1 Introduction

The KnightTour algorithm relies on the fact that any $n \times n$ board, where n is even and greater than 6, can be formed by some combination of 6×6, 6×8, 8×8, 8×10, 10×10, or 10×12 boards. If a knight's tour is found on all of the above base case boards, then it is guaranteed to exist on an $n \times n$ board. We will prove this statement by induction on n, the size of the board.

2 Proof

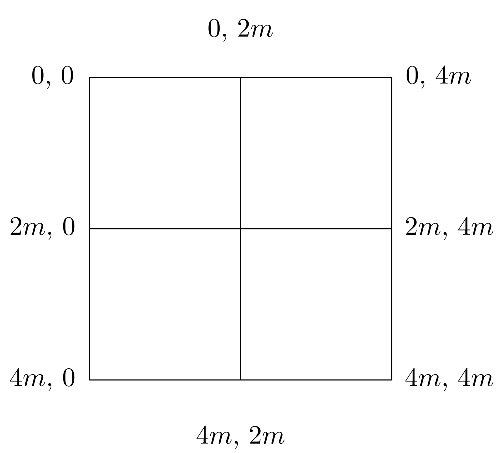
Theorem: Any $n \times n$ grid or $n \times (n+2)$ grid with $n \geq 12$ can be formed by some combination of 6×6, 6×8, 8×8, 8×10, 10×10, or 10×12 grids.

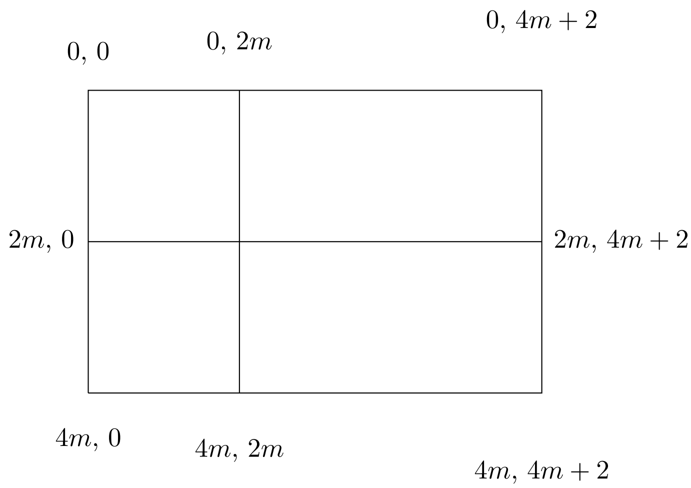
Proof by induction. We will use induction to prove the above theorem.

Base case: Let $n = 6$, 8, and 10. For each of the $n \times n$ and $n \times (n+2)$, we know that one of the above grids completely fills the grid we are looking for.

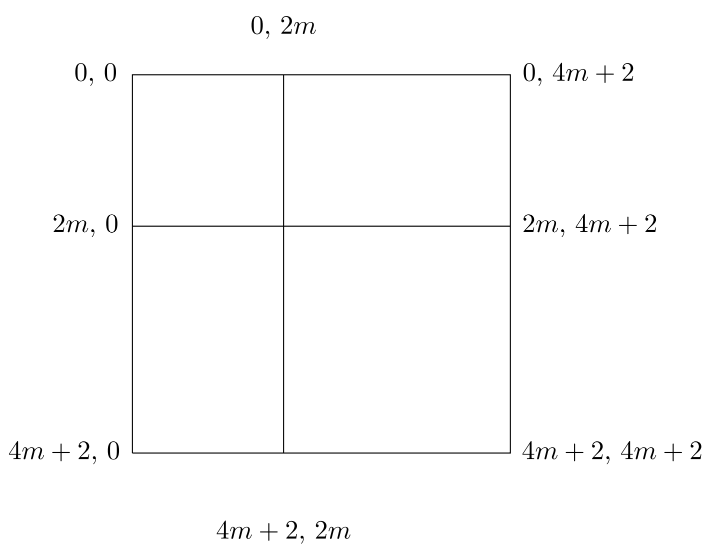
Inductive Hypothesis: Suppose there exists a k such that $10 \leq k < n$ and the claim holds for all k. Since n is even, we have 2 cases to check to prove the claim.

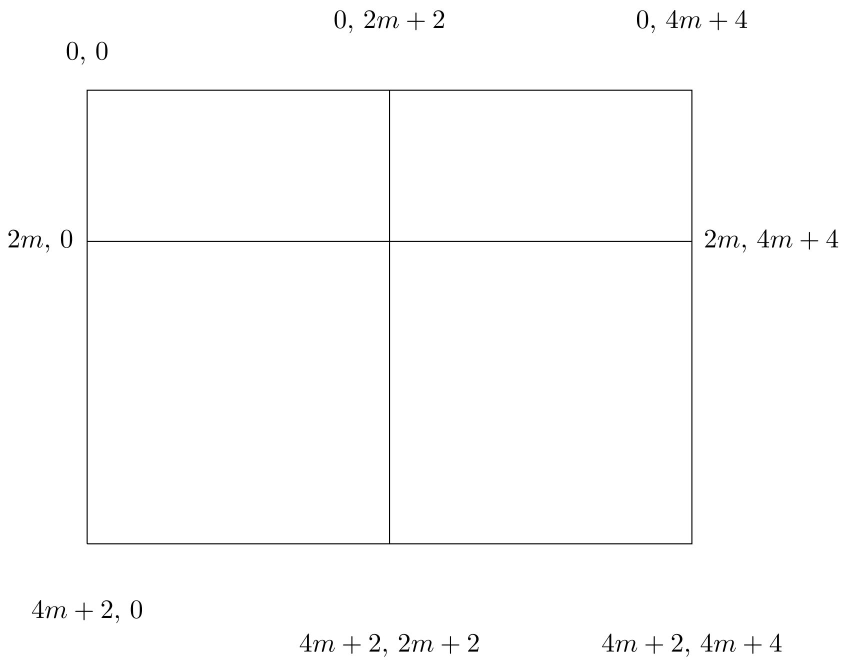
Case 1: $n = 4m$ for some $m \in \mathbb{N}$. For the $n \times n$ square, we divide it into four $2m \times 2m$ squares. For the $n \times (n+2)$ rectangle, we divide it into two $2m \times 2m$ squares and two $2m \times (2m+2)$ rectangles. By the inductive hypothesis, as $2m \geq 6$, the claim holds if $n = 4m$.





Case 2: $n = 4m+2$ for some $m \in \mathbb{N}$. For the $n \times n$ square, we divide it into one $2m \times 2m$ square, one $(2m+2) \times (2m+2)$ square, and two $2m \times (2m+2)$ rectangles. For the $n \times (n+2)$ rectangle, we divide it into two $(2m+2) \times (2m+2)$ squares and two $2m \times (2m+2)$ rectangles. By the inductive hypothesis, as $2m \geq 6$, the claim holds if $n = 4m+2$.





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