# Matrix theory - Assignment 16

Shreeprasad Bhat
AI20MTECH14011

***Abstract*—This document proves properties on transpose of linear transformation.**

Download latex-tikz from

```
https://github.com/shreeprasadbhat/matrix−theory/
    blob/master/assignment16/
```

## 1 Problem

Let $m, n, r$ be natural numbers. Let $A$ be an $m \times n$ matrix with real entries such that $(AA^t)^r = I$, where $I$ is the $m \times m$ is identity matrix and $A^t$ is the transpose of the matrix $A$. We can conclude that

1) $m = n$
2) $AA^t$ is invertible
3) $A^tA$ is invertible
4) if $m = n$, then $A$ is invertible

## 2 Solution

| **Option** | **Answer** |
|---|---|
| 1) $m = n$ | Let $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ and $r = 1$<br>$(\mathbf{AA^T})^r = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$<br>Since $m \neq n$<br>Option 1 is False. |
| 2) $AA^t$ is invertible | w.k.t $\det(A^n) = (\det(A))^n$<br>Since $(AA^t)^r = I$<br>So $\det((AA^T)^r) = \det(\mathrm{I})$<br>$(\det(AA^T))^r = 1$<br>$\implies \det(AA^T) \neq 0$<br>Hence $AA^T$ is invertible<br>Option 2 is True. |
| 3) $A^tA$ is invertible | Let $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ and $r = 1$<br>$(\mathbf{A^TA})^r = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$<br>But $\det(AA^T) = 0$.<br>$\implies AA^T$ is not invertible.<br>Hence Option 3 is False |
| 4) if $m = n$ then $A$ is invertible | Since $\det(AA^T) \neq 0$<br>$\det(\mathrm{A}).\det(A^T) \neq 0$<br>$\det(\mathrm{A}).\det(\mathrm{A}) \neq 0$<br>$\implies A$ is invertible.<br>Hence Option 4 is True |