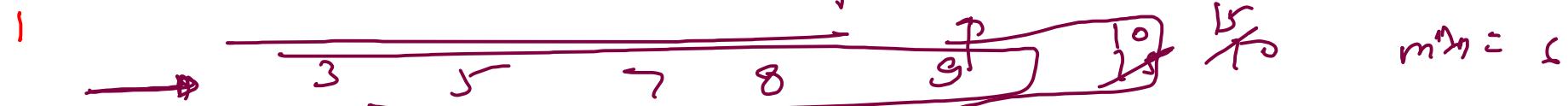
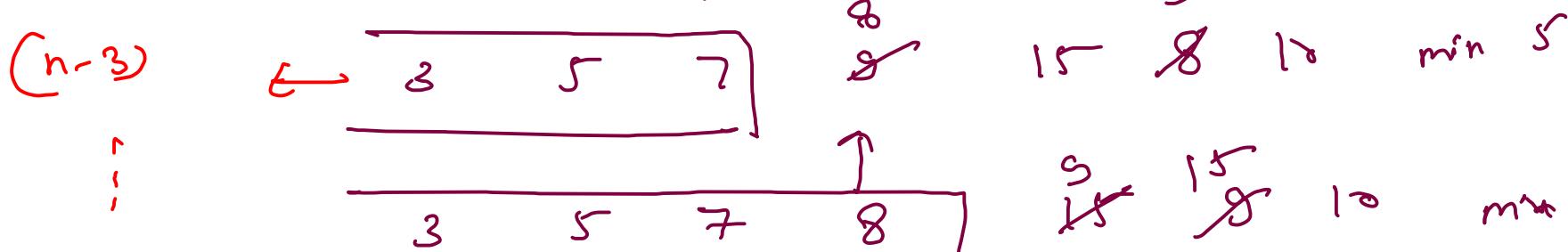
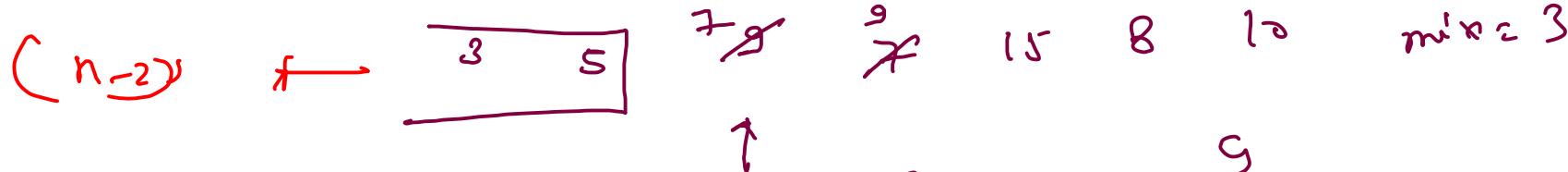


Bubble Sort → Adjacent Shifting

$$\text{Cost} = 1 + 2 + 3 + \dots + n$$

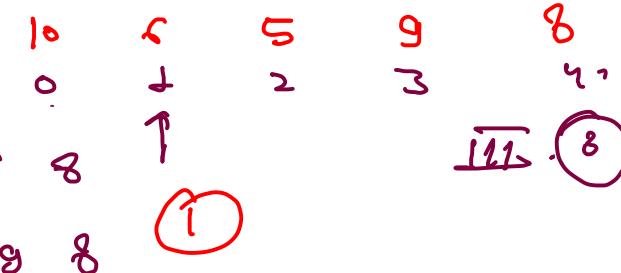
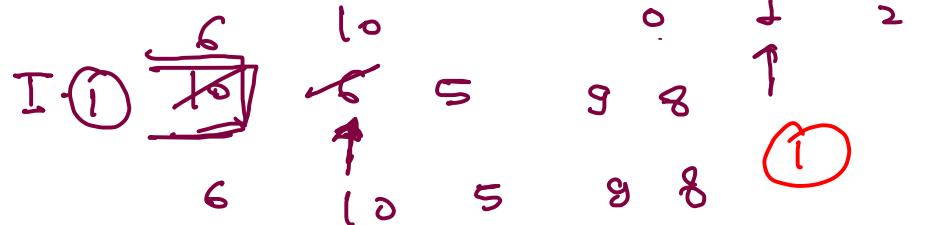
Selection Sort → Select Min. Element Grade
 $= \frac{n(n+1)}{2}$



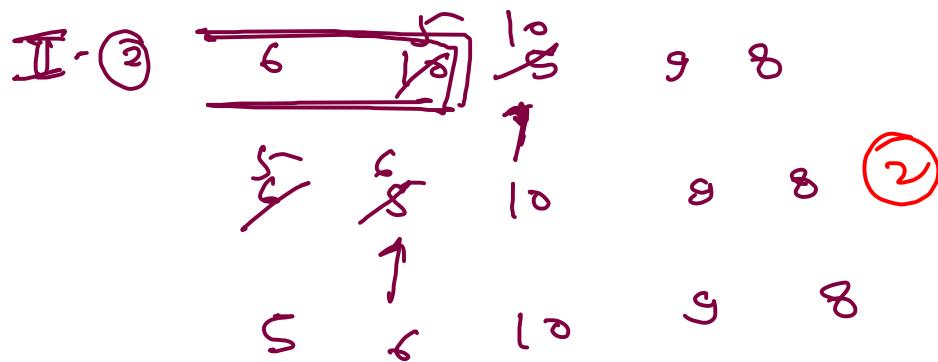
Insertion Sort

arr →

10 6 5 9 8 3 2 12 14 13
0 1 2 3 4 5 6 7 8 9

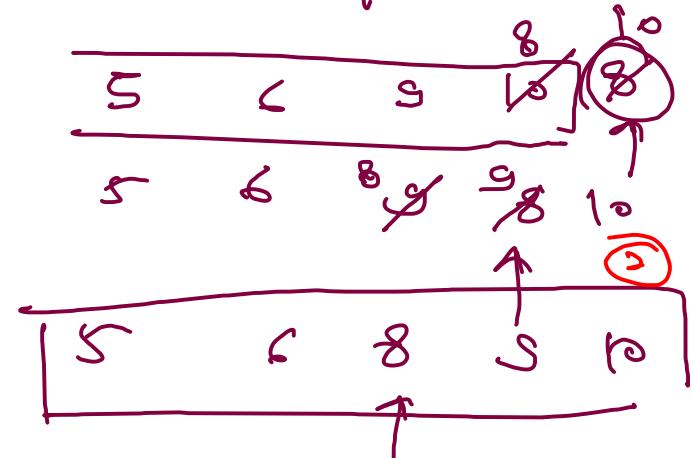
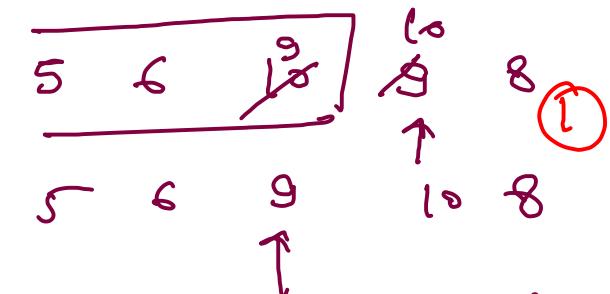


III 8



9 8
9 8
9 8
2

IV 4



= Complexity $\leq O(n^2)$

$50 \quad 40 \quad 30 \quad 20 \quad 10$ } worst case -
 $0 \quad 1 \quad 2 \quad 3 \quad 4$ } array Decreasing Order
 $\rightarrow O(n^2)$

$i=1, j=1, j+1$
~~50~~ ~~40~~ ~~30~~ ~~20~~ ~~10~~
~~40~~ ~~150~~ ~~30~~ ~~20~~ ~~10~~
 $= 1$
~~40~~ ~~50~~ ~~30~~ ~~20~~ ~~10~~

$$\text{cost} = 1 + 2 + 3 + 4 + \dots + n$$

$$= \frac{n(n+1)}{2} \in \frac{n^2}{2} + \frac{n}{2}$$

$$= \underline{\underline{O(n^2)}}$$

$i=2, j=i$
~~40~~ ~~80~~ ~~20~~ ~~10~~
~~80~~ ↑
~~30~~ ~~40~~ ~~50~~ ~~20~~ ~~10~~
 $= 2$
~~80~~ ~~40~~ ~~50~~ ~~20~~ ~~10~~

$i=4,$

~~20~~ ~~30~~ ~~40~~ ~~10~~ ~~50~~
~~10~~ ↑
~~40~~ ~~10~~ ~~50~~ ~~20~~ ~~30~~

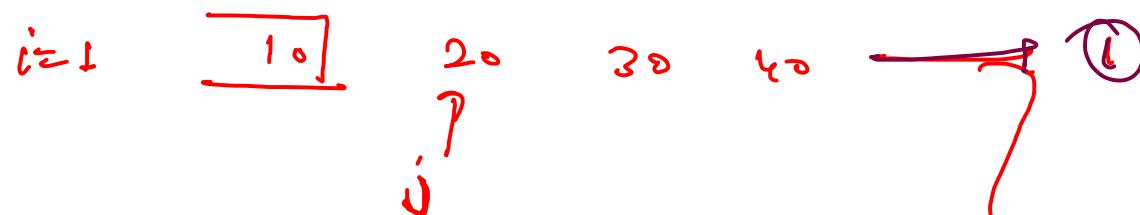
$= 4$

$i=3,$
~~30~~ ~~40~~ ~~20~~ ~~50~~ ~~10~~
~~20~~ ~~40~~ ~~40~~ ~~50~~ ~~10~~
~~20~~ ~~20~~ ~~40~~ ~~50~~ ~~10~~
~~20~~ ~~30~~ ~~40~~ ~~50~~ ~~10~~
 $= 3$
~~20~~ ~~30~~ ~~40~~ ~~50~~ ~~10~~

~~20~~ ~~30~~ ~~10~~ ~~40~~ ~~50~~
~~10~~ ↑
~~40~~ ~~10~~ ~~50~~ ~~20~~ ~~30~~
~~10~~ ~~20~~ ~~30~~ ~~40~~ ~~50~~
~~10~~ ↑
~~20~~ ~~30~~ ~~30~~ ~~40~~ ~~50~~
~~10~~ ~~20~~ ~~30~~ ~~40~~ ~~50~~

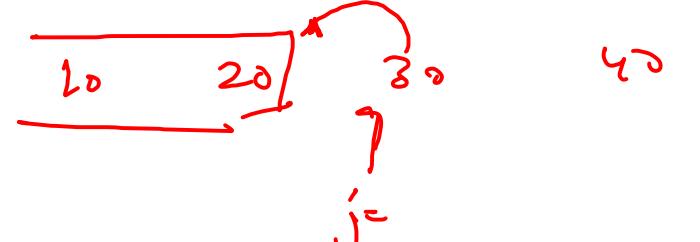
n

~~10~~ 0 20 L 30 2 40 3 } \Rightarrow Best case



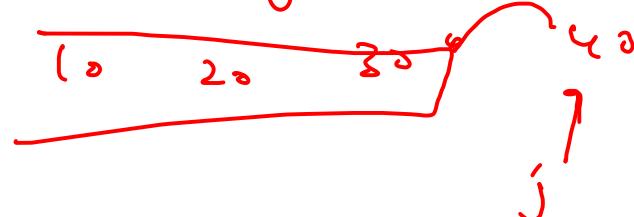
array-sorted

$\hookrightarrow O(n)$

$i=2$ 

cost = 1 + 1 + ... + n times

$$\approx n \times 1 = n$$

$i=3$ 

$\approx \underline{O(n)}$

$i=4 =$

Count Sort →

0 ≤ arr.length ≤ 100

0 ≤ arr[i] ≤ 9

array - sort →

4 2 2 1 5 1 2 1 1 5
↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

0 → ⚡ ✕ 2 (3)

1 → ⚡ ✕ ✕ 3 (4)

2 → ⚡ ✕ 2 (3)

3 → ⚡ ✕ 2 (2)

[0 6 0 1 1 1 2 2 2 2 3 4 4 5 6 6 7 7 7 8 9 9 9]

0 → ⚡ ✕ 1 (1)
1 → ⚡ ✕ 2 (2)
2 → ⚡ ✕ 3 (3)
3 → ⚡ ✕ 4 (4)
4 → ⚡ ✕ 5 (5)
5 → ⚡ ✕ 6 (6)
6 → ⚡ ✕ 7 (7)
7 → ⚡ ✕ 8 (8)
8 → ⚡ ✕ 9 (9)

9 , frequencies

$\Rightarrow O(n \cdot f_n) = O(2n) = O(n) =$

$\text{arr} = [7, -2, 4, -1, 3, -2, \cancel{1}, 7, 6, -1]$
 $\text{min} = -2$
 $\text{val} = 0 + 3 \rightarrow 3$ $\max \rightarrow 7$
 $\text{fmap} = \text{new int}[\max - \min + 1]$
 $\text{val} = \text{indx} + \min$
 $\text{Radix}' \left\{ \begin{array}{l} \text{stable} \\ \text{sorting} \end{array} \right.$

$\cancel{8} \rightarrow \cancel{-2} \cancel{\cancel{\cancel{1}}} \cancel{\cancel{\cancel{2}}} \cancel{\cancel{\cancel{3}}} \cancel{\cancel{\cancel{4}}} \cancel{\cancel{\cancel{5}}} \cancel{\cancel{\cancel{6}}} \cancel{\cancel{\cancel{7}}}$
 $\cancel{1} \rightarrow \cancel{-1} = \cancel{x} \cancel{\cancel{\cancel{2}}} \cancel{\cancel{\cancel{3}}} \cancel{\cancel{\cancel{4}}} \cancel{\cancel{\cancel{5}}} \cancel{\cancel{\cancel{6}}} \cancel{\cancel{\cancel{7}}}$
 $\cancel{2} \rightarrow \cancel{0} = 0$
 $\cancel{3} \rightarrow \cancel{1} = \cancel{x} \cancel{\cancel{\cancel{2}}} \cancel{\cancel{\cancel{3}}} \cancel{\cancel{\cancel{4}}} \cancel{\cancel{\cancel{5}}} \cancel{\cancel{\cancel{6}}} \cancel{\cancel{\cancel{7}}}$
 $\cancel{4} \rightarrow \cancel{2} = 0$
 $\cancel{5} \rightarrow \cancel{3} = \cancel{x} \cancel{\cancel{\cancel{2}}} \cancel{\cancel{\cancel{3}}} \cancel{\cancel{\cancel{4}}} \cancel{\cancel{\cancel{5}}} \cancel{\cancel{\cancel{6}}} \cancel{\cancel{\cancel{7}}}$
 $\cancel{6} \rightarrow \cancel{4} = \cancel{x} \cancel{\cancel{\cancel{2}}} \cancel{\cancel{\cancel{3}}} \cancel{\cancel{\cancel{4}}} \cancel{\cancel{\cancel{5}}} \cancel{\cancel{\cancel{6}}} \cancel{\cancel{\cancel{7}}}$
 $\cancel{7} \rightarrow \cancel{5} = 0$
 $\cancel{8} \rightarrow \cancel{6} = \cancel{x} \cancel{\cancel{\cancel{2}}} \cancel{\cancel{\cancel{3}}} \cancel{\cancel{\cancel{4}}} \cancel{\cancel{\cancel{5}}} \cancel{\cancel{\cancel{6}}} \cancel{\cancel{\cancel{7}}}$
 $\cancel{9} \rightarrow \cancel{7} = \cancel{x} \cancel{\cancel{\cancel{2}}} \cancel{\cancel{\cancel{3}}} \cancel{\cancel{\cancel{4}}} \cancel{\cancel{\cancel{5}}} \cancel{\cancel{\cancel{6}}} \cancel{\cancel{\cancel{7}}}$

$-2 \quad -2 \quad -1 \quad -1 \quad \cancel{1} \quad 3 \quad 4 \quad \cancel{5} \quad 7 \quad 7$

$\cancel{6} \rightarrow \cancel{8} = \cancel{x} \cancel{\cancel{\cancel{2}}} \cancel{\cancel{\cancel{3}}} \cancel{\cancel{\cancel{4}}} \cancel{\cancel{\cancel{5}}} \cancel{\cancel{\cancel{6}}} \cancel{\cancel{\cancel{7}}}$

```

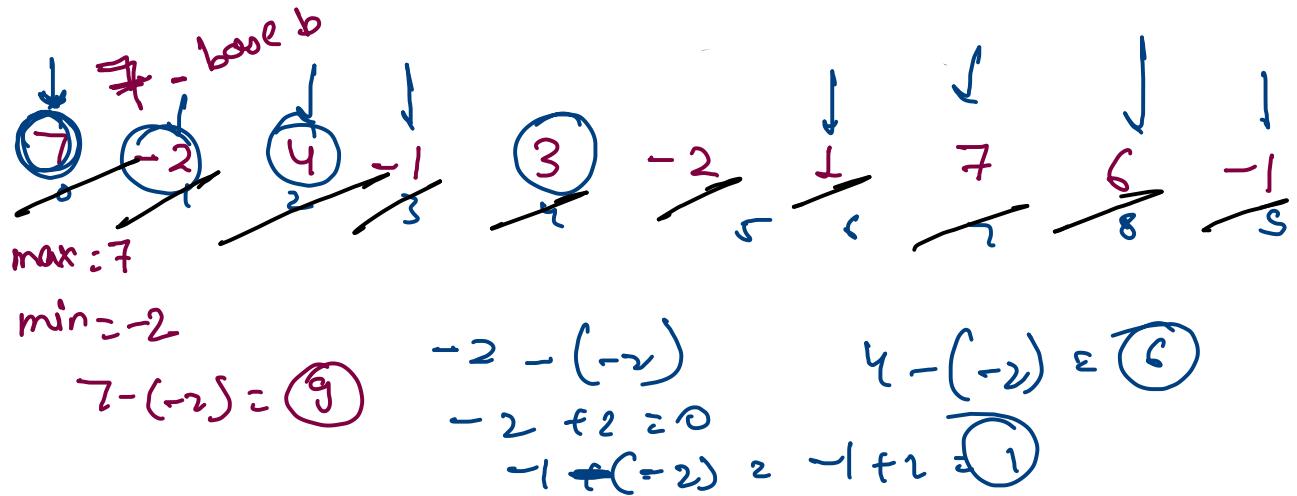
public static void countSort2(int[] arr, int min, int max) {
    int[] fmap = new int[max - min + 1];

    for(int i = 0; i < arr.length; i++) {
        int indx = arr[i] - min; // val - base value
        fmap[indx]++;
    }

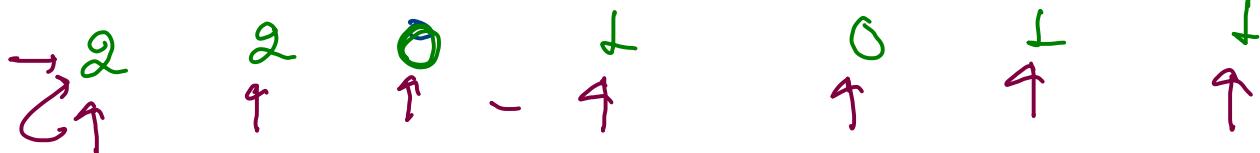
    int indx = 0;
    for(int i = 0; i < fmap.length; i++) {
        int val = i + min; // indx + base value
        int freq = fmap[i];

        for(int j = 0; j < freq; j++) {
            arr[indx] = val;
            indx++;
        }
    }
}

```



<u>val:</u>	-2	-1	0	1	2	3	4	5	6	7
	0	1	2	3	4	5	6	7	8	9



-2	-2	-1	-1	1	3	4	6	7	7
----	----	----	----	---	---	---	---	---	---

Indx + base val:

$$0 - 2 = \textcircled{-2}$$

$$1 - 2 = \textcircled{-1}$$

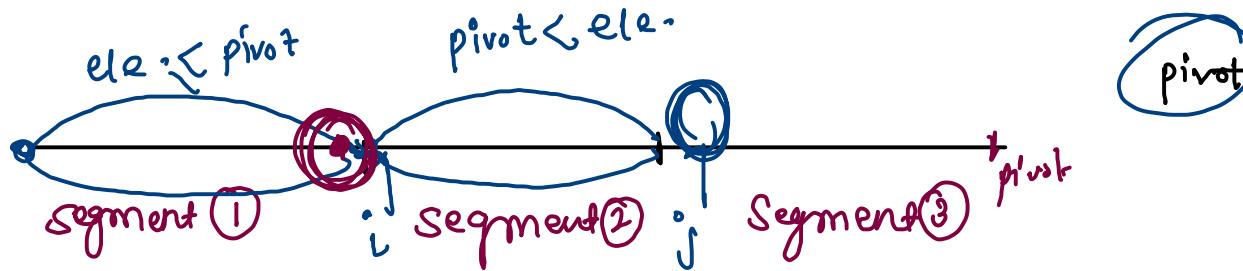
$$3 - 2 = \textcircled{1}$$

$$5 - 2 = \textcircled{3}$$

$$8 - 2 = \textcircled{6}$$

$$6 - 2 = \textcircled{4}$$

Partition of Array -



Segment ① → value \leq pivot

Segment ② → value $>$ pivot } { if i is first element of
Segment 2

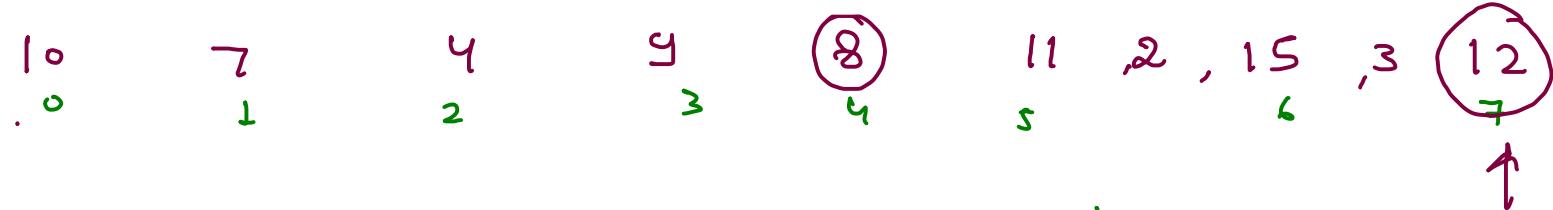
Segment ③ → value → unresolved → j is first element of Segment ③

$\text{arr}[j] > \text{pivot}$

$j++;$

$\text{arr}[j] \leq \text{pivot}$

Swap(i, j)
 $i++;$
 $j--;$



quicksort (int[] arr, int lo, int hi) {

High level

int pivot = arr[hi];
int pi = partitionIndex (arr, lo, hi, pivot);

quicksort (arr, lo, pi-1);

quicksort (arr, pi+1, hi);

}

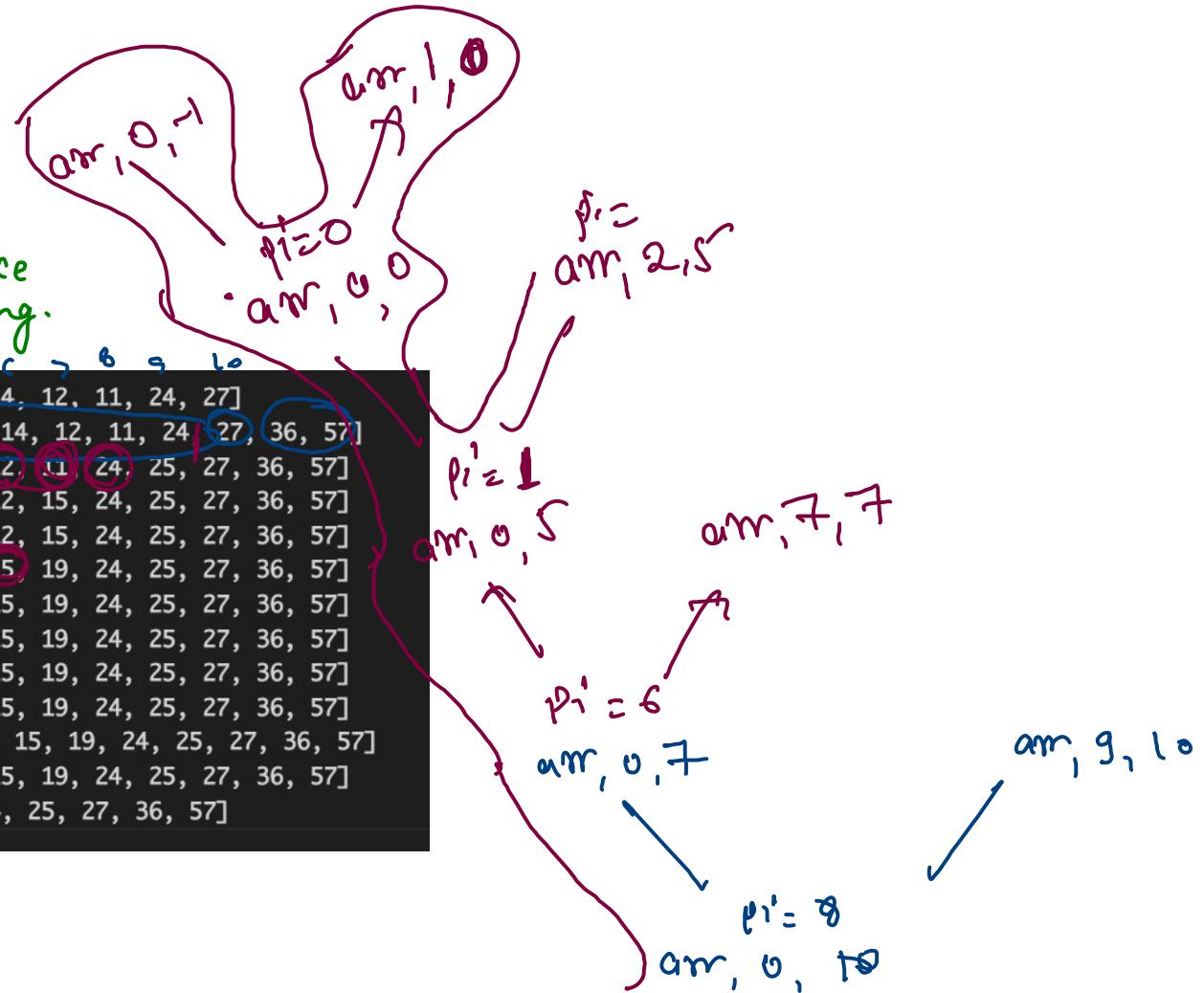
```

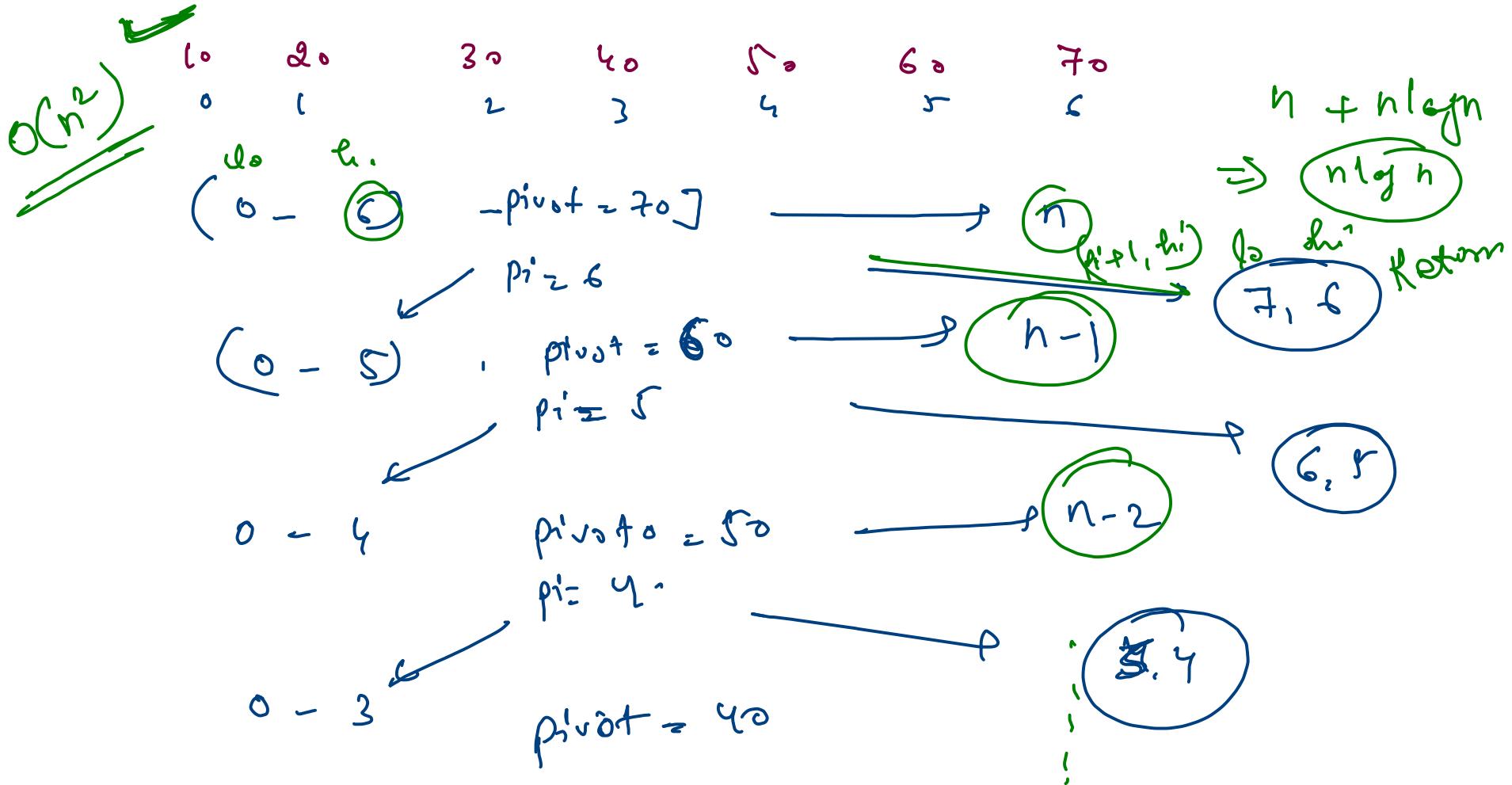
public static void quickSort(int[] arr, int lo, int hi) {
    int pivot = arr[hi];
    int pi = partitionIndx(arr, lo, hi, pivot);
    quickSort(arr, lo, pi - 1);
    quickSort(arr, pi + 1, hi);
}

```

quicksort → In place sorting.

Array Before sort : [10, 15, 19, 57, 25, 36, 14, 12, 11, 24, 27]
 lo : 0 hi : 10 pi : 8 Array : [10, 15, 19, 25, 14, 12, 11, 24, 27, 36, 57]
 lo : 0 hi : 7 pi : 6 Array : [10, 15, 19, 14, 12, 11, 24, 25, 27, 36, 57]
 lo : 0 hi : 5 pi : 1 Array : [10, 11, 19, 14, 12, 15, 24, 25, 27, 36, 57]
 lo : 0 hi : 0 pi : 0 Array : [10, 11, 19, 14, 12, 15, 24, 25, 27, 36, 57]
 lo : 2 hi : 5 pi : 4 Array : [10, 11, 14, 12, 15, 19, 24, 25, 27, 36, 57]
 lo : 2 hi : 3 pi : 2 Array : [10, 11, 12, 14, 15, 19, 24, 25, 27, 36, 57]
 lo : 3 hi : 3 pi : 3 Array : [10, 11, 12, 14, 15, 19, 24, 25, 27, 36, 57]
 lo : 5 hi : 5 pi : 5 Array : [10, 11, 12, 14, 15, 19, 24, 25, 27, 36, 57]
 lo : 7 hi : 7 pi : 7 Array : [10, 11, 12, 14, 15, 19, 24, 25, 27, 36, 57]
 lo : 9 hi : 10 pi : 10 Array : [10, 11, 12, 14, 15, 19, 24, 25, 27, 36, 57]
 lo : 9 hi : 9 pi : 9 Array : [10, 11, 12, 14, 15, 19, 24, 25, 27, 36, 57]
 Array after sort : [10, 11, 12, 14, 15, 19, 24, 25, 27, 36, 57]





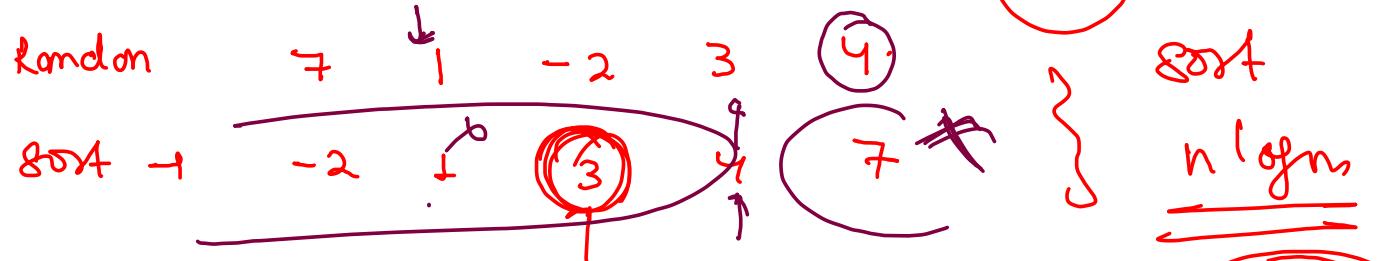
$$\text{Cost} = 1 + 2f_3 + \dots + n = \frac{n(n+1)}{2} = O(n^2) \quad \text{①}$$

k^{th} smallest



3rd smallest

, $k=3$



find correct position for $\text{Index} = k-1$, after sorting.

```
int quickselect (arr, lo, hi; int k) {  
    if (lo > hi) return -1;
```

```
    int pivot = arr [hi];
```

```
    int pi = partitionIndex (arr, pivot, lo, hi);
```

```
    if (pi > k) { // left call  
        return quickselect (arr, lo, pi-1);
```

```
    } else if (pi < k) {  
        return quickselect (arr, pi+1, hi);
```

```
    } else {  
        return pivot;
```