STEADY-STATE SEISMIC RESPONSE ANALYSIS OF SELF-CENTERING STRUCTURAL SYSTEMS WITH VISCOUS DAMPING

Digital Appendix

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In this appendix, the derivation of Eqs. (53) and (54) that was omitted in the main paper is detailed.

Derivation of Eq. (53):

We have formulated Eq. (47) based on Fig.10 in the paper:

$$m\ddot{u} + c_v\dot{u} + F_{\mathrm{Bi}}(u) + F_{\mathrm{SC}}(u) = -mA_G\, cos\,\omega\, t \tag{a1}$$

Note that the $F_{\mathrm{Bi}}(u)$ is described in Eq. (31) and the $F_{\mathrm{SC}}(u)$ is described in Eq. (8) in the paper. Substitution of Eqs. (8), (16), (18), (19) and (31) in the paper into Eq. (a1) gives:

$$-m\omega\dot{U}(t)\, sin\,\theta - m\omega^2 U(t)\, cos\,\theta + m\omega U(t)\dot{\varphi}(t)\, cos\,\theta + c_v\dot{U}(t)\, cos\,\theta - c_v U(t)\omega\, sin\,\theta + c_v\dot{\varphi}(t)U(t)\, sin\,\theta + k_e U(t)\, cos\,\theta + k_{\mathrm{Bi}}f_{\mathrm{Bi}} + k_{\mathrm{SC}}f_{\mathrm{SC}} = -mA_G\, cos\,\omega\, t \tag{a2}$$

The summation of Eq. (20) times $m\omega\sin\theta$ and Eq. (a2) times $\cos\theta$ gives the following equation:

$$m\omega\dot{U}(t)\sin\theta\cos\theta + m\omega\dot{\varphi}(t)U(t)\sin^2\theta - m\omega^2 U(t)\cos^2\theta + m\omega\dot{\varphi}(t)U(t)\cos^2\theta + c_{\mathrm{V}}\dot{U}(t)\cos^2\theta - c_{\mathrm{V}}U(t)\omega\sin\theta\cos\theta + c_{\mathrm{V}}\dot{\varphi}(t)U(t)\sin\theta\cos\theta + k_{\mathrm{e}}U(t)\cos^2\theta + k_{\mathrm{Bi}}f_{\mathrm{Bi}}\cos\theta + k_{\mathrm{SC}}f_{\mathrm{SC}}\cos\theta = -mA_{\mathrm{G}}\cos\omega t\cos\theta \tag{a3}$$

Since $U(t)$ and $\varphi(t)$ are assumed to be slowly varying functions of time t, these are treated as constants during one cycle of θ (θ: 0 to 2π). Thus, integrating Eq. (a3) over a cycle of θ gives (note: $\int_0^{2\pi}\varphi = \varphi$, $\int_0^{2\pi}\dot{\varphi} = \dot{\varphi}$, $\int_0^{2\pi}U = U$, $\int_0^{2\pi}\dot{U} = \dot{U}$):

$$2\omega\dot{\varphi}U(t) - \omega^2 U(t) + 2\beta_v\omega_{\mathrm{Bi}}\dot{U}(t) + (C_{\mathrm{Bi}} + \psi C_{\mathrm{SC}})\omega_{\mathrm{Bi}}^2 U(t) = -A_G\, cos\,\varphi \tag{a4}$$

The following parameters were defined:

$$\frac{k_{\mathrm{Bi}}}{m} = \omega_{\mathrm{Bi}}^2 \tag{a5}$$

$$\frac{c_v}{m} = 2\beta_v\omega_{\mathrm{Bi}} \tag{a6}$$

The equation describing steady state response is obtained by setting $\dot{U}(t)$ and $\dot{\varphi}(t)$ in Eq. (a4) equal to zero (i.e., slowly varying functions):

$$-\omega^2 U_0 + (C_{\mathrm{Bi}} + \psi C_{\mathrm{SC}})\omega_{\mathrm{Bi}}^2 U_0 = -A_G\, cos\,\varphi \tag{a7}$$

where the subscripts "0" to U and φ are used to denote steady state values. The Eq. (a7) corresponds to the Eq. (53) in the paper.

Derivation of Eq. (54):

Similar to the process that was conducted through Eqs. (a1) to (a7), the summation of Eq. (20) times $m\omega\cos\theta$ and Eq. (a2) times $\sin\theta$ gives the following equation:

$$m\omega\dot{U}(t)\,cos^2\theta + 2m\omega\dot{\varphi}(t)U(t)\,sin\,\theta\,cos\,\theta - m\omega\dot{U}(t)\,sin\,\theta\,cos\,\theta - m\omega^2\dot{U}(t)\,sin\,\theta\,cos\,\theta + c_v\dot{U}(t)\,sin\,\theta\,cos\,\theta - c_vU(t)\omega\,sin^2\theta + c_v\dot{\varphi}(t)U(t)\,sin^2\theta + k_eU(t)\,sin\,\theta\,cos\,\theta + k_{\mathrm{Bi}}f_{\mathrm{Bi}}\cos\theta + k_{\mathrm{SC}}f_{\mathrm{SC}}\,sin\,\theta = -mA_G\,cos\,\omega\,t\,cos\,\theta \quad \text{(a8)}$$

Since $U(t)$ and $\varphi(t)$ are assumed to be slowly varying functions of time t, these are treated as constants during one cycle of θ (θ: 0 to 2π). Thus, integrating Eq. (a3) over a cycle of θ gives (note: $\int_0^{2\pi}\varphi = \varphi$, $\int_0^{2\pi}\dot{\varphi} = \dot{\varphi}$, $\int_0^{2\pi}U = U$, $\int_0^{2\pi}\dot{U} = \dot{U}$):

$$-2\beta_v\omega_{\mathrm{Bi}}\omega U(t) + 2\beta_v\omega_{\mathrm{Bi}}\dot{\varphi}U(t) + (S_{\mathrm{Bi}} + \psi S_{\mathrm{SC}})\omega_{\mathrm{Bi}}^2U(t) = A_G\,sin\,\varphi \quad \text{(a9)}$$

The equation describing steady state response is obtained by setting $\dot{U}(t)$ and $\dot{\varphi}(t)$ in Eq. (a9) equal to zero (i.e., slowly varying functions):

$$-2\beta_v\omega_{\mathrm{Bi}}\omega U_0 + (S_{\mathrm{Bi}} + \psi S_{\mathrm{SC}})\omega_{\mathrm{Bi}}^2U_0 = A_G\,sin\,\varphi_0 \quad \text{(a10)}$$

where the subscripts "0" to U and φ are used to denote steady state values. The Eq. (a10) corresponds to the Eq. (54) in the paper.