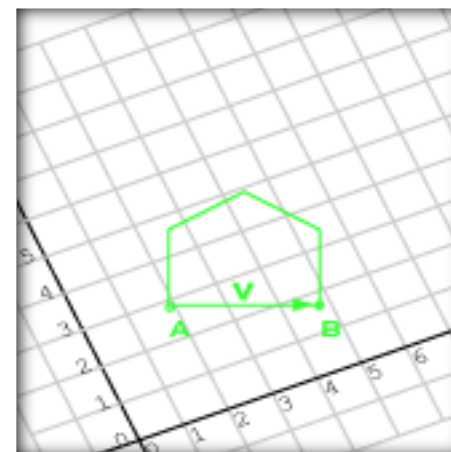
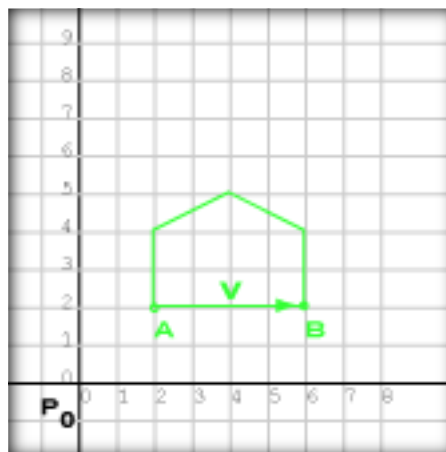
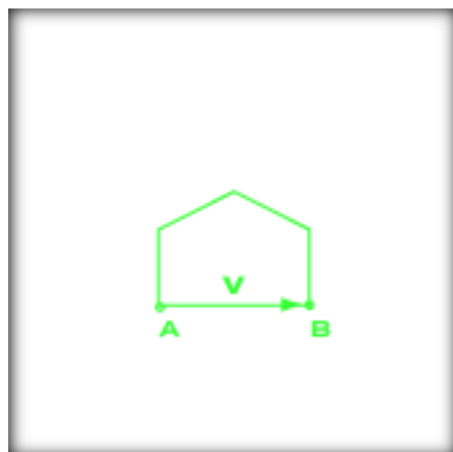


# **Računalniška grafika**

vektorji in matrike

# točke

matrična predstavitev



$$\begin{bmatrix} 2,9 \\ -4,6 \\ 0,0 \end{bmatrix}$$

**STOLPČNA MATRIKA**

$$[12,5 \quad -9,43]$$

**VRSTIČNA MATRIKA**

$$[1,3 \quad -4,1 \quad 0,0]^T$$

**TRANSPONIRANJE**

$$\begin{bmatrix} 2,9 \\ -4,6 \\ 0,0 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 2,9 \\ 0,0 \\ -4,6 \end{bmatrix}$$

**ENAKOST**

*“The devil is in  
the details”*

$$\begin{bmatrix} 2 & 4 & 12 & -1 \end{bmatrix}^T = \begin{bmatrix} 2 & 4 & 12 & -1 \end{bmatrix}^T$$

$$\begin{bmatrix} 2 & 4 & 12 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 12 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 12 & -1 \end{bmatrix} \neq \begin{bmatrix} 2 & 4 & 12 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 12 & -1 \end{bmatrix}^T \neq \begin{bmatrix} 2 & 4 & 12 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 12 & -1 \end{bmatrix}^T \neq \begin{bmatrix} 2 & 4 & 12 \end{bmatrix}^T$$

**ENAKOST**

$$\mathbf{a} = \begin{bmatrix} 2,9 \\ 0,0 \\ -4,6 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}$$

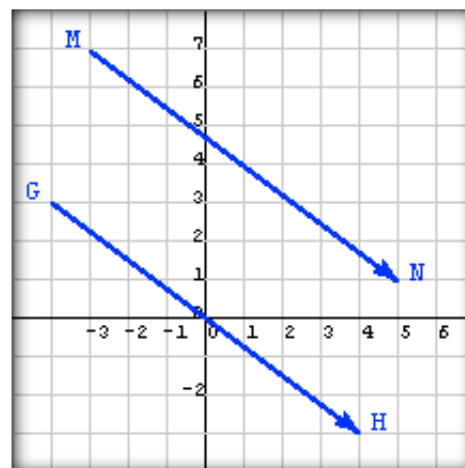
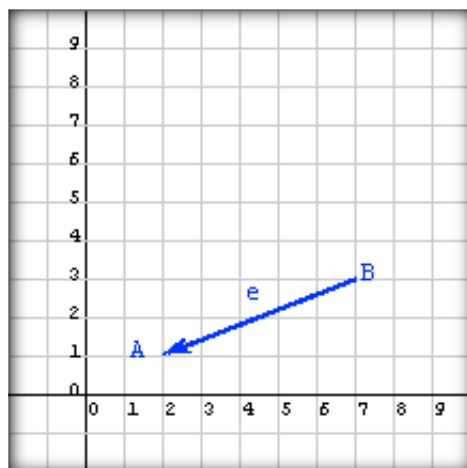
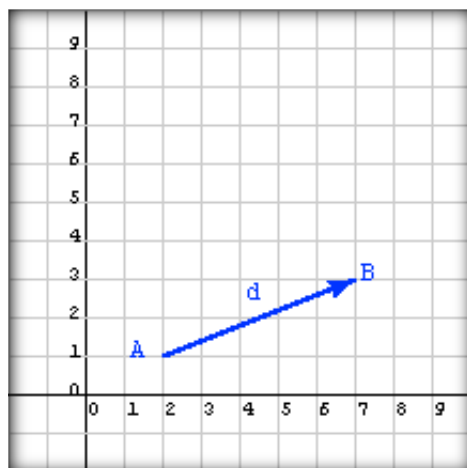
**NOTACIJA**

$$\mathbf{r} = \begin{bmatrix} r_\theta & r_1 \end{bmatrix}^\top$$



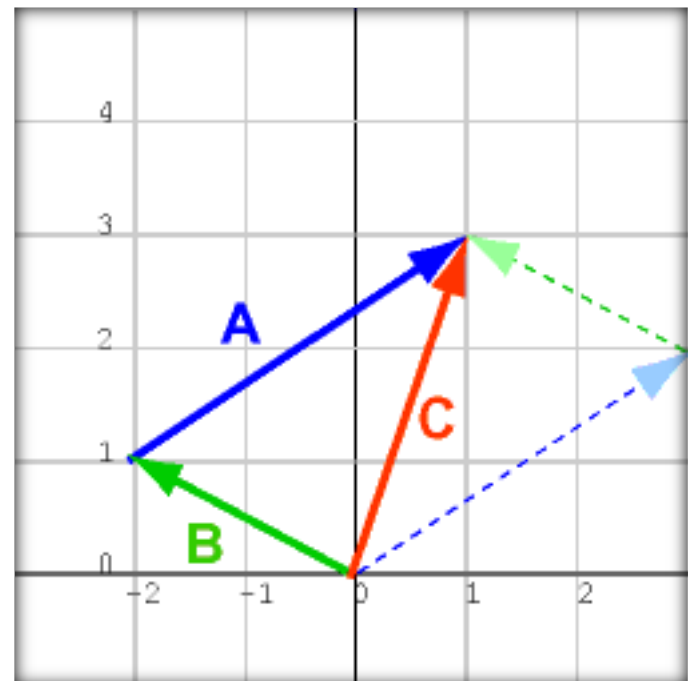
# vektorji

premik iz točke v točko  
vektorji nimajo lokacije



# vektori

geometrijsko seštevanje



$$\begin{bmatrix} 2,9 \\ 0 \\ -4,6 \end{bmatrix} + \begin{bmatrix} 1,1 \\ 4 \\ 8,6 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$$

$$\mathbf{a} + \mathbf{b} = \mathbf{c} \Leftrightarrow c_i = a_i + b_i$$

## SEŠTEVANJE MATRIK

$$\begin{bmatrix} 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 \end{bmatrix}$$

$$a + 0 = 0 + a = a$$

**ENOTA ZA SEŠTEVANJE**

$$\begin{bmatrix} 10 & 2 \end{bmatrix}^T - \begin{bmatrix} 1 & 3 \end{bmatrix}^T = \begin{bmatrix} 9 & -1 \end{bmatrix}^T$$

$$\mathbf{a} - \mathbf{b} = \mathbf{c} \Leftrightarrow c_i = a_i - b_i$$

## ODŠTEVANJE MATRIK

$$\begin{bmatrix} 2 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 5 \end{bmatrix} + \begin{bmatrix} -2 & -5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$\mathbf{a} - \mathbf{a} = \mathbf{a} + -\mathbf{a} = \mathbf{a} + (-1)\mathbf{a} = \mathbf{0}$$

**INVERZ ZA SEŠTEVANJE**

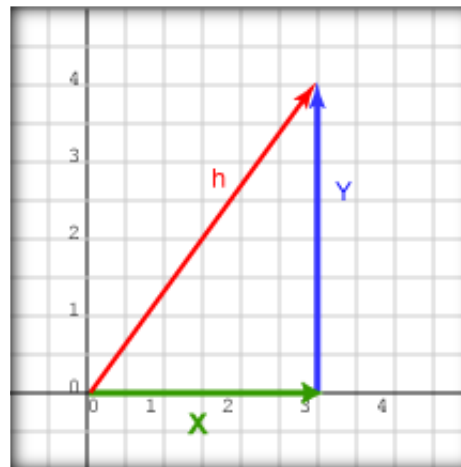
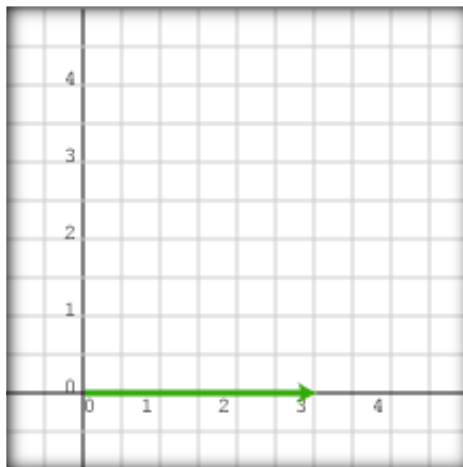
$$3 * \begin{bmatrix} -1,3 \\ u \\ v \end{bmatrix} = \begin{bmatrix} -3,9 \\ 3 * u \\ 3 * v \end{bmatrix}$$

$$\alpha \mathbf{a} = \mathbf{b} \Leftrightarrow b_i = \alpha a_i$$

## MNOŽENJE S SKALARJEM

# vektorji

dolžina vektorja



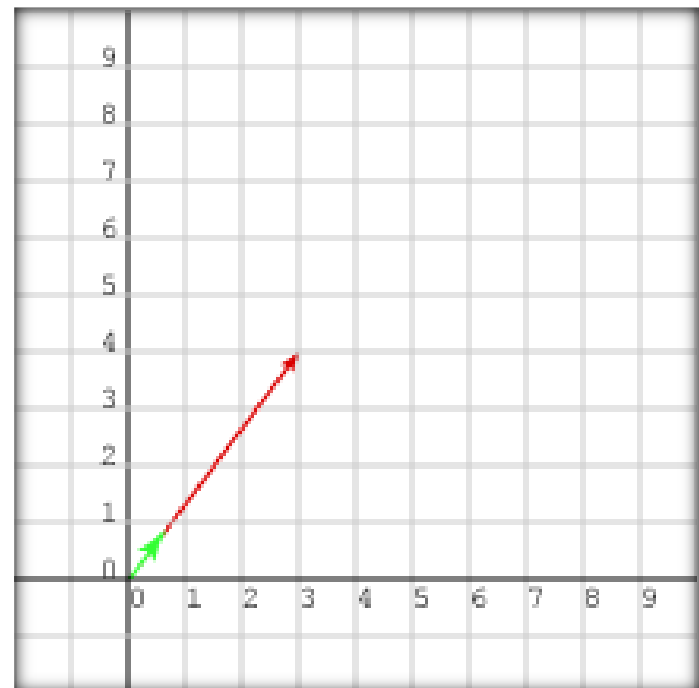


$$\mathbf{h} = \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \|\mathbf{h}\| = \sqrt{x^2 + y^2}$$
$$\|\mathbf{a}\| = \sqrt{\sum_{i=1}^n a_i^2}$$

**NORMA**

# vektorji

enotski vektor



$$\mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \Rightarrow \mathbf{v}_u = \mathbf{v} / \|\mathbf{v}\| = \begin{bmatrix} v_x / \|\mathbf{v}\| \\ v_y / \|\mathbf{v}\| \\ v_z / \|\mathbf{v}\| \end{bmatrix}$$

**NORMALIZACIJA**

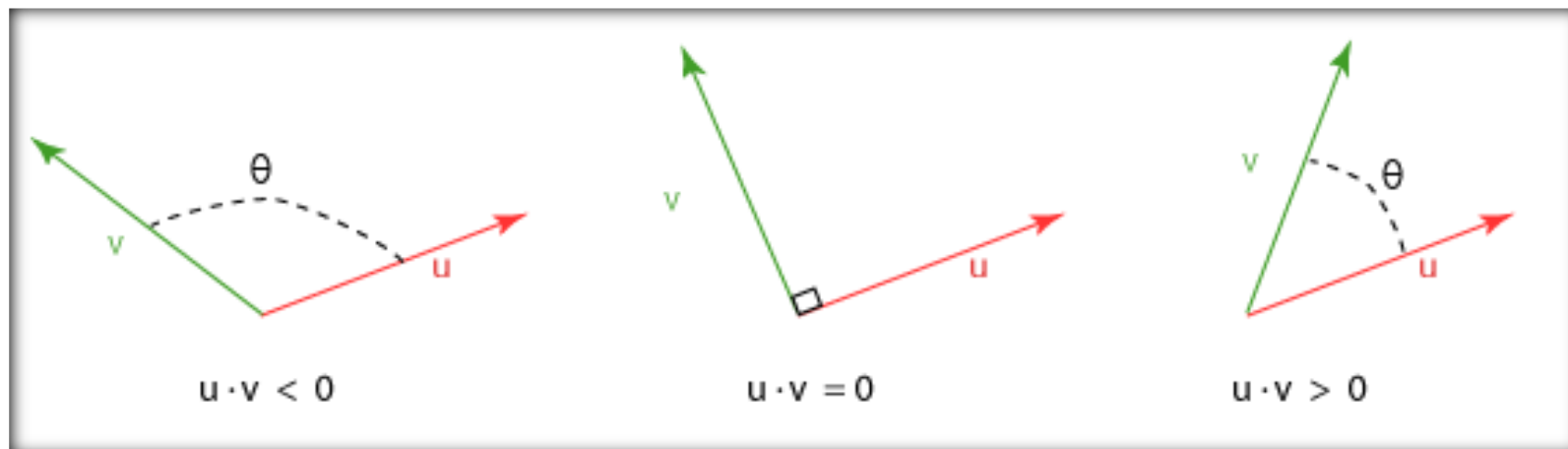
$$\mathbf{u} = \begin{bmatrix} u_0 \\ u_1 \\ u_2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} v_0 \\ v_1 \\ v_2 \end{bmatrix} \Rightarrow \mathbf{u} \cdot \mathbf{v} = u_0 v_0 + u_1 v_1 + u_2 v_2$$

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos \alpha$$

**SKALARNI PRODUKT**

# vektorji

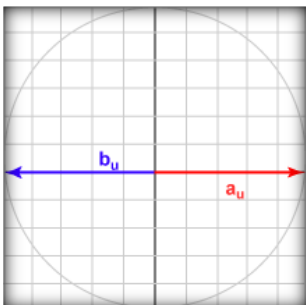
skalarni produkt in ortogonalnost



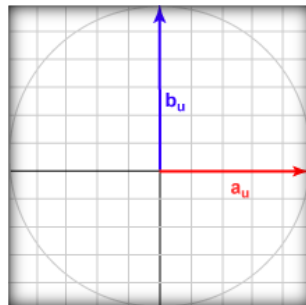
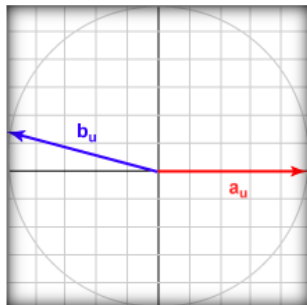
# vektorji

skalarni produkt enotskih vektorjev

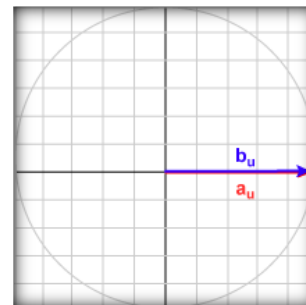
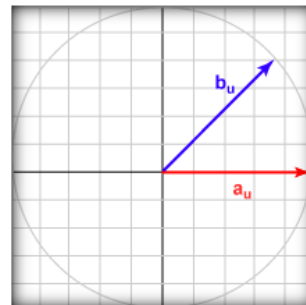
$$0 < \mathbf{a}_u \cdot \mathbf{b}_u < -1$$



$$\mathbf{a}_u \cdot \mathbf{b}_u = -1$$



$$\mathbf{a}_u \cdot \mathbf{b}_u = 0$$



$$\mathbf{a}_u \cdot \mathbf{b}_u = 1$$

## skalarni produkt

komutativnost

distributivnost za seštevanje

homogenost za množenje s skalarjem

asociativnost nedefinirana operacija

---

skalarni produkt ortogonalnih vektorjev

skalarni produkt vektorja 0

skalarni produkt z vektorjem 0

norma

$$\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$$

$$\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$$

$$(a\mathbf{u}) \cdot \mathbf{v} = \mathbf{u} \cdot (a\mathbf{v}) = a(\mathbf{u} \cdot \mathbf{v})$$

~~$$(\mathbf{u} \cdot \mathbf{v}) \cdot \mathbf{w} \neq \mathbf{u} \cdot (\mathbf{v} \cdot \mathbf{w})$$~~

$$\mathbf{u} \perp \mathbf{v} \Leftrightarrow \mathbf{u} \cdot \mathbf{v} = 0$$

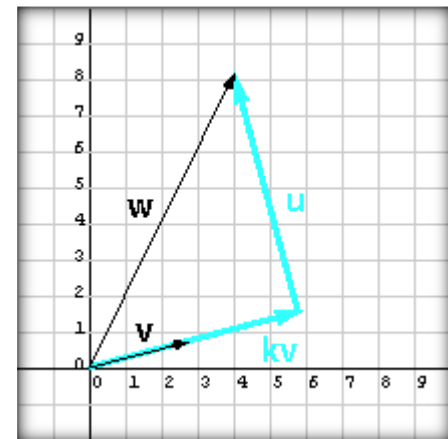
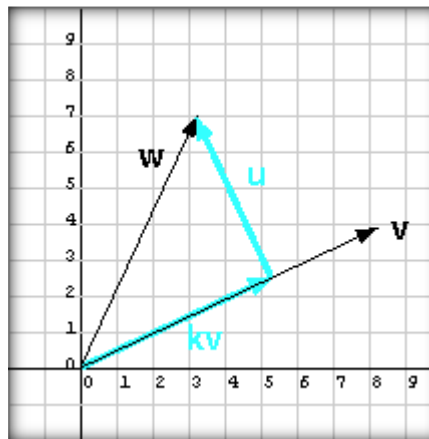
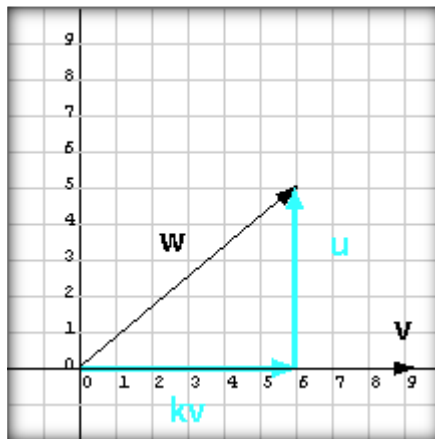
$$\mathbf{u} \cdot \mathbf{0} = \mathbf{0} \cdot \mathbf{v} = 0$$

$$\mathbf{0} \cdot \mathbf{0} = 0$$

$$\mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$$

# vektori

linearna neodvisnost  
projekcija vektorja na vektor



$$kv = \|\mathbf{w}\|(\mathbf{w}_u \cdot \mathbf{v}_u)\mathbf{v}_u$$



## vektorji

izračunaj dolžine vektorjev

izračunaj enotske vektorje

izračunaj kosinus kota med vektorji

združi v projekcijo

izračunaj ortogonalni vektor

$$\|\mathbf{w}\| = \mathbf{w} \cdot \mathbf{w}, \quad \|\mathbf{v}\| = \mathbf{v} \cdot \mathbf{v}$$

$$\mathbf{w}_u = \mathbf{w} / \|\mathbf{w}\|, \quad \mathbf{v}_u = \mathbf{v} / \|\mathbf{v}\|$$

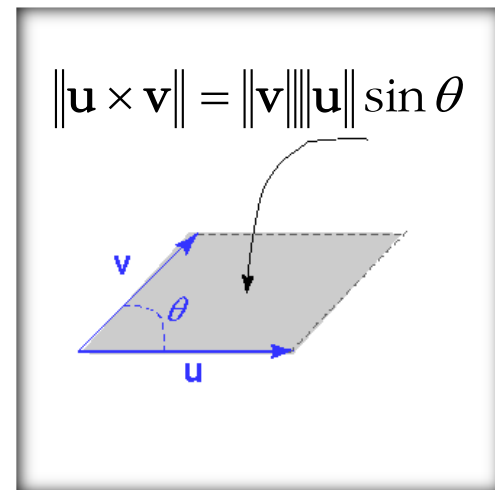
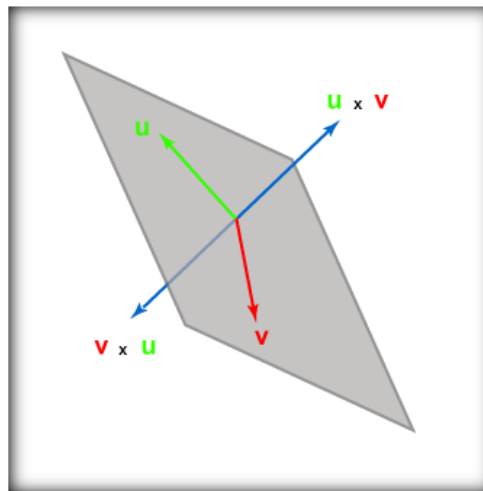
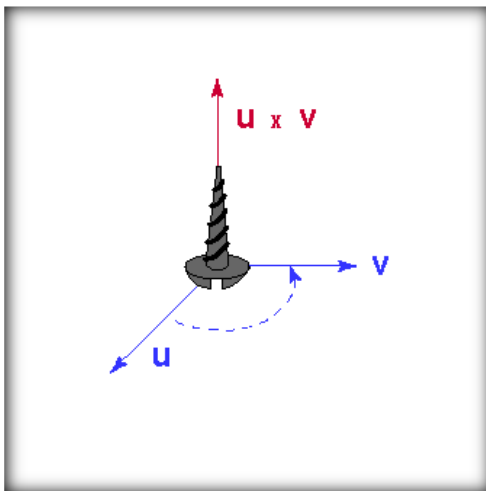
$$\cos \alpha = \mathbf{w}_u \cdot \mathbf{v}_u$$

$$k\mathbf{v} = \|\mathbf{w}\|(\mathbf{w}_u \cdot \mathbf{v}_u)\mathbf{v}_u$$

$$\mathbf{u} = \mathbf{w} - k\mathbf{v}$$

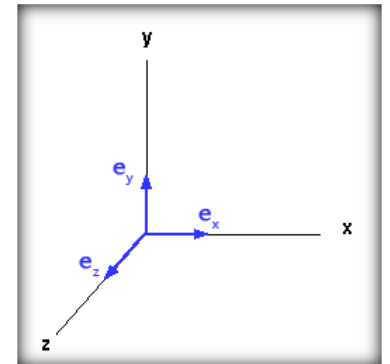
# vektorji

vektorski produkt



$$\mathbf{u} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}, \mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \Rightarrow \mathbf{u} \times \mathbf{v} = \begin{bmatrix} u_y v_z - u_z v_y \\ u_z v_x - u_x v_z \\ u_x v_y - u_y v_x \end{bmatrix}$$

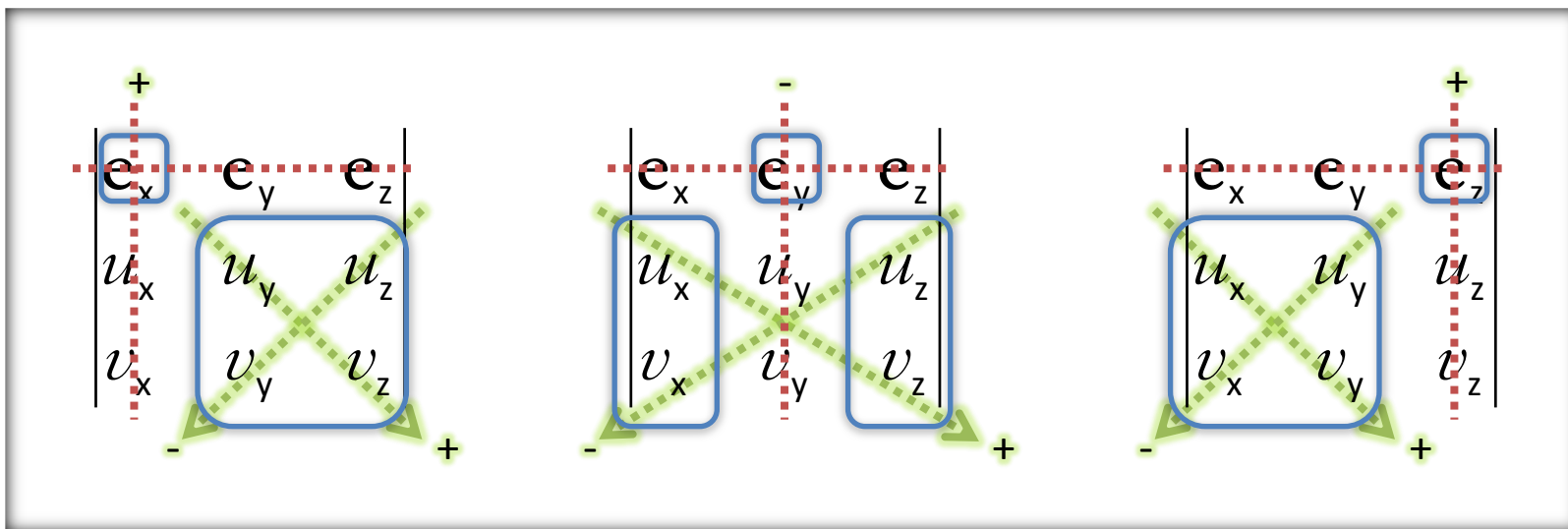
## VEKTORSKI PRODUKT



## vektorski produkt

miselni vzorec

$$\begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix}$$



$$\mathbf{u} \times \mathbf{v} = \mathbf{e}_x(u_y v_z - u_z v_y) - \mathbf{e}_y(u_x v_z - u_z v_x) + \mathbf{e}_z(u_x v_y - u_y v_x)$$

## vektorski produkt

antikomutativnost

distributivnost za seštevanje

homogenost za množenje s skalarjem

asociativnost ne velja

---

vektorski produkt kolinearnih vektorjev

vektorski produkt vektorja 0

vektorski produkt z vektorjem 0

vektorski produkt in koordinatne osi

desno pravilo

$$\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$$

$$\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w}$$

$$(a\mathbf{u}) \times \mathbf{v} = \mathbf{u} \times (a\mathbf{v}) = a(\mathbf{u} \times \mathbf{v})$$

$$(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} \neq \mathbf{u} \times (\mathbf{v} \times \mathbf{w})$$

$$\mathbf{u} \parallel \mathbf{v} \Leftrightarrow \mathbf{u} \times \mathbf{v} = \mathbf{0}$$

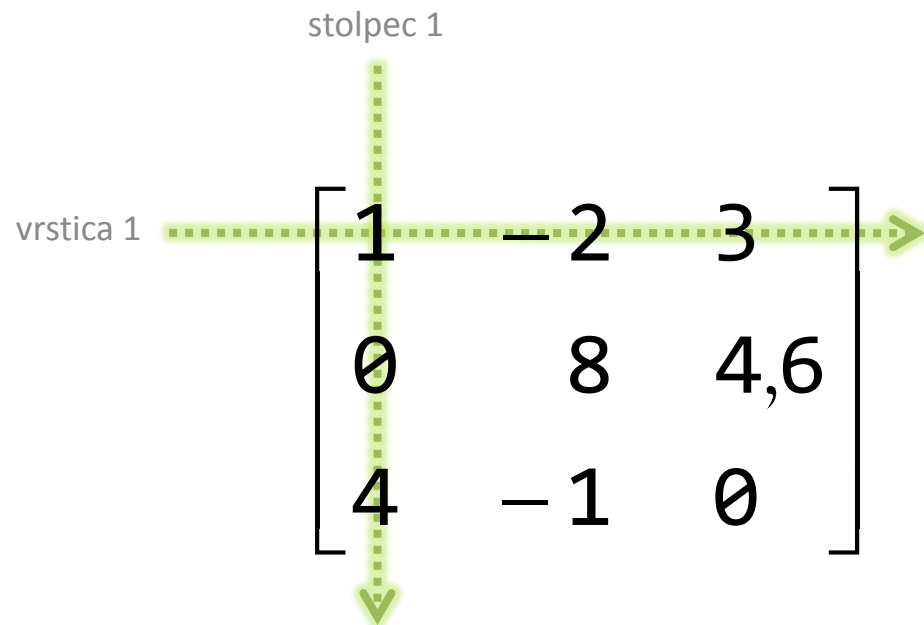
$$\mathbf{u} \times \mathbf{0} = \mathbf{0} \times \mathbf{v} = \mathbf{0}$$

$$\mathbf{0} \times \mathbf{0} = \mathbf{0}$$

$$\mathbf{e}_x \times \mathbf{e}_y = \mathbf{e}_z, \quad \mathbf{e}_y \times \mathbf{e}_z = \mathbf{e}_x, \quad \mathbf{e}_z \times \mathbf{e}_x = \mathbf{e}_y$$

stolpec 1

vrstica 1

$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 8 & 4,6 \\ 4 & -1 & 0 \end{bmatrix}$$


# SPLOŠNA MATRIKA

$$\mathbf{A}_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

**NOTACIJA**

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3 \times 2} \neq \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2 \times 3}$$

**ENAKOST**



$$\begin{bmatrix} 2 & 0 \\ -1 & 2 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ -2 & 4 \\ 5 & 4 \end{bmatrix}$$

$$\mathbf{A} + \mathbf{B} = \mathbf{C} \Leftrightarrow c_{ij} = a_{ij} + b_{ij}$$

## SEŠTEVANJE MATRIK

## seštevanje matrik

komutativnost

asociativnost

enota za seštevanje

inverz za seštevanje

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$

$$(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C})$$

$$\mathbf{A} + \mathbf{0} = \mathbf{0} + \mathbf{A} = \mathbf{A}$$

$$\mathbf{A} - \mathbf{A} = \mathbf{A} + (-\mathbf{1})\mathbf{A} = \mathbf{0}$$

$$3 * \begin{bmatrix} 2 & 0 \\ -1 & 2 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ -3 & 6 \\ 9 & 15 \end{bmatrix}$$

$$\alpha \mathbf{A} = \mathbf{B} \Leftrightarrow b_{ij} = \alpha a_{ij}$$

**MNOŽENJE MATRIK S SKALARJEM**

## množenje matrike s skalarjem

distributivnost seštevanja matrik in množenja s skalarjem

distributivnost seštevanja skalarjev in množenja matrike s skalarjem

množenje s skalarjem -1

$$\alpha(\mathbf{A} + \mathbf{B}) = \alpha\mathbf{A} + \alpha\mathbf{B}$$

$$(\alpha + \beta)\mathbf{A} = \alpha\mathbf{A} + \beta\mathbf{A}$$

$$(\alpha\beta)\mathbf{A} = \alpha(\beta\mathbf{A})$$

$$(-1)\mathbf{A} = -\mathbf{A}$$

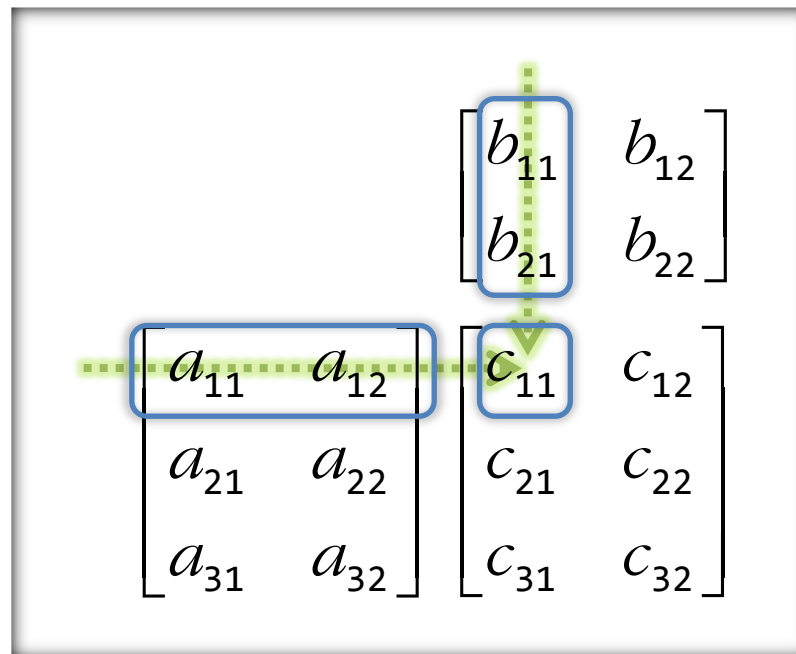
$$\begin{bmatrix} 2 & 0 \\ -1 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 8 & 10 \\ -3 & -3 \\ 11 & 13 \end{bmatrix}$$

$$\mathbf{A}_{n \times m} \mathbf{B}_{m \times p} = \mathbf{C}_{n \times p} \Leftrightarrow c_{ij} = \sum_{k=1}^m a_{ik} b_{kj}$$

# MNOŽENJE MATRIK

# množenje matrik

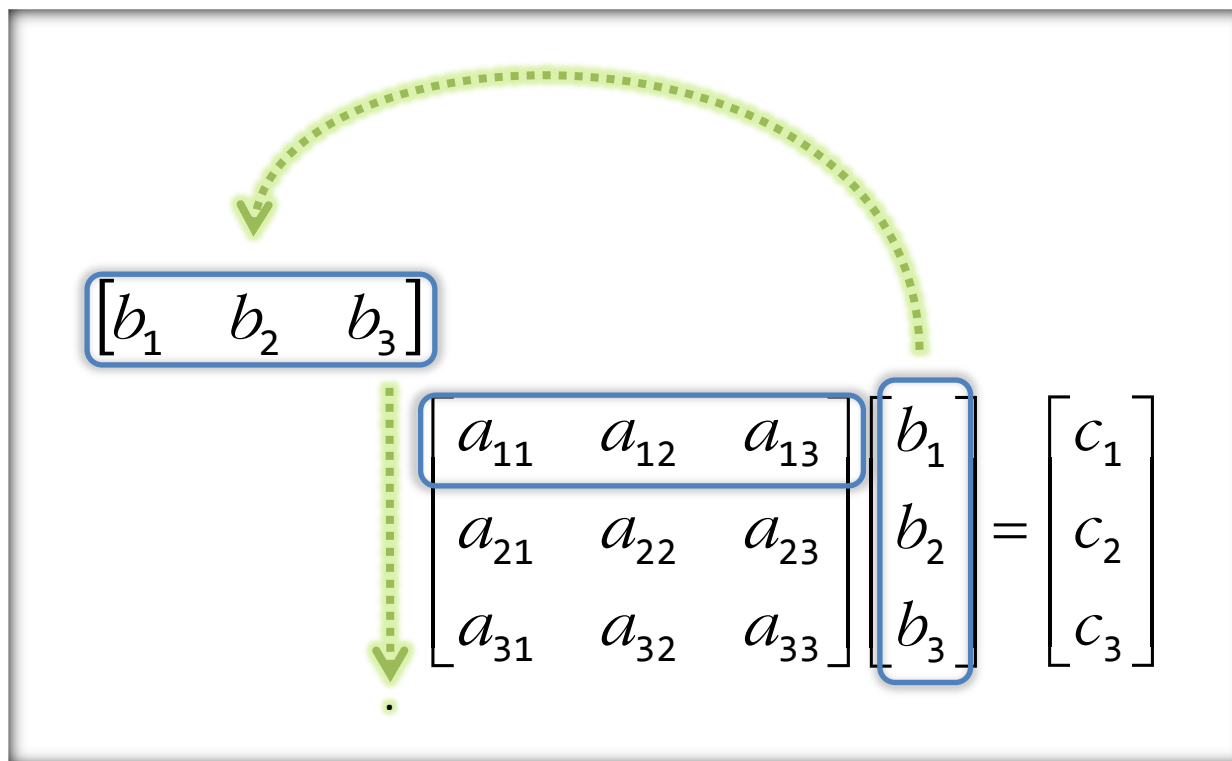
miselni vzorec za množenje splošnih matrik



$$c_{11} = a_{11}b_{11} + a_{12}b_{21}$$

## množenje matrik

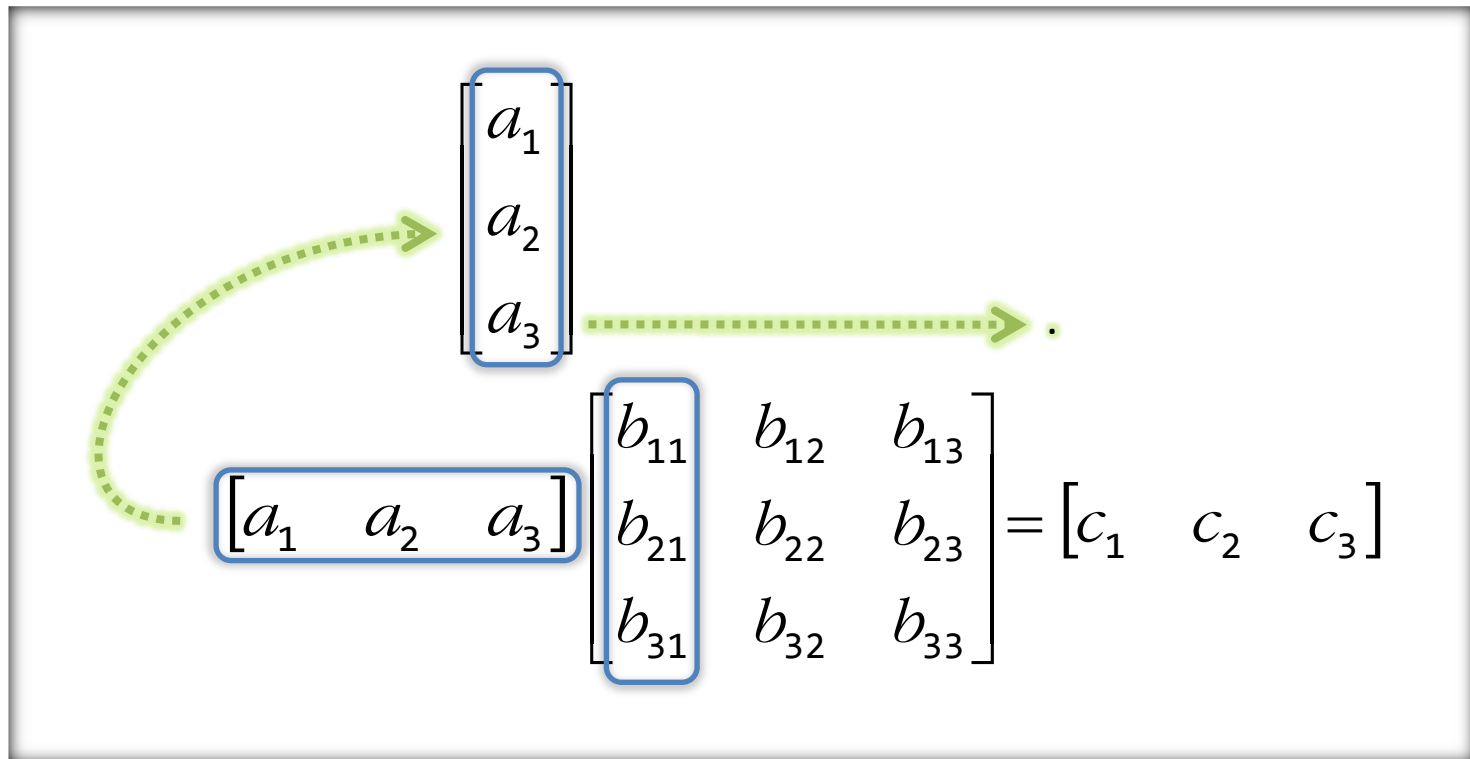
miselni vzorec za množenje s stolpčno matriko  
skalarni produkt vrstičnih matrik



$$\mathbf{c} = \mathbf{A}\mathbf{b}, \mathbf{a}_k = [a_{kj}] \Rightarrow c_k = \mathbf{a}_k \cdot \mathbf{b}$$

## množenje matrik

miselni vzorec za množenje vrstične matrike  
skalarni produkt stolpčnih matrik



$$\mathbf{aB} = \mathbf{c}, \mathbf{b}_k = [b_{jk}] \Rightarrow c_k = \mathbf{a} \cdot \mathbf{b}_k$$



## množenje matrik

komutativnost v splošnem ne velja

asociativnost

distributivnost za seštevanje

homogenost za množenje s skalarjem

---

množenje s skalarjem 0

množenje z matriko 0

$$\mathbf{AB} \neq \mathbf{BA}$$

$$(\mathbf{AB})\mathbf{C} = \mathbf{A}(\mathbf{BC})$$

$$\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC}$$

$$(\mathbf{A} + \mathbf{B})\mathbf{C} = \mathbf{AC} + \mathbf{BC}$$

$$(\alpha\mathbf{A})\mathbf{B} = \mathbf{A}(\alpha\mathbf{B}) = \alpha(\mathbf{AB})$$

$$\mathbf{0A} = \mathbf{0}$$

$$\mathbf{A0} = \mathbf{0A} = \mathbf{0}$$

$$\mathbf{I}_n = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}_{n \times n}$$

**ENOTA ZA MNOŽENJE**

$$AB = BA = I \Leftrightarrow B = A^{-1}$$

$$(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$$

**INVERZ ZA MNOŽENJE**

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\mathbf{A}^T = \mathbf{B} \Leftrightarrow b_{ij} = a_{ji}$$

**TRANSPONIRANJE**

# transponiranje matrik

lastnosti

$$\left(\mathbf{A}^{\top}\right)^{\top} = \mathbf{A}$$

$$\left(\alpha \mathbf{A}\right)^{\top} = \alpha \mathbf{A}^{\top}$$

$$\left(\mathbf{A} + \mathbf{B}\right)^{\top} = \mathbf{A}^{\top} + \mathbf{B}^{\top}$$

$$\left(\mathbf{ABC}\right)^{\top} = \mathbf{C}^{\top} \mathbf{B}^{\top} \mathbf{A}^{\top}$$

$$\left(\mathbf{A}^{-1}\right)^{\top} = \left(\mathbf{A}^{\top}\right)^{-1}$$

## vektori, točke, skalarji

točka – točka = vektor

točka + vektor = točka

*odmik, premik*

vektor + vektor = vektor

*seštevanje*

skalar \* vektor = vektor

*razteg*

vektor · vektor = skalar

*norma (dolžina), kot med vektorjema, pravokotnost*

vektor × vektor = vektor

*normala, usmerjenost, koordinatni sistem, vzporednost, površina*

<http://chortle.ccsu.edu/VectorLessons/vectorIndex.html>

<http://www.maths.surrey.ac.uk/explore/emmaspages/index.html>

<http://goo.gl/4gw7X>, <http://goo.gl/gUHp9>, <http://goo.gl/PjQfmZ>

Lengyel, *Mathematics for 3D Game Programming & Computer Graphics*, **ch 1-2**

Dunn, Parberry, *3D Math Primer for Graphics and Game Development*, **ch 2-5,7**

Akenine-Möller, Haines, Hoffman, *Real-Time Rendering*, 3rd Ed., **apx A**

Angel, *Interactive Computer Graphics*, 5th Ed., **apx A-B**

Parent, *Computer Animation Algorithms and Techniques*, **apx B.1-B.2**

dodatna literatura

# DO PRIHODNJIČ