Matrix theory - Assignment4

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***Abstract*—This document illustrates proving properties triangles using linear algebra**

Download all latex-tikz codes from

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https://github.com/shreeprasadbhat/matrix-theory/
    blob/master/assignment4/
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1 Problem

Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.

2 Solution

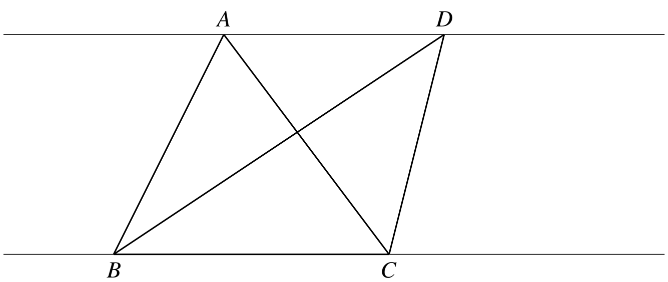

Fig. 0: $\triangle ABC$ and $\triangle BCD$ having common base BC

Given that,

$$\text{Area of } \triangle ABC = \text{Area of } \triangle BCD \tag{2.0.1}$$

We know that, area of triangle can be obtained by cross product.

$$\text{Area of } \triangle ABC = \frac{1}{2} \times [(\mathbf{B} - \mathbf{A}) \times (\mathbf{B} - \mathbf{C})] \tag{2.0.2}$$
$$= \frac{1}{2} \times [(\mathbf{B} - \mathbf{D}) + (\mathbf{D} - \mathbf{A})] \times (\mathbf{B} - \mathbf{C}) \tag{2.0.3}$$
$$= \text{Area of } \triangle BCD + \frac{1}{2} \times (\mathbf{D} - \mathbf{A}) \times (\mathbf{B} - \mathbf{C}) \tag{2.0.4}$$

From (2.0.1) and (2.0.4) we get,

$$(\mathbf{D} - \mathbf{A}) \times (\mathbf{B} - \mathbf{C}) = 0 \tag{2.0.5}$$
$$(\mathbf{D} - \mathbf{A}) \times (\mathbf{B} - \mathbf{C}) = k(\mathbf{B} - \mathbf{C}) \times (\mathbf{B} - \mathbf{C}) \tag{2.0.6}$$
$$\implies (\mathbf{D} - \mathbf{A}) = k(\mathbf{B} - \mathbf{C}) \tag{2.0.7}$$
$$\implies \mathbf{AD} \parallel \mathbf{BC} \tag{2.0.8}$$

Hence proved triangles on the same base and having equal areas lie between the same parallels.