* TOPIC : CATLAN NUMBERS :

Q: Count no. of BST's that can be formed using N nodes numbered from 1, 2, 3, ... N.

Follow up: (i) Count no. of Binary Trees.

(ii) Count no. of Unlabelled Binary Trees.

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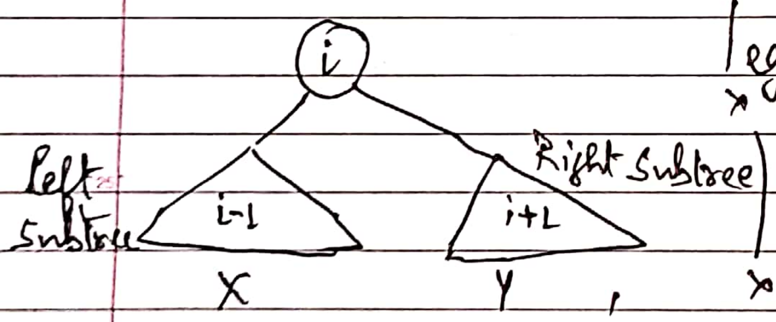
Soln: **Catlan Numbers:** **Formula:** $\frac{^{2n}C_n}{n+1}$

* **Dynamic Approach Formula / Recursive formula:**

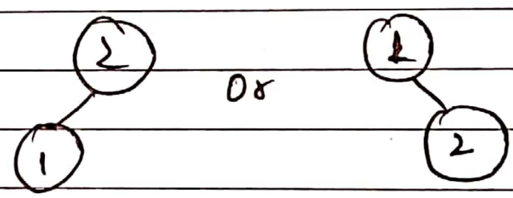
$$\boxed{\text{Ans} = \sum_{i=1}^{n} f(i-1) \cdot f(n-i)}$$

* **Intuition:**

For BST consider (i) is root node:



eg.



$$\underbrace{1 \ldots i\text{-}1}_{\substack{(i-1) \\ \text{nodes} \\ (X)}} \; i \; \underbrace{\ldots N}_{\substack{(N-i) \\ \text{nodes} \\ (Y)}}$$

Total trees:

$$\underbrace{^{X}C_1}_{\substack{\text{select one} \\ \text{out of } x \\ \text{(left)}}} \cdot 1 \cdot \underbrace{^{Y}C_1}_{\substack{\text{select one} \\ \text{out of } y \\ \text{(right)}}}$$

$= {}^{i-1}C_1 \cdot 1 \cdot {}^{N-i}C_1$

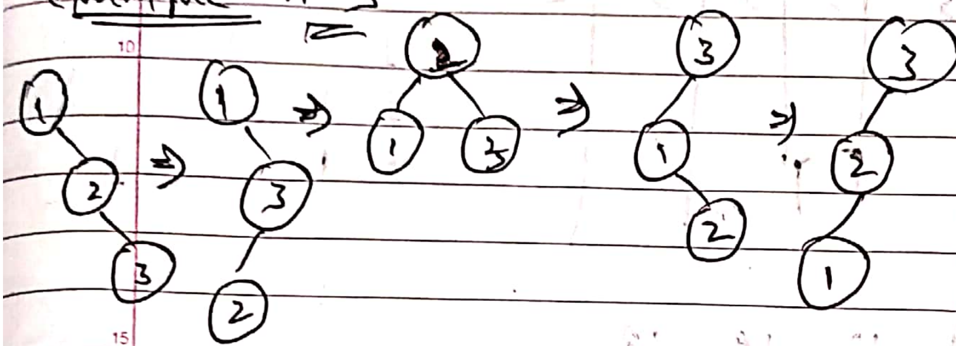
$= \frac{(i-1)!}{1!\,(i-2)!} \times \frac{(N-i)!}{(N-i-1)!\,1!}$

$= (i-1) \times (N-i)$

or: for all n: $\boxed{f(n) = \sum_{i=1}^{n} f(i-1)\cdot f(n-i)}$ where $f(0)=1$ (Null tree)

$f(1) = 1$ (one tree)

example $n=3$



$f(3)$ = for

$i=1$ $f(0)\,f(2) = 2$

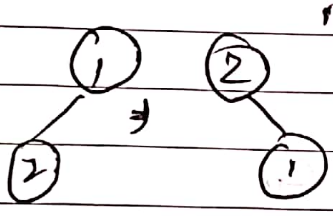
$i=2$ $f(1)\,f(1) = 1$ +

$i=3$ $f(2)\,f(0) = 2$

$5 = 5$

$\boxed{\text{So } f(3) = 5}$

$n=2$



$f(2) \Rightarrow$ for

$i=1$ $f(0)\,f(1) = 1\cdot 1$ +

$i=2$ $f(1)\,f(0) = 1\cdot 1$

$2 = 2$

$\boxed{\text{So } f(2) = 2}$

Follow up: (i) Binary Tree can be computed as all ~~com~~ permutation of tree with any node as root node.

So $f(n) = n! \times \frac{{}^{2n}C_n}{n+1}$

(ii) Unlabelled Binary Tree are just structures without labelling inside BST. So, $f(n) = \frac{{}^{2n}C_n}{n+1}$