Matrix theory - Assignment 14

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***Abstract*—This document proves properties on double dual**

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https://github.com/shreeprasadbhat/matrix−theory/
    blob/master/assignment14/
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1 Problem

Let S be a set, F a field, and $V(S;F)$ the space of all functions from S into F:

$$(f+g)(x) = f(x) + g(x)$$
$$(cf)(x) = cf(x)$$

Let W be any n-dimensional subspace of $V(S,F)$. Show that there exist points $x_1, x_2, \ldots, x_n$ in S and functions $f_1, \ldots, f_n$ in W such that $f_i(x_j) = \delta_{ij}$

2 Pictorial representation of problem

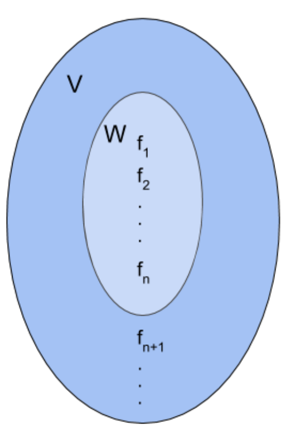

Fig. 0: Vector space of all function V and it's n-dimensional subspace W

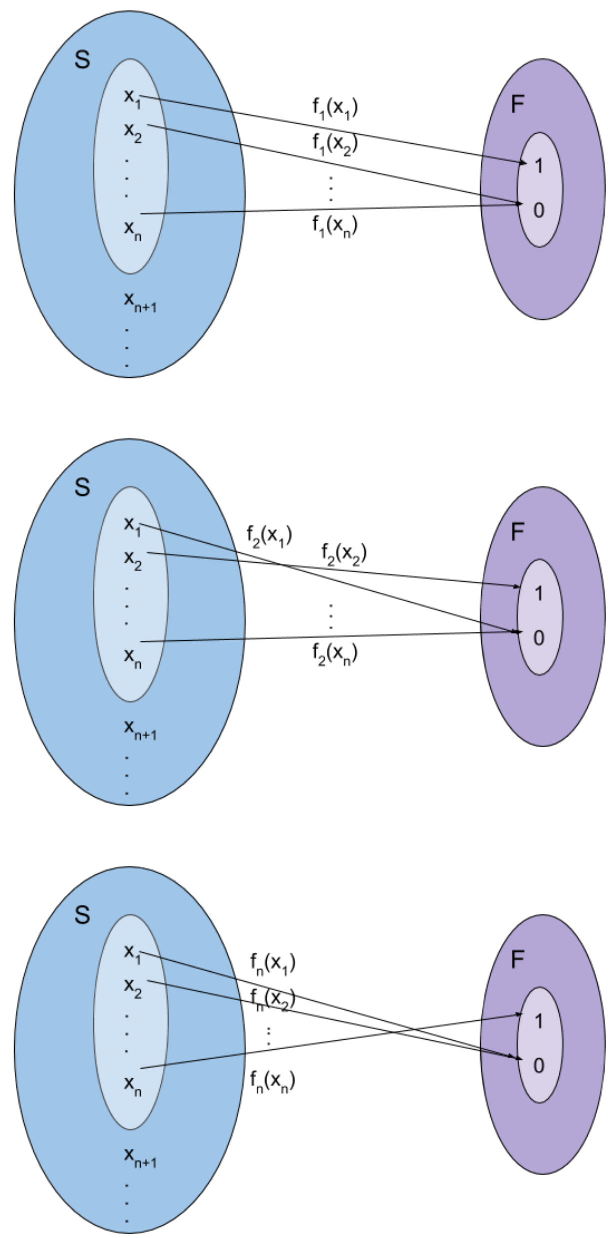

Fig. 0: Functions $f_1, f_2, \ldots, f_n$ where $f_i : S \to F$

3 Solution

Given	S is a set F is a field $V(S,F)$ is a linear functional such that W be n-dim subspace of $V(S,F)$. Also, $\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$
To prove	$f_i(x_j) = \delta_{ij}$ where $x_1, x_2, \ldots, x_n \in S$ and $f_1, f_2, \ldots, f_n \in W$
Proof	Let $\phi_x : W \to F$ Suppose $\phi_x(f) = 0 \ \ \forall x \in S \ \ \& \ \ f \in W$ $\implies f(x) = 0$ If $\forall x, \ \ \phi_x(f) \neq 0$ for some $f \in W$ If $n > 0 \ \exists \in S$ such that $\phi_x(f) \neq 0$ for some $f \in W$ $\implies f_1(x_1) \neq 0$ By scaling we can have $f_1(x_1) = 1$ Hence $f_i(x_j) = \delta_{ij}$

4 Example

Consider points $\{x_1, x_2\} \in S$, let

$$x_1 = 1 \tag{4.0.1}$$
$$x_2 = 0 \tag{4.0.2}$$

Also consider functions $\{f_1, f_2\} \in W$ where

$$f_1(x) = x \tag{4.0.3}$$
$$f_2(x) = 1 - x \tag{4.0.4}$$

Now, we have

$$f_1(x_1) = f_1(1) = 1 \tag{4.0.5}$$
$$f_1(x_2) = f_1(0) = 0 \tag{4.0.6}$$

Also,

$$f_2(x_1) = f_2(1) = 0 \tag{4.0.7}$$
$$f_2(x_2) = f_2(0) = 1 \tag{4.0.8}$$

Hence $f_i(x_j) = \delta_{ij}$.

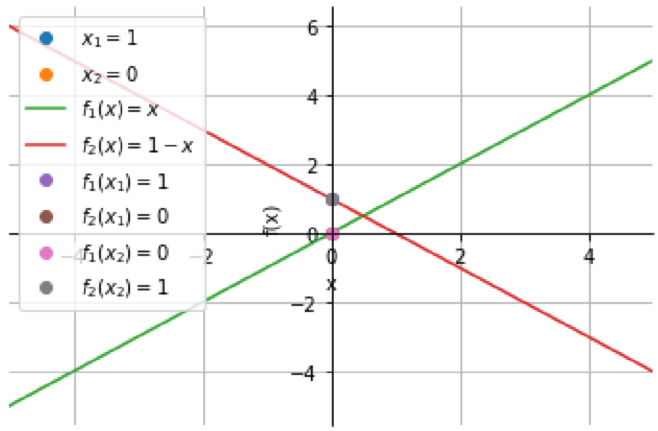

Fig. 0: Functions f_1, f_2 and points x_1, x_2