



Fairness of Exposure in Rankings

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Presentation Overview

- Background
- Ranking task
- Paper motivation
- Related work
- Framework for ranking under fairness constraints
- Three group fairness constraints
- Discussions
- Conclusions
- Questions
- Reviews



Background

- Discounted Cumulative Gain:
 - Highly relevant documents are more useful if appearing earlier in search result.
 - Highly relevant documents are more useful than marginally relevant documents which are better than non-relevant documents.
- Probability Ranking Principle: According to PRP, ideal ranking should order items in the decreasing order of their probability of relevance.

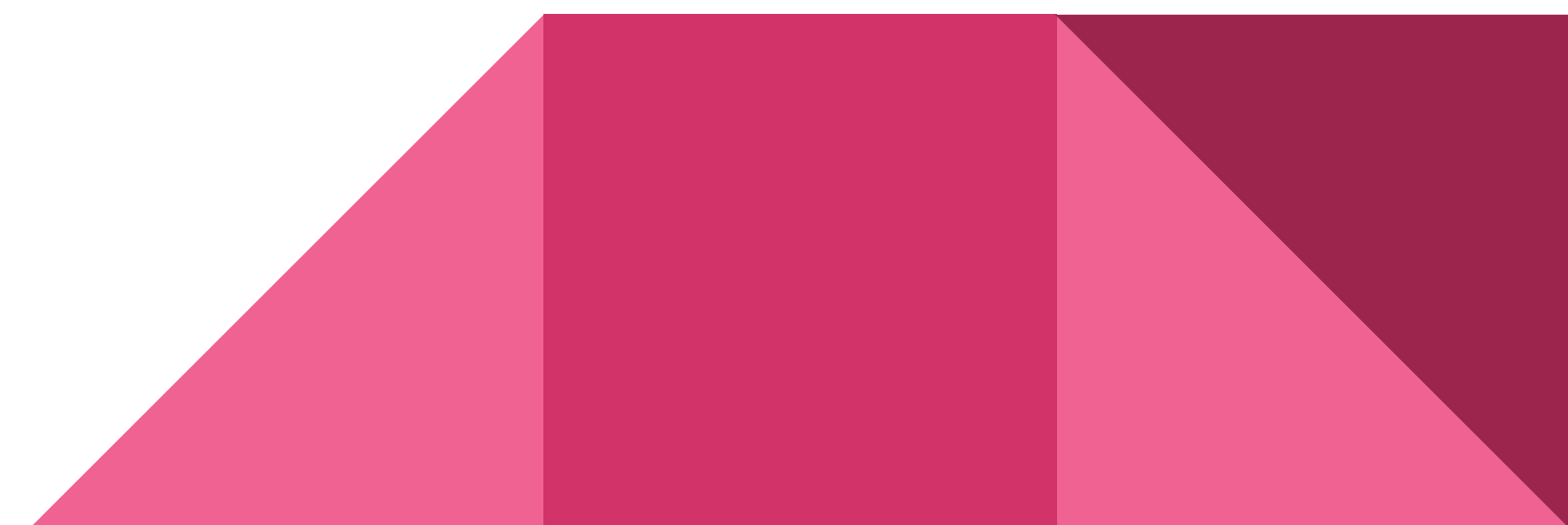


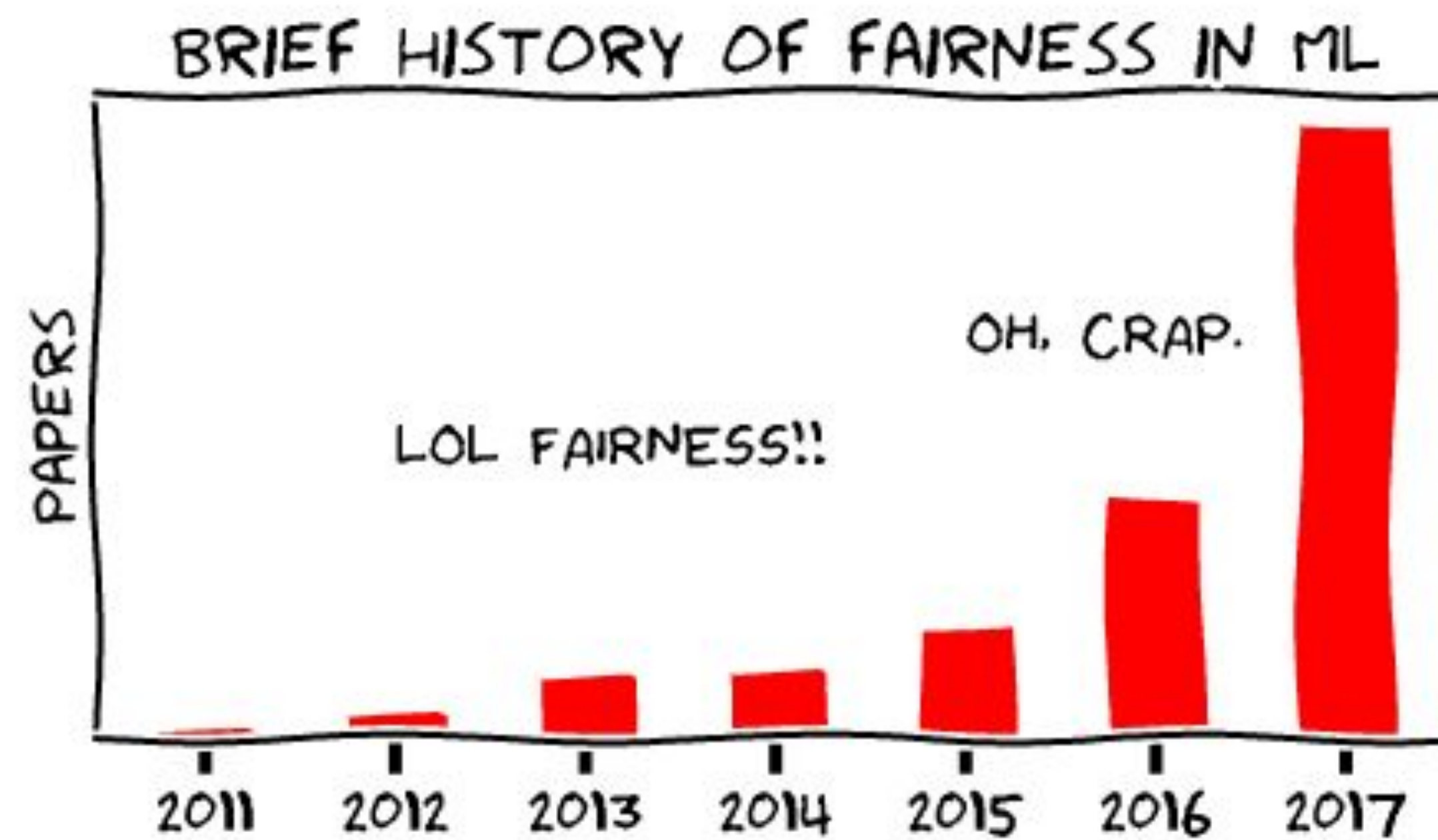
- Birkhoff-von Neumann decomposition:

A **doubly stochastic matrix** is a matrix in which each row and each column sum to one. A **permutation matrix** is a matrix in which each entry is either zero or one.

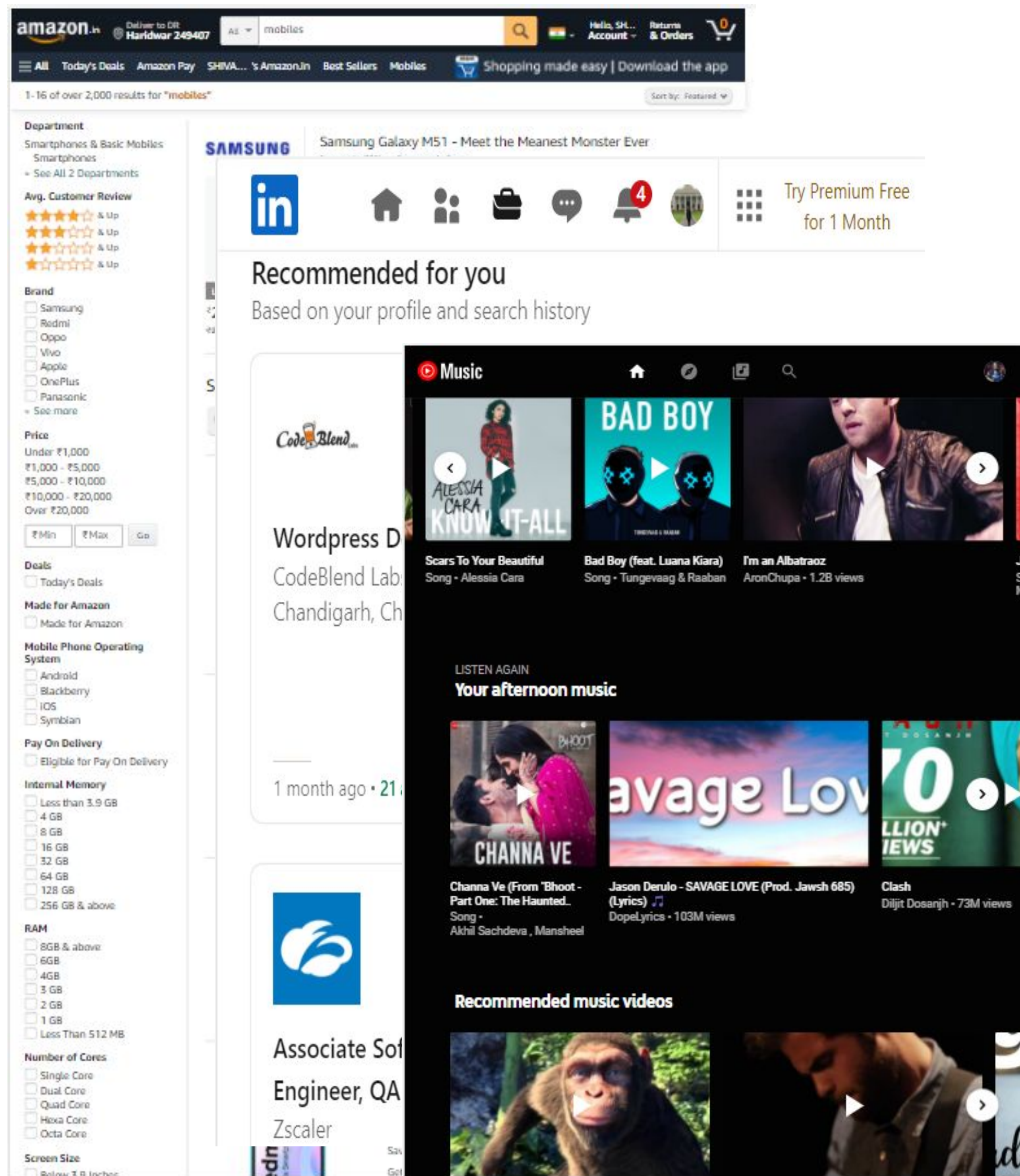
By the **Birkhoff–von Neumann Theorem**, each doubly stochastic matrix is a convex combination of permutation matrices. In other words, for each $N \times N$ doubly stochastic matrix A , there exists a decomposition of the form

$A = \theta_1 A_1 + \theta_2 A_2 + \cdots + \theta_n A_n$, where $0 \leq \theta_i \leq 1$, $\sum_i \theta_i = 1$, and where the A_i are permutation matrices.





The number of publications on fairness from 2011 to 2017



Ranking is Everywhere

Ranking is a dominant form of representation of information in the online web.

- Search engines
- Recommender systems (products, jobs, job seekers, friends on social media)
- News feeds

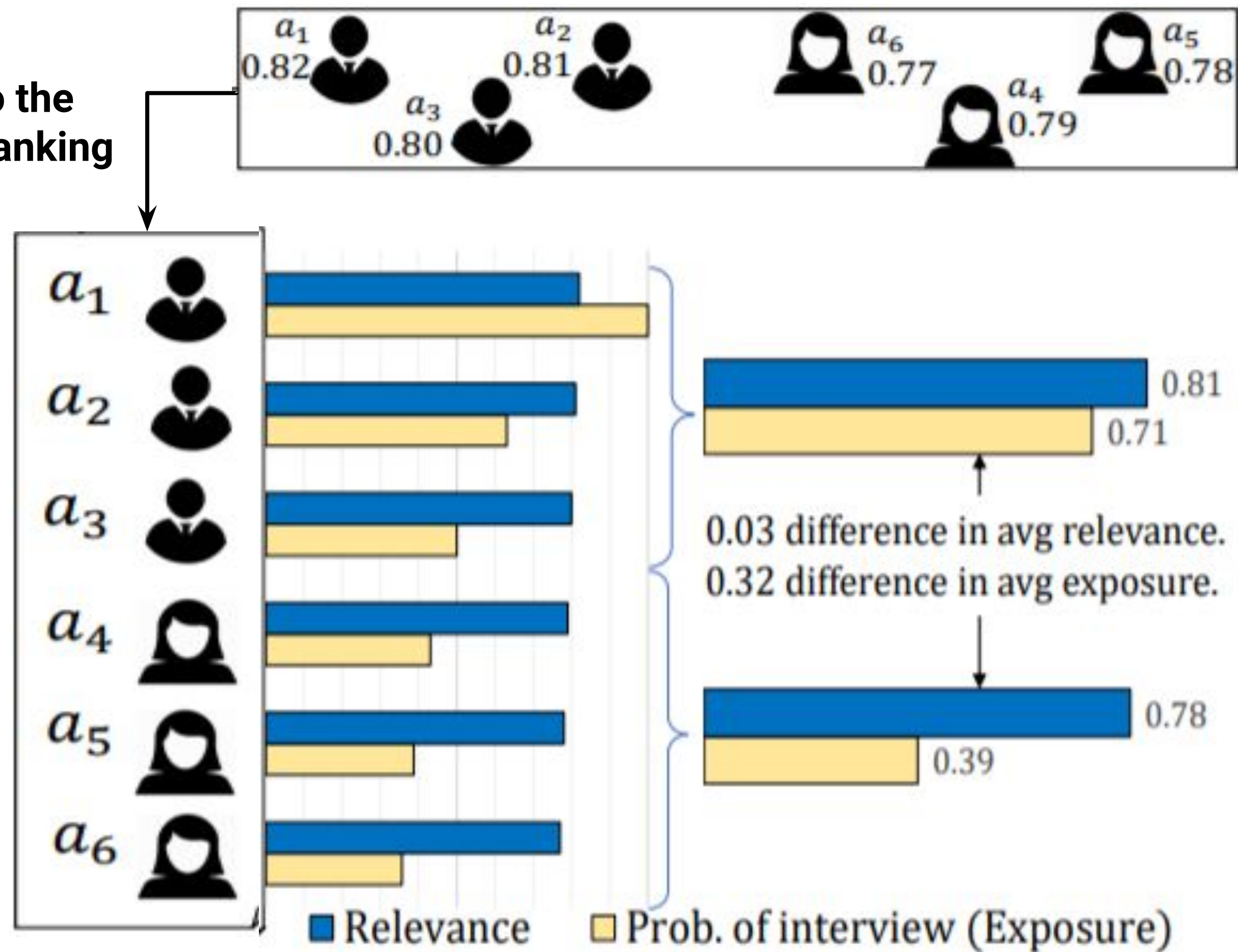
Paper Motivation

We are not only ranking documents these days in web but also people, businesses, or places thus it has become more important now to have a fairness in ranking as any sort of bias can lead to loss of opportunity for the protected groups.



Fairly Allocating Economic Opportunity

According to the
probability ranking
principle



$$\text{Exposure (Pos } j) = 1 / \text{Log } (1+j)$$

A job applicant ranking system

Fairly Representing a Distribution of Results




Figure 2: An image search result page for the query "CEO" showing a disproportionate number of male CEOs.

Giving Speakers Fair Access to Willing Listeners

- Consider an example of times during election in a country. There are many people posting their opinions on social media websites like Twitter or Facebook. Everyone has a freedom to speech and thus can post both positive and negative feedbacks about any party standing in election.
- But if a social media site is biased in its opinion towards one party that it might rank positive comments on top for one party and negative comments on top for other and this will influence a neutral person to also form a biased opinion during the election. The importance of equal exposure to both positive and negative comments is thus important for a social media site to be fair.




Related Work

- Algorithmic Fairness
 - Individual Fairness
 - Group Fairness
 - Fairness in Rankings
 - Information diversity in Retrieval
 - The motivation behind PRP diversified ranking is entirely based on to maximize the utility of the user while the approach in this paper can be used to balance the needs of both user and item.
 - Extrinsic diversity
 - Intrinsic diversity
 - Exploration diversity
- 

Framework for ranking under Fairness Constraints

Consider a single query q , ranking r over a set of documents $D = \{d_1, d_2, \dots, d_N\}$ and the utility $U(r|q)$. The problem of optimal ranking under fairness constraints can be formulated as the following optimization problem:

$$\begin{aligned} r = \operatorname{argmax}_r U(r|q) \\ \text{s.t. } r \text{ is fair} \end{aligned}$$


- Utility of Ranking
 - Probabilistic Rankings
 - Optimizing Fair Ranking via Linear Programming
 - Sampling Rankings
- 

Utility of Ranking

- Utility is derived from relevance of the individual items. For user u and query q , $\text{rel}(d|u,q)$ is the binary relevance value of document d . Different user may have different value for relevance even if they share the same q .
- Generic way to represent utility is:

$$U(r|q) = \sum_{u \in \mathcal{U}} P(u|q) \sum_{d \in \mathcal{D}} v(\text{rank}(d|r)) \lambda(\text{rel}(d|u, q)),$$

where v and λ are application dependent functions. Function $v(\text{rank}(d|r))$ models how much attention document d gets at rank $\text{rank}(d|r)$, and λ is a function that maps the relevance of a document for a user to its utility.



To incorporate the standard DCG metric in our framework:

$$DCG(r|q) = \sum_{u \in \mathcal{U}} P(u|q) \sum_{d \in \mathcal{D}} \frac{2^{\text{rel}(d|u, q)} - 1}{\log(1 + \text{rank}(d|r))}$$

where $v(\text{rank}(d|r)) = \frac{1}{\log(1 + \text{rank}(d|r))}$ and $\lambda(\text{rel}(d|u, q)) = 2^{\text{rel}(d|u, q)} - 1$

$$\begin{aligned} U(r|q) &= \sum_{d \in \mathcal{D}} v(\text{rank}(d|r)) \left(\sum_{u \in \mathcal{U}} \lambda(\text{rel}(d|u, q)) P(u|q) \right) \\ &= \sum_{d \in \mathcal{D}} v(\text{rank}(d|r)) u(d|q), \end{aligned}$$

where

$$u(d|q) = \sum_{u \in \mathcal{U}} \lambda(\text{rel}(d|u, q)) P(u|q)$$

In case of binary relevance and λ as the identity function, $u(d|q)$ is equivalent to the probability of relevance. Thus on sorting the documents by $u(d|q)$ will generate the ranking that will maximize the utility for any function v that decreases with rank. (Probability ranking principle)

$$\operatorname{argmax}_r U(r|q) \equiv \operatorname{argsort}_{d \in \mathcal{D}} u(d|q)$$



Probabilistic Rankings

If we search for utility-maximizing ranking under fairness constraints, then we will have to naively search for the complete ranking space which will take time exponential in $|D|$. Thus we are considering probabilistic rankings R instead of a single deterministic ranking r . R is a distribution over rankings.

$$\begin{aligned} U(R|q) &= \sum_r R(r) \sum_{u \in \mathcal{U}} P(u|q) \sum_{d \in \mathcal{D}} v(\text{rank}(d|r)) \lambda(\text{rel}(d|u, q)) \\ &= \sum_r R(r) \sum_{d \in \mathcal{D}} v(\text{rank}(d|r)) u(d|q) \end{aligned}$$



Let $P_{i,j}$ be the probability that R places document d_i at rank j , then P forms a doubly stochastic matrix of size $N \times N$, which means that the sum of each row and each column of the matrix is equal to 1. In other words, the sum of probabilities for each position is 1 and the sum of probabilities for each document is 1.

$$U(\mathbf{P}|q) = \sum_{d_i \in \mathcal{D}} \sum_{j=1}^N P_{i,j} u(d_i|q) v(j).$$

$u_i = u(d_i | q)$ and $v_j = v(j)$. In matrix multiplication, utility can be written as

$$U(\mathbf{P}|q) = \mathbf{u}^T \mathbf{P} \mathbf{v}$$



Optimizing Fair Ranking via Linear Programming

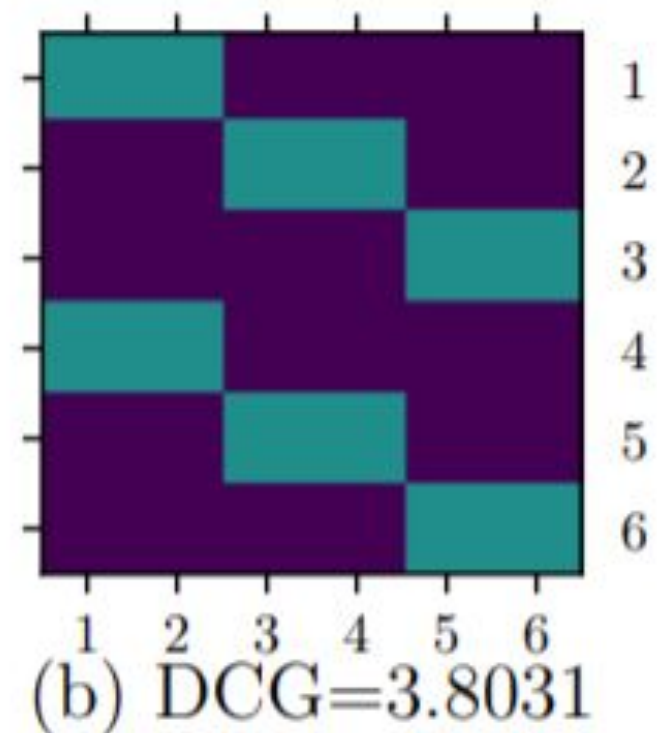
$$\begin{aligned} P = \operatorname{argmax}_P \quad & \mathbf{u}^T P \mathbf{v} && \text{(expected utility)} \\ \text{s.t.} \quad & \mathbf{1}^T P = \mathbf{1}^T && \text{(sum of probabilities for each position)} \\ & P \mathbf{1} = \mathbf{1} && \text{(sum of probabilities for each document)} \\ & 0 \leq P_{i,j} \leq 1 && \text{(valid probability)} \\ & P \text{ is fair} && \text{(fairness constraints)} \end{aligned}$$

$$\mathbf{f}^T P \mathbf{g} = h.$$

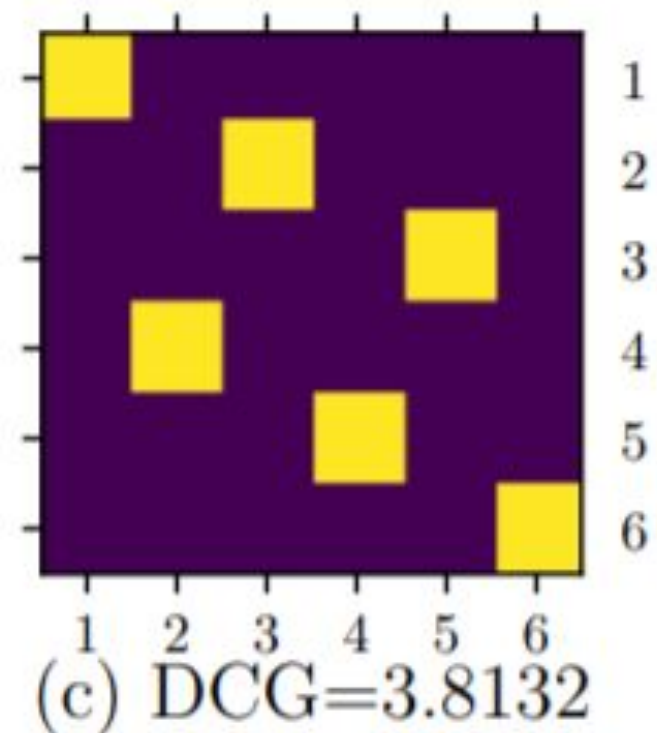


Sampling Rankings

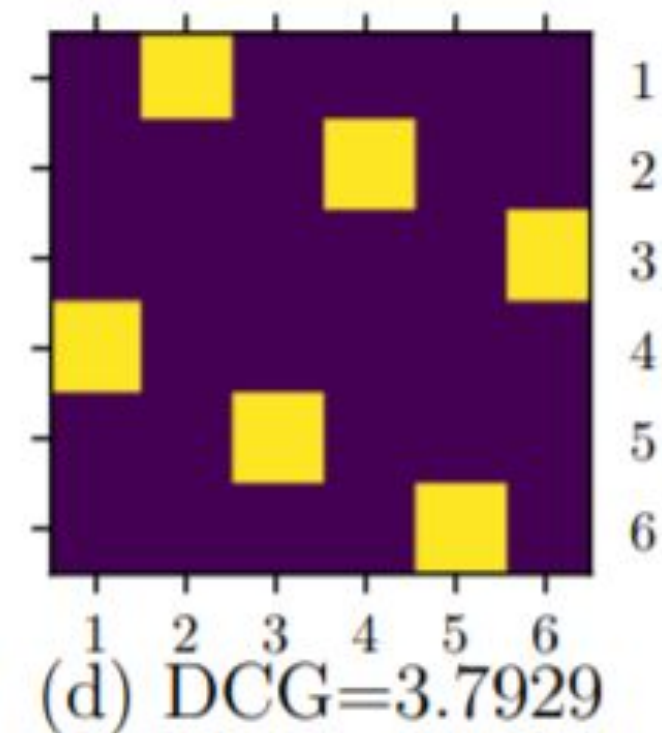
- We need to compute ranking R from our doubly stochastic matrix P and then sample r from R to present to the user.
- To compute R from P , we will use here Birkhoff-von Neumann decomposition.
- The permutation matrices correspond to deterministic rankings of the document set and the coefficients correspond to the probability of sampling each ranking.



$= 0.500 \times$

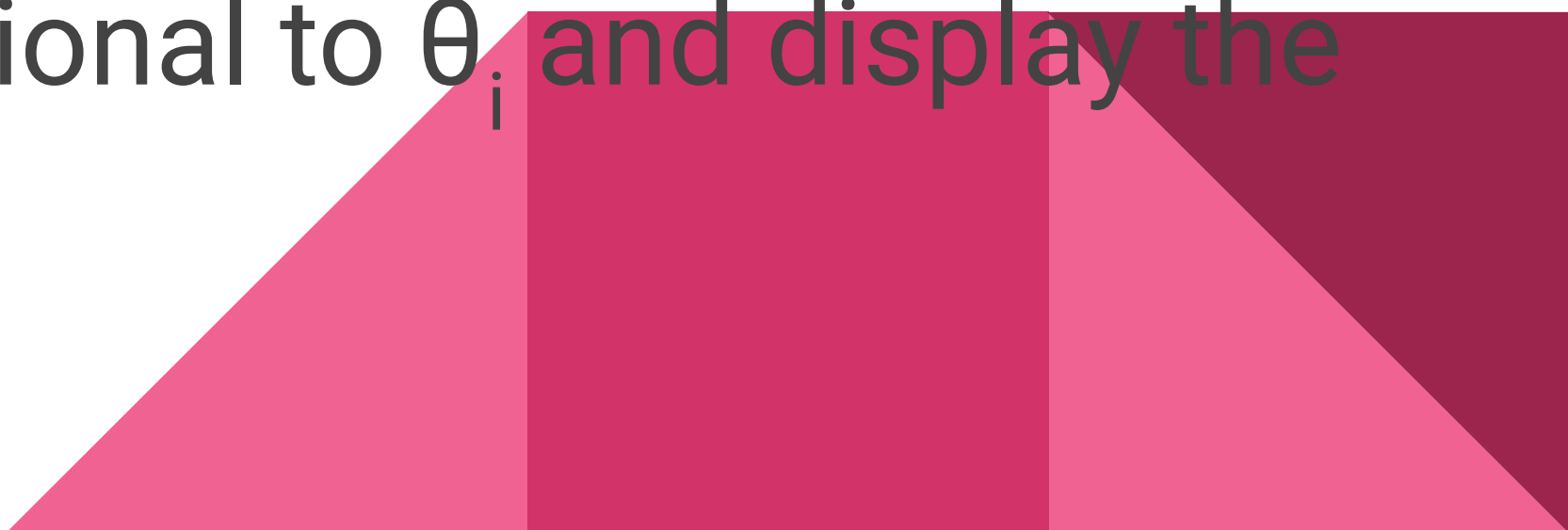


$+ 0.500 \times$



Summary of Algorithm

Assumption: the knowledge of the true relevances $u(d|q)$ (more in discussion)

- (1) Computing the utility vector u , the position discount vector v , as well as the vectors f and g , and the scalar h for the fairness constraints .
 - (2) Solve the linear program for P .
 - (3) Compute the Birkhoff-von Neumann decomposition $P = \theta_1 P_1 + \theta_2 P_2 + \cdots + \theta_n P_n$.
 - (4) Sample permutation matrix P_i with probability proportional to θ_i and display the corresponding ranking r_i .
- 

Constructing group fairness constraints

$$\text{Exposure}(d_i | \mathbf{P}) = \sum_{j=1}^N \mathbf{P}_{i,j} \mathbf{v}_j$$

We will construct our group fairness constraints on two examples

- Job-seeker example
- News recommendation dataset



Demographic Parity Constraints

$$\text{Exposure}(G_k | \mathbf{P}) = \frac{1}{|G_k|} \sum_{d_i \in G_k} \text{Exposure}(d_i | \mathbf{P}),$$

$$\text{Exposure}(G_0 | \mathbf{P}) = \text{Exposure}(G_1 | \mathbf{P})$$

$$\Leftrightarrow \frac{1}{|G_0|} \sum_{d_i \in G_0} \sum_{j=1}^N \mathbf{P}_{i,j} \mathbf{v}_j = \frac{1}{|G_1|} \sum_{d_i \in G_1} \sum_{j=1}^N \mathbf{P}_{i,j} \mathbf{v}_j$$

$$\Leftrightarrow \sum_{d_i \in \mathcal{D}} \sum_{j=1}^N \left(\frac{\mathbb{1}_{d_i \in G_0}}{|G_0|} - \frac{\mathbb{1}_{d_i \in G_1}}{|G_1|} \right) \mathbf{P}_{i,j} \mathbf{v}_j = 0$$

$$\Leftrightarrow \mathbf{f}^T P \mathbf{v} = 0 \quad \left(\text{with } \mathbf{f}_i = \frac{\mathbb{1}_{d_i \in G_0}}{|G_0|} - \frac{\mathbb{1}_{d_i \in G_1}}{|G_1|} \right)$$

Experiment 1

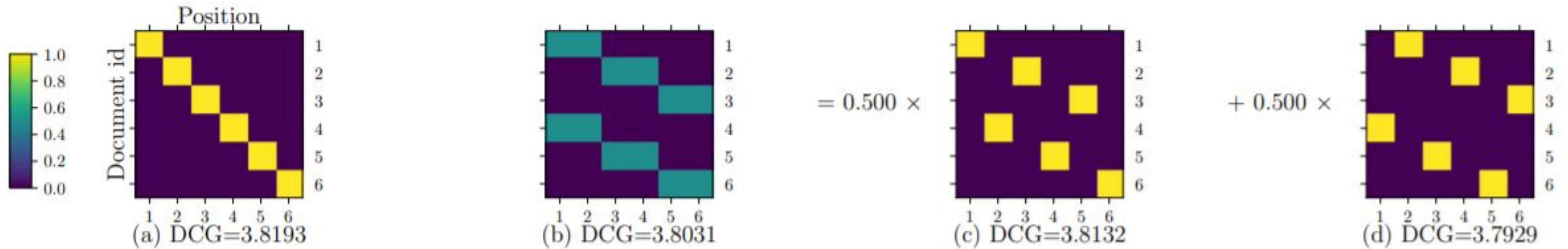


Figure 3: Job seeker example with demographic parity constraint. (a) Optimal unfair ranking that maximizes DCG. (b) Optimal fair ranking under demographic parity. (c) and (d) are the BvN decomposition of the fair ranking.



Experiment 2

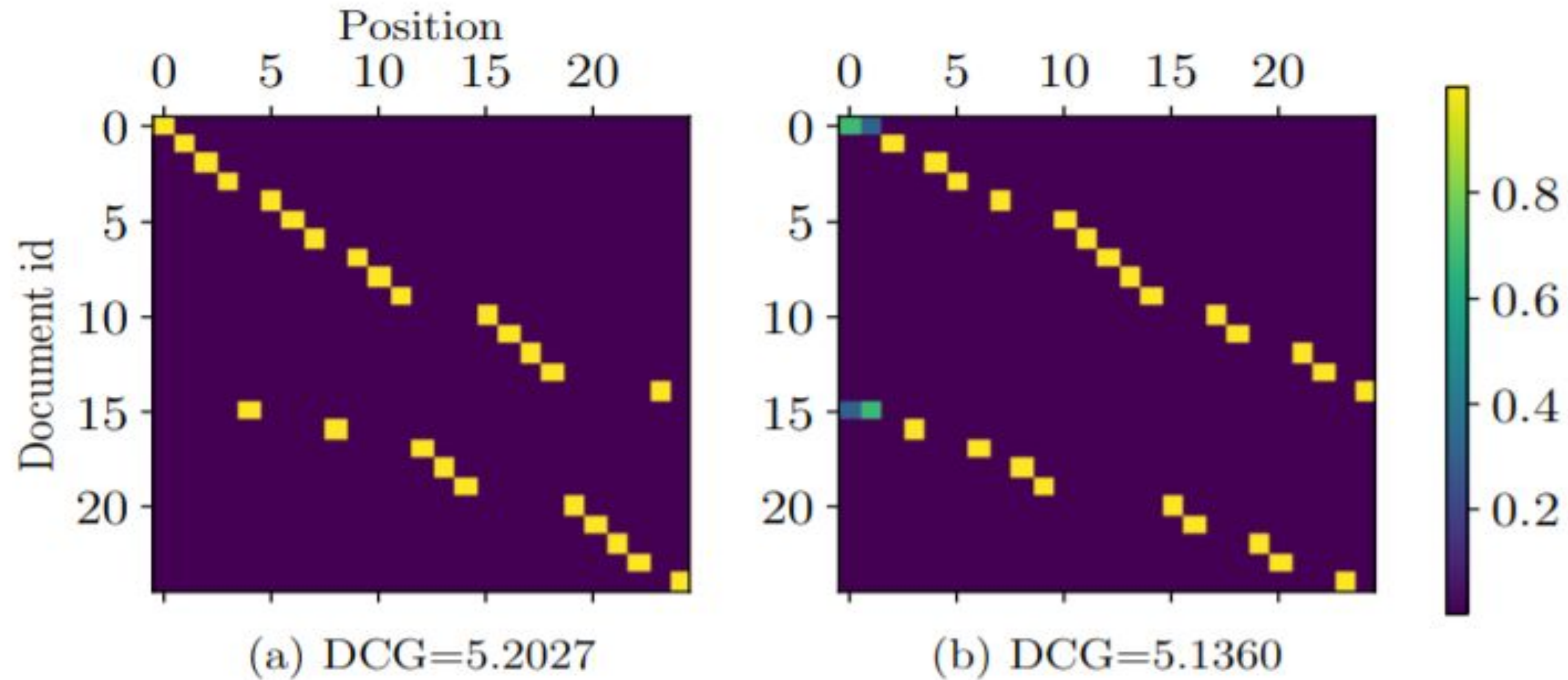


Figure 4: News recommendation dataset with demographic parity constraint. G_0 : Document id. 0-14, G_1 : 15-24 (a) Optimal unfair ranking that maximizes DCG. (b) Optimal fair ranking under demographic parity.

Disparate Treatment Constraints

$$U(G_k|q) = \frac{1}{|G_k|} \sum_{d_i \in G_k} \mathbf{u}_i,$$

$$\frac{\text{Exposure}(G_0|\mathbf{P})}{U(G_0|q)} = \frac{\text{Exposure}(G_1|\mathbf{P})}{U(G_1|q)}$$

$$\Leftrightarrow \frac{\frac{1}{|G_0|} \sum_{d_i \in G_0} \sum_{j=1}^N \mathbf{P}_{i,j} \mathbf{v}_j}{U(G_0|q)} = \frac{\frac{1}{|G_1|} \sum_{d_i \in G_1} \sum_{j=1}^N \mathbf{P}_{i,j} \mathbf{v}_j}{U(G_1|q)}$$

$$\Leftrightarrow \sum_{i=1}^N \sum_{j=1}^N \left(\frac{\mathbb{1}_{d_i \in G_0}}{|G_0|U(G_0|q)} - \frac{\mathbb{1}_{d_i \in G_1}}{|G_1|U(G_1|q)} \right) \mathbf{P}_{i,j} \mathbf{v}_j = 0$$

$$\Leftrightarrow \mathbf{f}^T \mathbf{P} \mathbf{v} = 0 \quad \left(\text{with } \mathbf{f}_i = \left(\frac{\mathbb{1}_{d_i \in G_0}}{|G_0|U(G_0|q)} - \frac{\mathbb{1}_{d_i \in G_1}}{|G_1|U(G_1|q)} \right) \right)$$

$$\text{DTR}(G_0, G_1|\mathbf{P}, q) = \frac{\text{Exposure}(G_0|\mathbf{P})/U(G_0|q)}{\text{Exposure}(G_1|\mathbf{P})/U(G_1|q)}$$

Experiment 1

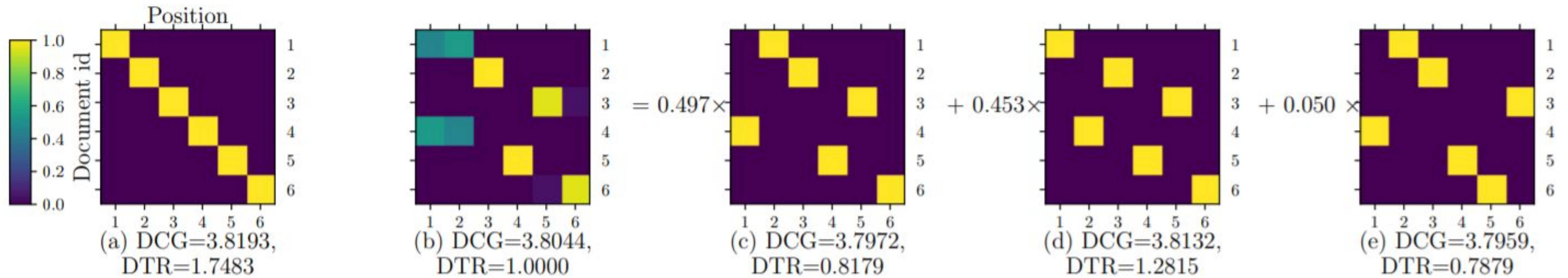


Figure 5: Job seeker example with disparate treatment constraint. (a) Optimal unfair ranking. (b) Fair ranking under disparate treatment constraint. (c), (d), (e) are the BvN decomposition of the fair ranking.

Experiment 2

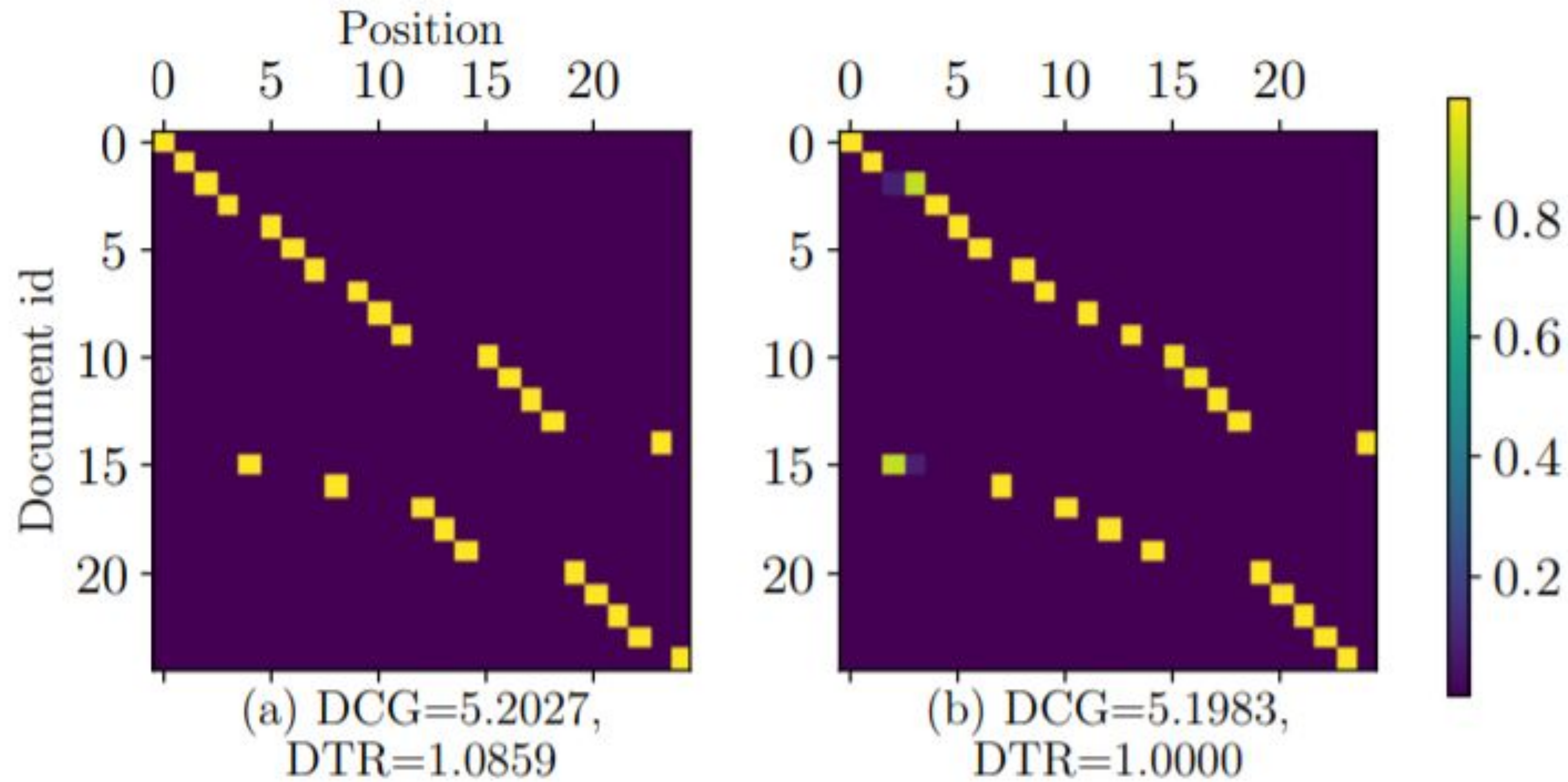


Figure 6: News recommendation dataset with disparate treatment constraint. (a) Optimal unfair ranking. (b) Fair ranking under disparate treatment constraint.

Disparate Impact Constraints

$$\begin{aligned} P(\text{click on document } i) &= P(\text{examining } i) \times P(i \text{ is relevant}) \\ &= \text{Exposure}(d_i | \mathbf{P}) \times P(i \text{ is relevant}) \\ &= \left(\sum_{j=1}^N \mathbf{P}_{i,j} \mathbf{v}_j \right) \times \mathbf{u}_i \end{aligned}$$

$$\begin{aligned} \frac{\text{CTR}(G_0 | \mathbf{P})}{U(G_0 | q)} &= \frac{\text{CTR}(G_1 | \mathbf{P})}{U(G_1 | q)} \\ \Leftrightarrow \frac{\frac{1}{|G_0|} \sum_{i \in G_0} \sum_{j=1}^N \mathbf{P}_{i,j} \mathbf{u}_i \mathbf{v}_j}{U(G_0 | q)} &= \frac{\frac{1}{|G_1|} \sum_{i \in G_1} \sum_{j=1}^N \mathbf{P}_{i,j} \mathbf{u}_i \mathbf{v}_j}{U(G_1 | q)} \end{aligned}$$

$$\Leftrightarrow \sum_{i=1}^N \sum_{j=1}^N \left(\frac{\mathbb{1}_{d_i \in G_0}}{|G_0| U(G_0 | q)} - \frac{\mathbb{1}_{d_i \in G_1}}{|G_1| U(G_1 | q)} \right) \mathbf{u}_i \mathbf{P}_{i,j} \mathbf{v}_j = 0$$

$$\Leftrightarrow \mathbf{f}^T \mathbf{P} \mathbf{v} = 0 \quad \left(\text{with } \mathbf{f}_i = \left(\frac{\mathbb{1}_{d_i \in G_0}}{|G_0| U(G_0 | q)} - \frac{\mathbb{1}_{d_i \in G_1}}{|G_1| U(G_1 | q)} \right) \mathbf{u}_i \right)$$

$$\text{CTR}(G_k | \mathbf{P}) = \frac{1}{|G_k|} \sum_{i \in G_k} \sum_{j=1}^N \mathbf{P}_{i,j} \mathbf{u}_i \mathbf{v}_j.$$

$$\text{DIR}(G_0, G_1 | \mathbf{P}, q) = \frac{\text{CTR}(G_0 | \mathbf{P}) / U(G_0 | q)}{\text{CTR}(G_1 | \mathbf{P}) / U(G_1 | q)}$$

Experiment 1

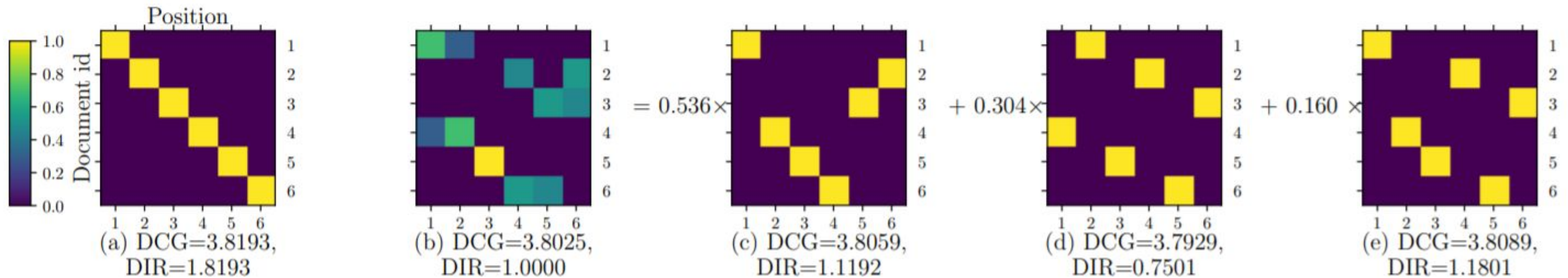


Figure 7: Job seeker example with disparate impact constraint. (a) Optimal unfair ranking. (b) Fair ranking under disparate impact constraint. (c), (d), (e) are the BvN decomposition of the fair ranking.

Experiment 2

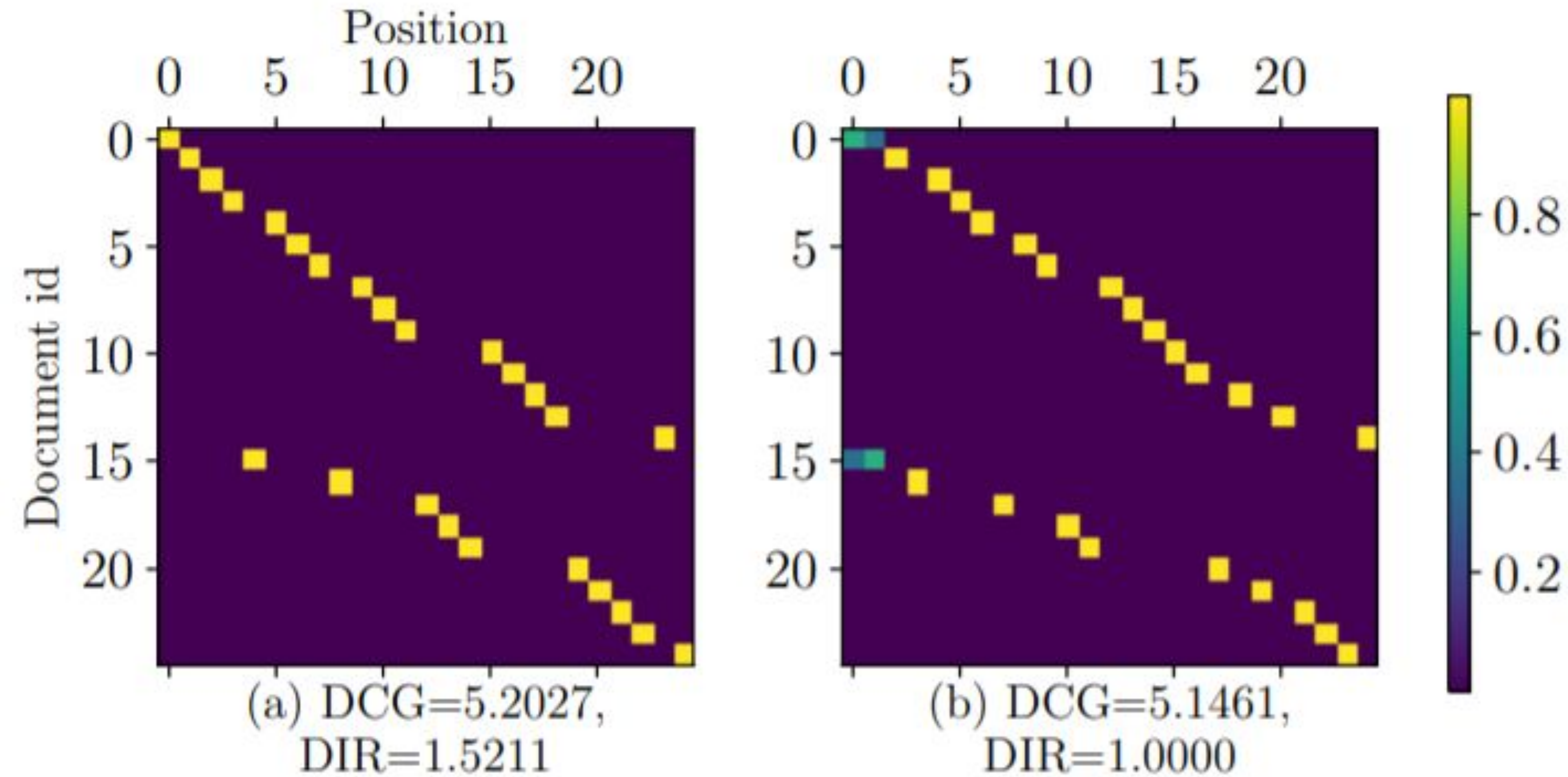


Figure 8: News recommendation dataset with disparate impact constraint. (a) Optimal unfair ranking. (b) Fair ranking under disparate impact constraint.

Discussion

- Fairness in rankings is inherently a trade-off between the utility of the users and the rights of the items that are being ranked, and that different applications require making this trade-off in different ways.
- The author created a flexible framework that covers a substantial range of fairness constraints. and thus we can define more than just these 3 constraints based on different application.



Group fairness vs. individual fairness

- Individual items within a group might still be considered to suffer from disparate treatment or impact
- We can include additional fairness constraints for sensitive attributes, like race, disability, and national origin within the group to refine the desired notion of fairness.
- In the case of Disparate Treatment, we can express individual fairness as a set of $N - 1$ constraints over N groups of size one.



Using estimated utilities

- We assumed that we have access to the true expected utilities (i.e. relevances) $u(d|q)$.
- In practice, these utilities are typically estimated via machine learning and thus are subject to some other biases that may lead to biased estimates $\hat{u}(d|q)$.
- Biased estimates might be the result of selection biases in click data.
- But as we have seen in our last presentation by Nishtha, with counterfactual learning techniques we can get unbiased ranking despite of biased click data.



Cost of fairness

$$\text{CoF} = \mathbf{u}^T (\mathbf{P}^* - \mathbf{P}) \mathbf{v}$$

Feasibility of fair solutions

$$\frac{\text{Exposure}(G_0|\mathbf{P})}{\text{Exposure}(G_1|\mathbf{P})} = \frac{U(G_0|q)}{U(G_1|q)}.$$

$$\frac{\sum_{j=|G_1|+1}^{|G_1|+|G_0|} \mathbf{v}_j}{\sum_{j=1}^{|G_1|} \mathbf{v}_j} \leq \frac{U(G_0|q)}{U(G_1|q)} \leq \frac{\sum_{j=1}^{|G_0|} \mathbf{v}_j}{\sum_{j=|G_0|+1}^{|G_0|+|G_1|} \mathbf{v}_j}$$

$$\max \left\{ \frac{\text{Exposure}(G_0|\mathbf{P})}{\text{Exposure}(G_1|\mathbf{P})} \right\} = \frac{\sum_{j=1}^{|G_0|} \mathbf{v}_j}{\sum_{j=|G_0|+1}^{|G_0|+|G_1|} \mathbf{v}_j},$$

(all G_0 documents in top $|G_0|$ positions)


$$\min \left\{ \frac{\text{Exposure}(G_0|\mathbf{P})}{\text{Exposure}(G_1|\mathbf{P})} \right\} = \frac{\sum_{j=|G_1|+1}^{|G_1|+|G_0|} \mathbf{v}_j}{\sum_{j=1}^{|G_1|} \mathbf{v}_j}$$

(all G_0 documents in bottom $|G_0|$ positions)

Conclusions

- In this paper, a general framework has been developed that employs probabilistic rankings and linear programming to compute the utility-maximizing ranking under a whole class of fairness constraints.
- To verify the expressiveness of this class, author demonstrated the concepts of fairness through demographic parity, disparate treatment, and disparate impact.
- We conjecture that the appropriate definition of fair exposure depends on the application, which makes this expressiveness desirable.



- Disparate treatment is intentional, where one is treating an individual “unfairly on account of membership in a protected class, such action relates to intent and maps onto disparate treatment doctrine.” For example, if your company is requiring assessments on particular skills for certain minority group applicants, it would be considered as disparate treatment.
 - Disparate impact is unintentional discrimination, where “the system treats someone unfairly [and] it is not necessarily because any person intended such a result.” For example, if your company uses technology or system that results in a disproportionate treatment “directed at a protected class, such as race, gender, age, disability, or in some cases, sexual orientation,” then we have a disparate impact present.
- 

Questions

Can the ranking distribution matrix model be extended to an $N \times K$ scenario, where we only care about rankings till K , instead of wanting to rank the entire list of documents?

Yes, I think this is possible to only compute rankings till top K . we just require to compute $P_{i,j}$, whose elements represent probability that R places document d_i at rank j , then P forms a doubly stochastic matrix of size $K \times K$, instead of $N \times N$.



The Disparate Treatment problem, can we apply a squeezing function (for example, sigmoid) to the utility/relevance function? This would decrease the winner-take-all factor.

Here the values follow the standard probabilistic definition of relevance, where 0.77 means that 77% of all employers issuing the query find that applicant relevant.



- Explain the process surrounding BvN decomposition and what the figures from the experiments show?
- Can complex clickthrough estimation model be also reduced to linear constraints?
- **How much is the probability of getting no feasible solution when the data is increased? Is adding extra documents (as discussed by authors) the only solution?**

So if we take our job seeker example here, we have 6 candidates, thus only 6 ranks are possible.

$v(j) = 1 / \text{Log}(1+j)$ thus $v(1) = 3.21$, $v(2)=2.095$, $v(3) = 1.660$, $v(4) = 1.430$, $v(5) = 1.285$, $v(6) = 1.183$

$$\frac{\sum_{j=|G_1|+1}^{|G_1|+|G_0|} \mathbf{v}_j}{\sum_{j=1}^{|G_1|} \mathbf{v}_j} \leq \frac{U(G_0|q)}{U(G_1|q)} \leq \frac{\sum_{j=1}^{|G_0|} \mathbf{v}_j}{\sum_{j=|G_0|+1}^{|G_0|+|G_1|} \mathbf{v}_j}$$

Maximum exposure = $7.076 / 3.898 = 1.815$

Minimum exposure = $3.898 / 7.076 = 0.0550$

$u = (0.81, 0.80, 0.79, 0.78, 0.77, 0.76)$

$\frac{U(G_0|q)}{U(G_1|q)} = 0.80/0.77 = 1.03$, thus feasible

$$\max \left\{ \frac{\text{Exposure}(G_0|\mathbf{P})}{\text{Exposure}(G_1|\mathbf{P})} \right\} = \frac{\sum_{j=1}^{|G_0|} \mathbf{v}_j}{\sum_{j=|G_0|+1}^{|G_0|+|G_1|} \mathbf{v}_j},$$

(all G_0 documents in top $|G_0|$ positions)

$$\min \left\{ \frac{\text{Exposure}(G_0|\mathbf{P})}{\text{Exposure}(G_1|\mathbf{P})} \right\} = \frac{\sum_{j=|G_1|+1}^{|G_1|+|G_0|} \mathbf{v}_j}{\sum_{j=1}^{|G_1|} \mathbf{v}_j}$$

(all G_0 documents in bottom $|G_0|$ positions)

but if we consider relevance as $= (0.82, 0.81, 0.80, 0.03, 0.02, 0.01)$

$\frac{U(G_0|q)}{U(G_1|q)} = 0.8/0.02 = 40$, Out of range and thus not feasible

Here as exposure depends on the $v(j)$ for feasibility, we have to add more documents only to increase the value of max exposure possible.

How do you account for the bias that comes from the ranking model itself - maybe some kind of ML model? The paper assumes that we have the true relevance score but in practice it would come from some ranking model.

So there are some possible sources of bias like bias in the training data, bias in user behaviour through click data, etc.

To account for the second type of bias which arises through clicks and user feedbacks, we have seen in our last presentation by Nishtha, that with counterfactual learning techniques we can get unbiased ranking despite of biased click data.



Time complexity analysis of the algorithm in section 3.3

In section 3.3 we got a linear program which can be solved using **Interior-point method** as mentioned in the paper as well. There is a polynomial time algorithm called Karmarkar's Algorithm which can solve linear programming problem very efficiently. Its running time is $O(n^{3.5} L^2 \cdot \log L \cdot \log \log L)$

where n is number of variables and L is number of bits of input to algorithm.



Is the BvN decomposition of matrix P unique? If not, then there are probably many R functions that can satisfy a given linear programming problem and the paper chooses one arbitrarily. Is there scope for improvement here? Can it be better to break ties more smartly?

BvN decomposition of a doubly stochastic matrix may not be unique - both in terms of permutation matrices used and also in the number of matrices used to produce such a decomposition.

A new paper is under review for ICLR 2021 which proposes a new algorithm

Approximate Birkhoff-von-Neumann decomposition: a differentiable approach

The author claims this to be an improvement over previous greedy methods to solve BnvD and this paper considers our paper of today, fairness in exposure of ranking to explain its approach

How big is the dataset on which they experiment?

Around 8000 documents

<https://users.soe.ucsc.edu/~yiz/papers/data/YOWStudy/>



Reviews

Overall everyone found the paper easy to understand.

There were few concerns regarding the scalability of the framework as the matrix method store N^2 values, so the space complexity will be high if N is large.

Also if add more constraints to refine the fairness against discrimination of not only gender but race, disability, etc then how the complexity of algorithm changes and how will the formulas be formulated in that sense.

The dataset taken was very small and does not reflect the feasibility of algorithm in real world online setting.



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Thank You