

Print zigzag. →

Input1 -> 1

Output1 -> 111

Input2 -> 2

Output2 -> 211121112

Input2 -> 3

Output3 -> 321112111232111211123

pzz(n) ?

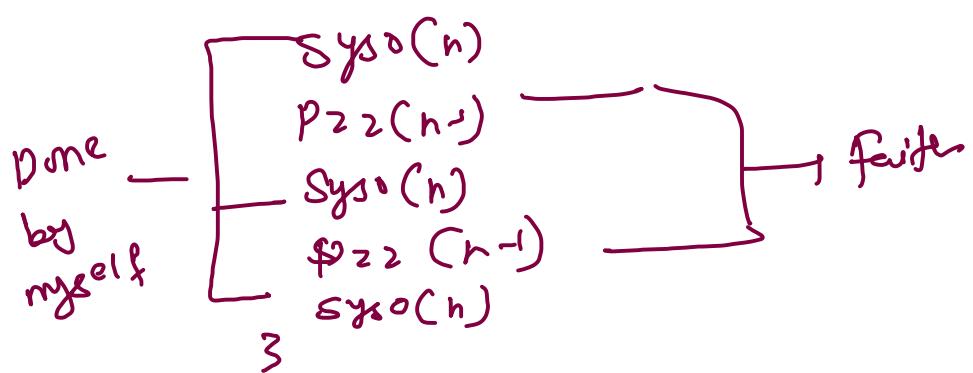
$$\text{Expectation } pzz(n) = n \ (n-1) \ \dots \ n \ (n-1) \ n$$

$$\underline{\text{faith}} \rightarrow pzz(n-1) = \underbrace{n-1}_{\text{faith}} \ (n-2) \ n-1 \ (n-2) \ n$$

Merging ↗

$$pzz(n) = \underbrace{n}_{\text{faith}} \ \underbrace{(n-1)}_{\text{faith}} \ \dots \ n \ \underbrace{(n-1)}_{\text{faith}} \ n$$

$$pzz(n) = \text{Sys}(n) \ pzz(n-1) \ \text{Sys}(n) \ pzz(n-1) \\ \text{Sys}(0)$$



low level analysis (print zig-zag)

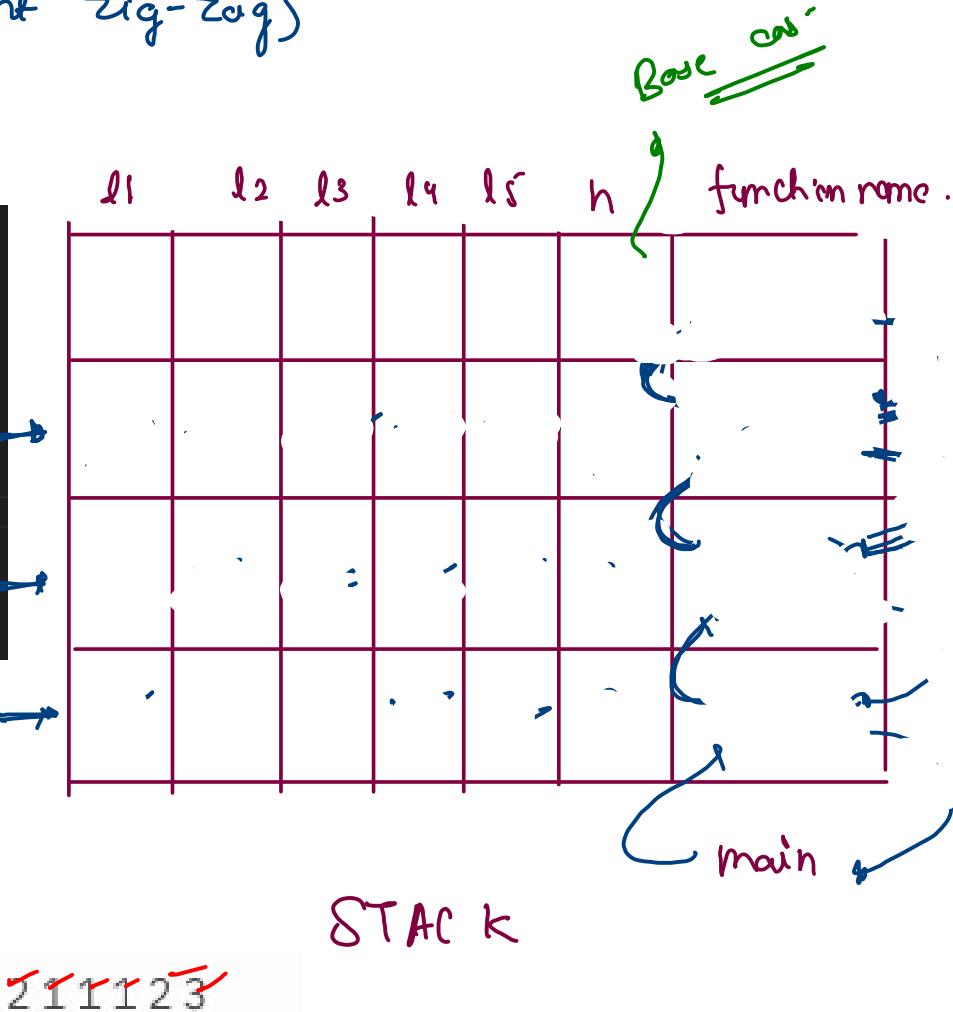
$n = 3$

```

public static void pzz(int n) {
    // pre area
    1 System.out.println("Pre : " + n);
    2 pzz(n - 1);
    // In area
    3 System.out.println("In : " + n);
    4 pzz(n - 1);
    // post area
    5 System.out.println("Post : " + n);
}

```

if($n == 0$)
return;



> 3 2 1 1 1 2 1 1 1 2 3 2 1 1 1 2 1 1 1 2 3

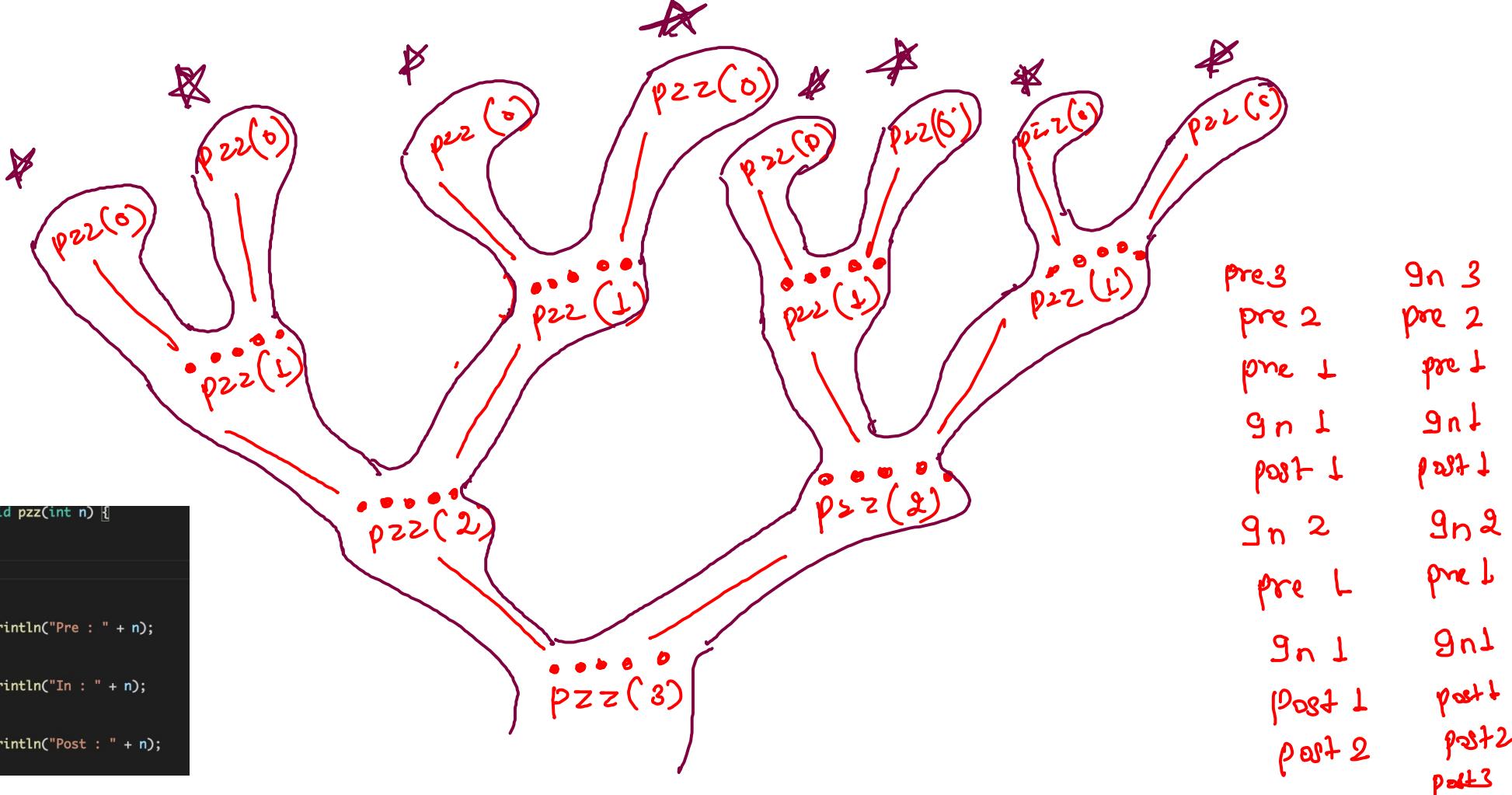
= 3 2 1 1 1 2 1 1 1 2 3 2 1 1 1 2 1 1 1 2 3

console

Pre: S	gn: 3
pre: 2	pre: 2
pre: L	pre: L
gn: L	gn: L
post: L	post: L
gn: 2	gn: 2
pre: L	pre: L
gn: L	gn: L
post: L	post: L
post: 2	post: 2
	post: 3

Recursion Tree / Euler Diagram / Euler traversal

3 2 1 1 1 2 1 1 1 2 3 2 1 1 1 2 1 1 2 3



```
public static void pzz(int n) {
    // base case
    if(n == 0)
        return;

    // pre area
    System.out.println("Pre : " + n);
    1. pzz(n - 1);
    // In area
    2. System.out.println("In : " + n);
    pzz(n - 1);
    // post area
    3. System.out.println("Post : " + n);
}
```

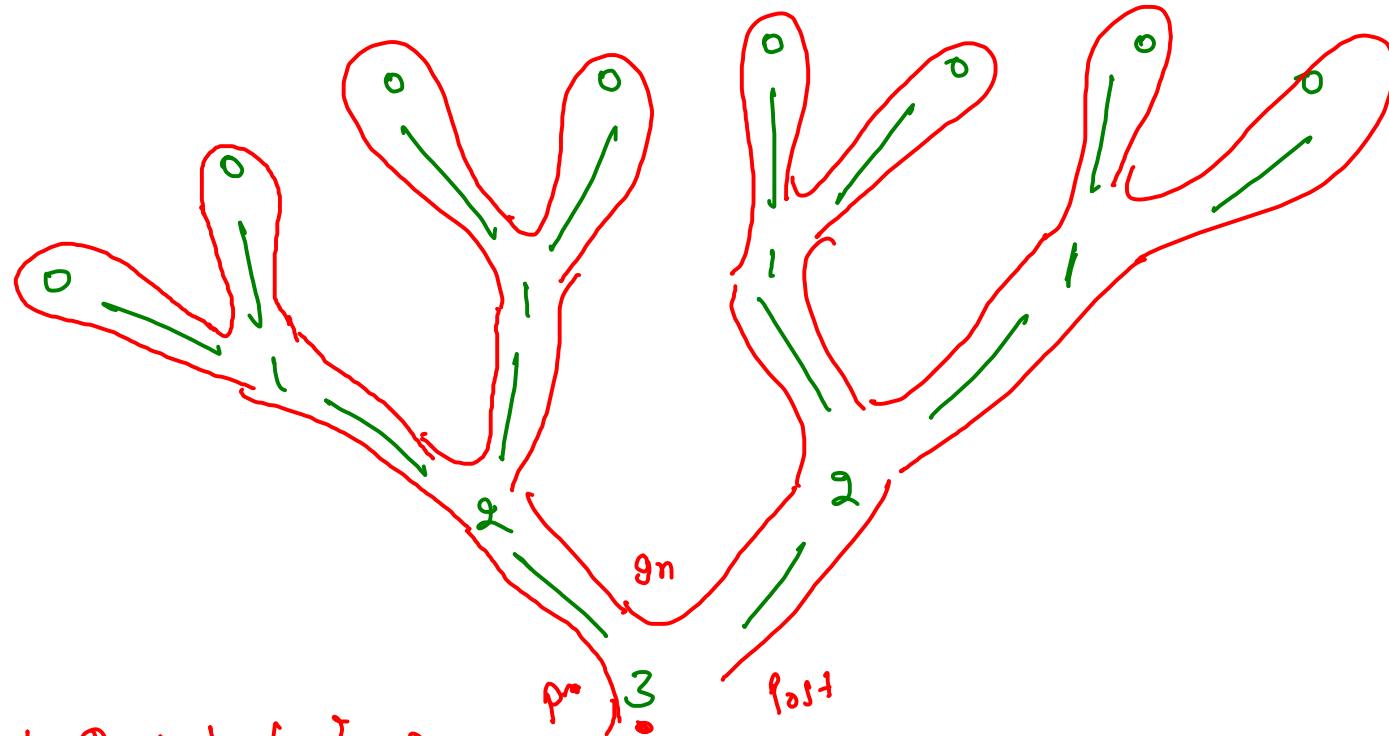
$\text{sys}^{\alpha}(n)$

$P_{\Sigma}(n-1)$

$\text{sys}^{\beta}(n)$

$P_{\Sigma}(n-1)$

$\text{sys}^{\alpha}(n)$



3 2 1 1 1 2 2 1 1 2 3

2 1 1 1 2 2 1 1 2 3

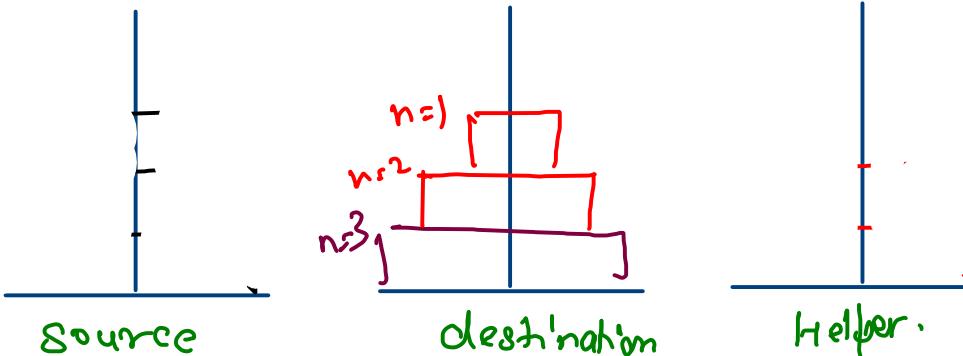
✓ $\text{toh}(n)$, source, destination, Helper

Expectation →

✓ $\text{toh}(3, A, B, C) \rightarrow$ print
 8 steps to move 3 discs from A to C using B.
 src dst helper
 ↑ ↑ ↑
 8 steps to move 3 discs from A to C using B.

(n-1) disc - Recursion

$\text{toh}(3, A, B, C)$

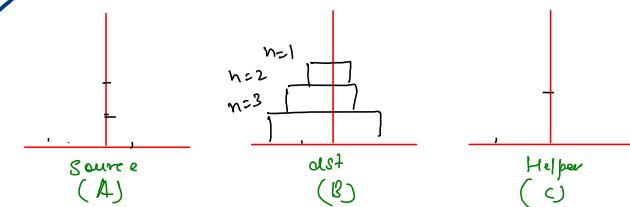
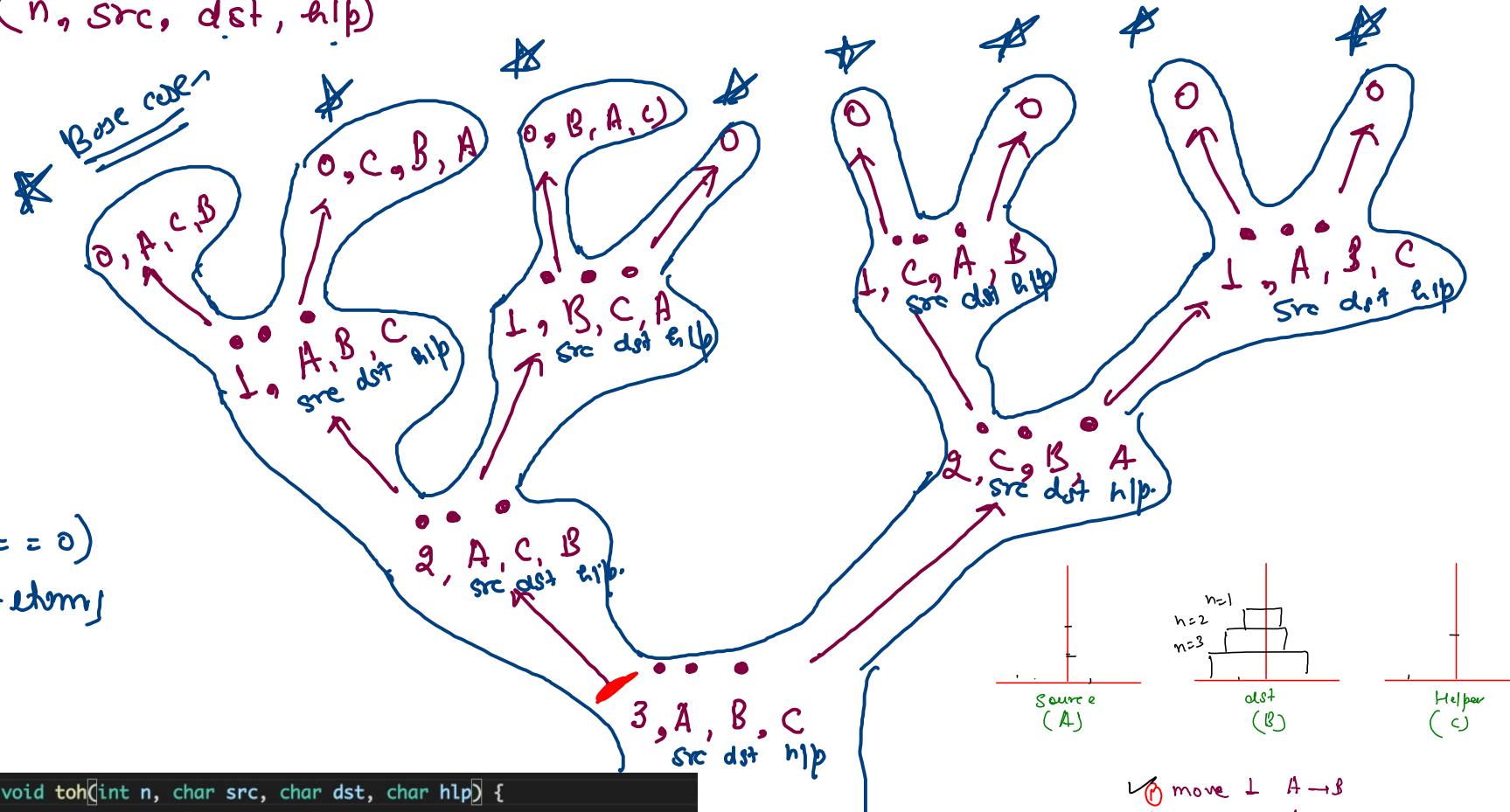


// forth → $\text{toh}(n-1, \text{src}, \text{helper}, \text{dst})$
 // my work
 → $\text{sys0}(\text{"move"}, \text{n}^{\text{th}} \text{disc from src to dst});$
 // forth → $\text{toh}(n-1, \text{helper}, \text{dst}, \text{src})$
 ↑ ↑ ↑
 src dst helper

$toh(n, src, dst, hlp)$

$if(n == 0)$
return;

```
public static void toh(int n, char src, char dst, char hlp) {
    // faith
    1' toh(n - 1, src, hlp, dst);
    // my work
    2' System.out.println("move " + n + "th disc from " + src + " to " + dst);
    // faith
    3' toh(n - 1, hlp, dst, src);
}
```



- ✓ ① move 1 A → B
- ✓ ④ move 2 A → C
- ✓ ⑩ move 1 B → C
- ✓ ⑫ move 3 A → B
- ✓ ⑪ move 1 C → A
- ✓ ⑮ move 2 C → B
- ✓ ⑯ move 1 A → B

$t_{0h}(n)$

$$\xrightarrow{t_{0h}(n)} \rightarrow c_1$$

$-sys()$

$$\xrightarrow{t_{eh}(n)} \rightarrow c_2$$

}

$$[c_1 + c_2 + 1]$$

$$T(n) = T(n-1) + T(n-1) + 1 \quad \left. \begin{array}{l} \text{Resource} \\ \text{Relation} \end{array} \right\}$$

$$T(n) = \cancel{2T(n-1)} + 1 \rightarrow ①$$

$$\cancel{2^2 T(n-2)} = \cancel{2^2 T(n-2)} + 2 \rightarrow ②$$

$$\cancel{2^3 T(n-3)} = \cancel{2^3 T(n-3)} + 2^2 \rightarrow ③$$

$$\cancel{2^4 T(n-4)} = \cancel{2^4 T(n-4)} + 2^3 \rightarrow ④$$

$$\frac{n-x=1}{x=n-1}$$

$$\cancel{2^{x-1} T(n-(x-1))} = \cancel{2^x T(n-x)} + \cancel{2^{x-1}} \rightarrow ②$$

$$T(n) = 2^x T(n-x) + [2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^{n-1}]$$

$$T(n) = 2^n + 1 \left(\frac{2^n - 1}{2 - 1} \right)$$

$n=3$

$$T(n-x) = T(x)$$

$$n-x=0, \boxed{n=x}$$

$\frac{a(r^n - 1)}{r-1} \rightarrow \text{sum of GP}$

$$\underbrace{a, ar, ar^2, ar^3, \dots, ar^{n-1}}$$

$$= 2^n + 2^n - 1$$

$$T(n) = \frac{2^{n+1} - 1}{2 - 1} \xrightarrow{\text{Reduce}} \underbrace{T(n)}_{O(2^n)} = 2^n - 1$$

$O(2^n) =$