Radix Sort →

digit wise sort → stability maintain

7 4 9 6 3	8 7 2 4 1	1 7 9 2 8	5 4 1 3 2	4 2 3 8 6	1 6 3 9 7
8 7 2 4 1	8 6 2 7 1	5 4 1 3 2	8 7 2 4 1	5 4 1 3 2	1 7 9 2 8
1 6 3 9 7	5 4 1 3 2	8 7 2 4 1	8 6 2 7 1	7 4 9 6 3	4 2 3 8 6
4 2 3 8 6	7 4 9 6 3	7 4 9 6 3	4 2 3 8 6	8 6 2 7 1	5 4 1 3 2
1 7 9 2 8	4 2 3 8 6	8 6 2 7 1	1 6 3 9 7	1 6 3 9 7	7 4 9 6 3
8 6 2 7 1	1 6 3 9 7	4 2 3 8 6	1 7 9 2 8	8 7 2 4 1	8 6 2 7 1
5 4 1 3 2	1 7 9 2 8	1 6 3 9 7	7 4 9 6 3	1 7 9 2 8	8 7 2 4 1

(74963/1) % 10 = 3

(74963/10) % 10 = 6 (74963/100) % 10 = 9

74963 } → String – "74963"

'9' – '0'

'9' – 48

Complexities	Reocuurence Relation	Example
$O(1)$	—	Arithmatic operator, conditional checks
$O(\log n)$	$T(n) = T(n/2) + K$	Binary Search, Smart power
$O(\sqrt{n})$	—	Prime
$O(n)$	$T(n) = T(n-1) + k$ $T(n) = 2T(n/2) + K$ $T(n) = T(n/2) + n + K$	Normal Power, fake Smart Power quick select
$O(n \log n)$	$T(n) = 2T(n/2) + n + K$	Mergesort, quicksort
$O(n^2)$	$T(n) = T(n-1) + n + k$	Bubble, Selection, Insertion, worst case of quicksort
$O(2^n)$	$T(n) = 2T(n-1) + K$ $T(n) = T(n-1) + T(n-2) + K$	Subseq, Fibonacci
$O(n)!$	$T(n) = n \times T(n-1) + K$	Permutation.

$\underline{O(\log n)}$

$n \to n/2$ $\quad \cancel{T(n/2)} = \cancel{T(n/2^2)} + k \longrightarrow (2)$

$n \to n/2$ $\quad \cancel{T(n/2^2)} = \cancel{T(n/2^3)} + k \longrightarrow (3)$

$n \to n/2$ $\quad \cancel{T(n/2^3)} = \cancel{T(n/2^4)} + k \longrightarrow (4)$

$T(n) = \cancel{T(n/2)} + k \longrightarrow (1)$

$\vdots \qquad \vdots \qquad \vdots \qquad x$ times.

Base case $\Longrightarrow T(1)$

$\cancel{T(n/2^{x-1})} = T(n/2^x) + k$

add all Eq^n

$T(n) = T(n/2^x) + kx \longrightarrow (5)$

$\Rightarrow T(n/2^x) = T(1) \Rightarrow 1$

$\Rightarrow \dfrac{n}{2^x} = 1$

$n = 2^x$

take $\log_2$ both side.

$\log_2 n = \log_2 2^x$

$\Rightarrow \boxed{x = \log_2 n}$

put value of 'x' in eq (5)

$T(n) = T(1) + kx \Rightarrow T(n) = 1 + k\log n$

$\boxed{T(n) = O(\log n)}$

Power Analysis

Normal power	Fake Smart Power	Smart Power.

pow(x, n) {
 if(n==0) return 1;
 return x * pow(x, n-1);
 }
	pow(x, n) {
	 if(n==0) return 1;
	 if(n%2==0)
	 return pow(x, n/2) * pow(x, n/2);
	 else (
	 x * pow(x, n/2) * pow(x, n/2);
		pow(x, n) {
		 if(n==0) return 1;
		 half = pow(x, n/2);
		 if(n % 2 == 0)
		 return half * half;
		 else
		 return half * half * x;

$T(n) = T(n-1) + k$ $O(n)$	$T(n) = T(n/2) + T(n/2) + K$	$T(n) = T(n/2) + k$

$\underline{O(n)}$ →

$n \to n-1$ $\qquad T(n) = \cancel{T(n-1)} + k$ —— 1

$n \to n-1$ $\qquad \cancel{T(n-1)} = \cancel{T(n-2)} + k$ —— 2

$n \to n-1$ $\qquad \cancel{T(n-2)} = \cancel{T(n-3)} + k$ ⟶ 3

$\qquad \cancel{T(n-3)} = \cancel{T(n-4)} + k$ ⟶ 4

$\vdots \qquad \vdots \qquad \vdots$ $\quad x$ times

$\qquad \cancel{T(n-(x-1))} = T(n-x) + k$ ⟶ Base case $T(1)$

add all Equations

$$T(n) = T(n-x) + kx \longrightarrow 5$$

$$T(n-x) = T(1)$$

$$\Rightarrow n - x = 1$$

$$\Rightarrow \boxed{x = n-1}$$

put the value of 'x' in eq of 5

$$T(n) = T(1) + kx$$

$$\Rightarrow T(n) = k(n-1) + 1$$

$$T(n) = kn \underbrace{-k+1}$$

$$T(n) = kn + C$$

$$\Rightarrow \boxed{T(n) = O(n)}$$

$\underline{\underline{O(n)}}$

Fake smart p

$$T(n) = T(n/2) + T(n/2) + k$$

$$\Rightarrow \quad T(n) = \cancel{2T(n/2)} + k \quad \text{———} \quad (1)$$

$$n \to n/2 \,]\times 2 \qquad \cancel{2T(n/2)} = \cancel{2^2 T(n/2^2)} + 2k \quad \text{———} \quad (2)$$

$$n \to n/2 \,]\times 2 \qquad \cancel{2^2 T(n/2^2)} = \cancel{2^3 T(n/2^3)} + 2^2 k \quad \text{———} \quad (3)$$

$$n \to n/2 \,]\times 2 \qquad \cancel{2^3 T(n/2^3)} = \cancel{2^4 T(n/2^4)} + 2^3 k \quad \text{———} \quad (4)$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad x \text{ times,}$$

$$\cancel{2^{x-1} T(n/2^{x-1})} = 2^x T(\underbrace{n/2^x}_{\to T(1)}) + 2^{x-1} k$$

$$T(n) = 2^x T(n/2^x) + k + 2k + 2^2 k + 2^3 k + \dots + 2^{x-1} k \to (5)$$

$$T(n/2^x) = T(1)$$

$$\Rightarrow \frac{n}{2^x} = 1$$

$$\boxed{n = 2^x}$$

$$\log_2 n = \log_2 2^x \quad \Rightarrow \boxed{x = \log n}\ , \text{ put value of } x \text{ in eq (5)}$$

$$T(n) = 2^x \underbrace{T(1)}_{1} + k(\underbrace{1 + 2 + 2^2 + \dots}_{\text{G.P.},\ a=1,\ r=2,\ 1 \Rightarrow \text{sum} = \frac{a(r^n-1)}{r-1}} + 2^{x-1})$$

$$T(n) = 2^x + k(2^x - 1) \quad \Rightarrow \quad 2^x(k+1) - k$$

$$T(n) = n(k+1) - k \quad \Rightarrow \quad \boxed{T(n) = O(n)}$$

~~Merge Sort~~

quick Sort

$$T(n) = \underbrace{T(n/2)} + \underbrace{T(n/2)} + \underline{n} + \underline{k}$$

$$T(n) = \cancel{2T(n/2)} + n + k \longrightarrow 1$$

$n \to n/2 \;] \times 2$ $\qquad \cancel{2T(n/2)} = 2^2 \cancel{T(n/2^2)} + \cancel{2}n/\cancel{2} + 2k \longrightarrow 2$

$n \to n/2 \;] \times 2$ $\qquad \cancel{2^2 T(n/2^2)} = 2^3 \cancel{T(n/2^3)} + \cancel{2^2} n/\cancel{2^2} + 2^2 k \longrightarrow 3$

$n \to n/2 \;] \times 2$ $\qquad \cancel{2^3 T(n/2^3)} = 2^4 \cancel{T(n/2^4)} + \cancel{2^3} n/\cancel{2^3} + 2^3 k \longrightarrow 4$

x times

$$\cancel{2^{x-1} T(n/2^{x-1})} = 2^x \underbrace{T(n/2^x)}_{\to T(1)} + \cancel{2^{x-1}} n/\cancel{2^{x-1}} + 2^{x-1} k$$

$$T(n) = \underline{2^x} T(n/2^x) + nx + k[1 + 2 + 2^2 + \dots + 2^{x-1}] \longrightarrow 5$$

$$T(n/2^x) = T(1) \Rightarrow \frac{n}{2^x} = 1 \Rightarrow \boxed{x = \log n}$$ put value of 'x' in eq 5

$$\Rightarrow T(n) = n \underset{1}{T(1)} + nx + k(2^x - 1)$$

$$T(n) = n + n\log n + k(n-1)$$

$$T(n) = n\log n + n(k+1) - k$$

$$\Rightarrow \boxed{T(n) = O(n \log n)}$$

Selection Sort →

$T(n)$

```
SelectionSort(arr, i){
    if(i == arr.length) return;

    // find min index from i to n
    minIndx = ] using loop.

    swap(i, minIndx, arr);

    selection( arr, i+1);
}
```

n + $T(n-1)$

Re. Relation →

$$T(n) = T(n-1) + n + K$$

QuickSort worst →

$T(n)$

pivot = some index 'n'

left sort $(0 - n-1)$ $T(n-1)$

~~Right sort (n, n)~~

worst $T(n) = T(n-1) + n + K$

Selection Sort

$n \to n-1$

$n \to n-1$

$n \to n-1$

$$T(n) = \cancel{T(n-1)} + n + k \longrightarrow (1)$$

$$\cancel{T(n-1)} = \cancel{T(n-2)} + n-1 + k \longrightarrow (2)$$

$$\cancel{T(n-2)} = \cancel{T(n-3)} + n-2 + k \longrightarrow (3)$$

$$\cancel{T(n-3)} = \cancel{T(n-4)} + n-3 + k \longrightarrow (4)$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

x time.

$$\cancel{T(n-(x-1))} = T(\underbrace{n-x}_{T(1)}) + n-(x-1) + k$$

add all Eqⁿ =

$$T(n) = T(n-x) + nx - (1+2+3+\cdots+(x-1)) + kx \longrightarrow (5)$$

$$T(n-x) = T(1) \Rightarrow n-x = 1 \Rightarrow \boxed{x = n-1}$$

$$T(n) = T(1) + nx - \frac{x(x-1)}{2} + kx$$

$$\Rightarrow T(n) = 1 + nx - \frac{x(x-1)}{2} + kx.$$

put the value of 'x'.

$$T(n) = 1 + n(n-1) + k(n-1) - \frac{(n-1)(n-2)}{2}.$$

$$T(n) = 1 + n^2 - n + kn - k - \left[\frac{n^2 - 2n - n + 2}{2}\right]$$

$$T(n) = an^2 + bn + c$$

$$\Rightarrow \boxed{T(n) = O(n^2)}$$

Concept →

First 'n' natural number.

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$$

$$1 + 2 + 3 + \cdots + n-1 = ?$$

$$\underbrace{1 + 2 + 3 + \cdots + n-1}_{F} + n = \frac{n(n+1)}{2}$$

$$F + n = \frac{n(n+1)}{2}$$

$$F = \frac{n(n+1)}{2} - n = n\left[\frac{n}{2} + \frac{1}{2} - 1\right]$$

$$= n\left[\frac{n}{2} - \frac{1}{2}\right]$$

$$\Rightarrow \boxed{F = \frac{n(n-1)}{2}}$$

Subseq.

$T(n) \to$ Yes call $(i+1)$ $T(n-1)$; No call $(i+1)$ $T(n-1)$

$$T(n) = T(n-1) + T(n-1) + \boxed{K}$$

$$T(n) = 2\cancel{T(n-1)} + K \quad \text{--- (1)}$$

$n \to n-1$] $\times 2$

$$\cancel{2T(n-1)} = \cancel{2^2 T(n-2)} + 2K \quad \text{--- (2)}$$

$n \to n-1$] $\times 2$

$$\cancel{2^2 T(n-2)} = \cancel{2^3 T(n-3)} + 2^2 K \quad \text{--- (3)}$$

$n \to n-1$] $\times 2$

$$\cancel{2^3 T(n-3)} = \cancel{2^4 T(n-4)} + 2^3 K \quad \text{--- (4)}$$

x times

$\Rightarrow T(1)$

$$\cancel{2^{x-1} T(n-(x-1))} = 2^x T(n-x) + 2^{x-1} K$$

add all eq^n

$$T(n) = 2^x T(n-x) + K(1 + 2 + 2^2 + 2^3 + \dots + 2^{x-1}) \quad \text{--- (5)}$$

$T(n-x) = T(1) \Rightarrow n - x = 1 \Rightarrow \boxed{x = n-1}$

put value of 'x' in eq (5)

$$T(n) = 2^x \underset{1}{T(1)} + K(2^x - 1)$$

$$T(n) = 2^x + K(2^x - 1)$$

$$T(n) = 2^x (K+1) - K \qquad x \to n-1$$

$$T(n) = 2^{n-1}(K+1) - K \Rightarrow a \cdot 2^n + b.$$

$$\boxed{T(n) = O(2^n)}$$

$T(n)$

```
fib(int n){
    if(n==0 || n==1) return n;
  a = fib(n-1)
  b = fib(n-2);
  return a+b;
}
```

$\underline{T_1(n)} = T(n-1) + T(n-2) + k \longrightarrow$ eq 1

$T_2(n) = T(n-1) + T(n-1) + k \longrightarrow$ eq 2

$T_3(n) = T(n-2) + T(n-2) + k \longrightarrow$ eq 3

from eq 1, 2 & 3

$T_3(n) \leq T_1(n) \leq T_2(n)$

$T(n-2) + T(n-2) + k \leq T(n) \leq T(n-1) + T(n-1) + k$

$\left(2^{n-2}(k+1) - k\right) \leq T(n) \leq \left(2^{n-1}(k+1) - k\right)$

$\Rightarrow \boxed{T(n) = O(2^n)}$

$T(n) = \cancel{n\,T(n-1)} + k \longrightarrow (1)$

$n \to n-1 \,]\times n \quad \cancel{n\,T(n-1)} = \cancel{n(n-1)\,T(n-2)} + nk \longrightarrow (2)$

$n \to n-1 \,]\times n \quad \cancel{n(n-1)\,T(n-2)} = n(n-1)(n-2)\,T(n-3) + n(n-1)k \longrightarrow (3)$

$n \to n-1 \,]\times n \quad \cancel{n(n-1)(n-2)\,T(n-3)} = \cancel{n(n-1)(n-2)}(n-3)\,T(n-4) + n(n-1)(n-2)k \longrightarrow (4)$

x time

$x = n-1$

$x - n - 1$

$n - (n-1-1)$

$n - n + 2$

$\cancel{n(n-1)(n-2)\ldots[n-(x-2)]\,T(n-(x-1))} \quad n(n-1)(n-2)(n-3)\ldots[n-(x+1)]\,T(n-x) + n(n-1)(n-2)\ldots(n-(x-2))k$

$T(n) = \underbrace{n(n-1)(n-2)\ldots(n-(x+1))\,T(n-x)}_{1} + k\left[\,1 + n + n(n-1) + n(n-1)(n-2) + n(n-1)(n-2)\ldots(n-(x-2))\right] \frac{}{2}$

$T(n-x) = T(1)$

$\Rightarrow \boxed{n = x+1} \quad \boxed{x = n-1}$

$T(n) = n! + k\,[\text{sum}]$

$T(n) = n! + k\left[\frac{n!}{2} + C\right]$

$T(n) = n!\,(\text{expr}) + C \quad \Rightarrow \boxed{T(n) = O(n!)}$

1. `for(int i=1; i<n, i++)` → $O(n)$

$i^2 \le n \Rightarrow i \le \sqrt{n}$

2. `for(int i=0; i*i <= n; i++)` → $O(\sqrt{n})$

3. `for(int i=1; i<n; i*=2)` → $O(\log n)$

4. `for(int i=n; i>1; i=i/2)` → $O(\log n)$

5.
```
for( int i=1; i<=n; i+=m)
    for(int j=1; j<m; j++)
```

6.
```
j=1
for(int i=1; i<=n; ) {
    if(j==i) {
        j=1;
        i++;
    } j++;
}
```

$n \to n/2 \to n/4 \to \cdots \to 1$

$\underbrace{\quad\quad}_{x}$ x time $= \log n$.

$1 \to 2 \to 4 \to 8 \to \cdots n/2 \to n$

$x \Rightarrow \log n$

7.
```
int i=1;
s=1;
while( s <= n) {
    i++;
    s= s+i;
}
```

```
for( i = 1 ; i < n ; i += m) {
    for(int j=1; j < m; j++ {

    }
}
```

$O(n)$

$i = 1$ — m_1 — m

$i = m$ — m_2 — $2m$

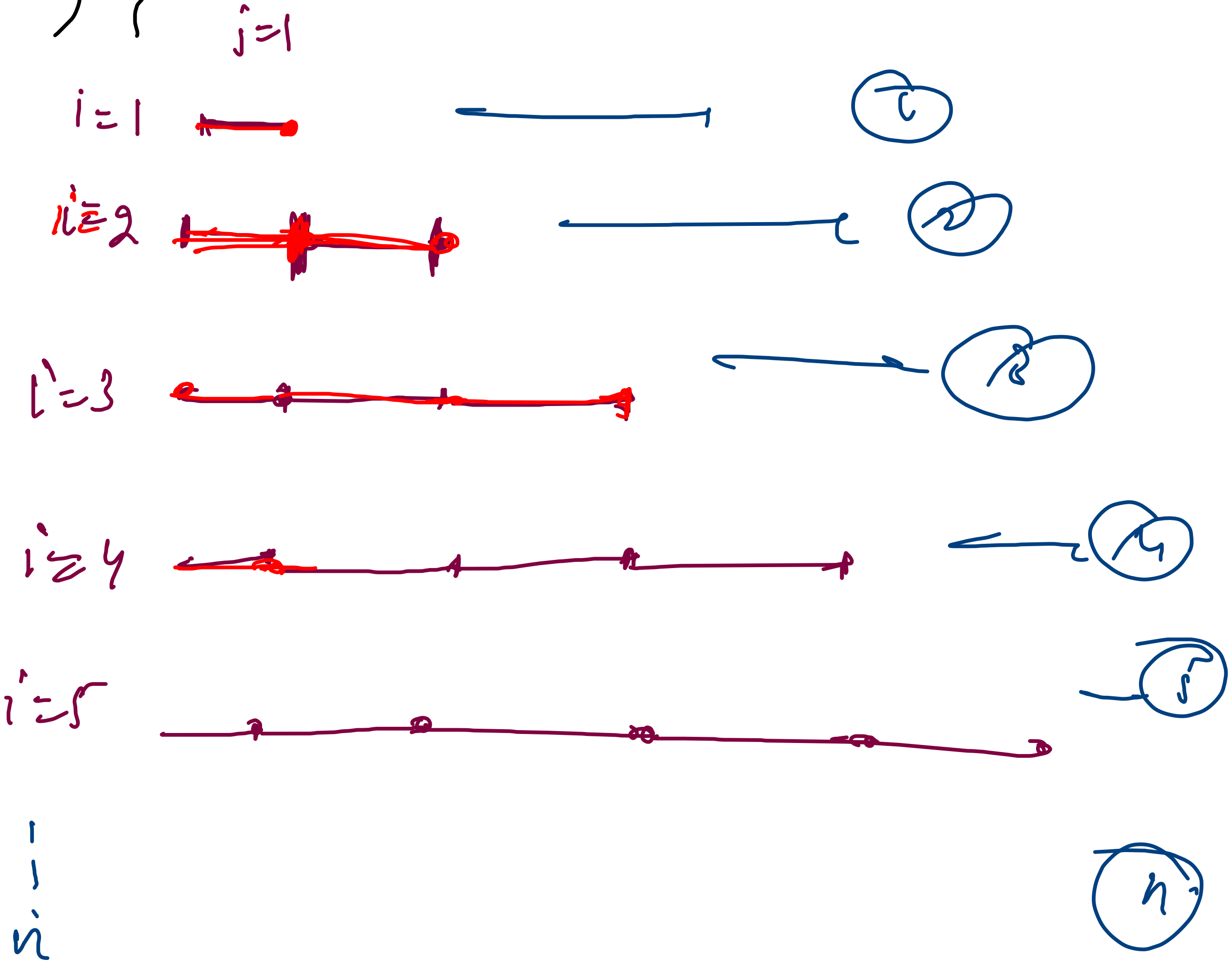
$i = 2m$ — m_3 — $3m$

$i =$

$m_1 + m_2 + m_2 = n$

n

```
j=1
for(int i=1; i<=n; ){
    if(j==i){
        j=1;
        i++;
    }
    j++;
}
```



$1+2+3+\dots+n$

$= \frac{n(n+1)}{2}$

$= O(n^2)$

Sum=1

(7) int i=1; S=1;

i: n=50

while(S ≤ n) {

i=1 2 3 4 5

i++;

S=1+2+3+4+5+.. S=S+i;

}

x=?

i=1 2 3 4 5 6 x

$$S = 1 + 2 + 3 + 4 + 5 + 6 + \cdots + x.$$

$$1 + 2 + 3 + \cdots \cdots + x \le n.$$

$$\frac{x(x+1)}{2} \le n.$$

x = no. of itr.

$$x^2 + x - 2n \le 0$$

$$ax^2 + bx + c = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1+8n}}{2}$$

$$\Rightarrow x = \frac{-1 + \sqrt{1+8n}}{2}$$

$$\Rightarrow x \propto \sqrt{n}$$

$$\Rightarrow \boxed{T(n) = O(\sqrt{n})}$$