Predicting and analyzing the COVID-19 pandemic in Italy using SEIR-type and deep learning models: a comparative study

Simonetta Bodojra S279635
Silvia Giammarinaro S269991

A.Y. 2020/2021

1.
Introduction

COVID-19 pandemic

- First cases at the end of 2019 in Wuhan, China

- 30th January 2020: OMS declares Public Health Emergency

- 24th February 2020: first “red zone” areas in Italy

Our work

- Studying the pandemic in Italy

- Mathematical model: SEIR-type model

- Deep learning: LSTM

- Results and comparison

2.

Models

SIR model

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

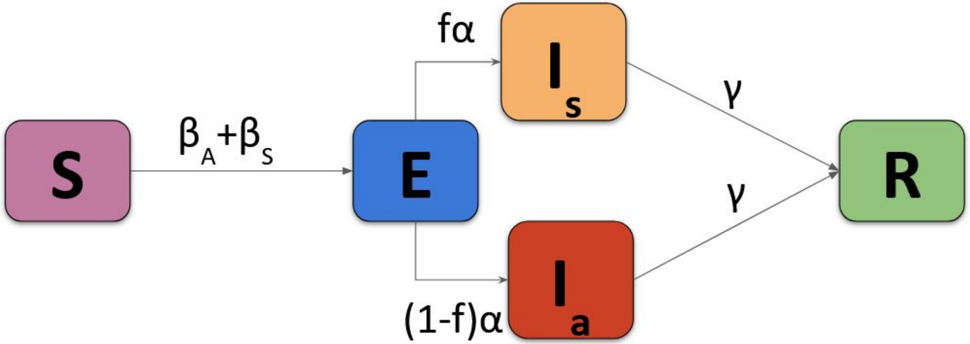
SEIR model

$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dE}{dt} = \beta SI - \alpha E$$
$$\frac{dI}{dt} = \alpha E - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

$$S \xrightarrow{\beta} E \xrightarrow{\alpha} I \xrightarrow{\gamma} R$$

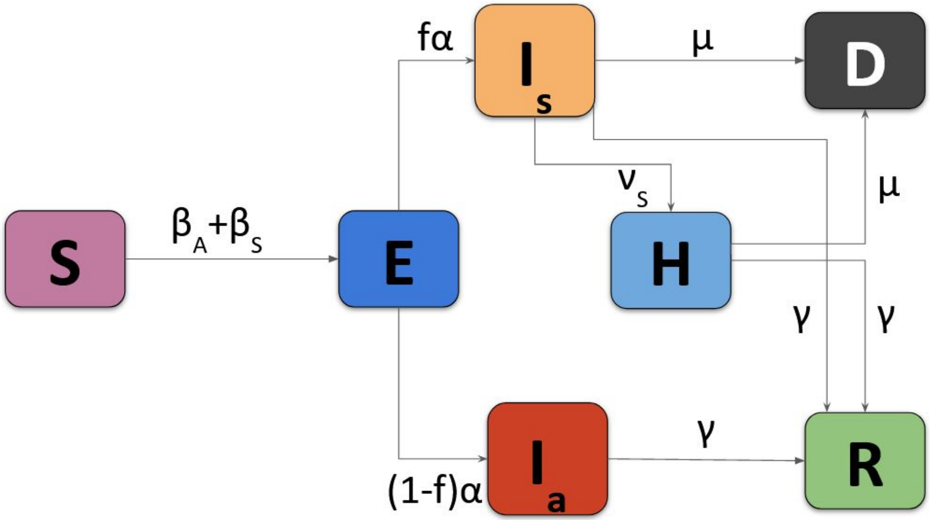
SEIIR model

$$\frac{dS}{dt} = -(\beta_s I_s + \beta_a I_a) S$$
$$\frac{dE}{dt} = (\beta_s I_s + \beta_a I_a) S - \alpha E$$
$$\frac{dI_a}{dt} = (1 - f)\alpha E - \gamma I_a$$
$$\frac{dI_s}{dt} = f\alpha E - \gamma I_s$$
$$\frac{dR}{dt} = \gamma (I_s + I_a)$$



SEIIRHD model

$$\frac{dS}{dt} = -(\beta_s I_s + \beta_a I_a)S$$
$$\frac{dE}{dt} = (\beta_s I_s + \beta_a I_a)S - \alpha E$$
$$\frac{dI_a}{dt} = (1-f)\alpha E - \gamma I_a$$
$$\frac{dI_s}{dt} = f\alpha E - (\gamma + \mu + \nu_s)I_s$$
$$\frac{dR}{dt} = \gamma(I_s + I_a + H)$$
$$\frac{dH}{dt} = \nu_s I_s - (\gamma + \mu)H$$
$$\frac{dD}{dt} = \mu(I_s + H)$$



2.2 Qualitative Analysis

The SEIIRHD model has one equilibrium point:

- Disease free equilibrium point (DFE): no disease is present in the population.

Qualitative Analysis: SEIIRHD

- Feasible Region for the SEIIRHD model:

$$\Omega_{\mathtt{SEIIRHD}} = \{(S(t), E(t), I_a(t), I_s(t), H(t), R(t), D(t)) \in R^7_+ : 0 \leq N(t) \leq N_0\}$$

- Disease-free equilibrium point

$$(S^*_{DFE}, E^*_{DFE}, I^*_{aDFE}, I^*_{sDFE}, R^*_{DFE}, H^*_{DFE}, D^*_{DFE}) = (1, 0, 0, 0, 0, 0, 0)$$

$$\mathcal{F} = \begin{bmatrix} \beta_a I_a S + \beta_s I_s S \\ 0 \\ 0 \end{bmatrix} \quad \mathcal{V} = \begin{bmatrix} \alpha E \\ \gamma I_a - (1-f)\alpha E \\ (\gamma + \mu + \nu_s) I_s - f\alpha E \end{bmatrix}$$

Vector of new infection rates

Vector of other rates

Qualitative Analysis: SEIIRHD

Next Generation Matrix

$$FV^{-1} = \begin{bmatrix} \frac{f\beta_s}{\gamma+\mu+\nu_s} + \frac{(1-f)\beta_a}{\gamma} & \frac{\beta_s}{\gamma+\mu+\nu_s} & \frac{\beta_a}{\gamma} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{R}_0 = \rho(FV^{-1}) = \frac{f\beta_s}{\gamma+\mu+\nu_s} + \frac{(1-f)\beta_a}{\gamma}$$

Theorem. The DFE point is asymptotically stable if $\mathcal{R}_0 < 1$

3.
Results

3 Results

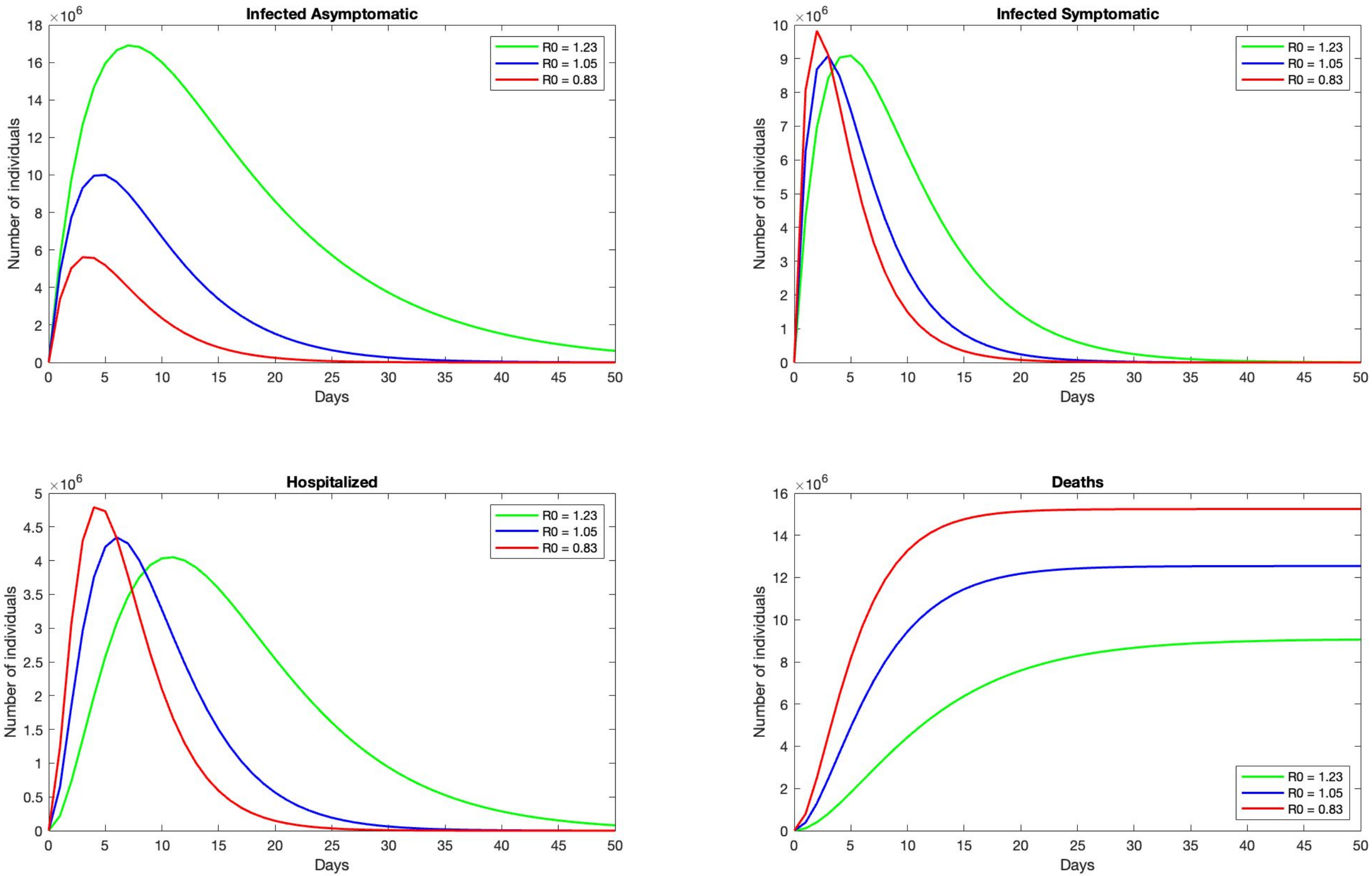
- 3.1 Model Simulation
- 3.2 Sensitivity analysis
- 3.3 Estimating parameters from real data with the SEIIRHD model
- 3.4 LSTM model
- 3.5 A comparison between SEIIRHD and LSTM models

3.1 Model simulations

- Generate a sample of size 1000 from a uniform distribution U(a; b);
- Compute first and third quartile and median for each sample;
- Plot the results.

Parameter	Description	Distribution interval
f	Probability of being symptomatic	(0.3, 0.9)
β_a	Transmission rate from S to E from contact with I_a	(0.0, 0.6)
β_s	Transmission rate from S to E from contact with I_s	(0.0, 0.6)
γ	Recovery rate	(0.0, 0.4)
α	Inverse of the incubation period	(0.15, 0.35)
ν_s	Hospitalization rate from state I_s	(0.0, 0.4)
μ	Death Rate	(0.0, 0.2)

3.1 Model simulations



3.1 Model simulations

Parameters			
	$\mathcal{R}_0 = 1.23$	$\mathcal{R}_0 = 1.05$	$\mathcal{R}_0 = 0.83$
f	0.4528	0.6013	0.7479
β_a	0.1587	0.3176	0.4434
β_s	0.1504	0.3210	0.4442
γ	0.0919	0.1911	0.2912
α	0.1982	0.2515	0.3003
ν_s	0.0991	0.2027	0.3096
μ	0.0462	0.1018	0.1498

Maximum value for each compartment			
I_a	16.90M	9.99M	5.61M
I_s	9.09M	9.08M	9.82M
H	4.05M	4.34M	4.79M
D	9.05M	12.54M	15.24M

3.2 Sensitivity Analysis

Sensitivity index: correlation between each parameter and the basic reproduction number $\mathcal{R}_0$

$$C_p^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial p} \times \frac{p}{\mathcal{R}_0}$$

$C_{\beta_a}^{\mathcal{R}_0}$	$C_{\beta_s}^{\mathcal{R}_0}$	$C_{f}^{\mathcal{R}_0}$	$C_{\gamma}^{\mathcal{R}_0}$	$C_{\nu_s}^{\mathcal{R}_0}$	$C_{\mu}^{\mathcal{R}_0}$
0.7670	0.2330	-0.4017	-0.0728	-0.0066	-0.0031

3.3 Numerical Simulations

Two periods are considered:

- Second wave from October 8th 2020 to November 23rd 2020;
- Third wave from January 21st to February 21th 2021.

Algorithm 1: Concatenated SEIIRHD fitting

Result: f, α, γ, β_a, β_s, nu_s, μ

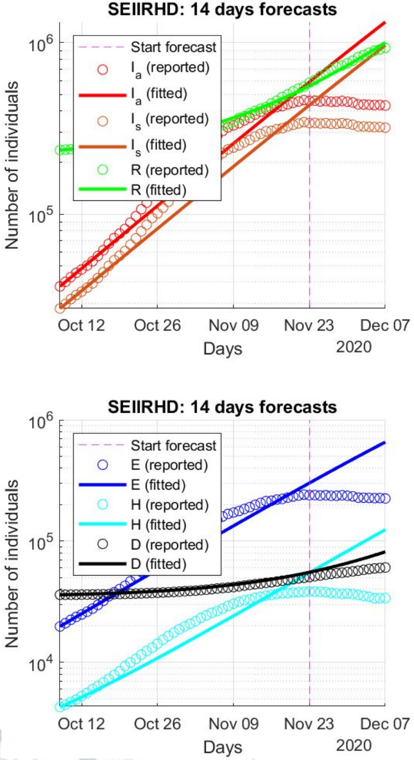
Initialize β_0, γ_0;
$[\beta_{SIR}, \gamma_{SIR}] = \text{SIR}\ (\beta_0, \gamma_0)$;

Initialize α_0;
$[\beta_{SEIR}, \gamma_{SEIR}, \alpha_{SEIR}] = \text{SEIR}(\beta_{SIR}, \gamma_{SIR}, \alpha_0)$;

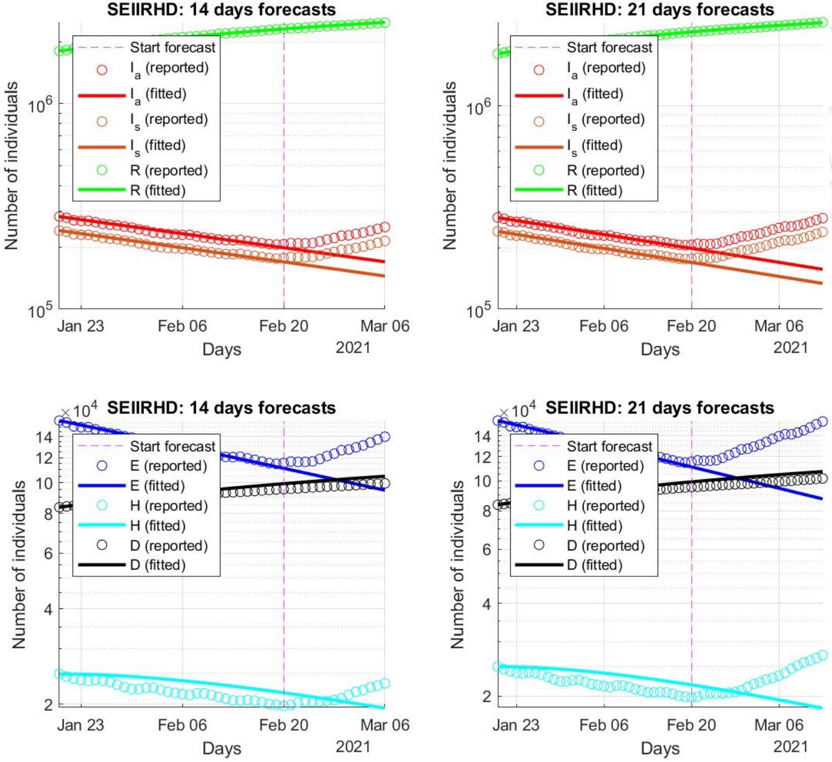
Initialize f_0, $\beta_{a,0}$, $\beta_{s,0}$;
$[f_{SEIIR}, \alpha_{SEIIR}, \gamma_{SEIIR}, \beta_{a,SEIIR}, \beta_{s,SEIIR}] = \text{SEIIR}\ (f_0, \alpha_{SEIR}, \gamma_{SEIR}, \beta_{a,0}, \beta_{s,0})$;

Initialize ν_0, μ_0 ;
$\text{SEIIRHD}\ (f_{SEIIR}, \alpha_{SEIIR}, \gamma_{SEIIR}, \beta_{a,SEIIR}, \beta_{s,SEIIR}, \nu_0, \mu_0\ )$;

3.3.1 Italy

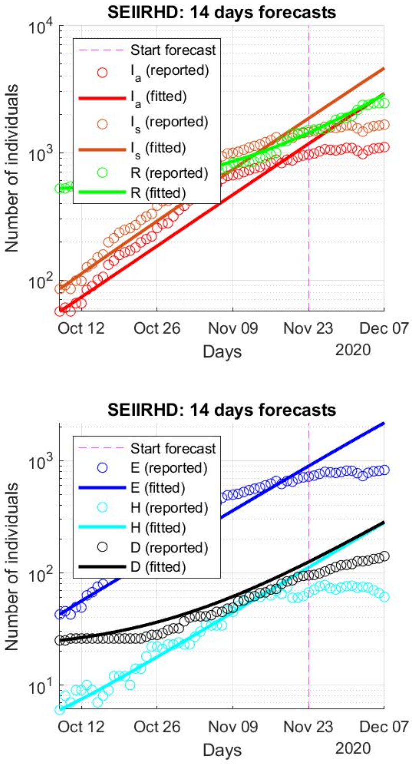


Second wave

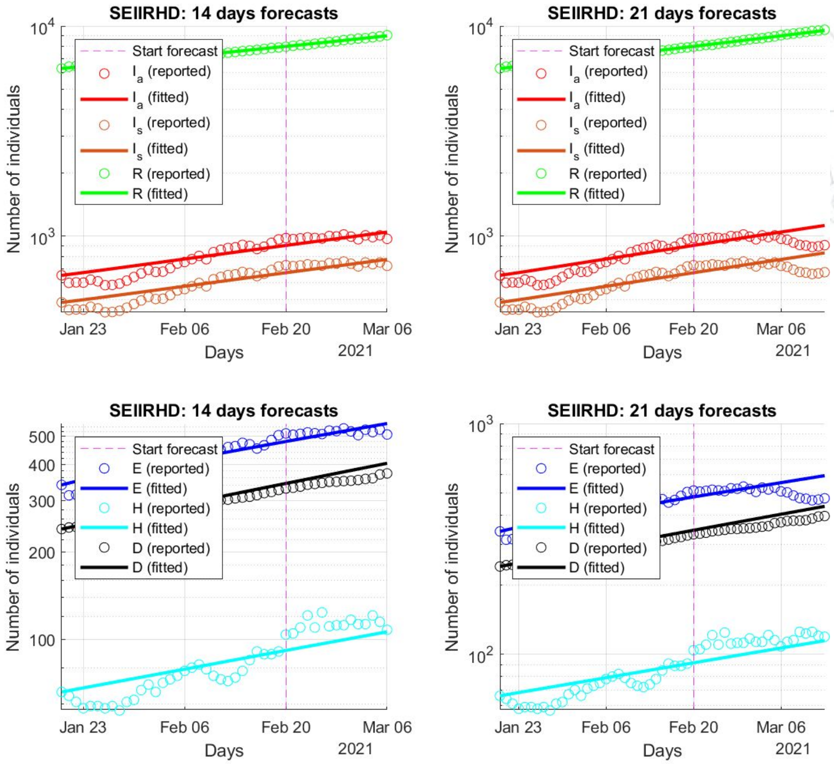


Third wave

3.3.2 Molise

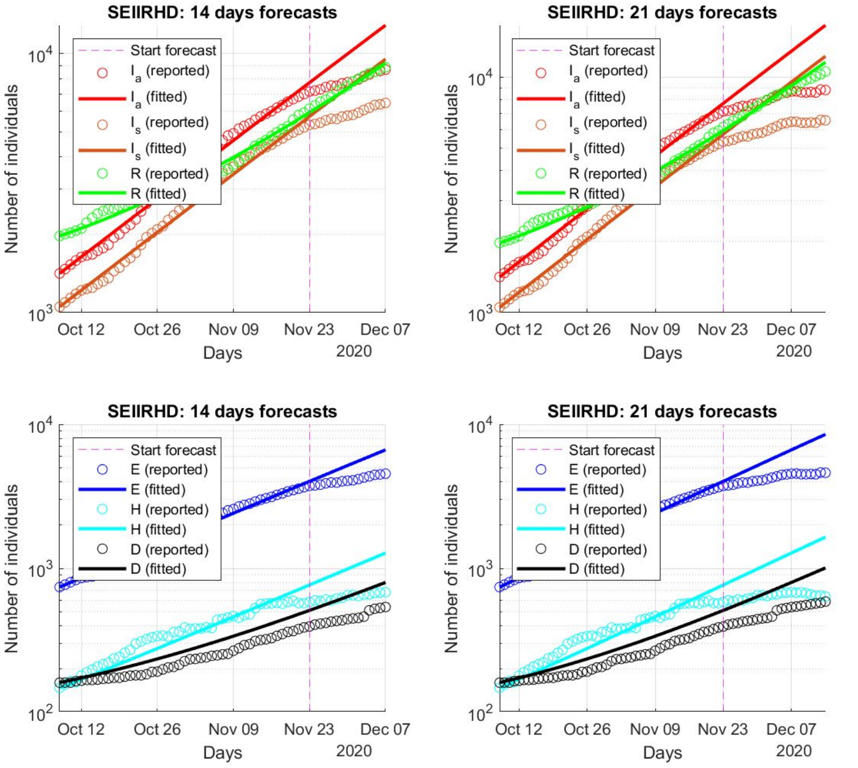


Second wave

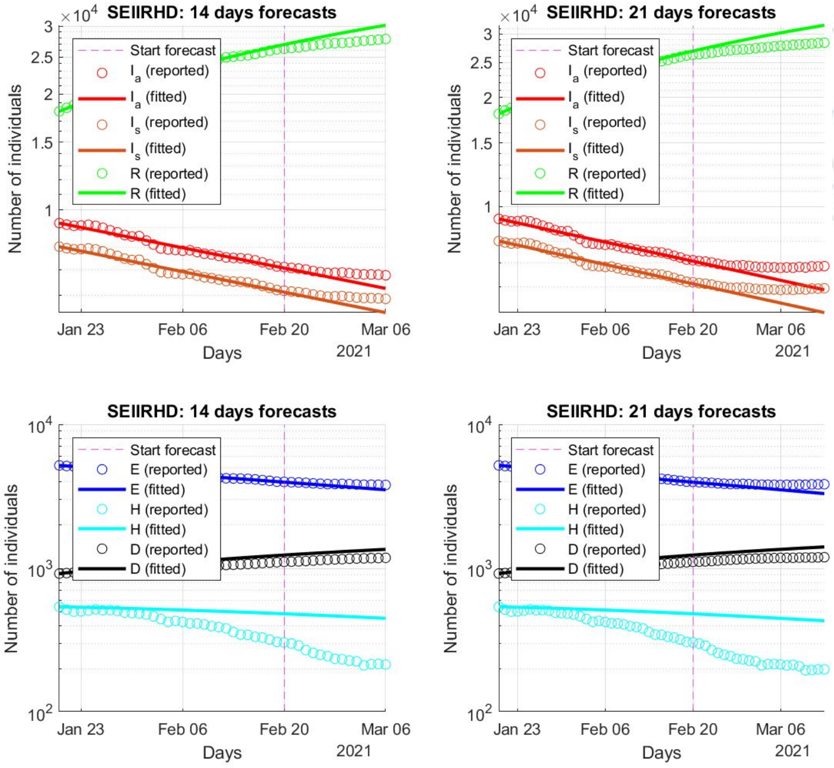


Third wave

3.3.3 Sardegna



Second wave



Third wave

3.3.4 Summary of numerical simulations

Parameters	Second wave			Third wave		
	Italy	Molise	Sardegna	Italy	Molise	Sardegna
f	0.4600	0.6363	0.4679	0.5164	0.4820	0.5211
β_a	0.0833	0.0564	0.0708	0.0217	0.0508	0.0076
β_s	0.1281	0.0754	0.0103	0.0253	0.0692	0.0087
γ	0.0200	0.0198	0.0132	0.0351	0.0391	0.0183
α	0.2741	0.3061	0.1774	0.0884	0.1803	0.0356
$\tau(\frac{1}{\alpha})$	3.6483	3.2662	5.6369	11.3122	5.2500	28.080
ν_s	0.0102	0.0053	0.0068	0.0039	0.0075	0.0012
μ	0.0025	0.0035	0.0024	0.0022	0.0052	0.0014
$\mathcal{R}_0$	4.8531	5.5499	4.4216	0.6197	1.4386	0.4199

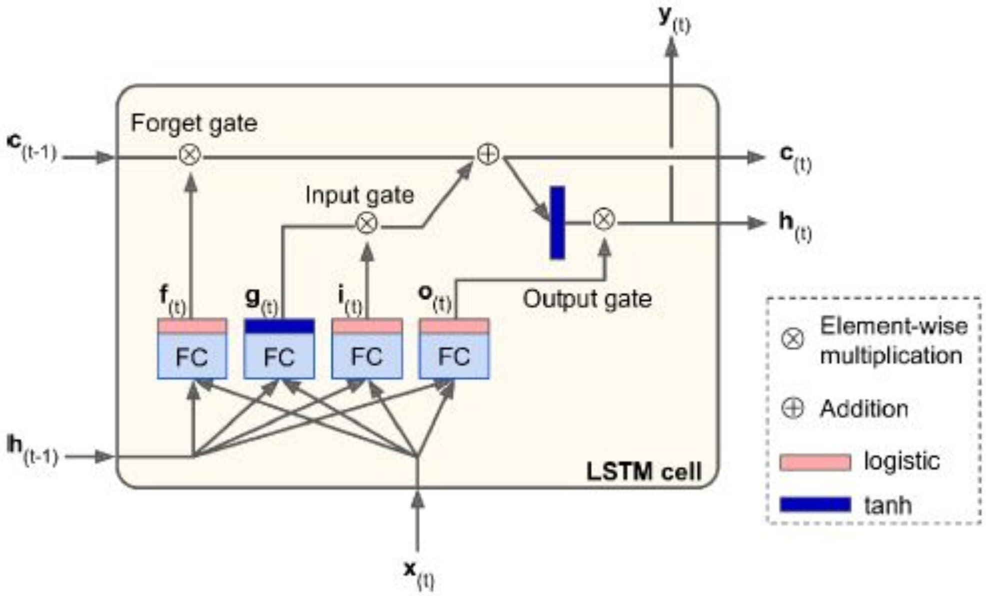
3.4 LSTM: a deep learning model for predictions

Vectors:

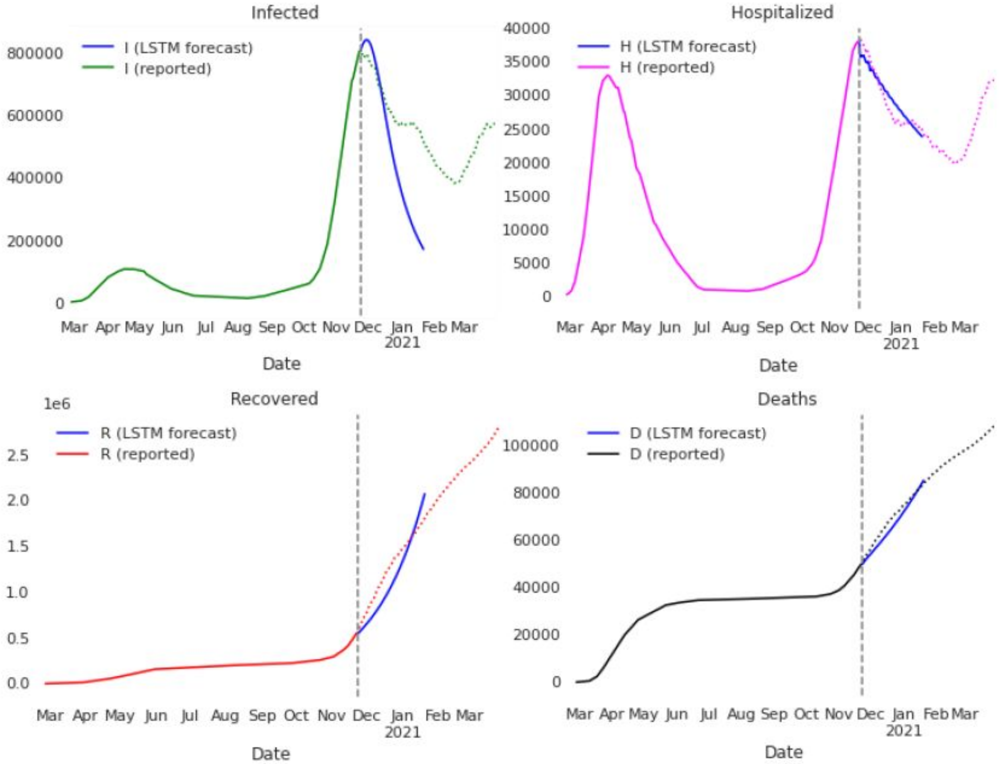
- Inputs: X
- Long-term state: H
- Short-term state: C

Functions:

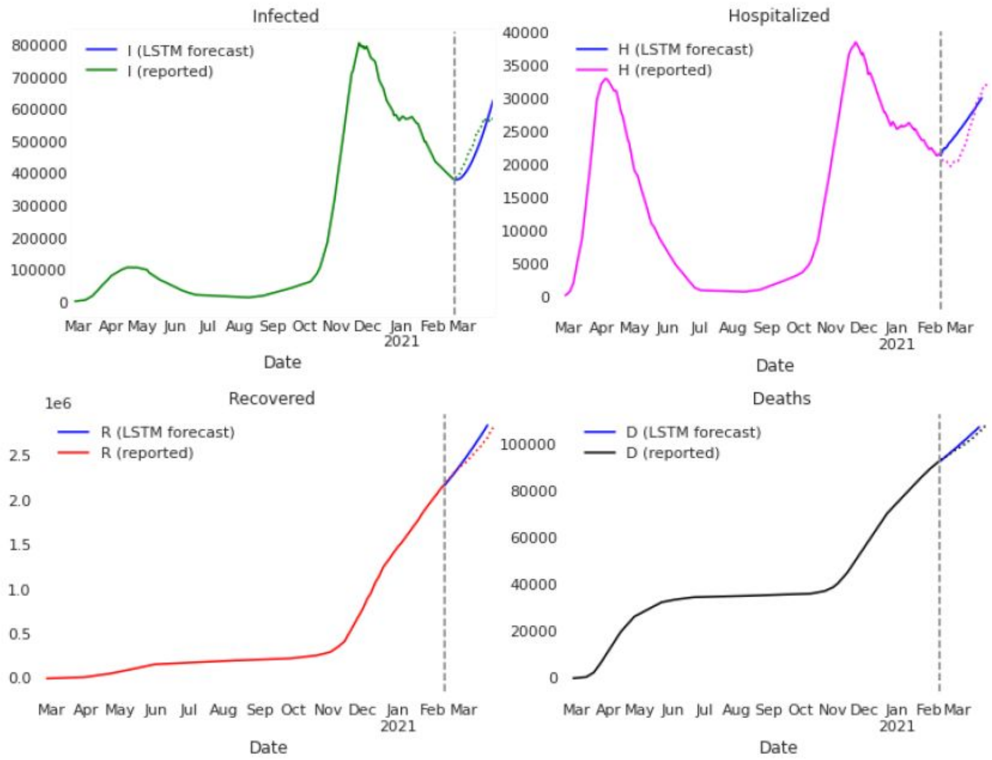
- Output $g_{(t)}$
- Forget Gate $f_{(t)}$
- Input Gate $i_{(t)}$
- Output Gate $o_{(t)}$



Results on second wave



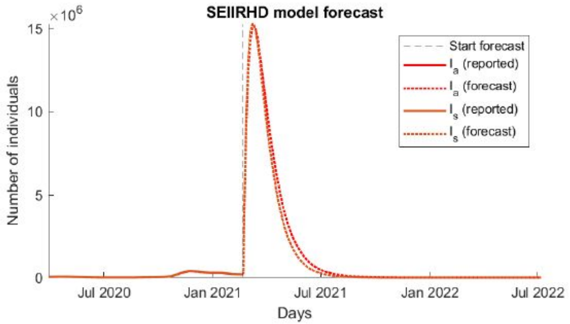
Results on third wave

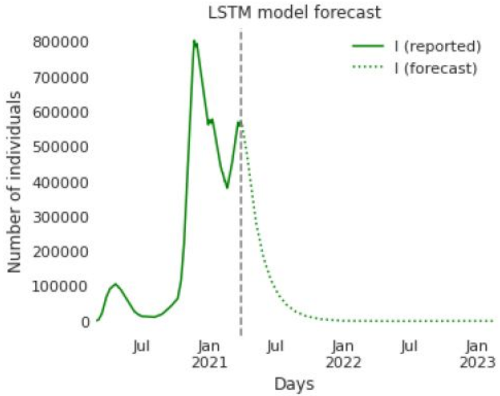


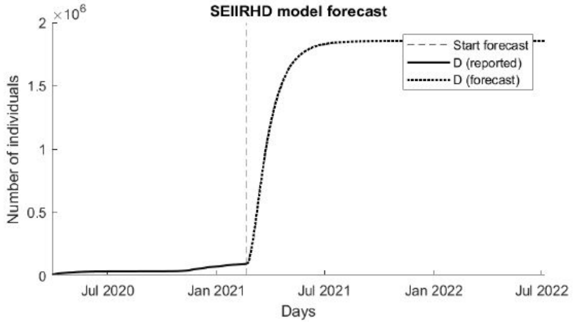
3.5 A comparison between SEIIRHD and LSTM model

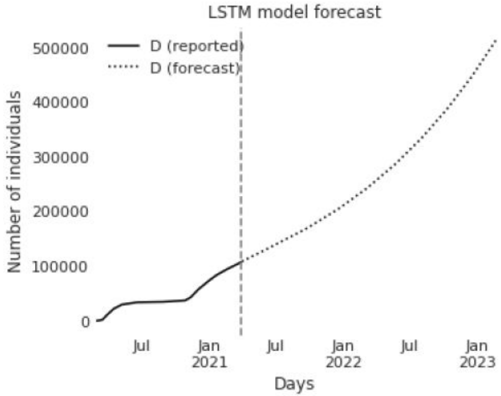
Compartment	Second wave					
	SEIIRHD			LSTM		
	MAE	RMSE	R_2	MAE	RMSE	R_2
Infected	2489632.79	3235121.77	-0.86	156992.10	201420.69	0.27
Hospitalized	378468.57	508199.33	-0.78	3702.55	3935.55	-0.25
Deaths	125326.99	178037.58	0.91	2171.30	2819.07	0.94
Recovered	1655269.79	2550386.67	0.91	138311.86	186830.11	0.87
	Third wave					
Infected	71694.57	83659.48	-0.99	186830.11	94799.61	0.56
Hospitalized	5784.88	7492.38	-0.98	1945.79	2302.81	0.55
Deaths	4603.20	4629.50	0.99	2671.78	3393.29	0.67
Recovered	17408.67	25839.88	0.99	20282.58	20282.58	0.98

3.6 When the pandemic will end?









3.6 When the pandemic will end?

Date	Infected		Deaths	
	SEIIRHD	LSTM	SEIIRHD	LSTM
June 2021	2.82M	172.24K	1.85M	129.99K
September 2021	63.06K	24.00K	1.85M	1.62M
December 2022	1469	3560	1.85M	1.97M
March 2022	35	2589	1.85M	2.24M
June 2022	0	1328	1.85M	2.93M
September 2022	0	873	1.85M	3.57M
December 2022	0	726	1.85M	4.33M

4.

Conclusions

Conclusions and future work

Asymptomatic individuals play a huge role in the epidemic

LSTM has a better forecast performance in the short term

Include vaccination, treatment strategies, NPI measures to the SEIIRHD model

Merge the SEIIRHD model with the LSTM

Thanks for your attention!

Any questions?