Matrix theory - Assignment 9

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***Abstract*—This document proves result on linear transformations**

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https://github.com/shreeprasadbhat/matrix−theory/blob/master/assignment9/

1 Problem

Let $\mathbf{V}$ be the space of $n \times 1$ matrices over F and let $\mathbf{W}$ be the space of $m \times 1$ matrices over F. Let $\mathbf{A}$ be a fixed $m \times n$ matrix over F and let T be the linear transformation from $\mathbf{V}$ into $\mathbf{W}$ defined by $T(X) = \mathbf{AX}$. Prove that T is the zero transformation if and only if $\mathbf{A}$ is the zero matrix.

2 Proof

If $\mathbf{A}$ is a zero transformation, then $\mathbf{AX} = 0$ where $\mathbf{X}$ can take any value. Let's take $\mathbf{X_j}$ as vector with jth entry 1 and remaining 0. Now $\mathbf{A}X_j = 0 \implies \mathbf{A_j} = 0$. Since $\mathbf{A_j} = 0$, for j=1,2,...n in the linear transformation $\mathbf{AX} = 0$, $\mathbf{A}$ is zero matrix.

Let us assume $\mathbf{A}$ is a zero matrix, then $0.\mathbf{X} = 0, \forall \mathbf{X} \in F$. Hence the if $\mathbf{A}$ is zero matrix, linear transformation $T(X) = \mathbf{AX}$ is the zero transformation.