Kinematic Car Model

Matt Schmittle

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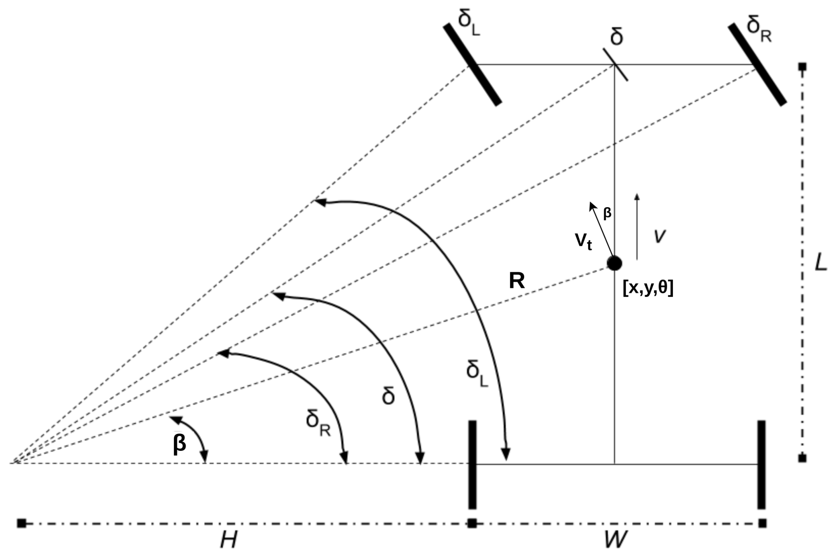

Figure 1: Visual representation of the kinematic car model

The Kinematic Model of a robot car. State consists of position and heading angle $\mathbf{x} = (x, y, \theta)$. Controls consist of velocity and steering angle $\mathbf{u} = (v, \delta)$. This model is also called the bicycle model because we model the car as a bicycle where the wheels are centered on the vehicle. Without any control the state changes like so:

$$\dot{x} = vcos(\theta)$$
$$\dot{y} = vsin(\theta)$$
$$\dot{\theta} = \omega$$

This can be seen visually in Figure 2. ω is the angular velocity from the center of rotation to the center of the back axle and can further be defined as follows.

$$\omega = \frac{v}{(H + \frac{W}{2})}$$

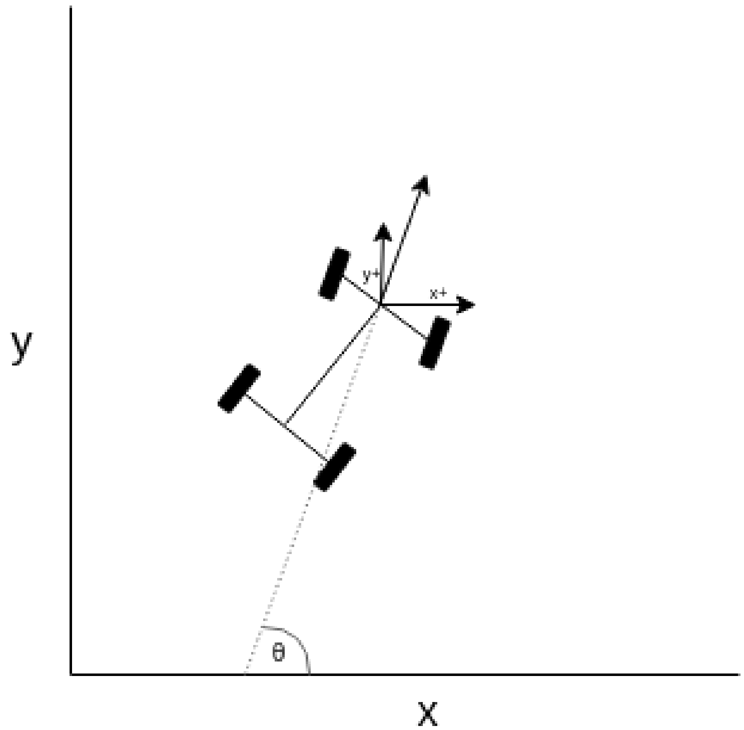

Figure 2: Visual representation of the movement of the car given no control.

Where v in this case is the tangential velocity. And we know by definition, $\omega = \frac{v}{(H+\frac{W}{2})}$. Figure 1 shows the tangential velocity v_t.

$$\dot{\theta} = \frac{v}{(H + \frac{W}{2})}$$

$$= \frac{v}{\frac{L}{tan(\delta)}}$$

$$= \frac{V}{L} tan(\delta)$$

Now if we apply a control u_t that affects the state at $t + 1$. We can integrate over the time step to see how the state changes as theta changes.

$$\frac{\partial x}{\partial t} = v cos(\theta)$$
$$\frac{\partial y}{\partial t} = v sin(\theta)$$
$$\frac{\partial \theta}{\partial t} = \frac{v}{L} tan(\delta)$$

Change in θ:

$$\int_{\theta_t}^{\theta_{t+1}} d\theta = \int_t^{t+\Delta t} \frac{v}{L} tan(\delta) dt$$

$$\theta_{t+1} - \theta_t = \frac{v}{L} tan(\delta)[t + \Delta t - t] = \frac{v}{L} tan(\delta)\Delta t$$

Change in x:

$$\int_{x_t}^{x_{t+1}} dx = \int_t^{t+\Delta t} vcos(\theta)dt = \int_t^{t+\Delta t} vcos(\theta) \frac{d\theta}{\frac{v}{L} tan(\delta)} = \frac{L}{tan(\delta)} \int_{\theta_t}^{\theta_{t+1}} cos(\theta)d\theta$$

$$x_{t+1} - x_t = \frac{L}{tan(\delta)}[sin(\theta_{t+1}) - sin(\theta_t)]$$

Change in y:

$$\int_{y_t}^{y_{t+1}} dy = \int_t^{t+\Delta t} vsin(\theta)dt = \int_{\theta_t}^{\theta_{t+1}} vsin(\theta) \frac{d\theta}{\frac{v}{L} tan(\delta)} = \frac{L}{tan(\delta)} \int_{\theta_t}^{\theta_{t+1}} sin(\theta)d\theta$$

$$y_{t+1} - y_t = \frac{L}{tan(\delta)}[-cos(\theta_{t+1}) + cos(\theta_t)]$$

Putting it all together we have:

$$x_{t+1} = x_t + \frac{L}{tan(\delta)}(sin(\theta_{t+1}) - sin(\theta_t))$$

$$y_{t+1} = y_t + \frac{L}{tan(\delta)}(-cos(\theta_{t+1}) + cos(\theta_t))$$

$$\theta_{t+1} = \theta_t + \frac{v}{L} tan(\delta)\Delta t \quad \square$$