

Matrix theory - Assignment 14

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Abstract—This document proves properties on double dual

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<https://github.com/shreeprasadbhat/matrix-theory/blob/master/assignment14/>

1 PROBLEM

Let S be a set, F a field, and $V(S; F)$ the space of all functions from S into F :

$$(f + g)(x) = f(x) + g(x)$$

$$(cf)(x) = cf(x)$$

Let W be any n -dimensional subspace of $V(S, F)$. Show that there exist points x_1, x_2, \dots, x_n in S and functions f_1, \dots, f_n in W such that $f_i(x_j) = \delta_{ij}$

2 PICTORIAL REPRESENTATION OF PROBLEM

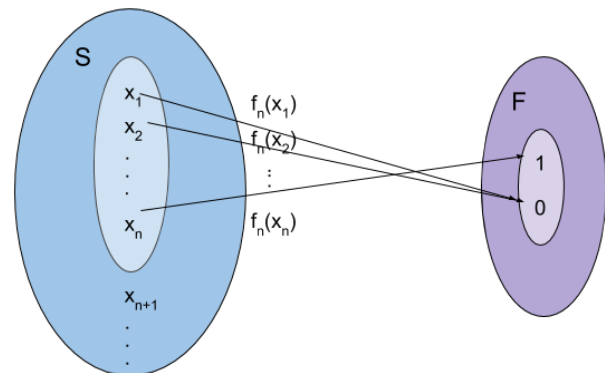
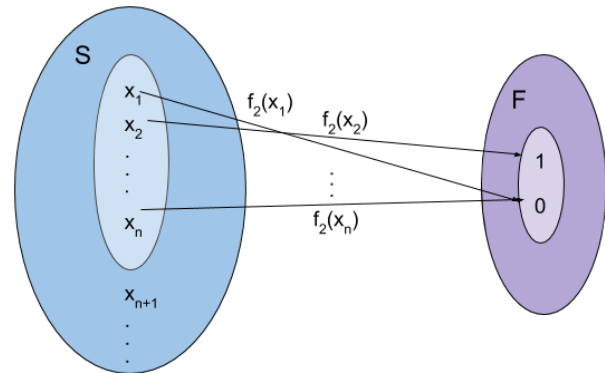
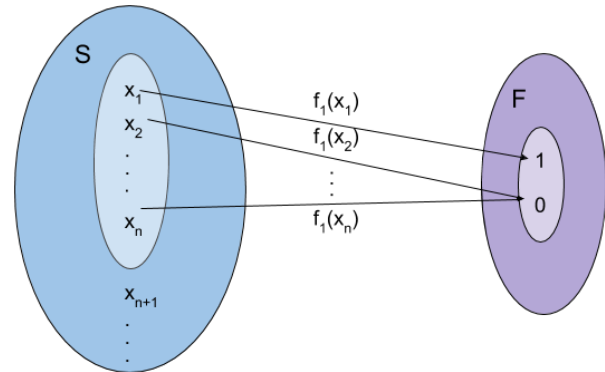
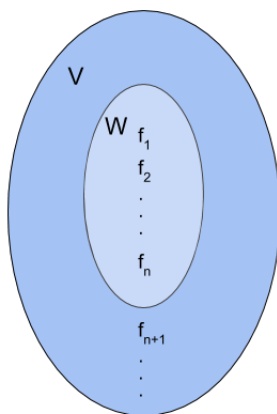


Fig. 0: Functions f_1, f_2, \dots, f_n where $f : S \rightarrow F$ and $f_i(x_j) = \delta_{ij}$ for $i, j = 1, \dots, n$

Fig. 0: Vector space of all function V and it's n -dimensional subspace W

3 SOLUTION

Given	<p>S is a set F is a field $V(S, F)$ is a linear functional such that W be n-dim subspace of $V(S, F)$.</p> <p>Also, $\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$</p>
To prove	<p>$f_i(x_j) = \delta_{ij}$</p> <p>where $x_1, x_2, \dots, x_n \in S$ and $f_1, f_2, \dots, f_n \in W$</p>
Proof	<p>Let $\phi_x : W \rightarrow F$</p> <p>Suppose $\phi_x(f) = 0 \quad \forall x \in S \quad \& \quad f \in W$ $\implies f(x) = 0$</p> <p>If $\forall x, \phi_x(f) \neq 0$ for some $f \in W$ If $n > 0 \exists x \in S$ such that $\phi_x(f) \neq 0$ for some $f \in W$ $\implies f_1(x_1) \neq 0$</p> <p>By scaling we can have $f_1(x_1) = 1$</p> <p>Hence $f_i(x_j) = \delta_{ij}$</p>

Now, we have

$$f_1(x_1) = f_1(1) = 1 \quad (4.0.5)$$

$$f_1(x_2) = f_1(0) = 0 \quad (4.0.6)$$

Also,

$$f_2(x_1) = f_2(1) = 0 \quad (4.0.7)$$

$$f_2(x_2) = f_2(0) = 1 \quad (4.0.8)$$

Hence $f_i(x_j) = \delta_{ij}$.

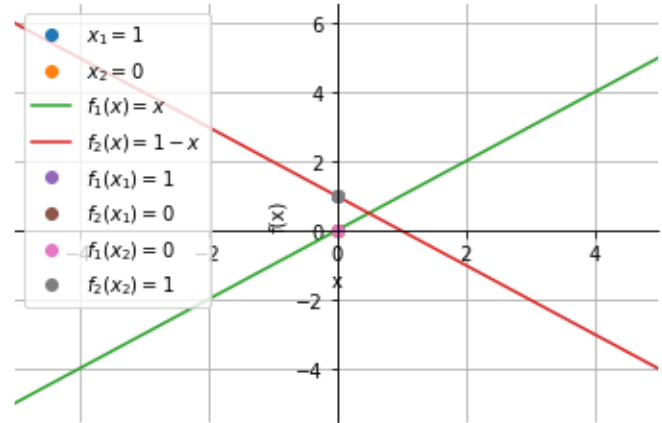


Fig. 0: Functions f_1, f_2 and points x_1, x_2

4 EXAMPLE

Consider points $\{x_1, x_2\} \in S$, let

$$x_1 = 1 \quad (4.0.1)$$

$$x_2 = 0 \quad (4.0.2)$$

Also consider functions $\{f_1, f_2\} \in W$ where

$$f_1(x) = x \quad (4.0.3)$$

$$f_2(x) = 1 - x \quad (4.0.4)$$