Starlings Murmuration Simulation

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1 Introduction

Starlings are a species of small birds that show tendency to migrate in flocks. These flocks are called murmurations and can contain thousands of starlings. Due to their small size and coordinated movements, they can often put up very beautiful displays while moving. In this project, we try to mathematically mode this behaviour of theirs and run a Computer Simulation of the same

2 Modelling the Starling

We seek to model each bird as an individual agent which reacts to its surroundings in order to determine its flying direction. In the most simple model, its environment consists only of other birds. The bird must therefore negotiate three types of constraints in order to determine its flying direction: External forces, Internal forces and Personal limitations.

2.1 External Forces

External forces are the forces of mother nature which the bird act on the bird. We assume that the bird does not need to spend energy to generate these forces, and also cannot have any direct control over them. While there are many such forces, we seek to model only the major ones(this selection is similar to what is done for modelling airplanes):

- **Gravity:** Gravity is perhaps the simplest yet most significant force to model. The concept of flight is fascinating essentially since it seems to overcome gravity.
- **Buoyancy:** Buoyancy may not be too important in air, but it is extremely easy to adjust within gravity. It can become significant for birds with very low density, so it cannot harm to model it.
- **Lift:** The lift force is the main contributor towards overcoming gravity. In fact the bird can maintain its altitude only if the sum of Lift and Buoyancy is same as Gravity.

- **Drag:** The drag force arises due to air resistance which tries to prevent the motion of any body travelling through a fluid. But for this force, the bird could keep flying without applying any effort.

Birds, unlike airplanes can flap their wings in order to generate upward force. This force is more like a buoyant force, but needs the bird to spend energy to get it and can also control its magnitude to some extent. Therefore, we do not include it here.

2.2 Internal Forces

Once the external forces have been decided, the bird may decide to fly in a certain direction. It is somewhat misleading to call this decision a force, these are actually factors based on which the bird decides what kind of force to apply on itself(actually the bird applies the force on the environment and it gets it force by Newtons Third Law). These, however are all vector quantities and it is rather intuitive to consider them as components of the net force that the bird applies.

In the current model, we consider three forces, which are commonly used to simulate flocking behaviour:

- **Cohesion:** The cohesive force is the attractive force that a bird experiences towards other members of the flock. It ensures that the flock stays together and does not start moving in random directions.

- **Separation:** The separative force acts opposite to the cohesive force and ensures that the birds keep a minimum distance between them and don't start colliding with each other

- **Alignment:** This force tries to coordinate the direction of motion of the entire flock. Each bird has the urge to fly in the same direction as its neighbours.

The above mentioned factors present a somewhat idealistic view of how a certain bird desires to fly. However, there are other factors effecting how the bird really flies.

To start off, a bird is not likely to know about another bird which is flying right behind it. In fact, it can only see those birds which are in its **field of view**, ie the maximum angle from its line of motion which it can see. This is actually quit large for birds since they have eyes on the sides of their heads. Further, two birds which are far off from each other are less likely to affect each other than two birds that are close by. This phenomenon is modelled by the (near)**sightedness** of the bird. Finally, some birds may be more **adventurous** than others, giving them urges to fly off in random directions rather than those specified by the above forces.

Finally, a bird can apply only a limited amount of force, in which it would try to overcome the External forces and move in its desired direction. This limit would be determined both by an overall **maximum acceleration** and the individual **strength** of the bird.

3 Mathematical Formulation

To begin our mathematical formulation, we need to first fix a coordinate system. We fix the origin at the flock centroid(this is beneficial for bounding box creation during implementation) and the y axis upwards. The x and z axis are arbitarily chosen since the system has cylindrical symmetry.

The mathematical values of the external forces are more or less standard and do not require much explanation

$$\vec{F_{gravity}} = \rho_{bird} V \vec{g}$$

$$\vec{F_{buoyancy}} = -\rho_{air} V \vec{g}$$

$$\vec{F_{lift}} = C_{lift} \rho_{air} \vec{v}^2 W \hat{j}$$

$$\vec{F_{drag}} = -\frac{1}{2} C_{drag} \vec{v}^2 A \hat{v}$$

$$\vec{F_{external}} = \vec{F_{gravity}} + \vec{F_{buoyancy}} + \vec{F_{lift}} + \vec{F_{drag}}$$

ρ_{bird}, ρ_{air} are the densities of the bird and air,

C_{lift}, C_{drag} are the lift and drag coefficients,

g is the acceleration due to gravity $= 0\hat{i} - 9.8\hat{j} + 0\hat{k}$

W, A are wing area and cross sectional area respectively,

$\vec{v}$ is the current velocity of the bird,

V is the volume of the bird

The internal forces are more involved. To begin with, field of view simply filters out the birds which are beyond the specified angle, so all statements made hereafter are wrt the rest of the birds.

3.1 Internal Forces

Two different approaches are possible to deal with the sightedness issue. One is to say that only a fixed number of nearest neighbours affect the bird. This partially relies on the fact that the bird would not want to over complicate its life and would approximate the world by a fixed number of birds around it. However, the evidence for this is quite emperical and needs to be judge in light of the fact that the number of birds in a certain region around the bird is held more or less constant by the Cohesion and separation forces. This also does not account for the chance that the bird might look at a very distant bird and adjust itself according to it.

The other view is to limit the distance till which a bird can see. This seems more logical, except for the fact that there is unlikely to be a hard boundary at a given distance. Therefore, we choose an exponential decay function to model the sightedness of the birds:

$$W_i = e^{-sd_i}$$

d_i is the distance to the i^{th} bird

s is the sightedness of the bird

At this point, it may seem strange that a bird would look at like a thousand other birds before deciding which direction to move, which is clearly illogical. One way of explaining this would be that the bird looks at only a certain number of birds at each moment, and the quantity mentioned here is the over time expectation of looking at each bird.

This done, we can move on to the three forces. We calculate for each bird separately(call this the target bird and then aggregate to get the net forces.

It is intuitive to keep Alignment proportional to the velocity of the target bird. As for Separation and Cohesion, they should logically be proportional to the inverse of the distance between the two birds(Actually, cohesion can be constant, allowing for the sightedness to take care of the distance factor. Ignoring this fact is more of a personal intuition). Also, the separation force needs to grow faster that the cohesion force, as we want to prevent collisions. We chose to take the simplest expressions that satisfy these constraints.

$$\vec{F_{cohesion,i}} = \frac{C_{coh}}{d_i}\hat{d_i}$$

$$\vec{F_{separation,i}} = -\frac{C_{sep}}{d_i^2}\hat{d_i}$$

$$\vec{F_{align,i}} = C_{ali}\vec{v_i}$$

$\hat{d_i}$ is the unit vector towards the i^{th} bird

$\vec{v_{targ}}$ is the velocity of i^{th} bird

$C_{coh}, C_{sep}, C_{ali}$ are respective coefficients

Once these forces are calculated due to each bird, we need to aggregate them to get net internal force. Aggregation can be done in many ways, but the two most common ways are summation and averaging. Summation is good for aggregating quantities that are likely to add up destructively. This is clearly true for the separation force. Averaging, on the other hand, make sense for quantities that would add up constructively, like the alignment force. Cohesion force can go either way, but it is somewhat safer to consider averaging since a zero is less dangerous than an infinity. The Weights assigned to the different birds is taken care of here.

$$\vec{F_{cohesion}} = \frac{\sum_i W_i \vec{F_{cohesion,i}}}{\sum_i W_i}$$

$$\vec{F_{alignment}} = \frac{\sum_i W_i \vec{F_{alignment,i}}}{\sum_i W_i}$$

$$\vec{F_{separation}} = \sum_i W_i \vec{F_{separation,i}}$$

Finally, we need to add the adventure component to these forces to calculate the net internal force on the bird. Now, while an adventurous bird may not want to align itself with the other birds or get attracted to them, it will certainly not want to collide with other birds. Therefore, the random component will only replace the alignment and cohesion forces and not the separation force.

$$\vec{F_{internal}} = \vec{F_{separation}} + \frac{1000 - Adv}{1000}(\vec{F_{cohesion}} + \vec{F_{alignment}}) + \frac{Adv}{1000}\vec{R}$$

$Adv \in \{0, ..., 1000\}$ is the adventurousness parameter

$\vec{R}$ is a suitable random vector(suitability depends on range of magnitudes)

3.2 Putting it Together

Now that we have both the external and internal force vectors, we need to put them together to calculate the acceleration, velocity and power of the bird. It is here that we need to take into account the maximum force that the bird can apply is limited.

If the bird was an inanimate object, it would simply drift in the direction of the external force. What the bird really does is try its best to change this to its desired direction of flight as much as possible. This difference is simply given by $\vec{v_{diff}} = \vec{v} + \vec{F_{external}} - \vec{F_{internal}}$. Since we can only take discrete time, the desired acceleration is $\vec{a_{desired}} = \vec{v_{diff}}/P$ where P is the clock period. The magnitude of $\vec{a_{desired}}$ needs to be bounded by a $a_{bound} = a_{max} \times strength$ where a_{max} is the global maximum acceleration and strength is a property of the bird.

The bounding mechanism itself is somewhat non trivial. The naive way of doing it would be to cut off any value higher that the required bound. But that would be very unrealistic as in a real scenario, the bird would start facing difficulty gradually till it reaches a point where it can no longer fly faster. A good function to approximate this is the *tanh* function, which has derivative of 1 around zero, but is bounded by 1 on the positive side. However, its derivative never falls to 0 for any finite parameter.

Once the bounding is done, we can calculate Power easily using laws of physics.

$$\vec{a_{final}} = tanh(\frac{|\vec{a_{desired}}|}{a_{bound}})\frac{a_{bound}}{|\vec{a_{desired}}|}\vec{a_{desired}}$$

$$\vec{v_{final}} = \vec{v} + \vec{v} + \vec{F_{external}} - \vec{a_{final}}$$

$$Power = m\vec{a_{final}}.\vec{v_{final}}$$

Position and Energy follow trivially.

4 Implementation

5 Evaluation and Parameter Tuning

6 Conclusion