

Differentiated Bertrand Competition

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What are differentiated goods?

- ▶ They are goods that are similar, but each is different to the other in some important way
- ▶ They are **not** *homogeneous* or *standardized* goods so they are **not** perfect substitutes for each other.
- ▶ For example, iPhones and Pixel phones are both in the cell phone industry, but they're different to each other.
- ▶ Or, Toyota Corolla and Honda Civic cars.
- ▶ How can we model these kinds of markets with *price competition*?



Figure 1: iPhone and Pixel

Let's use a model with LOCATIONS

- ▶ A unit measure of consumers are evenly distributed on the interval $[0, 1]$.
- ▶ Firm A's product is located at one end, point 0 (iPhone).
- ▶ Firm G's product is located at the other end, point 1 (Pixel).
- ▶ The location represents the consumer's tastes. Let $d(x, y) = |x - y|$ be the euclidean distance function.
- ▶ The distances between a consumer at point x and the two products are $d(x, iPhone) = x$ and $d(x, Pixel) = 1 - x$

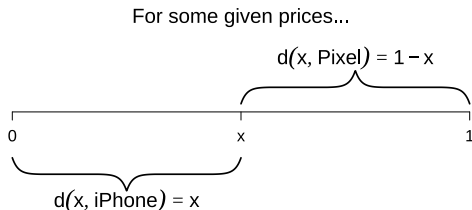


Figure 2: A model of locations and demand

What about consumers' preferences? And the rules of competition?

For a consumer at point $x \in [0, 1]$:

- ▶ Utility from iPhone:

$$U_x^{iPhone} = v - d(x, iPhone) - p_1 = v - x - p_1$$

- ▶ Utility from Pixel:

$$U_x^{Pixel} = v - d(x, Pixel) - p_2 = v - (1 - x) - p_2$$

Bertrand competition:

- ▶ The two firms simultaneously **post prices** $p_1 = p_{iPhone}$ and $p_2 = p_{Pixel}$.
- ▶ Each consumer purchases their most preferred product (highest utility), assuming they receive a positive utility (otherwise they don't buy).
- ▶ To simplify, we can focus on the region where $p_1, p_2 < v - 1$ and so every consumer purchases one of the two products, i.e. they buy either an iPhone or a Pixel.

Question 1

For every pair of prices p_1, p_2 such that $0 < p_2 - p_1 < 1$ find the location $x \in [0, 1]$ of the consumer who is indifferent between purchasing the iPhone and the Pixel.

- ▶ Which product is purchased by the consumers to the left of the indifferent consumer?
- ▶ Which product is purchased by the consumers to the right of the indifferent consumer?

Question 1: Answer

For a consumer at location $x \in [0, 1]$ (as in Figure 2):

- Purchase iPhone if

$$U^{iPhone} = v - x - p_1 > v - (1 - x) - p_2 = U^{Pixel}$$

$$\iff \frac{1}{2} + \frac{p_2 - p_1}{2} \equiv \frac{1}{2} + \frac{\Delta_p}{2} > x$$

- Purchase Pixel if

$$U^{iPhone} = v - x - p_1 < v - (1 - x) - p_2 = U^{Pixel}$$

$$\iff \frac{1}{2} + \frac{p_2 - p_1}{2} \equiv \frac{1}{2} + \frac{\Delta_p}{2} < x$$

- Indifferent if

$$x = \frac{1}{2} + \frac{\Delta_p}{2}$$

Question 1: Answer as a figure

Conceived of slightly differently...

For some given prices...

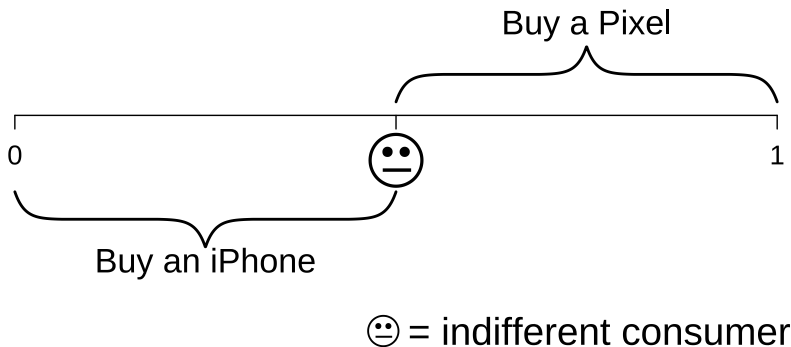


Figure 3: The indifferent consumer and purchases on either side

Question 2: Demand

- ▶ Remember that the measure of consumers is 1 and they are uniformly distributed on $[0, 1]$.
- ▶ For every pair of prices p_1, p_2 , derive the measure (or fraction) of consumers that purchase the iPhone and the measure (or fraction) of consumers that purchase the Pixel.
- ▶ We will denote the measure (fraction) of agents that purchase the iPhone by the demand function $Q_1(p_1, p_2)$ (demand for iPhone) and the measure (fraction) of agents that prefer the Pixel by $Q_2(p_1, p_2)$ (demand for Pixel).

Question 2: Demand - Answer

Sell goods such that you reach the indifferent consumer!

Demand for iPhone:

$$Q_1(p_1, p_2) = \frac{1}{2} + \frac{\Delta p}{2}$$

Remember that the demands sum to 1, that is:

$$Q_1 + Q_2 = 1 \Rightarrow Q_2 = 1 - Q_1$$

We can use this to find the demand for Pixels given that we've already found the demand for iPhones.

Demand for Pixel:

$$Q_2(p_1, p_2) = 1 - Q_1(p_1, p_2)$$

$$1 - \left(\frac{1}{2} + \frac{\Delta p}{2} \right) = \frac{1}{2} - \frac{\Delta p}{2}$$

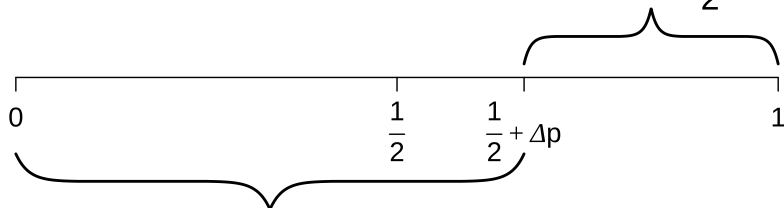
- What happens in the different cases for the prices, $p_1 > p_2$, $p_2 > p_1$, or $p_1 = p_2$? Think about the figures we drew earlier.

Question 2: Answer as figures a

When $\Delta p = p_2 - p_1 > 0$

Demand for Pixels

$$Q_2(p_1, p_2) = \frac{1}{2} - \Delta p$$



Demand for iPhones

$$Q_1(p_1, p_2) = \frac{1}{2} + \Delta p$$

Figure 4: Demand and a price difference

Question 2: Answer as figures b

When Pixels are more expensive than iPhones

$$\Delta p = p_2 - p_1 > 0$$

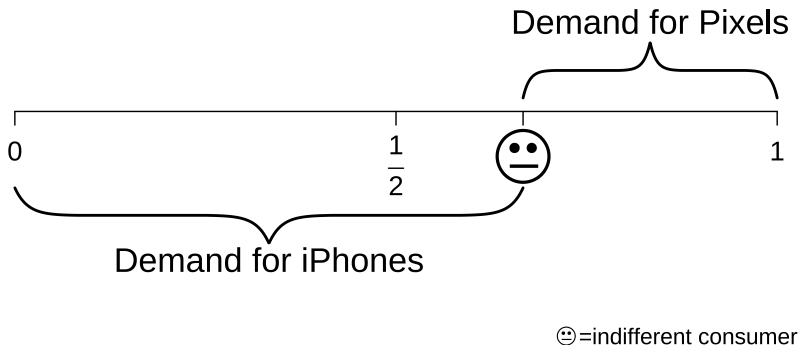
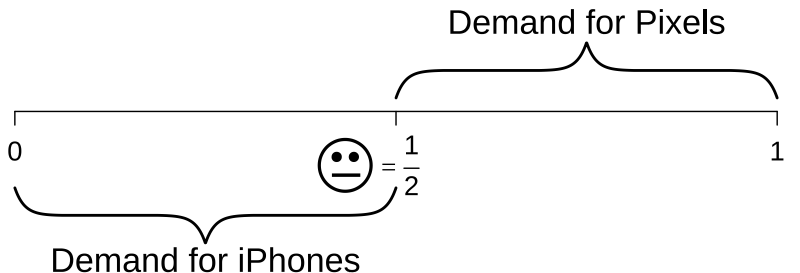


Figure 5: Greater demand for iPhones

Question 2: Answer as figures c

When the prices of Pixels and iPhones are equal

$$\Delta p = p_2 - p_1 = 0$$



😊=indifferent consumer

Figure 6: Equal demand for each

Question 2: Answer as figures d

When iPhones are more expensive than Pixels

$$\Delta p = p_2 - p_1 < 0$$

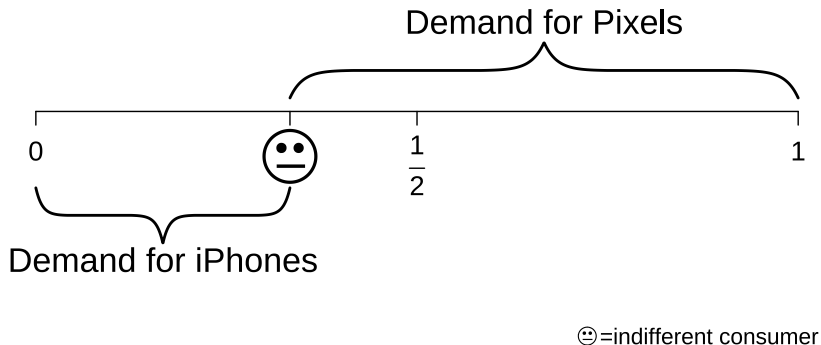


Figure 7: Greater demand for Pixels

Question 3: Profits

Assume that Firm i has constant marginal costs $c_i > 0$ and $c_i < v - 1$. 1. Remember that profits are total revenues minus total costs. 2. Revenues are given by the price at which the good is sold multiplied by the quantity sold. 3. Therefore, for every pair of prices, the profit of firm i is:

$$\underbrace{\pi_i(p_i, p_j)}_{\text{Total profits}} = \underbrace{p_i Q_i(p_i, p_j)}_{\text{Total revenues}} - \underbrace{c_i Q_i(p_i, p_j)}_{\text{Total costs}} = \underbrace{(p_i - c_i)}_{\text{Profit per unit}} \underbrace{Q_i(p_i, p_j)}_{\text{Units sold}}$$

Write the profit of each firm only as a function of the prices (hint: use the demands from previous question).

Question 3: Profits - Answer

- ▶ We know that $Q_1(p_1, p_2) = \frac{1}{2} + \frac{\Delta_p}{2}$.
- ▶ And that $Q_2(p_1, p_2) = \frac{1}{2} - \frac{\Delta_p}{2}$.
- ▶ $\Delta_p = p_2 - p_1$, which means that $-\Delta_p = p_1 - p_2$.

Therefore the profits are as follows:

- ▶ iPhone: $\pi_1(p_1, p_2) = (p_1 - c_1) \left(\frac{1}{2} + \frac{p_2 - p_1}{2} \right)$
- ▶ Pixel: $\pi_2(p_1, p_2) = (p_2 - c_2) \left(\frac{1}{2} + \frac{p_1 - p_2}{2} \right)$

Question 4: Formulate as a game and solve for the NE

- ▶ Remember that the firms simultaneously choose prices.
- ▶ Formulate this situation as a game and solve for the Nash Equilibrium.

Guidance: To solve this question we need to do the following.

- ▶ Specify the players, their actions, and their payoffs.
- ▶ Find the players' best responses.
- ▶ Having found the best responses, use them to find the mutual best response or Nash equilibrium.
- ▶ Extension: Comment on the profits and isoprofits at the Nash.

Question 4: Answer

Initial description of the game

- ▶ **Players:** 2 Firms, such as Apple (A) and Google (G)
- ▶ **Actions:** Firms simultaneously choose prices $p_i \in [0, \infty]$
- ▶ **Payoffs:**
 - ▶ $\pi_1(p_1, p_2) = (p_1 - c_1) \left(\frac{1}{2} + \frac{p_2 - p_1}{2} \right)$
 - ▶ $\pi_2(p_1, p_2) = (p_2 - c_2) \left(\frac{1}{2} + \frac{p_1 - p_2}{2} \right)$

Now we have to solve for the Nash equilibrium.

1. Find each firm's best-response function by imposing FOCs and checking SOC.
2. Solve for the NE prices with those BRFs.
3. We will also graph these and check against ideas from Year 1.

Question 4: Answer - Finding the best responses

Firm i chooses p_i to maximize $\pi_i(p_i, p_j)$

$$\max_{p_i \geq 0} (p_i - c_i) \frac{1}{2} (1 + p_j - p_i)$$

$$FOC : \frac{\partial \pi_i}{\partial p_i} = 0 \Rightarrow \frac{1}{2} (c_i + 1 + p_j - 2p_i) = 0$$

$$\Rightarrow p_i^*(p_j) = \frac{1}{2} (c_i + 1 + p_j)$$

$$SOC : \frac{\partial^2 \pi_i}{\partial p_i^2} < 0 : \frac{\partial^2 \pi_i}{\partial p_i^2} = \frac{1}{2} (-2) = -1 < 0$$

Therefore, the two firms's best-response functions are:

$$p_1^*(p_2) = \frac{1}{2} (1 + p_2 + c_1)$$

$$p_2^*(p_1) = \frac{1}{2} (1 + p_1 + c_2)$$

Question 4: Answer - Finding the NE

1. Graph the two best-response functions in (p_1, p_2) space.
2. Find the Nash equilibrium graphically and explain it.
3. Find the Nash equilibrium algebraically and explain it.

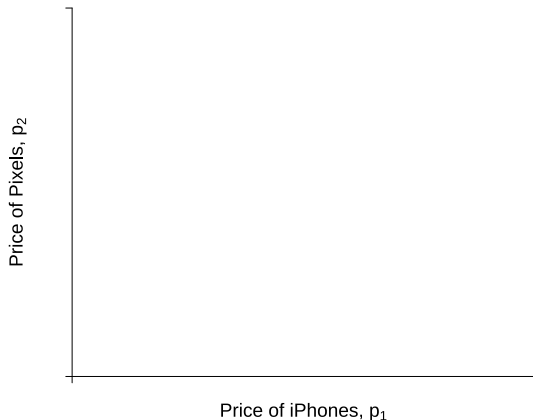


Figure 8: Axes for the BRFs

Question 4: Answer - Finding the NE - Graphical Solution

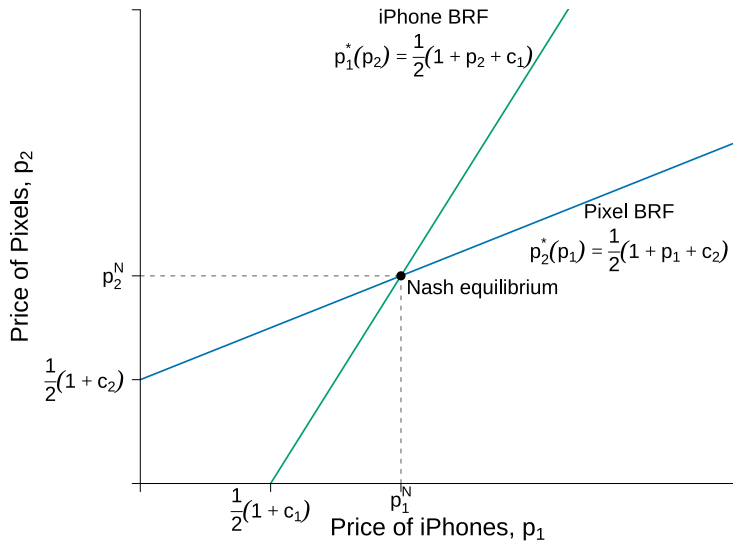


Figure 9: Using the BRFs to find the NE

Question 4: Answer - Finding the NE - Algebraic Solution

At the NE, $p_1 = p_1^*(p_2)$ and $p_2 = p_2^*(p_1)$:

$$\begin{cases} p_1 = \frac{1}{2}(1 + p_2 + c_1) \\ p_2 = \frac{1}{2}(1 + p_1 + c_2) \end{cases}$$

Because $p_1 = p_2$ at the intersection of the BRFs, we know that:

$$p_1 = \frac{1}{2}(1 + c_1) + \frac{1}{2} \left(\frac{1}{2}(1 + p_1 + c_2) \right)$$

$$p_1 = \frac{1}{2} + \frac{1}{2}c_1 + \frac{1}{4}p_1 + \frac{1}{4} + \frac{1}{4}c_2$$

Collecting terms:

$$p_1 - \frac{1}{4}p_1 = \frac{3}{4}p_1 = \frac{3}{4} + \frac{1}{2}c_1 + \frac{1}{4}c_2$$

Multiply through by $\frac{4}{3}$:

$$\left(\frac{4}{3}\right) \left(\frac{3}{4}\right) p_1 = p_1 = 1 + \frac{2}{3}c_1 + \frac{1}{3}c_2 = 1 + \frac{c_2 + 2c_1}{3}$$

Question 4: Answer - Finding the NE - Algebraic Solution

We therefore have the Nash equilibrium solution for p_1 and p_2 . This is a pair of strategies (p_1^N, p_2^N) (N indicates Nash equilibrium – sometimes used in notation).

$$p_1^N = 1 + \frac{c_2 + 2c_1}{3}$$

$$p_2^N = 1 + \frac{2c_2 + c_1}{3}$$

Hence the NE (p_1^N, p_2^N) is $(1 + \frac{c_2+2c_1}{3}, 1 + \frac{2c_2+c_1}{3})$.

Question 4: Answer - Profits & clarifying understanding

- a. What do the FOCs mean?
- b. What happens at each point along a best-response function and how do we *construct* it?

Question 4: Answer - Profits & clarifying understanding

- a. When we impose FOCs it means that *marginal revenues* equal *marginal costs*; that is, the first derivative of the total cost function equals the derivative of the revenue function. The firm cannot make more profits by deviating from $MR = MC$.
- b. Lots to say here:
 - ▶ The BRF is a locus of points where, for every price the opponent chooses, the firm is maximizing profits at that point (i.e. $MR=MC$ at every point along the BRF).
 - ▶ So for every point along the BRF, there exists an isoprofit that would be tangent to a given price of the other firm (the other firm's action or strategy).
 - ▶ We can also think about the firm's MRS. MRS comes from the *objective function*, the profits; for Firm 1, $mrs(p_1, p_2) = -\frac{dp_2}{dp_1} = 0$ when an isoprofit is tangent to a given price of Firm 2 where the isoprofits intersect the BRF).

Question 4: Answer - Profits & clarifying understanding

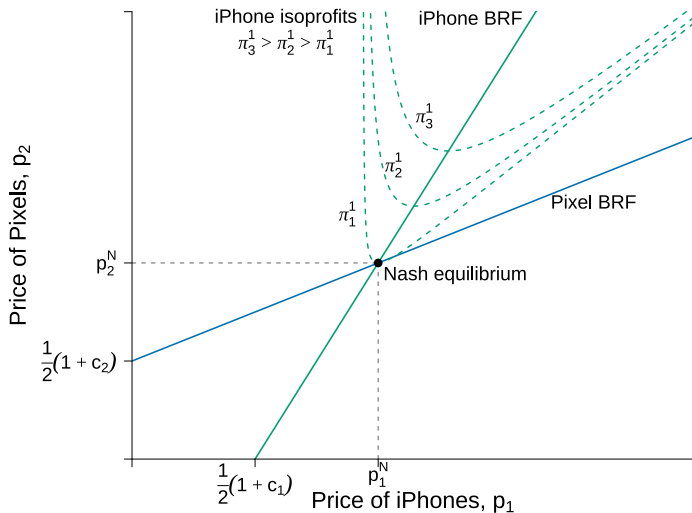


Figure 10: BRFs and isoprofits; what would Google's isoprofits look like?

Question 4: Addendum - Reminder about the mrs (part 1)

- ▶ When an economic agent—a person, the owners of a firm—optimize some objective function, then the contours of that objective function (indifference curves for utility or isoprofits for profit) imply that there is a marginal rate of substitution.
- ▶ The marginal rate of substitution is the negative of the slope of the contour.
- ▶ In year 1, when you did this with utilities, this was:
$$mrs_1(x, y) = -\frac{dy}{dx} = \frac{\text{marginal utility of } x}{\text{marginal utility of } y} \text{ for indifference curves.}$$
- ▶ Where the marginal utilities were $u_x = \frac{\partial u}{\partial x}$ and $u_y = \frac{\partial u}{\partial y}$.
- ▶ How would this look here in Bertrand competition?

Question 4: Addendum - Reminder about the mrs (part 2)

- ▶ In the case of Bertrand Competition in (p_1, p_2) space,

$$mrs_1(p_1, p_2) = -\frac{dp_2}{dp_1} = \frac{\frac{\partial \pi_1}{\partial p_1}}{\frac{\partial \pi_1}{\partial p_2}}$$

- ▶ Notice that when we impose FOCs, $\frac{\partial \pi_1}{\partial p_1} = 0$. This is the numerator of the mrs and equal to zero.
- ▶ Hence, the slope of the isoprofit curve $\frac{dp_2}{dp_1} = 0$ at each point where the isoprofit curve intersects the BRF because at that point the owners of the firm are maximizing profits *given the price set by the other firm*.
- ▶ You should see a similar pattern with Cournot competition, except the isoprofit curves move in a different direction as profit increases. (Task: Can you explain why this is?)