Matrix theory - Assignment 6

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***Abstract*—This document illustrates on how to perform QR decomposition of a matrix**

Download all python codes from

```
https://github.com/shreeprasadbhat/matrix-theory/
    tree/master/assignment6/codes
```

and latex codes from

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https://github.com/shreeprasadbhat/matrix-theory/
    blob/master/assignment6/
```

1 Problem

Find QR decomposition for matrix

$$\mathbf{V} = \begin{pmatrix} 6 & \frac{17}{2} \\ \frac{17}{2} & 12 \end{pmatrix} \tag{1.0.1}$$

2 Construction

$$\text{Let } \mathbf{V} = \begin{pmatrix} \mathbf{c_1} & \mathbf{c_2} \end{pmatrix} = \begin{pmatrix} a & h \\ h & b \end{pmatrix} \tag{2.0.1}$$

Let $\mathbf{Q}$ be an orthogonal and $\mathbf{R}$ be an upper triangular matrix such that,

$$\mathbf{V} = \mathbf{QR} \tag{2.0.2}$$

$$= \begin{pmatrix} \mathbf{q_1} & \mathbf{q_2} \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} \\ 0 & r_{22} \end{pmatrix} \tag{2.0.3}$$

$$\begin{pmatrix} \mathbf{c_1} & \mathbf{c_2} \end{pmatrix} = \begin{pmatrix} \mathbf{q_1} r_{11} & \mathbf{q_1} r_{12} + \mathbf{q_2} r_{22} \end{pmatrix} \tag{2.0.4}$$

$$\implies \mathbf{c_1} = \mathbf{q_1} r_{11} \tag{2.0.5}$$

$$r_{11} = \|\mathbf{c_1}\| = \sqrt{a^2 + h^2} \tag{2.0.6}$$

$$\mathbf{q_1} = \frac{\mathbf{c_1}}{r_{11}} \tag{2.0.7}$$

$$\mathbf{c_2} = \mathbf{q_1} r_{12} + \mathbf{q_2} r_{22} \tag{2.0.8}$$

$$r_{21} = \mathbf{q_1}^T c_2 \tag{2.0.9}$$

$$r_{22} = \|\mathbf{c_2} - \mathbf{q_1} r_{12}\| \tag{2.0.10}$$

$$\mathbf{q_2} = \frac{\mathbf{c_2} - \mathbf{q_1} r_{12}}{r_{22}} \tag{2.0.11}$$

3 Solution

Given,

$$\mathbf{V} = \begin{pmatrix} 6 & \frac{17}{2} \\ \frac{17}{2} & 12 \end{pmatrix} \tag{3.0.1}$$

From (2.0.6),

$$r_{11} = \sqrt{6^2 + \left(\frac{17}{2}\right)^2} = \frac{\sqrt{433}}{2} \tag{3.0.2}$$

Substitute (3.0.2) in (2.0.7)

$$\mathbf{q_1} = \begin{pmatrix} \frac{12}{\sqrt{433}} \\ \frac{17}{\sqrt{433}} \end{pmatrix} \tag{3.0.3}$$

Substitute (3.0.3) in (2.0.9)

$$r_{21} = \frac{306}{\sqrt{433}} \tag{3.0.4}$$

Substitute (2.0.10) in (3.0.4)

$$r_{22} = \frac{\sqrt{433}}{866} \tag{3.0.5}$$

Substitute (3.0.5) in (2.0.11)

$$\mathbf{q_2} = \begin{pmatrix} \frac{17}{\sqrt{433}} \\ -\frac{12}{\sqrt{433}} \end{pmatrix} \tag{3.0.6}$$

Hence QR decomposition of V is,

$$\mathbf{V} = \begin{pmatrix} \frac{12}{\sqrt{433}} & \frac{17}{\sqrt{433}} \\ \frac{17}{\sqrt{433}} & -\frac{12}{\sqrt{433}} \end{pmatrix} \begin{pmatrix} \frac{\sqrt{433}}{2} & \frac{306}{\sqrt{433}} \\ 0 & \frac{\sqrt{433}}{866} \end{pmatrix} \tag{3.0.7}$$

Verify (3.0.7) using python code from

```
https://github.com/shreeprasadbhat/matrix-theory/
    tree/master/assignment6/codes/
    find_QR_decomposition.py
```