

## 1. Basics of Bit Manipulation:

- ✓ How data stores i.e. Numbers
- ✗ types of Data (Number), Range
  - byte nibble - 4 bits
  - short
  - int
  - long
- ✗ 3. Decimal to Binary | Binary to Decimal
- ✗ 4. Operators (Bits)
- ✗ 5. How to ON a bit
- ✗ 6. How to OFF a bit
- ✗ 7. toggle
- ✗ 8. Check if bit is ON or OFF

$$(57)_{10} \rightarrow (x)_2$$

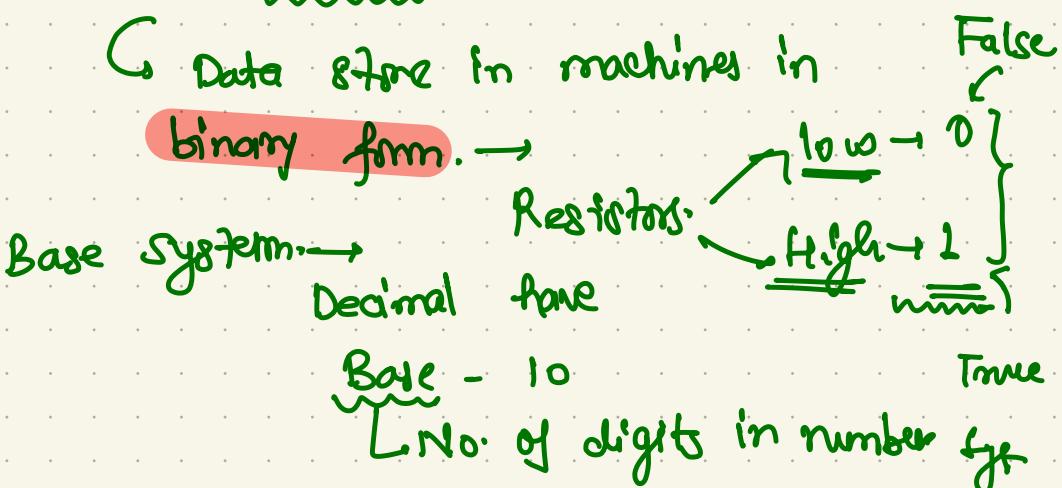
$x = (111001)_2$

|   |    |   |
|---|----|---|
| 2 | 57 |   |
| 2 | 28 | 1 |
| 2 | 14 | 0 |
| 2 | 7  | 0 |
| 2 | 3  | 1 |
| 2 | 1  | 1 |
|   | 0  |   |

## ✓ Basics of bit Mani.

2. Print values of RSB
3. Kernighan's Algorithm
4. Josephus Special
5. Gray code

$$n = 57$$



Decimal →  $\underbrace{10}_{\text{2}}$  → 0, 1, 2, ..., 9

Binary →  $\underbrace{2}_{\text{2}}$  → 0, 1

Store → Binary form

point → Decimal form

let discuss about nibble [data have only 4 bits allowed].  
suppose it is a data type have 4 bit

most significant Bit

|             |              |
|-------------|--------------|
| 0 0 0 0 → 0 | 1 0 0 0 → 8  |
| 0 0 0 1 → 1 | 1 0 0 1 → 9  |
| 0 0 1 0 → 2 | 1 0 1 0 → 10 |
| 0 0 1 1 → 3 | 1 0 1 1 → 11 |
| 0 1 0 0 → 4 | 1 1 0 0 → 12 |
| 0 1 0 1 → 5 | 1 1 0 1 → 13 |
| 0 1 1 0 → 6 | 1 1 1 0 → 14 |
| 0 1 1 1 → 7 | 1 1 1 1 → 15 |

from 4 bits, → we can represent number from 0 to 15.

NOTE:

- \* we can store 16 numbers from 0 to 15, But -ve numbers are not there

Approach '0' for negative numbers:→

If msb for a binary represent is 0 then no. if +ve-

If msb for a binary is 1 then number is -ve-

$x = \boxed{\underline{\quad} \underline{\quad}}$   
msb      considered bit for number

MSB check for +ve, -ve

msb → 0 → +ve

msb → 1 → -ve

msb

0 0 0 0 → +0 \*

0 0 0 1 → +1

0 0 1 0 → +2

0 0 1 1 → +3

0 1 0 0 → +4

0 1 0 1 → +5

0 1 1 0 → +6

0 1 1 1 → +7

msb

1 0 0 0 → -0 \*

1 0 0 1 → -1

1 0 1 0 → -2

1 0 1 1 → -3

1 1 0 0 → -4

1 1 0 1 → -5

1 1 1 0 → -6

1 1 1 1 → -7

Problem arises for +ve 0 and -ve 0.

Discarded method because  
of +ve 0 and -ve 0

Possibility from 4 bits are following  
these, but we make a change  
 $\downarrow$  to understand the +ve and -ve numbers.

Possibility  
for msb

|             |              |
|-------------|--------------|
| 0 0 0 0 → 0 | 1 0 0 0 → -8 |
| 0 0 0 1 → 1 | 1 0 0 1 → -7 |
| 0 0 1 0 → 2 | 1 0 1 0 → -6 |
| 0 0 1 1 → 3 | 1 0 1 1 → -5 |
| 0 1 0 0 → 4 | 1 1 0 0 → -4 |
| 0 1 0 1 → 5 | 1 1 0 1 → -3 |
| 0 1 1 0 → 6 | 1 1 1 0 → -2 |
| 0 1 1 1 → 7 | 1 1 1 1 → -1 |

[MSB → 0 → simple conversion  
msb → 1 → take 2's compliment

and attach a negative sign to a number

represented by style tick-

2's complement → 1's complement + 1

↳ represented by double tick.

toggle of all

bits is 1's

complement of a number

Example.

$$(1001)'' \rightarrow (1001)' + 1$$

$$\rightarrow (0110) + 1$$

$$\rightarrow \underline{\underline{0111}} \rightarrow \text{conversion}$$

$$\rightarrow \underline{\underline{-7}}$$

4 bits → -8 to 7

double 0's

problem is  
resolved now.

|         |                  |
|---------|------------------|
| 1 0 0 0 | $\rightarrow -8$ |
| 1 0 0 1 | $\rightarrow -7$ |
| 1 0 1 0 | $\rightarrow -6$ |
| 1 0 1 1 | $\rightarrow -5$ |
| 1 1 0 0 | $\rightarrow -4$ |
| 1 1 0 1 | $\rightarrow -3$ |
| 1 1 1 0 | $\rightarrow -2$ |
| 1 1 1 1 | $\rightarrow -1$ |
| 0 0 0 0 | $\rightarrow 0$  |
| 0 0 0 1 | $\rightarrow 1$  |
| 0 0 1 0 | $\rightarrow 2$  |
| 0 0 1 1 | $\rightarrow 3$  |
| 0 1 0 0 | $\rightarrow 4$  |
| 0 1 0 1 | $\rightarrow 5$  |
| 0 1 1 0 | $\rightarrow 6$  |
| 0 1 1 1 | $\rightarrow 7$  |

Range of number  $\rightarrow -8$  to 7  
cyclic behaviour 4 bits.

of N.S.  
Number add 1 in 7  $\rightarrow 7+1$   
in Binary System  $\Rightarrow 7 \rightarrow 0111$   
System  $\begin{array}{r} 1 \\ 0 \\ 1 \\ 1 \\ \hline 0 \\ 1 \\ 0 \\ 1 \\ \hline 1 \\ 0 \\ 1 \\ 0 \\ \hline \end{array}$   $\rightarrow -6$

if we add 1 in  
max. possible value  
of data type, then

it become change in  
minimum possible value  
of data type

machine

$$\left\{ \begin{array}{l} (-8) \\ + (-1) \\ \hline 0 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} 1000 \\ 1111 \\ \hline 0111 \end{array} \right. \xrightarrow{\text{Addition}} \text{COA} \rightarrow \text{computer Architecture}$$

Addition

$$\left\{ \begin{array}{l} (-8) \\ + (-1) \\ \hline 0 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} 1000 \\ 1111 \\ \hline 0111 \end{array} \right. = -9 \xrightarrow{\text{ALU}} \text{addition} \rightarrow \text{perf}$$

### Binary Subtraction-

$$\begin{array}{r} (10)_2 \xrightarrow{(2)_n} \\ -8 \\ 1000 \\ \hline 0000 \\ \text{msb} \rightarrow 1000 \\ \text{to integers} \\ \hline +7 \end{array}$$

integer  $\rightarrow \infty + 1 \Rightarrow -\infty$

integer, MaxValue + 1

computer Architecture

$$-\infty + 1 \Rightarrow$$

for any  
number system

(suppose here for nibble)

$$\begin{array}{r} -8 \\ 1 \\ \hline \end{array} \rightarrow \begin{array}{l} -\infty \text{ for nibble-} \\ \text{Add both of these. } -8 \rightarrow +000 \end{array}$$

$$\begin{array}{r} 1 \rightarrow 001 \\ \hline 1001 \end{array}$$

$\circlearrowleft -7$

4 bits  
allowed

MSB  $\begin{cases} 0 & \rightarrow \text{Normal conversion} \rightarrow \text{number is} \\ & \quad +ve \\ 1 & \rightarrow \text{take } 2's \text{ complement and attach -ve sign to number} \end{cases}$

Example  $\rightarrow$

$$\begin{array}{r} 1101 \\ \hline 1 \end{array}$$

$2's \text{ complement}$

$\begin{cases} 1 & \rightarrow 1's \text{ complement} + 1 \\ & \quad \text{or} \\ & \quad \text{toggle to all bits.} \end{cases}$

MSB = 1  
→ -ve Number

$$\begin{aligned} (1101) &\rightarrow (1101)' + 1 \\ &= 0010 \end{aligned}$$

$$\begin{array}{r} 1 \\ \hline 0011 \\ \hline \end{array}$$

MSB = 1 equal to 3

$\circlearrowleft -3$

because MSB is 1

Range of Numbers: →

| Data type | no. of bits. | Range                     |
|-----------|--------------|---------------------------|
| nibble    | 4            | $-2^3$ to $2^3 - 1$       |
| byte      | 8            | $-2^7$ to $2^7 - 1$       |
| short     | 16           | $-2^{15}$ to $2^{15} - 1$ |
| int       | 32           | $-2^{31}$ to $2^{31} - 1$ |
| long      | 64           | $-2^{63}$ to $2^{63} - 1$ |

$$12 \rightarrow (01100)_{10} \rightarrow (12)_{10}$$

gnt  $\xrightarrow{\quad}$

$\xrightarrow{\quad}$  nibble → 4  
 Higher bit to lower bit → lossy conversion?

n-bit number-

$$-2^3 - 2^3 - 1$$

.....

$$0000 \rightarrow 0$$

$$0001 \rightarrow 1$$

$$0010 \rightarrow 2$$

$$0011 \rightarrow 3$$

$$0100 \rightarrow 4$$

$$0101 \rightarrow 5$$

$$0110 \rightarrow 6$$

$$0111 \rightarrow 7$$

$$1000 \rightarrow -8$$

$$1001 \rightarrow -7$$

$$1010 \rightarrow -6$$

$$1011 \rightarrow -5$$

$$1100 \rightarrow -4$$

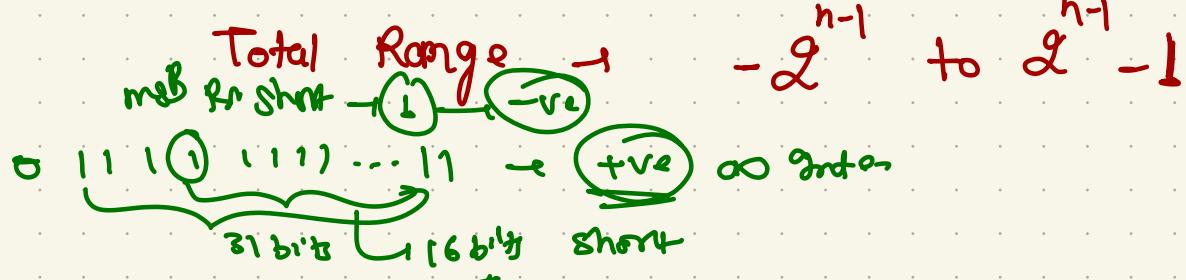
$$1101 \rightarrow -3$$

$$1110 \rightarrow -2$$

$$1111 \rightarrow -1$$

+ve Range →  $2^{n-1} - 1$

-ve Range →  $-2^{n-1}$



## operators in Bit Manipulation :

time complexity  $\rightarrow O(1)$

|   | &  | $\wedge$   | <<                            | >>   | >>>                     | $\sim$                | $-x$                  |
|---|--|--|-------------------------------|--|-------------------------|-----------------------|-----------------------|
| OR  | AND  | XOR  | left shift                    | right shift  | triple right shift      | I's compliment        | 2's compliment        |
| $\downarrow$  | $\downarrow$   | $\downarrow$   | $\downarrow$                  | $\downarrow$   | $\downarrow$            | $\downarrow$          | $\downarrow$          |
| { a   1 $\rightarrow$ 1<br>a   0 $\rightarrow$ a<br>1 -> Dom'nt<br>OR<br>0 0 - 0<br>0 1 - 1<br>1 0 - 1<br>1 1 - 1<br><u>bits ON</u><br>bit off<br>useful. | a & 1 $\rightarrow$ a<br>a & 0 $\rightarrow$ 0<br>0 & 0 $\rightarrow$ 0<br>0 & 1 $\rightarrow$ 0<br>1 & 0 $\rightarrow$ 0<br>1 & 1 $\rightarrow$ 1<br><u>bit off</u><br><u>useful.</u> | a $\wedge$ 1 = ~a<br>a $\wedge$ 0 = a<br>0 $\wedge$ 0 $\rightarrow$ 0<br>0 $\wedge$ 1 $\rightarrow$ 0<br>1 $\wedge$ 0 $\rightarrow$ 0<br>1 $\wedge$ 1 $\rightarrow$ 1<br><u>some 0</u><br><u>otherwise 1</u><br><u>toggle</u><br><u>toggle</u> | shift bits toward left side.  | shift bits toward right                              | shift bits toward right | I's compliment<br>new | 2's compliment<br>new |
|   |  |  | x 0 0 1 1<br>After left shift | ignoring bit some as MSB                             | ignoring bit always '0' | toggle all bits,      | <u>(s+)</u>           |
|   |  |  | 0 1 1 0                       | 4 bit number<br>msb $\rightarrow$ LL00<br>rightshift | G 0 1 1 0<br>G 0 0 1 1  | LL00                  |                       |
|   |  |  |                               | LL10<br>msb $\rightarrow$ 0 1 1 0<br>G 0 0 1 1       | 0 1 1 0<br>G 0 0 1 1    |                       |                       |

Bit ON

 $x \rightarrow 10110\boxed{0}101$ 

↳ on this bit

 $x \rightarrow 10110101$ 

mask  $\rightarrow 0000\boxed{1}\overbrace{000}$

$x \mid \text{mask}$      $\underline{\underline{10111101}}$

OR

$\underbrace{7 \ll 3}_{(111)_2} \rightarrow (111000)_2$

Bit OFF

 $x = 101\boxed{1}0101$ 
 $x = 101\boxed{0}101$ 

mask =  $\underline{\underline{11101111}}$

$x \& \text{mask} = \underline{\underline{10100101}}$

on third bit on number 'x'

How to prepare mask

Can we prepare  $\underline{\underline{000000...1}}$

mask = 1     $\begin{array}{l} \text{Binary} \\ (\text{in decimal}) \end{array}$

Shift toward left in mask

mask =  $\underbrace{(1 \times \underbrace{< 3}_{\text{left shift}})}_{\substack{\text{no. of shift} \\ \text{no. of number}}} \leftarrow$

which we are performing shifting.

mask =  $\underbrace{(1 \times \underbrace{< 4}_{\text{number}})}_{\text{it's combin.}}$

$0000\dots 1$

$00\dots 10000$

$15 \rightarrow \underbrace{111\dots 01111}$

Complexity  $\rightarrow O(1)$

O(1)

toggle a bit →

$$x = 101 \boxed{1} 0101 \quad x = 1\boxed{0} 1101$$

\*

$$\begin{array}{r} \text{mask} \rightarrow 00010000 \\ \hline x \& \text{mask} = \boxed{1} 1101 \end{array}$$

mask = 010000  
x & mask = 1101

$$x \& \text{mask} = 10100101$$

Let toggle.

Check if bit is ON or OFF →

$$x = 110\boxed{1}0011$$

$$\begin{array}{r} \text{mask} = 00010000 \\ \hline x \& \text{mask} = 00010000 \end{array}$$

$$x \& \text{mask} = 00010000$$

$(x \& \text{mask}) == \text{mask} \rightarrow$  bit is ON

$(x \& \text{mask}) == 0 \rightarrow$  bit is OFF

| Java Operator Precedence                  |   |
|---|---|
| Operators                                 | Precedence                                |
| postfix increment and decrement           | ++ -- ✓                                   |
| prefix increment and decrement, and unary | ++ -- + - ~ !                             |
| multiplicative                            | * / %                                     |
| additive                                  | + -                                       |
| shift                                     | << >> >>>                                 |
| relational                                | < > <= >= instanceof                      |
| equality                                  | == !=                                     |
| bitwise AND                               | &   |
| bitwise exclusive OR                      | ^K  |
| bitwise inclusive OR                      |   |
| logical AND                               | &&  |
| logical OR                                |   |
| ternary                                   | ? :                                       |
| assignment                                | = += -= *= /= %=<br>&= ^=  = <<= >>= >>>= |