

Matrix theory - Assignment 6

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Abstract—This document illustrates on how to perform QR decomposition of a matrix

Download all python codes from

<https://github.com/shreeprasadbhat/matrix-theory/tree/master/assignment6/codes>

and latex codes from

<https://github.com/shreeprasadbhat/matrix-theory/blob/master/assignment6/>

1 PROBLEM

Find QR decomposition for matrix

$$\mathbf{V} = \begin{pmatrix} 6 & \frac{17}{2} \\ \frac{17}{2} & 12 \end{pmatrix} \quad (1.0.1)$$

2 CONSTRUCTION

$$\text{Let } \mathbf{V} = (\mathbf{c}_1 \quad \mathbf{c}_2) = \begin{pmatrix} a & h \\ h & b \end{pmatrix} \quad (2.0.1)$$

Let \mathbf{Q} be an orthogonal and \mathbf{R} be an upper triangular matrix such that,

$$\mathbf{V} = \mathbf{QR} \quad (2.0.2)$$

$$= (\mathbf{q}_1 \quad \mathbf{q}_2) \begin{pmatrix} r_{11} & r_{12} \\ 0 & r_{22} \end{pmatrix} \quad (2.0.3)$$

$$(\mathbf{c}_1 \quad \mathbf{c}_2) = (\mathbf{q}_1 r_{11} \quad \mathbf{q}_1 r_{12} + \mathbf{q}_2 r_{22}) \quad (2.0.4)$$

$$\Rightarrow \mathbf{c}_1 = \mathbf{q}_1 r_{11} \quad (2.0.5)$$

$$r_{11} = \|\mathbf{c}_1\| = \sqrt{a^2 + h^2} \quad (2.0.6)$$

$$\mathbf{q}_1 = \frac{\mathbf{c}_1}{r_{11}} \quad (2.0.7)$$

$$\mathbf{c}_2 = \mathbf{q}_1 r_{12} + \mathbf{q}_2 r_{22} \quad (2.0.8)$$

$$r_{21} = \mathbf{q}_1^T \mathbf{c}_2 \quad (2.0.9)$$

$$r_{22} = \|\mathbf{c}_2 - \mathbf{q}_1 r_{12}\| \quad (2.0.10)$$

$$\mathbf{q}_2 = \frac{\mathbf{c}_2 - \mathbf{q}_1 r_{12}}{r_{22}} \quad (2.0.11)$$

3 SOLUTION

Given,

$$\mathbf{V} = \begin{pmatrix} 6 & \frac{17}{2} \\ \frac{17}{2} & 12 \end{pmatrix} \quad (3.0.1)$$

From (2.0.6),

$$r_{11} = \sqrt{6^2 + \left(\frac{17}{2}\right)^2} = \frac{\sqrt{433}}{2} \quad (3.0.2)$$

Substitute (3.0.2) in (2.0.7)

$$\mathbf{q}_1 = \begin{pmatrix} \frac{12}{\sqrt{433}} \\ \frac{\sqrt{433}}{17} \end{pmatrix} \quad (3.0.3)$$

Substitute (3.0.3) in (2.0.9)

$$r_{21} = \frac{306}{\sqrt{433}} \quad (3.0.4)$$

Substitute (2.0.10) in (3.0.4)

$$r_{22} = \frac{\sqrt{433}}{866} \quad (3.0.5)$$

Substitute (3.0.5) in (2.0.11)

$$\mathbf{q}_2 = \begin{pmatrix} \frac{17}{\sqrt{433}} \\ -\frac{\sqrt{433}}{12} \end{pmatrix} \quad (3.0.6)$$

Hence QR decomposition of \mathbf{V} is,

$$\mathbf{V} = \begin{pmatrix} \frac{12}{\sqrt{433}} & \frac{17}{\sqrt{433}} \\ \frac{\sqrt{433}}{17} & -\frac{\sqrt{433}}{12} \end{pmatrix} \begin{pmatrix} \frac{\sqrt{433}}{2} & \frac{306}{\sqrt{433}} \\ 0 & \frac{\sqrt{433}}{866} \end{pmatrix} \quad (3.0.7)$$

Verify (3.0.7) using python code from

`codes/find_QR_decomposition.py`