Matrix theory - Assignment 16

Shreeprasad Bhat
AI20MTECH14011

***Abstract*—This document proves properties on transpose of linear transformation.**

Download latex-tikz from

https://github.com/shreeprasadbhat/matrix−theory/blob/master/assignment16/

1 Problem

Let m, n, r be natural numbers. Let A be an $m \times n$ matrix with real entries such that $(AA^t)^r = I$, where I is the $m \times m$ is identity matrix and A^t is the transpose of the matrix A. We can conclude that

1) $m = n$

2) AA^t is invertible

3) A^tA is invertible

4) if $m = n$, then A is invertible

2 Solution

Options 2) and 4) are correct.

Option	Answer
1) $m = n$	Let $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ and $r = 1$ $(\mathbf{AA^T})^r = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ Since $m \neq n$ Option 1 is False.
2) AA^t is invertible	w.k.t $\det(A^n) = (\det(A))^n$ Since $(AA^t)^r = I$ So $\det((AA^T)^r) = \det(\mathrm{I})$ $(\det(AA^T))^r = 1$ $\implies \det(AA^T) \neq 0$ Hence AA^T is invertible Option 2 is True.
3) A^tA is invertible	Let $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ and $r = 1$ $(\mathbf{A^TA})^r = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ But $\det(AA^T) = 0$. $\implies AA^T$ is not invertible. Hence Option 3 is False
4) if $m = n$ then A is invertible	Since $\det(AA^T) \neq 0$ $\det(\mathrm{A}).\det(A^T) \neq 0$ $\det(\mathrm{A}).\det(\mathrm{A}) \neq 0$ $\implies A$ is invertible. Hence Option 4 is True