

Matrix theory - Assignment4

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Abstract—This document illustrates proving properties triangles using linear algebra

Download all latex-tikz codes from

<https://github.com/shreeprasadbhat/matrix-theory/blob/master/assignment4/>

Hence proved triangles on the same base and having equal areas lie between the same parallels.

1 PROBLEM

Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.

2 SOLUTION

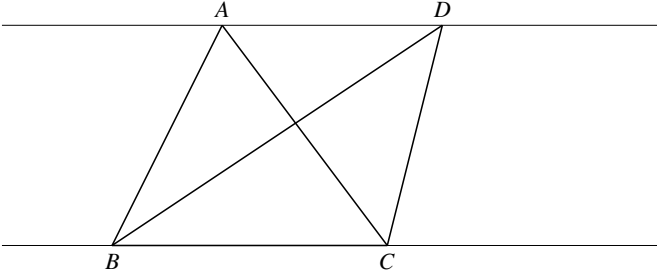


Fig. 0: $\triangle ABC$ and $\triangle BCD$ having common base BC

Given that,

$$\text{Area of } \triangle ABC = \text{Area of } \triangle BCD \quad (2.0.1)$$

We know that, area of triangle can be obtained by cross product.

$$\text{Area of } \triangle ABC = \frac{1}{2} \|(\mathbf{B} - \mathbf{A}) \times (\mathbf{B} - \mathbf{C})\| \quad (2.0.2)$$

$$= \frac{1}{2} \|[(\mathbf{B} - \mathbf{D}) + (\mathbf{D} - \mathbf{A})] \times (\mathbf{B} - \mathbf{C})\| \quad (2.0.3)$$

$$= \text{Area of } \triangle BCD + \frac{1}{2} \|(\mathbf{D} - \mathbf{A}) \times (\mathbf{B} - \mathbf{C})\| \quad (2.0.4)$$

From (2.0.1) and (2.0.4) we get,

$$\|(\mathbf{D} - \mathbf{A}) \times (\mathbf{B} - \mathbf{C})\| = 0 \quad (2.0.5)$$

$$\|(\mathbf{D} - \mathbf{A}) \times (\mathbf{B} - \mathbf{C})\| = \|k(\mathbf{B} - \mathbf{C}) \times (\mathbf{B} - \mathbf{C})\| \quad (2.0.6)$$

$$\implies (\mathbf{D} - \mathbf{A}) = k(\mathbf{B} - \mathbf{C}) \quad (2.0.7)$$

$$\implies \mathbf{AD} \parallel \mathbf{BC} \quad (2.0.8)$$