Enhanced Hierarchical Modulation

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***Abstract*—Two schemes enhancing hierarchical modulation are presented and analyzed in this paper. One scheme is to optimize the enhancement-layer signal constellation(s) for higher spectral efficiency. The other one is to optimize Gray bits-to-symbol mapping for less demodulation errors. Both schemes are simple and efficient. They can be used to recover the throughput loss of regular hierarchical modulations with little complexity increase. The rationales behind the proposed approaches are presented with the analysis of achievable rate, effective signal-to-noise ratio, modulation efficiency, Voronoi decomposition and minimum Euclidean distance and the comparison with regular modulations. Computer simulations are also provided to support our conclusions.**

I. Introduction

Broadcast multicast service (BCMCS) has increasingly been popular for delivering multimedia content to mobile users. BCMCS can be offered through either a 3rd generation and beyond radio access network like WCDMA or EV-DO network or a dedicate digital broadcast infrastructure like DVB-T/H/S2, MediaFLO and DMB. Traditional digital broadcast system is designed with the tradeoff between maximum achievable rate and intended coverage in mind. Their capacities are limited by maximum transmit power and worst channel conditions so that every user in an intended coverage area can reliably receive services. The users under good reception condition and with advanced receiver may not have many advantages, even though their achievable throughput can be much higher.

Recently there are lots of interests in upgrading existing digital broadcast systems with more services for new users while keep existing users unchanged, delivering additional or better quality of service (QoS) to users with advanced receivers while still guaranteing others' services, and providing unequal protection on digital contents [1, 2, 3, 4]. Many technologies are under investigation for these goals, such as rateless coding, hierarchical modulation, multiple-input multiple-output (MIMO) and selective retransmission. However, backward compatibility is one of the major concerns in upgrading existing systems with additional service channels since there are a large number of users already served by existing systems and it is prohibitively expensive to simply replace their existing user equipments by next-generation ones. It is

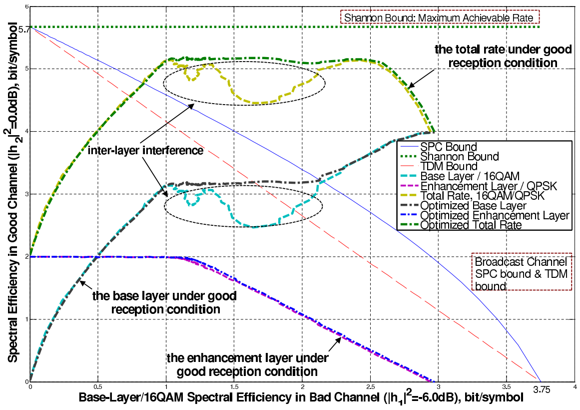


Fig. 1. Achievable capacity of hierarchical modulations with 16QAM base layer and QPSK enhancement layer.

also expected that existing receivers can continue to operate in upgraded systems, even though they are not able to receive supplemental services provided by upgraded networks. Hierarchical modulation is one of the promising technologies for upgrading existing systems while maintaining strictly backward compatibility. One of the key advantages of hierarchical modulation is the added complexity and cost are low. It has been proven and included in DVB-T [1], MediaFLO [2] and UMB (Ultra Mobile Broadband), a 3.5th generation mobile network standard developed by 3GPP2 [4].

Hierarchical modulation is a signal precoding technique for multiplexing multiple data streams into single symbol stream in which each symbol consists of one base layer and one or multiple enhancement layers. When hierarchical-modulated signals are transmitted, users with good reception and advanced receiver can demodulate multiple layers while others with conventional receiver or poor reception may only demodulate the data stream embedded in base layer. Therefore network operator can target different types of users with different services or QoS. But traditional hierarchical modulation may suffer from serious inter-layer interference (ILI) so that the achievable rate by low-layer signal(s), e.g. base-layer signals, is dented by interference from high-layer signal(s). One example of this is shown in Fig. 1, where the 16QAM-modulated base layer suffers from the existence of QPSK-modulated enhancement layer. In order to recover the capacity loss due to the interference from enhancement layer(s), two approaches are presented in this paper. One approach is to optimize the signal constellation of enhancement layer(s) so that higher throughput is achievable when demodulating low-layer signals. The other one is to extend traditional single-layer Gray bits-to-symbol mapping to multi-layer Gray mapping for minimizing bit-error rates (BER). In Fig. 1, it shows that the rate loss by regular hierarchical modulation can be restored by using the proposed schemes. Another important advantage of the proposed schemes is the incurred implementation complexity increase is low. Partial of our schemes is adopted in UMB by 3GPP2 [4].

II. Sinal Model And Problem Description

We will limit our discussions to two-layer signal constellations with the enhancement layer QPSK-modulated and the base layer QPSK- or 16QAM-modulated in this paper, although the concepts proposed here can be generalized to most hierarchical modulations. The reason for this is not only because of the simplicity of QPSK and 16QAM modulations but also because QPSK and 16QAM are of the most popular signal constellations adopted in various digital communication systems and standards. It is also shown that the addition of QPSK as enhancement layer may yield significant performance gain by using our approaches. Since many high-order regular or hierarchical signal constellations may be decomposed into multiple QPSK signals adding together, many analysis and conclusions presented in this paper can be straightforwardly extended.

The signal constellations of regular square-shaped QPSK/QPSK and 16QAM/QPSK hierarchical modulation are shown in Fig. 2 [1]. Obviously, the regular 16QAM can be taken as a special case of QPSK/QPSK hierarchical modulation, in which both base layer and enhancement layer are QPSK-modulated. The minimum Euclidean distance (MED) of base layer and enhancement layer are denoted by 2α and 2β [2], individually. With superimposing base-layer signal and enhancement layer signal together, the MED of resulted hierarchical modulation becomes

$$d_{\min} \;=\; \min\left\{2\left(\alpha-\beta\right),\, 2\alpha,\, 2\beta\right\} \;<\; 2\alpha \;. \quad (1)$$

Smaller minimum Euclid distance usually results in more ambiguity and more demodulation errors. For demodulation BER, another important fact is the employed bits-to-symbol mapping rule. The bits-to-symbol mapping for hierarchical modulation shown in Fig. 2 is a interleaved

[1] In this paper, a hierarchical modulation is denoted by *layer 0 (or base layer) constellation / layer 1 constellation / ...*, where the signal constellation of different layers are separated by backslash from the lowest layer (also called base layer) to the highest layer.

[2] Without loss of generality, it is assumed that $\alpha \geq \beta$.

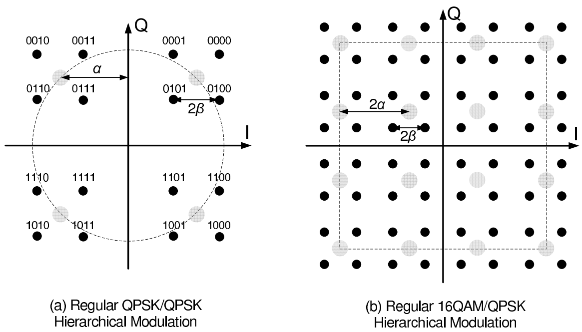


Fig. 2. Regular hierarchical modulation examples: the base layer is QPSK/16QAM and the enhancement layer is QPSK.

Gray mapping, where the bits $b_0 b_1$ from base layer and $e_0 e_1$ from enhancement layer are interleaved in one codeword $b_0 e_0 b_1 e_1$. The Gray mapping shown in Fig. 2 for hierarchical signal constellation is a kind of one-dimension Gray mapping, in which the bits-to-symbol mapping rules for each layer are same and independent to each other. And it is fixed regardless the power-splitting ratio ζ between layers, which is defined by

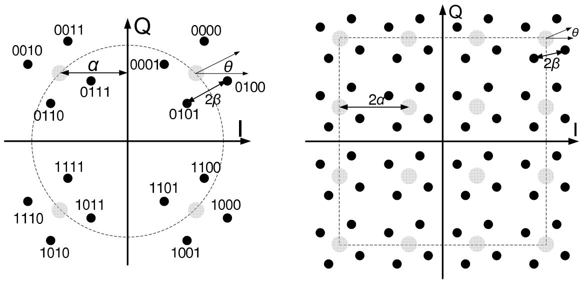
$$\zeta \;=\; \frac{P_{\mathrm{E}}}{P_{\mathrm{B}}} \;, \quad (2)$$

with $\zeta < 1$ in most cases; Otherwise we think two signal constellations exchanged layers in hierarchical signal constellation. For QPSK/QPSK hierarchical modulation, the power-splitting ratio is $\zeta_{\text{QPSK/QPSK}} = \frac{\beta^2}{\alpha^2}$. For 16QAM/QPSK modulation, $\zeta_{\text{16QAM/QPSK}} = \frac{\beta^2}{4\alpha^2}$. When $\zeta_{\text{QPSK/QPSK}} = \frac{1}{4}$, the QPSK/QPSK modulation in Fig. 2(a) becomes square-shaped 16QAM. In general, the enhancement-layer signal can be taken as additional noise by base layer. At this time, most existing conventional receivers can continue to demodulate base-layer signals with no additional change but at a lower signal-to-noise/interference ratio (SINR) $\hat{\gamma}$, which is written by

$$\hat{\gamma} \;=\; \frac{P_{\mathrm{B}}}{P_{\mathrm{E}}+\sigma^2} \;<\; \gamma \;=\; \frac{P_{\mathrm{B}}}{\sigma^2} \quad (3)$$

with σ^2 denoting the power of the background additive Gaussian white noise (AWGN), especially when ζ is small. On the other hand, signals of both base layer and enhancement layer(s) can be demodulated by a advanced receiver. This is called the strictly backward compatibility of hierarchical modulation, which makes it attractive for providing seamless upgrading, unequal protection, additional services or differentiated QoS with little change on existing digital broadcast systems.

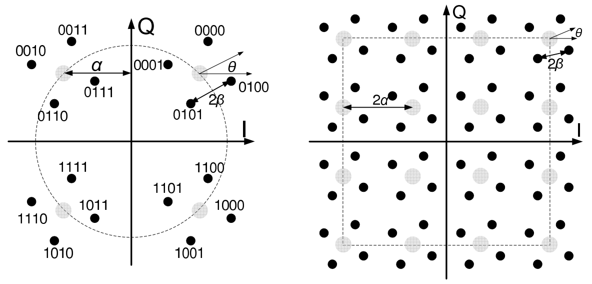
However, regular hierarchical modulation may seriously suffer from ILI, which not only decreases the base-layer SINR from γ to $\hat{\gamma}$ but also lowers the achievable spectral efficiency. This is observed from Fig. 1. On the other hand, it is well-known that the achievable throughput of a received signal essentially depends on the



(a) Enhanced QPSK/QPSK Hierarchical Modulation

(b) Enhanced 16QAM/QPSK Hierarchical Modulation

Fig. 3. Enhancing hierarchical modulation by rotating enhancement layer.



(a) Enhanced QPSK/QPSK Hierarchical Modulation

(b) Enhanced 16QAM/QPSK Hierarchical Modulation

Fig. 4. Enhancing hierarchical modulation by multi-dimension Gray mapping when $2\beta > \max\{\Delta_1,\ \Delta_2\}$

power distribution profile of the signal [5] in signal space instead of the power-splitting ratio ζ. This is similar to channel coding. From a channel coding point of view, higher throughput is achievable by the *i.i.d. Gaussian code* defined in coding space, even though it may not be implementable from an engineering standpoint [6]. How to transmit a signal close to Shannon channel capacity and implementable in a relatively easy way is not only critical for the signal constellation design block but also every other component in a communication system.

III. The Enhanced Hierarchical modulations

The first approach is to optimize the signal constellation of hierarchical modulations. This can help improve the spectral efficiency of hierarchical modulation. There are many ways to do it. The one we proposed is to optimally rotate the enhancement-layer(s) of a signal constellation. For the QPSK/QPSK hierarchical modulation shown in Fig. 2(a), the QPSK signal constellation of the enhancement layer is rotated in the anti-clock direction by θ, $0 \leq \theta \leq \frac{1}{4}\pi$, and resulted signal constellation is shown in Fig. 3(a). For 16QAM/QPSK, the regular and enhanced hierarchical modulations are shown in Fig. 2(b) and Fig. 3(b), respectively. With optimal rotation angle θ_{opt}, most of the lost base-layer capacity can be recovered without scarifying the throughput of enhancement layer(s). As shown in Fig. 1, the achievable rate of QPSK-modulated enhancement layers is unchanged before and after the rotation.

Besides signal design for higher spectral efficiency, coding can help minimize demodulation errors and therefore achieve the throughput. In reality, most capacity-achieving codes are designed to balance the implementation complexity and achievable performance. Gray codes is one of the examples of this. Gray code, also known as reflective binary code, is a binary numeral system where two successive value differ in only one digit. Even though it was original designed to prevent spurious output from electromechanical switches, Gray code for bits-to-symbol mapping, mostly called Gray mapping and being implemented with other channel coding, is generally accepted as the optimal mapping rule for minimizing BER. Gray mapping for regular QPSK/QPSK hierarchical modulation is shown in Fig. 2, where the codewords with minimum Euclid distance have minimum Hamming distance as well. However, the Euclid distance profile will change when the enhancement-layer signal constellation is rotated and the power-splitting ratio is changed. This means the original Gray mapping in Fig. 2 may not always be optimal. In this case, it may be necessary to do bits-to-symbols remapping based on each Euclidean distance file instance. One example of Gray remapping for QPSK/QPSK hierarchical modulation is shown in Fig. 4, in which the bits-to-symbol mapping is re-arranged when the inter-layer MED is enough smaller.

IV. Achievable Rate of Modulated Signals

More three decades ago Cover showed that higher sum capacity is achievable if messages for two users of different reception conditions are superpositedly precoded [6]. Hierarchical modulation is one of the practical implementations of superposition precoding (SPC) for providing different rates and protections for users with different receptions. In general, the achievable rate of a N-ary modulated signal, either of regular signal constellation or of hierarchical signal constellation, through AWGN channel is given by [5]

$$\begin{aligned} R &= \log_2(N) - \\ &\frac{1}{N}\sum_{i=0}^{N-1} \mathrm{E}\left\{\log_2\left[\sum_{j=0}^{N-1} e^{-\frac{|s_j+n-s_i|^2-|n|^2}{2\sigma^2}}\right]\right\}, \end{aligned} \tag{4}$$

which is the achievable rate when a receiver try to decode the whole hierarchically modulated symbol. With (4), the AWGN capacity of regular QPSK and 16QAM can be plotted as in Fig 5. Though the rate in (4) is achievable by users with advanced receiver, it is more than achievable for a user with a conventional demodulation receiver which usually detects the base-layer signals only. The achievable rate of either base layer or enhancement layer

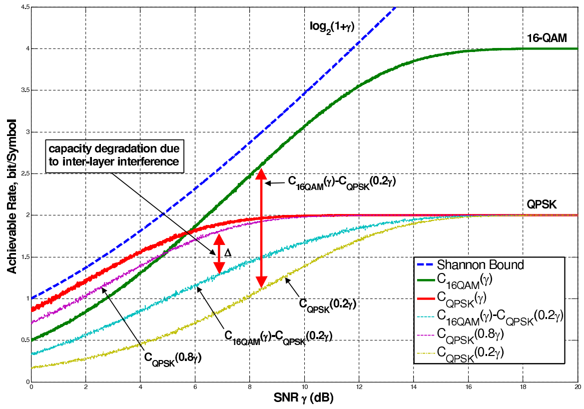


Fig. 5. Achievable rates of regular 16QAM modulation: a hierarchical modulation perspective.

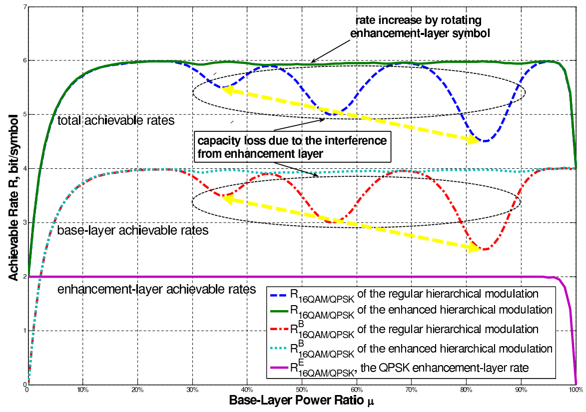


Fig. 6. Achievable rates of 16QAM/QPSK hierarchical modulation with different power splitting and $\frac{P}{\sigma^2} = 20$dB.

is lower than (4). Following the concept of the successive interference cancellation, the achievable rate, also termed *equivalent capacity*, for a receiver decoding up to l layers of a hierarchical modulated symbol is [7]

$$\tilde{R}_l \quad = \quad \sum_{i=0}^{l-1} R_i \quad = \quad R - \sum_{j=l}^{L} R_j \quad . \tag{5}$$

Let's take the regular 16QAM as an example. A regular 16QAM can be take as a special case of QPSK/QPSK hierarchical modulation with $\zeta_{\text{QPSK/QPSK}} = \frac{1}{4}$. This means the achievable rate of the enhancement layer is the same as the regular QPSK capacity and the achievable rate of the base layer becomes

$$\begin{aligned} \mathrm{R}^{\mathrm{B}}_{\text{QPSK/QPSK}}\left(\gamma,\ \tfrac{1}{5}\right) &= \mathrm{R}_{\text{QPSK/QPSK}}\left(\gamma,\ \tfrac{1}{5}\right) - \mathrm{R}_{\text{QPSK}}\left(\tfrac{1}{5}\gamma\right) \\ &= \mathrm{R}_{\text{16QAM}}\left(\gamma\right) - \mathrm{R}_{\text{QPSK}}\left(\tfrac{1}{5}\gamma\right) \ , \end{aligned} \tag{6}$$

which are plotted in Fig 5. Due to the ILI from the QPSK-modulated enhancement layer, the actual throughput of the QPSK base layer $\mathrm{R}^{\mathrm{B}}_{\text{QPSK/QPSK}}\left(\gamma,\ \frac{1}{5}\right)$ is lower than the corresponding QPSK rate $\mathrm{R}_{\text{QPSK}}\left(\frac{4}{5}\gamma\right)$, i.e.,

$$\mathrm{R}^{\mathrm{B}}_{\text{QPSK/QPSK}}\left(\gamma,\ \tfrac{1}{5}\right) \quad \leq \quad \mathrm{R}_{\text{QPSK}}\left(\tfrac{4}{5}\gamma\right) \quad . \tag{7}$$

In Fig 5, it shows that the degradation of the base-layer capacity can be up to around $\Delta = 0.56$ bits/symbol, which is about 14% of the maximum total achievable rate 2 bits/symbol for the QPSK base-layer. This kind of degradation can be further illustrated in Fig. 6, where the hierarchical modulation is 16QAM/QPSK-modulated. In Fig. 6, the total SNR is fixed at $\frac{P}{\sigma^2} = 20$dB but the power of the 16QAM sublayer is changed from 0% to 100% of the total power P. The achievable rates of each layer and the whole constellation are plotted in Fig. 6. One of the interesting things shown in Fig. 6 is the equivalent capacity of the 16QAM base layer changes periodically instead of monotonically with increase the power ratio between the base layer and the whole signal. The good things in Fig. 6 is this kind of capacity loss can be recovered by optimally rotating the enhancement layer. This is one of the advantages of the proposed enhanced hierarchical modulations.

V. Effective Signal-to-Noise Ratio and Modulation Efficiency

Besides the above information-theoretical point of view of hierarchical modulation, it is also interesting to understand hierarchical modulation from a signal-processing perspective. At this time, the performance of hierarchical modulation will be evaluated through actual implementations, where demodulation BER is one of the major concerns. In general, it is difficult to give a simple closed-form BER expression for hierarchical signal constellation. The BER of square-shaped M-QAM constellation and a hierarchical QAM constellation can be computed by using recursive algorithms [8]. It is known that the BER expression for QPSK is

$$\mathrm{P}_{e,\text{QPSK}}\left(\gamma\right) \quad = \quad \mathrm{Q}\left(\sqrt{\tfrac{\gamma}{2}}\right) \ , \tag{8}$$

where $\mathrm{Q}\left(x\right) = \frac{1}{2}\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$ denotes the Q-function. From a signal processing standpoint, the BER and capacity degradation may happen when there is a change in noise and/or interference distribution, even though the received SNR γ is the same. For example, the BER performance of regular QPSK/QPSK hierarchical modulation becomes deteriorated in Fig. 7 with increasing $\zeta_{\text{QPSK/QPSK}}$. But, if we optimally rotate the enhancement-layer signal constellation, the performance loss can be recovered. This kind of recovery can be more significant with large ζ. There are many ways for quantifying and understanding this kind of BER performance loss due to either interference or receiver design. One approach for capturing this kind of degradation is to calculate the effective signal-to-

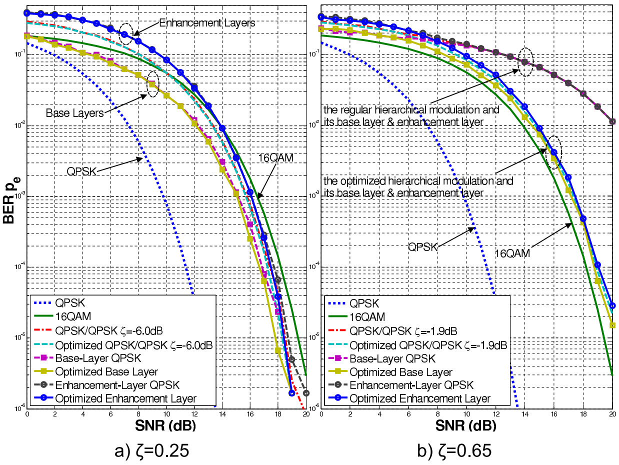


Fig. 7. Bit-error rate of uncoded QPSK/QPSK hierarchical modulations using maximum likelihood demodulation .

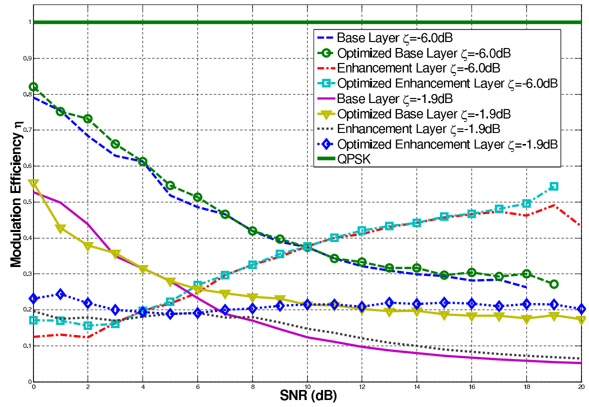


Fig. 8. Hierarchical modulation efficiency of QPSK/QPSK hierarchical modulation using maximum likelihood demodulation.

noise ratio (ESNR) for the receiver output, which is defined by

$$\tilde{\gamma}(\gamma) \quad \equiv \quad \Psi^{-1}(p_e(\gamma)) \ , \tag{9}$$

where $p_e(\gamma)$ is the demodulation BER of the signal with SNR γ and $\Psi^{-1}(*)$ denotes the inverse function of $\Psi(\cdot)$, the demodulation error probability function with no ILI. For example, the ESNR for the QPSK-modulated base layer or enhancement layer of any hierarchical modulation can be calculated by

$$\tilde{\gamma}_{\text{QPSK/QPSK}} \quad = \quad 2\left[\mathrm{Q}^{-1}(p_e(\gamma))\right]^2 \ . \tag{10}$$

More specifically, the ESNR for the base layer of regular QPSK/QPSK hierarchical modulation with maximum likelihood demodulator is given by

$$\tilde{\gamma}^{\mathrm{B}}_{\text{QPSK/QPSK}}(\gamma) = 2\left[\mathrm{Q}^{-1}\left(\frac{\mathrm{Q}((1-\sqrt{\zeta})\gamma)+\mathrm{Q}((1+\sqrt{\zeta})\gamma)}{2}\right)\right]^2 . \tag{11}$$

By normalizing ESNR by γ, we can obtain hierarchical modulation efficiency (ME) η by

$$\eta(\gamma) \quad = \quad \frac{\tilde{\gamma}}{\gamma} \quad = \quad \frac{1}{\gamma}\Psi^{-1}(p_e(\gamma)) \ . \tag{12}$$

Without interference, $\eta(\gamma) = 1$; otherwise, $\eta(\gamma) < 1$. η is also the measure of inter-layer resistance for hierarchical modulation. Higher ME is, stronger interference-resistance the signal has. As an example, the ME of QPSK/QPSK hierarchical modulation are plotted in Fig. 8. We can see the enhanced hierarchical modulation has higher ME than the regular modulation. The difference is more obvious when ζ becomes large. This means enhanced hierarchical modulation has stronger inter-layer interference resistance than regular hierarchical modulation.

VI. Asymptotic Modulation Efficiency and Voronoi Decomposition

The asymptotic modulation efficiency (AME) η_∞ is given by

$$\eta_\infty \quad = \quad \lim_{\gamma\to\infty}\eta(\gamma) \quad = \quad \lim_{\sigma\to 0}\frac{\sigma^2}{P}\Psi^{-1}(p_e) \ . \tag{13}$$

For the example in (11), the AME can be calculated by

$$\eta_\infty \quad = \quad \lim_{\gamma\to\infty}\frac{2\left[\mathrm{Q}^{-1}\left(\frac{\mathrm{Q}((1-\sqrt{\zeta})\gamma)+\mathrm{Q}((1+\sqrt{\zeta})\gamma)}{2}\right)\right]^2}{\gamma} . \tag{14}$$

From (13) and (14), it shows that AME basically shows how fast ESNR is approaching SNR when $\gamma\to\infty$. This can be expressed by

$$\eta_\infty \quad = \quad \frac{\partial\eta(\gamma)}{\partial\gamma}\Big|_{\gamma=\infty} \ . \tag{15}$$

The AME for QPSK/QPSK hierarchical modulation can also be found in Fig. 8, in which they are the points that the ME are approaching to when SNR becomes larger and larger.

Since AME measures the effects of ILI when there is no noise, it essentially reflects the power distribution profile of a signal constellation in signal space, which can be further illustrated by the Voronoi decomposition. For example, the Voronoi decomposition for QPSK and 16QAM is show in Fig. 9, where each Voronoi cells are rectangular. But, the Voronoi boundary for the enhanced hierarchical modulations are different. The Voronoi decomposition of enhanced QPSK/QPSK is shown in Fig. 10, where the Voronoi boundary becomes polygons and the area of each Voronoi region is changed. This kind of changes certainly affect the demodulation performance of the signals.

VII. Euclid Distance and Hamming Distance

When the receiver selects $\mathbf{s}_i$ instead of the transmitted symbol $\mathbf{s}_j$, there is a demodulation error happened.

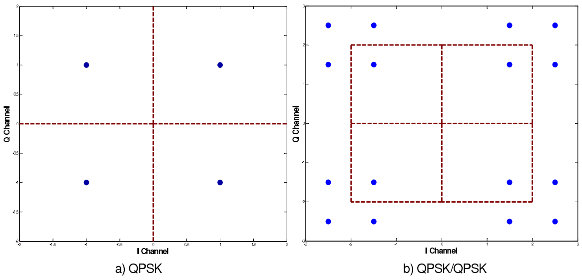


Fig. 9. Voronoi diagram for regular modulations

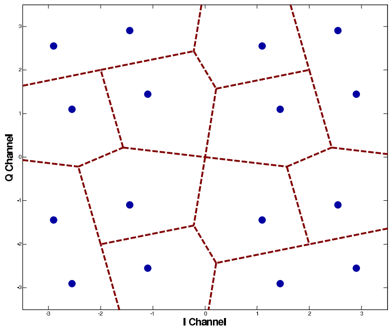


Fig. 10. Voronoi diagram for enhanced QPSK/QPSK modulation

The conditional demodulation error for AWGN channel is

$$Pr\left(\mathbf{s}_j \to \mathbf{s}_i | \mathbf{s}_j\right) = Q\left(\frac{\|\mathbf{s}_i - \mathbf{s}_j\|}{\sqrt{2}\sigma}\right) \leq Q\left(\frac{d_{\min}}{\sqrt{2}\sigma}\right). \tag{16}$$

The BER performance of a signal constellation is dominated by symbol pairs with EMD $d_{\min}$ especially when SNR is high. Therefore it is interesting to find optimal bits-to-symbol mapping rules, in which the codes for the closest two signals have minimum difference. In general, Gray mapping in two-dimensional signals worked with channel coding is accepted as optimal for minimizing BER for equally likely signals. Gray mapping for regular hierarchical signal constellations is shown in Fig. 2, where the codes for the closest two signals are different in only one bit. However, this kind of Euclidean distance profile may not be fixed in hierarchical modulation. An example of the minimum Euclidean distance of 16QAM/QPSK hierarchical modulation with different rotation angles is shown in Fig. 11. It is easy to find the inter-layer Euclidean distance may become shortest when the power splitting ratio ζ increase in a two-layer hierarchical modulation. This happens especially when the enhancement layer is rotated. In order to minimize BER when Euclidean distance profile is changed in hierarchical modulation, it is necessary to re-do bits-to-symbol mapping. One example of Gray remapping is shown in Fig. 4. Their BER performance is shown in Fig. 12. It shows that different mapping rules may have different BER performance when the power splitting ratio ζ changes.

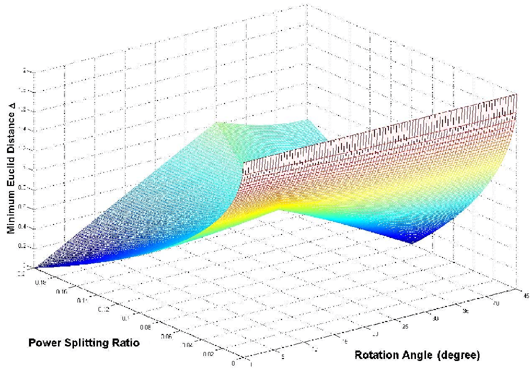


Fig. 11. Minimum Euclidean distance of 16QAM/QPSK modulations

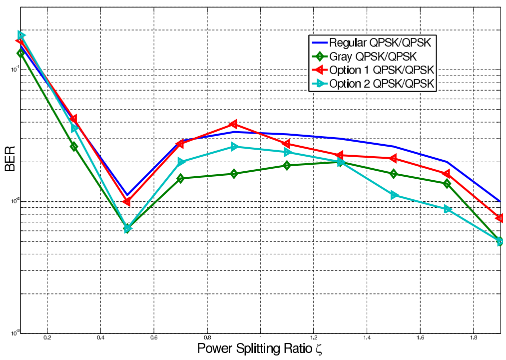


Fig. 12. BER Performance of QPSK/QPSK hierarchical modulations with different mapping rules and SNR=14dB.

VIII. Conclusions

In this paper, two schemes for enhancing hierarchical modulations are presented for higher throughput and less error rate. One approach is to optimize the signal constellation and the other one is to optimize the bits-to-symbol mapping. The rationales as well as the performance of the proposed approaches are analyzed. They can be used for helping upgrade and design BCMCS systems with minimum complexity increase. Some of them is adopted in the 3.5G standard UMB by 3GPP2.

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