3)
Elasticity of wage WRT size	0.18	0.11	0.03
Dependent variable	$log(w_{ij})$	Log annual earnings	Log annual earnings
Independent variable	$log(n_{ij})$	Log firm employees	Log firm employees

- Model implies 10% larger firm pays 1.8% more

Back - Pass-through

Discussion - Wage bill shares and MRPL

Identifying *MRPL*

$$\frac{s^{wn}_{ijt}}{s^{wn}_{ikt}} = \left(\frac{\mu\left(s^{wn}_{ijt}\right)}{\mu\left(s^{wn}_{ikt}\right)}\right)^{1+\eta} \left(\frac{MRPL_{ijt}}{MRPL_{ikt}}\right)^{1+\eta}$$

- Up to a normalization, $\{s^{wn}_{ijt}\}$ can be used to infer $\{MRPL_{ijt}\}$

Back - Equilibrium

Discussion - Wage bill shares and MRPL

Identifying *MRPL*

$$\frac{s_{ijt}^{wn}}{s_{ikt}^{wn}} = \left(\frac{\mu\left(s_{ijt}^{wn}\right)}{\mu\left(s_{ikt}^{wn}\right)}\right)^{1+\eta} \left(\frac{MRPL_{ijt}}{MRPL_{ikt}}\right)^{1+\eta}$$

- Up to a normalization, $\{s_{ijt}^{wn}\}$ can be used to infer $\{MRPL_{ijt}\}$

Implications for measurement in LBD

- Labor markets relatively easy to define
- Wage bill shares observed s_{ijt}^{wn}
- Construct wage bill Herfindahl indices $HHI_{jt} = \sum_i s_{ijt}^{wn2}$
- Contrast with studies of competition in goods markets which do not have local measures of sales shares

  Autor, Dorn, Katz, Patterson, Van Reenen (2018), Phillipon Gutierrez (2018)

Back - Equilibrium

2. Pass-through - Corporate tax effects

After-tax profits with a corporate profit tax

$$
\begin{aligned}
\pi_{ij} &= \pi_{ij}^{Econ.} - \tau_C \pi_{ij}^{Acc.} \\
\pi_{ij}^{Econ.} &= z_{ij} n_{ij}^{\alpha} k_{ij}^{1-\alpha} - w_{ij} n_{ij} - R k_{ij} - \delta k_{ij} \\
\pi_{ij}^{Acc.} &= z_{ij} n_{ij}^{\alpha} k_{ij}^{1-\alpha} - w_{ij} n_{ij} - \lambda_K R k_{ij} - \delta k_{ij}
\end{aligned}
$$

- Can only write off fraction λ_K of capital financed by debt

Result

$$
\begin{aligned}
\pi_{ij} &= MRPL\left(z_{ij}, r, \tau_C\right) n_{ij} - w_{ij} n_{ij} \\
MRPL\left(z_{ij}, r, \tau_C\right) &= \frac{1}{1+\tau_C} \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} \left(\frac{\widetilde{z}_{ij}}{\widetilde{R}}\right)^{\frac{1-\alpha}{\alpha}} \widetilde{z}_{ij} \\
\widetilde{z}_{ij} &= (1-\tau_C) z_{ij} \\
\widetilde{R} &= (1+\lambda_K \tau_C) r + (1+\tau_C)\delta
\end{aligned}
$$

Back to corporate tax exercise. Calibration overview

Product market discussion

- Labor market power μ_{ijt} identified from product market power in tradeable goods market (the focus of our paper)
- Tradeable goods prices that are set non-competitively by a firm enter the marginal revenue product, $MRPL_{ijt}$
- $MRPL_{ijt}$ is distinct from what we call the labor market markdown
- We recover μ_{ijt} by comparing local labor market responses to corporate tax changes *within* a NAICS3 code
- If tradeable good prices (e.g. furniture prices) do not differ across local labor markets within a state, our estimate of μ_{ijt} only captures labor market power.

▸ Back

Evidence of upward sloping labor supply curves

Generation 1: Exogenous variation in employment demand or wages

- Staiger, Spetz, Phibbs (2010): mandated pay changes in registered nurse market (from national payscale to local)
  - LS elast of 0.10
- Ashenfelter, Farber, Ransom (2010) provide summary

Generation 2: Vacancy applications and wages

- Banfi and Villena-Roldan (2018): controlling for firm size, job title, and all available observables, higher wage offering attracts more workers
- Belot, M., P. Kircher, and P. Muller (2015): in actual UI sponsored job search office, post fake vacancies with higher wages, those vacancies draw more job searchers

Strong evidence for upward sloping LS curve faced by individual firms, conditional on size

back

Is monopsony power generated by outside options

Theory: Zhu (2011) provides framework with N firms, agents understand outside option is to match with remaining $N-1$ firms

- Wages (asset prices) fall if bargaining breaks down

Empirics: Does outside option affect wages?

- Jager, Schoefer, Young, Zweimüller (2018): outside options not strong determinant of wages
  - Four large reforms of UI in Austria.
  - Wage response less than 1 cent per 1.00 dollar UI increase
  - Nash-bargaining implies 39 cent per 1.00 dollar UI increase in calibrated model
- Hagedorn, Karahan, Manovskii, Mitman (2014): important determinants
  - County border-pair identification strategy

back

Welfare - Output decomposition - Details

- Compute counterfactual output with scale effect only:

$$n^s_{ij} = n_{ij} \frac{\int \sum n^c_{ij}\, dj}{\int \sum n_{ij}\, dj}$$

$$y^s_{ij} = \widetilde{z}_{ij} \widetilde{Z} (n^s_{ij})^{\widetilde{\alpha}}$$

- We then compute the share of gains due to reallocation:

$$\frac{\frac{\int \sum y^c_{ij}\, dj}{\int \sum y_{ij}\, dj} - \frac{\int \sum y^s_{ij}\, dj}{\int \sum y_{ij}\, dj}}{\frac{\int \sum y^c_{ij}\, dj}{\int \sum y_{ij}\, dj} - 1}$$

- Share output gains due to reallocation: 26%
- Share output gains due to scale: 74%

▸ Back - Welfare - Output decomposition

Minimum wage - Appendix

Household - Additional constraint: Labor supply less than labor demand:

$$n_{ijt} \leq \underline{n}_{ijt}$$

- Define $\lambda_t \nu_{ijt}$ as associated multiplier
- λ_t is the multiplier on the budget constraint
- ν_{ijt} is marginal utility of sending a worker to firm with a binding $w_{ij} = \underline{w}$
- $\widetilde{w}_{ijt} = w_{ijt} - \nu_{ijt}$ is the *perceived wage*

Firm - Problem as before with added constraint:

$$w_{ijt} = \begin{cases} \overline{\varphi}^{-\frac{1}{\varphi}} \mathbf{N}_t^{\frac{1}{\varphi}} \left(\frac{\mathbf{N}_{jt}}{\mathbf{N}_t} \right)^{\frac{1}{\theta}} \left(\frac{n_{ijt}}{\mathbf{N}_{jt}} \right)^{\frac{1}{\eta}} & , \quad \text{if} \quad n_{ijt} > \underline{n}_{ijt} \\ \underline{w} & , \quad \textit{otherwise} \end{cases}$$

Result - Equilibrium can be solved in perceived wages $\widetilde{w}_{ijt}$

Back

Minimum wage - Appendix

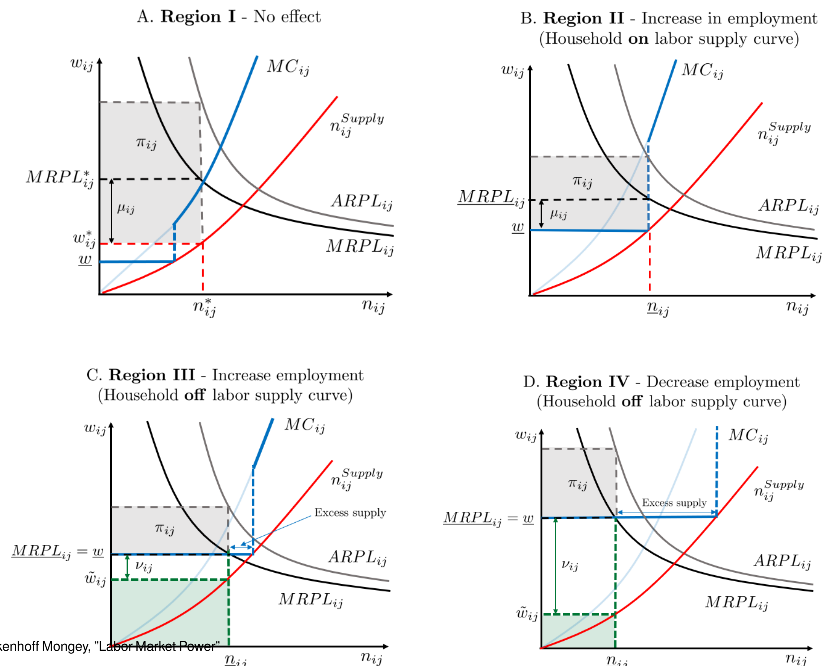
Define the *perceived* wage-bill share:

$$\widetilde{s}_{ijt} = \frac{(w_{ijt} - \nu_{ijt})n_{ijt}}{\sum_{i\in j}(w_{ijt} - \nu_{ijt})n_{ijt}}$$

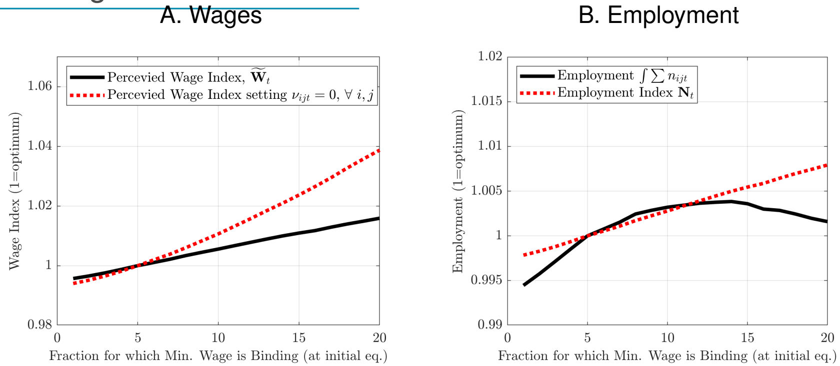
Define the *perceived* sectoral and aggregate wage indexes:

$$\widetilde{\mathbf{W}}_{jt} := \left[\sum_{i\in j}\left(w_{ijt} - \nu_{ijt}\right)^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \quad \widetilde{\mathbf{W}}_{t} := \left[\int \widetilde{\mathbf{W}}_{jt}^{1+\theta}\, dj\right]^{\frac{1}{1+\theta}} .$$

Back


A. Region I - No effect
w_{ij}
MC_{ij}
n_{ij}^{Supply}
π_{ij}
$MRPL^*_{ij}$
μ_{ij}
$ARPL_{ij}$
w^*_{ij}
$\underline{w}$
$MRPL_{ij}$
n^*_{ij}
n_{ij}
B. Region II - Increase in employment
(Household on labor supply curve)
w_{ij}
MC_{ij}
n_{ij}^{Supply}
π_{ij}
$MRPL_{ij}$
μ_{ij}
$ARPL_{ij}$
$\underline{w}$
$MRPL_{ij}$
$\underline{n}_{ij}$
n_{ij}
C. Region III - Increase employment
(Household off labor supply curve)
w_{ij}
MC_{ij}
n_{ij}^{Supply}
Excess supply
π_{ij}
$MRPL_{ij} = \underline{w}$
$ARPL_{ij}$
ν_{ij}
$\tilde{w}_{ij}$
$MRPL_{ij}$
$\underline{n}_{ij}$
n_{ij}
D. Region IV - Decrease employment
(Household off labor supply curve)
w_{ij}
MC_{ij}
π_{ij}
n_{ij}^{Supply}
Excess supply
$MRPL_{ij} = \underline{w}$
ν_{ij}
$ARPL_{ij}$
$MRPL_{ij}$
$\tilde{w}_{ij}$
$\underline{n}_{ij}$
n_{ij}

Minimum wage - Scale effects



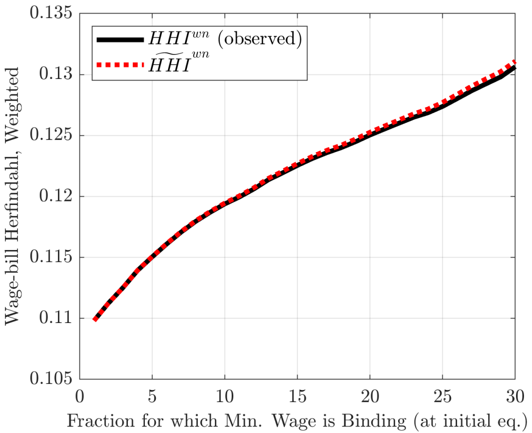
(i) Perceived wages, which determine n_{ij}, do not increase as much

(ii) Small firms shrink, (enter Region IV), employment falls

(iii) HHI monotonically increases, implying falling labor share

Back

Minimum wage - Concentration



- E.g. Would imply decline in labor share of 2 ppt over this range

Back

Minimum wage - Appendix

- Initialize the algorithm by (i) guessing a value for $\widetilde{\mathbf{W}}_t^{(0)}$, (ii) assuming all firms are in *Region I*, which implies guessing $\nu_{ijt}^{(0)} = 0$. These will all be updated in the algorithm.

1. Solve the sectoral equilibrium:

   1.1 Guess perceived shares $\widetilde{s}_{ijt}^{(0)}$.

   1.2 In *Region I*, where minimum wage does not bind, solve for the firm's wage as before, except with the perceived aggregate wage index $\widetilde{\mathbf{W}}_t$ instead of $\mathbf{W}_t$:

$$w_{ijt} = \left[\omega\mu\left(\widetilde{s}_{ijt}\right)\widetilde{\mathbf{W}}_t^{(1-\widetilde{\alpha})(\theta-\varphi)}\widetilde{z}_{ijt}\widetilde{s}_{ijt}^{(l)-\frac{(1-\widetilde{\alpha})(\eta-\theta)}{\eta+1}}\right]^{\frac{1}{1+(1-\widetilde{\alpha})\theta}}$$

   1.3 In all other regions *Region II, III, IV*, set $w_{ijt} = \underline{w}$.

   1.4 Compute perceived wages using the guess $\nu_{ijt}^{(k)}$: $\widetilde{w}_{ijt} = w_{ijt} - \nu_{ijt}^{(k)}$

   1.5 Update shares using $\widetilde{w}_{ijt}$:

$$\widetilde{s}_{ijt}^{(l+1)} = \frac{\widetilde{w}_{ijt}^{1+\eta}}{\sum_{i\in j}\widetilde{w}_{ijt}^{1+\eta}} \quad \left(:= \frac{\widetilde{w}_{ijt}n_{ijt}}{\sum_{i\in j}\widetilde{w}_{ijt}n_{ijt}} = \frac{\widetilde{w}_{ijt}\overline{\varphi}\left(\frac{\widetilde{w}_{ijt}}{\widetilde{\mathbf{W}}_{jt}}\right)^{\eta}\left(\frac{\widetilde{\mathbf{W}}_{jt}}{\widetilde{\mathbf{W}}_t}\right)^{\theta}\widetilde{\mathbf{W}}_t^{\varphi}}{\sum_{i\in j}\widetilde{w}_{ijt}\overline{\varphi}\left(\frac{\widetilde{w}_{ijt}}{\widetilde{\mathbf{W}}_{jt}}\right)^{\eta}\left(\frac{\widetilde{\mathbf{W}}_{jt}}{\widetilde{\mathbf{W}}_t}\right)^{\theta}\widetilde{\mathbf{W}}_t^{\varphi}}\right)$$

   1.6 Iterate over (b)-(e) until $\widetilde{s}_{ijt}^{(l+1)} = \widetilde{s}_{ijt}^{(l)}$.

1. Recover employment n_{ijt} according to the current guess of firm region. First use $\widetilde{w}_{ijt}$ to compute $\widetilde{\mathbf{W}}_{jt}$, $\widetilde{\mathbf{W}}_{t}$. Then by region:
   (I) Firm is unconstrained:

$$n_{ijt} = \overline{\varphi} \left(\frac{w_{ijt}}{\widetilde{\mathbf{W}}_{jt}} \right)^{\eta} \left(\frac{\widetilde{\mathbf{W}}_{jt}}{\widetilde{\mathbf{W}}_{t}} \right)^{\theta} \widetilde{\mathbf{W}}_{t}^{\varphi}$$

   (II) Firm is constrained and employment is determined by the household labor supply curve at $\underline{w}$:

$$n_{ijt} = \overline{\varphi} \left(\frac{\underline{w}}{\widetilde{\mathbf{W}}_{jt}} \right)^{\eta} \left(\frac{\widetilde{\mathbf{W}}_{jt}}{\widetilde{\mathbf{W}}_{t}} \right)^{\theta} \widetilde{\mathbf{W}}_{t}^{\varphi}$$

   (III),(IV) Firm is constrained and employment is determined by firm $MRPL_{ij}$ curve at $\underline{w}$:

$$n_{ijt} = \left(\frac{\widetilde{\alpha} \widetilde{Z} \widetilde{z}_{ijt}}{\underline{w}} \right)^{\frac{1}{1-\widetilde{\alpha}}}$$

2. Update $\nu_{ijt}^{(k)}$:
   2.1 Use n_{ijt} to compute $\mathbf{N}_{jt}$, $\mathbf{N}_{t}$.
   2.2 Update ν_{ijt} from the *household's* first order conditions:

$$\nu_{ijt}^{(k+1)} = w_{ijt} - \overline{\varphi}^{-\frac{1}{\varphi}} \left(\frac{n_{ijt}}{\mathbf{N}_{jt}} \right)^{\frac{1}{\eta}} \left(\frac{\mathbf{N}_{jt}}{\mathbf{N}_{t}} \right)^{\frac{1}{\theta}} \mathbf{N}_{t}^{\frac{1}{\varphi}}$$

3. Update $\widetilde{\mathbf{W}}_{t}^{(k)}$:
   3.1 Compute $\widetilde{w}_{ijt} = w_{ijt} - \nu_{ijt}^{(k+1)}$
   3.2 Use $\widetilde{w}_{ijt}$ to update the aggregate wage index to $\widetilde{\mathbf{W}}_{t}^{(k+1)}$.
4. Update firm regions:
   4.1 Compute profits for all firms: $\pi_{ijt} = \widetilde{Z} \widetilde{z}_{ijt} n_{ijt}^{\widetilde{\alpha}} - \underline{w} n_{ijt}$.
   4.2 If in sector j there exists a firm with $w_{ijt} < \overline{w}$, then move the firm with the lowest wage into *Region II*.
   4.3 If in sector j there exists a firm that was initially in *Region II* and has negative profits $\pi_{ijt} < 0$, move that firm into *Region III*.[1]
5. Iterate over (1) to (5) until $\nu_{ijt}^{(k+1)} = \nu_{ijt}^{(k)}$ and $\widetilde{\mathbf{W}}_{t}^{(k+1)} = \widetilde{\mathbf{W}}_{t}^{(k)}$.