Matrix theory - Assignment 14

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***Abstract*—This document proves properties on double dual**

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https://github.com/shreeprasadbhat/matrix-theory/blob/master/assignment14/

1 Problem

Let S be a set, F a field, and $V(S;F)$ the space of all functions from S into F:

$$(f+g)(x) = f(x) + g(x)$$
$$(cf)(x) = cf(x)$$

Let W be any n-dimensional subspace of $V(S,F)$. Show that there exist points $x_1, x_2, \dots, x_n$ in S and functions $f_1, \dots, f_n$ in W such that $f_i(x_j) = \delta_{ij}$

2 Solution

Given	S is a set F is a field $V(S,F)$ is a linear functional such that W be n-dim subspace of $V(S,F)$. Also, $\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$
To prove	$f_i(x_j) = \delta_{ij}$ where $x_1, x_2, \dots, x_n \in S$ and $f_1, f_2, \dots, f_n \in W$
Proof	Let $\phi_x : W \to F$ Suppose $\phi_x(f) = 0 \;\; \forall x \in S \;\; \& \;\; f \in W$ $\implies f(x) = 0$ If $\forall x, \;\; \phi_x(f) \neq 0$ for some $f \in W$ If $n > 0 \; \exists \in S$ such that $\phi_x(f) \neq 0$ for some $f \in W$ $\implies f_1(x_1) \neq 0$ By scaling we can have $f_1(x_1) = 1$ Hence $f_i(x_j) = \delta_{ij}$

3 Figure

Consider $x_1 = 1, x_2 = 0$, where $\{x_1, x_2\} \in S$. Let $f_1(x) = x, f_2(x) = 1 - x$ where $\{f_1, f_2\} \in W$.

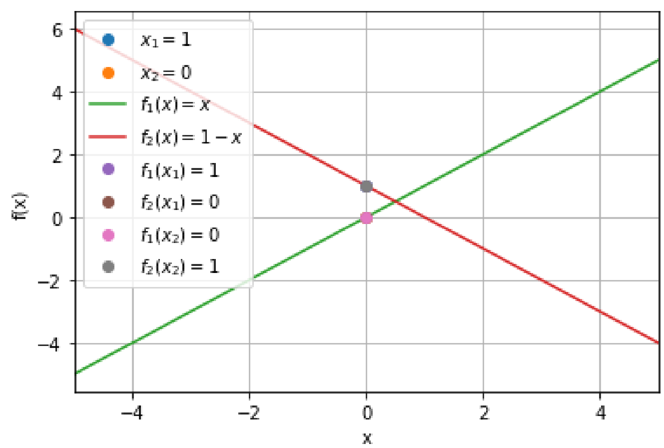

Fig. 0: Pictorial representation of functions and points in vector space