Matrix theory - Assignment4

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***Abstract*—This document illustrates proving properties triangles using linear algebra**

Download all latex-tikz codes from

https://github.com/shreeprasadbhat/matrix−theory/blob/master/assignment4/

1 Problem

Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.

2 Solution

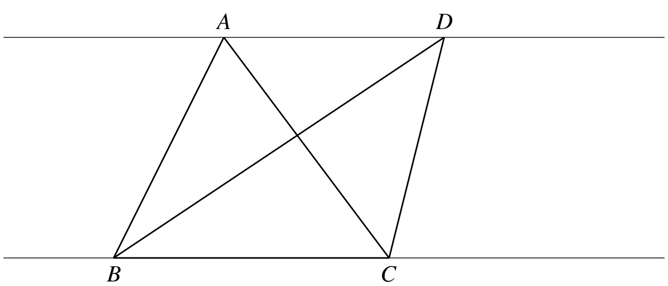

Fig. 0: $\triangle ABC$ and $\triangle BCD$ having common base BC

Given that,

$$\text{Area of } \triangle ABC = \text{Area of } \triangle BCD \tag{2.0.1}$$

We know that, area of triangle can be obtained by cross product.

$$\text{Area of } \triangle ABC = \frac{1}{2}\left\|(\mathbf{B}-\mathbf{A})\times(\mathbf{B}-\mathbf{C})\right\| \tag{2.0.2}$$
$$= \frac{1}{2}\left\|((\mathbf{B}-\mathbf{D})+(\mathbf{D}-\mathbf{A}))\times(\mathbf{B}-\mathbf{C})\right\| \tag{2.0.3}$$
$$= \text{Area of } \triangle BCD + \frac{1}{2}\left\|(\mathbf{D}-\mathbf{A})\times(\mathbf{B}-\mathbf{C})\right\| \tag{2.0.4}$$

From (2.0.1) and (2.0.4) we get,

$$\left\|(\mathbf{D}-\mathbf{A})\times(\mathbf{B}-\mathbf{C})\right\| = 0 \tag{2.0.5}$$

We know that, two nonzero vectors are **parallel** if and only if their cross product is zero.

$$\implies (\mathbf{A}-\mathbf{D}) = k(\mathbf{B}-\mathbf{C}) \tag{2.0.6}$$

Hence proved triangles on the same base and having equal areas lie between the same parallels.