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International Journal of Control

ISSN: 0020-7179 (Print) 1366-5820 (Online) Journal homepage: http://www.tandfonline.com/loi/tcon20

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To cite this article: Abhinav Sinha & Rajiv Kumar Mishra (2018): Consensus in first order nonlinear heterogeneous multi-agent systems with event-based sliding mode control, International Journal of Control, DOI: 10.1080/00207179.2018.1531147

To link to this article: https://doi.org/10.1080/00207179.2018.1531147

Accepted author version posted online: 01 Oct 2018.
Published online: 10 Oct 2018.

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INTERNATIONAL JOURNAL OF CONTROL
https://doi.org/10.1080/00207179.2018.1531147

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Consensus in first order nonlinear heterogeneous multi-agent systems with event-based sliding mode control

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ABSTRACT
This article is aimed at solving leader-following consensus problem with an event-based sliding mode controller. The proposed control technique is partitioned into two parts – a finite time consensus problem and an event-based control mechanism. Leader-following heterogeneous multi-agents of first order having inherent nonlinear dynamics have been analysed to demonstrate efficacy and robustness of the proposed controller. In the first part, states of the follower agents have been attempted to be kept in consistence with those of the virtual leader using sliding mode control. In the second part, an event-based implementation of the control law has been incorporated to minimise the computational load on, and energy expenditure of the computational device equipped with the agents, while ensuring that the desired closed-loop performance of the system is not compromised. The advantage of using such a scheme, i.e. an event-based sliding mode controller, lies in the robustness capabilities of sliding mode controller and reduced computational expense of event-based mechanism. Numerical simulations and mathematical foundations ascertain the effectuality of the controller proposed herein.

ARTICLE HISTORY
Received 22 July 2017
Accepted 25 September 2018

KEYWORDS
Heterogeneous multi-agent systems (MAS); event-triggered leader–follower tracking control; event-triggered sliding modes; inter-event execution time; formation keeping; consensus

1. Introduction

Over past years, research in multi-agent systems (MAS) has caught interest and attention of diverse group of researchers around the globe from control and allied sciences fraternity in view of solving many engineering and non-engineering problems, some of which include attitude alignment of spacecraft, odour source localisation (Sinha, Kumar, Kaur, & Bhondekar, 2018), control of unmanned aerial vehicles (UAV) for rescue operations, intelligent transportation systems, formation control of robots, and distributed computing and distributed sensor networks. In a networked system, such as multi-agent systems or large-scale interconnected systems, there are many sub-systems. Even a miniscule perturbation in a single agent can affect the entire network adversely. Decentralised control approaches prove to be effective under such scenarios. A high degree of autonomy is achievable if every agent can be controlled via robust design techniques, and at the same time, an efficient coordination control is also possible. Multiple autonomous agents form a class of dynamical system where they interact with each other over a shared network to reach a common goal. Compared to a single autonomous agent, MAS provides relatively higher redundancy and improved operational efficiency. A particularly challenging problem in this area is commonly known as consensus tracking, i.e. consensus with a virtual leader (agent in swarm whose dynamics has to be imitated by other agents). A group of followers attempt to track the states of the virtual leader in finite time via local interactions. This group should corner the ability to interact with neighbours and score the task together as a single entity. When every agent in the group has come to a common ground over some state-dependent quantity of interest, we say that consensus has been achieved. A consensus algorithm or protocol, thus, becomes an interaction rule specifying the exchange of information among agents on the shared network (Olfati-Saber, Fax, & Murray, 2007). Another related and challenging problem is formation control of agents, i.e. stabilising the relative distance or velocity amongst agents to a certain value (sometimes also referred as formation keeping).

Observations in nature such as flock of birds, schools of fish, and swarm of insects have led to the development of bio-inspired algorithms for a collective behaviour of MAS. The first flocking behaviour was simulated by Reynolds (1987). Subsequently, various contributions have been marked in this field. In a leader-following consensus problem, there is a leader and some informed agents which can receive the leader's information directly, along with the other uninformed agents which only follow their neighbours, and the goal is to match states of the followers with those of the leader. Ren, Beard, and Atkins (2007) studied the asymptotic consensus problem for MAS having single integrator dynamics. Authors in Ren and Beard (2005) proposed that asymptotic consensus can be obtained in MAS having first order dynamics only if the network contains a directed spanning tree. In practical applications, fast convergence rate is an important criterion for evaluating consensus protocol. The convergence rate is dependent on algebraic connectivity, i.e. the second smallest eigenvalue of the interacting Laplacian matrix decides the convergence rate. It has been proposed in Zhou and Wang (2009) that the convergence speed can be improved by maximising the

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algebraic connectivity of interaction graph, but the consensus can be achieved only asymptotically. However, any practical applications require consensus to be reached in finite time. In Hui, Haddad, and Bhat (2008), finite-time consensus problem for networked systems has been addressed, wherein the agents are modelled by continuous time integrators. In Wang and Xiao (2010), to solve consensus problem for a MAS with connected topology, some protocol has been proposed which requires sufficiently large time interval. A finite-time consensus protocol for agents modelled as double integrators has been studied in Li, Du, and Lin (2011). To achieve finite-time consensus in first order MAS, authors in Chen, Lewis, and Xie (2011) have utilised a binary consensus protocol. Consensus of linear MAS with second order integrator dynamics was proposed in Ren (2008). A combined study of consensus in first and second order MAS with integrator dynamics has been put forward in Ferrari-Trecate, Galbusera, Marciandi, and Scattolini (2009) using model predictive control. Authors in Li, Duan, and Huang (2009) proposed an observer-based algorithm for the problem of consensus tracking in MAS with general linear dynamics. Consensus in a leader–follower MAS having linear dynamics and switching topology has been studied in Su and Huang (2011, 2012). Similarly, a group consensus control for double-integrator dynamic MAS with fixed communication topology has been studied in Feng, Xu, and Zhang (2012).

It is worthy to note that in most of the studies, either the dynamics is of pure integrator type or a linear one. In practice, this assumption may not suffice due to ubiquitous complex nonlinearity. Moreover, authors explicitly assumed that the set of agents is homogeneous, i.e., agents are similar and exhibit same dynamics. Truly homogeneous agents are very difficult to obtain. In many cases, it has been observed that even truly homogeneous agents inevitably drift towards heterogeneity over time and continued operation. Authors in Wang, Wu, Huang, and Wang (2008) proposed that heterogeneous multi-robot systems with agents having different features are more applicable to a certain problem as opposed to their homogeneous counterparts. Thus, it is of more importance to study the consensus protocols in heterogeneous set of agents. A nonlinear cooperative control for consensus in nonlinear and heterogeneous MAS has been designed in Qu, Chunyu, and Wang (2007). Cooperative control of nonlinear and non-identical MAS using an input–output feedback linearisation technique has been demonstrated in Bidram, Lewis, Davoudi, and Guerrero (2013). Consensus problem of heterogeneous MAS with directed network topologies has been analysed in Meiling, Yong, and Rui (2017). Distributed consensus problem of a heterogeneous MAS with fixed and switching topologies has been studied in Zheng and Wang (2012). A group consensus control protocol for heterogeneous MAS with fixed and switching topologies has been put forward in Wen, Huang, Wang, Chen, and Peng (2016). The global synchronisation criteria for heterogeneous complex networks with geometric characterisations have been studied in Liang, She, Wang, Chen, and Wang (2017)

In a networked system, a small perturbation in any agent can adversely affect the entire interconnected network. The dynamic consensus tracking for directed network topology using proportional-derivative algorithm has been proposed in Douligeris and Develekos (2010). Nonlinear dynamics of MAS has been approximated using adaptive neural network (Liu, Tong, Wang, Li, & Chen, 2011) and fuzzy logic (Liu, Tong, & Chen, 2013) in early works and consensus has been investigated. The leader–follower consensus control has also been proposed where dynamics of a virtual leader is controlled and followers are made to follow desired trajectory (Lu, Lu, Chen, & Lu, 2013). However, these model-free techniques become incommodious coping with rapidly changing fast dynamics of some MAS, and are not insensitive to external perturbations. Recently, robust control techniques have been widely incorporated into the consensus control of MAS due to their better disturbance rejection potencies. Distributed H_∞ consensus and control have been investigated in Lin and Jia (2008). Sliding mode control is widely used robust control scheme which completely rejects the matched uncertainty and bounded disturbances. The consensus problem of a second order MAS with directed communication topology, as opposed to undirected communication topology, has also been addressed in literature using terminal sliding mode control (Zou, Kumar, & Hou, 2013). The sliding mode controllers are applied for finite-time distributed tracking of second order MAS in Ren and Chen (2015). Authors in Ren and Chen (2015) proposed distributed asymptotic consensus controller and a finite-time consensus control protocol for leader–follower second order nonlinear MAS with directed communication topology based on sliding mode. An observer-based higher order sliding mode controller for the consensus of nonlinear MAS has been proposed in Ghayoomi and Ghasemi (2017). In Zhang, Yang, Zhao, and Wen (2013), a sliding mode controller has been designed to solve the finite-time tracking problem for systems with external disturbances and Lipschitz-type inherent dynamics. Yu and Long proposed integral sliding mode for finite-time consensus of second order MAS with disturbances (Yu & Long, 2015). An adaptive finite-time leaderless consensus control of MAS consisting of nonlinear mechanical systems with parametric uncertainties has been studied in Huang, Wen, Wang, and Song (2015) under an undirected graph topology. A backstepping-based fully distributed adaptive consensus tracking control scheme for a class of higher order nonlinear systems with unknown control directions has been addressed in Huang, Song, Wang, Wen, and Li (2018) where communications among the agents have been represented by a directed graph. Authors in Huang, Song, Wang, Wen, and Li (2017) investigated distributed adaptive leader-following consensus control for higher order nonlinear MAS with time-varying reference trajectory under directed topology subjected to mismatched unknown parameters and uncertain external disturbances. In Huang et al. (2017), the local estimators have been used for the bounds of the reference trajectory and a backstepping-based smooth distributed adaptive control protocol has been proposed.

With the advancement of embedded systems, almost all the controllers are realised in sampled data approach owing to easy implementation properties. Therefore, the practical approach to solve leader follower consensus problem is to use sampled data control especially in bandwidth and energy constrained environments. The traditional sampled data control system considers periodic update of the controllers even after achieving control objective. The measurements are sampled and control is updated periodically even if the system may tolerate fluctuations in some allowable range. This results in wastage of significant computational and communication resources. An efficient way to reduce communication and computational burden is to use event-triggering scheme. The samples are obtained only when an event is triggered. The objective of the event-triggering scheme is to sample and update controller only when the local measurement error crosses a predefined threshold while ensuring satisfactory closed-loop performance of the system. An event-triggering scheme for linear quadratic control has been proposed in Antunes and Khashooei (2017), in which the system model was linear and performance was measured by an average quadratic cost. If the Euclidean norm of the error between the state and a state prediction exceeded a threshold then an event was triggered to update the controller. An event-driven consensus problem of linear MAS under fixed topology has been investigated in Zhu, Jiang, and Feng (2014). The problem of event-triggered leader–follower consensus for a second order MAS with a fixed topology as well as a switching topology has been studied in Li, Liao, Huang, and Zhu (2015). Authors in Guo and Dimarogonas (2013) proposed event-triggered nonlinear consensus protocol for homogeneous MAS having single integrator dynamics. Garcia, Cao, and Casbeer (2017) investigated event-based consensus of leader–follower heterogeneous MAS with linear dynamics. In Antunes and Khashooei (2017), the triggering rule is static and in Garcia et al. (2017), the triggering rule is time varying but not dynamic. It should be noted that the communication among follower agents in Rezaei and Menhaj (2017), Zhou, Yu, Sun, and Yu (2015), Zhu et al. (2014), Li et al. (2015), Huang et al. (2018), and Huang et al. (2017) are continuous, eventually leading to significant wastage of available resources, as well as the consensus protocol is also conservative.

Motivated by the studies presented above and aiming to design a robust controller that does not place a significant burden of computation and communication in a networked system, we have synthesised a controller on the archetype of sliding modes implemented in event-triggered fashion. The contributions of this work are threefold.

- Dynamics of the MAS in this study are general nonlinear, locally Lipschitz functions. When the nonlinear functions are zero, the dynamics reduce to those of pure integrator systems, and when the dynamics are linearised, linear MAS can be dealt with in the same methodology. Thus, this work generalises the work of designing finite-time robust consensus control of MAS.
- The proposed controller is an event-based controller designed on the archetype of sliding modes, utilising the inherent robustness of sliding modes under minimum computational and communication requirements while ensuring desirable closed-loop performance. The event triggering rule used in this work is novel and dynamic in nature as opposed to many static or time-varying triggering used in literature. Moreover, the system is *Zeno* free, i.e. sampling is admissible.
- The proposed controller is robust to exogenous perturbations that are matched and bounded. Formation keeping of MAS has also been demonstrated to aid the proposition made in this work.

The remainder of this manuscript is organised as follows. After introduction in Section 1, a threshold of discussion has been carried out in the preliminaries of Section 2. Dynamics of MAS and the problem formulation are presented in Section 3, followed by the synthesis of the proposed controller. Results of numerical simulation have been discussed in Section 5, and recommendations and outlook are provided in concluding remarks in Section 6.

2. Preliminaries

2.1 Spectral graph theory for MAS

A directed graph, also known as digraph (Chartrand, Lesniak, & Zhang, 2015), is represented here by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. $\mathcal{V}$ is the non-empty set that contains finite number of vertices or nodes (Bella, 1998; Deo, 1974) such that $\mathcal{V} = \{1, 2, \ldots, N\}$. $\mathcal{E}$ denotes edges that are directed and are represented as $\mathcal{E} = \{(i, j)\ \forall\ i, j \in \mathcal{V} \,\&\, i \neq j\}$. $\mathcal{A}$ is the weighted adjacency matrix such that $\mathcal{A} = a(i, j) \in \mathbb{R}^{\mathrm{N}\times\mathrm{N}}$. The existence of an edge (i, j) is only possible if and only if the vertex i receives the information supplied by the vertex j, i.e. $(i, j) \in \mathcal{E}$. Hence, i and j are referred to as neighbours. Let us consider a set $\mathcal{N}_i$ that contains labels of vertices that are neighbours of the vertex i. For the adjacency matrix $\mathcal{A}$, $a(i, j) \in \mathbb{R}^+ \cup \{0\}$. If $(i, j) \in \mathcal{E} \Rightarrow a(i, j) > 0$. If $(i, j) \notin \mathcal{E}$ or $i = j \Rightarrow a(i, j) = 0$. The Laplacian matrix $\mathcal{L}$ (Chartrand et al., 2015; Chung, 1997; Gross, Yellen, & Zhang, 2013) lies at the heart of the consensus problem and is given by $\mathcal{L} = \mathcal{D} - \mathcal{A}$ where $\mathcal{D}$ is a diagonal matrix, i.e. $\mathcal{D} = \mathrm{diag}(d_1, d_2, \ldots, d_n)$ whose entries are $d_i = \sum_{j=1}^{n} a(i, j)$. $\mathcal{D}$ is known as degree matrix in the notions of graph theory. A directed path from vertex j to vertex i defines a sequence comprising edges $(i, i_1), (i_1, i_2), \ldots, (i_l, j)$ with distinct vertices $i_k \in \mathcal{V}, k = 1, 2, 3, \ldots, l$. $\mathcal{B}$ is also a diagonal matrix with entries 1 or 0 and is commonly referred as incidence matrix. If there exists an edge between leader agent and any other agent, the entry is 1, otherwise the entry is 0. Furthermore, it can be inferred that the path between two distinct vertices is not uniquely determined. However, if a distinct node in $\mathcal{V}$ contains directed path to every other distinct node in $\mathcal{V}$, then the directed graph $\mathcal{G}$ is said to have a spanning tree. Physically, each agent in MAS is represented by a vertex or a node and the line of communication between any two agents is represented as a directed edge. The relationship between $\mathcal{G}$ and $\mathcal{V}$ establishes the following lemmas whose proofs can be found in Chung (1997) and thus, are omitted here.

Lemma 2.1: *Consider a directed graph $\mathcal{G}$ and its Laplacian matrix $\mathcal{L}$. The set of eigenvalues of $\mathcal{L}$ contains at least one zero eigenvalue. Other nonzero eigenvalues of $\mathcal{L}$ have positive real parts. $\mathcal{L}$ has a simple zero eigenvalue only when $\mathcal{G}$ has a spanning tree. Also, $\mathcal{G}$ is said to be balanced if the following criterion is met*:

$$\mathcal{L}\mathbf{1}_N = \mathbf{1}_N^{\mathrm{T}}\mathcal{L} = \mathbf{0}_N. \tag{1}$$

Here $\mathbf{1}_N$ denotes a column vector of all 1 s, *i.e.* $[1, 1, \ldots, 1]^{\mathrm{T}}$ *and* $\mathbf{0}_N$ *denotes a column vector of all* 0 s, *i.e.* $[0, 0, \ldots, 0]^{\mathrm{T}}$. *Both*

$\mathbf{1}_N$ *and* $\mathbf{0}_N \in \mathbb{R}^{\mathrm{N}}$. *The elements of* $\mathcal{L}$ *are denoted as* $l(i,j)$ *such thatl*$(i,j) \in \mathbb{R}^{\mathrm{N}\times\mathrm{N}}$.

Lemma 2.2: *The matrix* $\mathcal{L}+\mathcal{B}$ *has full rank when* $\mathcal{G}$ *has a spanning tree with leader as the root. This implies non singularity of* $\mathcal{L}+\mathcal{B}$.

2.2 Event-triggered sliding mode control

Sliding Mode Control (SMC) (David Young, Utkin, & Ozguner, 1999; Utkin, 1992) is known for its inherent robustness. The switching nature of the control is used to nullify bounded disturbances and matched uncertainties. The switching happens about a surface (hyperplane) in state space known as sliding surface (hyperplane). The control forces the system monotonically towards the sliding surface and this phase is regarded as reaching phase. When the system reaches the sliding surface, it remains there for all future time, thereby ensuring that the system dynamics remain independent of bounded disturbances and matched uncertainties. The controller has a reaching phase (trajectories in phase plane emanate and move towards the switching surface) and a sliding phase (trajectories in the phase plane that reach the switching surface try to remain there).

2.2.1 Reaching phase

Let the hyperplane discussed be given as $\sigma(x)$. In order to drive state trajectories onto this manifold, a proper discontinuous control effort $u(t,x)$ needs to be synthesised that satisfies the following inequality:

$$\sigma^{\mathrm{T}}(x)\dot{\sigma}(x) \leq -\eta\|\sigma(x)\|, \tag{2}$$

with η being positive and is called the reachability constant.

$$\because\ \dot{\sigma}(x) = \frac{\partial\sigma}{\partial x}\dot{x} = \frac{\partial\sigma}{\partial x}f(t,x,u) \tag{3}$$

$$\therefore\ \sigma^{\mathrm{T}}(x)\frac{\partial\sigma}{\partial x}f(t,x,u) \leq -\eta\|\sigma(x)\|. \tag{4}$$

2.2.2 Sliding phase

The motion of state trajectories confined on the switching manifold is known as *sliding*. A sliding mode is said to exist if the state velocity vectors are directed towards the manifold in its neighbourhood (David Young et al., 1999; Zak, 2003). Under this circumstance, the manifold is called attractive (Zak, 2003), i.e. trajectories starting on it remain there for all future time and trajectories starting outside it tend to it in an asymptotic manner.

$$\because\ \dot{\sigma}(x) = \frac{\partial\sigma}{\partial x}f(t,x,u),$$

Hence, in sliding motion

$$\frac{\partial\sigma}{\partial x}f(t,x,u) = 0. \tag{5}$$

Then $u = u_{eq}$ (say) be a solution and is generally referred to as the equivalent control. This u_{eq} is not the actual control applied to the system but can be thought of as a control that must be applied on an average to maintain sliding motion. It

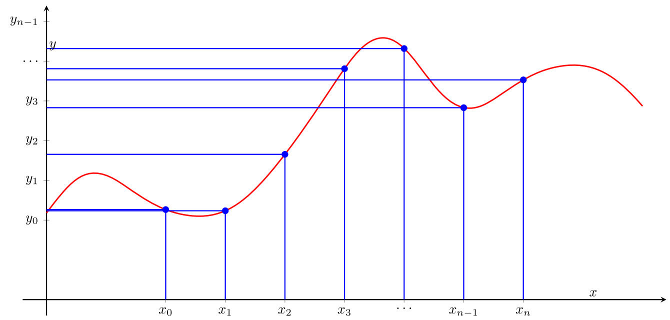

Figure 1. Riemann Sampling.

is mainly used for analysis of sliding motion (Yan, Spurgeon, & Edwards, 2017).

In practice, individual autonomous agents in MAS are often equipped with small digital microcontrollers to reduce the cost. These microcontrollers have limited computing and communication capabilities. According to traditional sampled data control systems theory, samples of a measured output are obtained in periodic fashion with a fixed sampling rate. In addition to this, a zero order hold operator is needed to maintain the control input signal constant between successive sample instants. This sampling technique is known as periodic sampling and is generally done along the time axis, also known as *Riemann* sampling (Astrom & Bernhardsson, 2002). An alternative and more efficient way to obtain samples along dependent variable axis (vertical axis), known as *Lebesgue* sampling (Astrom & Bernhardsson, 2002). Under this technique, sampling interval is no longer periodic and samples are obtained only when a *noticeable change*, also referred to as an *event* occurs. As a result, the controller does not need to update itself periodically, but a hold type operator is still needed to maintain the value constant between successive sample instants. The continuous sampling and transmission, along with the occupancy of central processing unit to perform computations when the signal is constant (not changing too frequently), lead to significant waste of available resources. The optimum utilisation of communication, computing and energy expenses is a concern in various applications with increasing number of systems getting networked. One mitigation strategy adopted is event-based control wherein control is applied only when the system calls for it depending upon some *event*. Event-based sampling is a tradeoff between performance and sampling frequency. See Astholfi and Marconi (2007), Shi, Shi, and Chen (2016), Mazo and Tabuada (2011), Tabuada (2007), Aström (2008), Anta and Tabuada (2010), Tallapragada and Chopra (2013), Lemmon (2010), Sinha and Mishra (2018b), Sinha and Mishra (2018a), and Majumder, Mishra, Sinha, Singh, and Sahu (2018) and references therein. Figures 1 and 2 show a comparison between the two sampling schemes discussed above, and Figure 3 is a simple illustration of event-based control. The dotted lines from the event generator to the digital network, and from the digital network to the control signal generator, represent signal at event times. The solid lines represent continuous time signals.

As a consequence of combining event-based strategies with sliding mode control, the robustness of the system has been

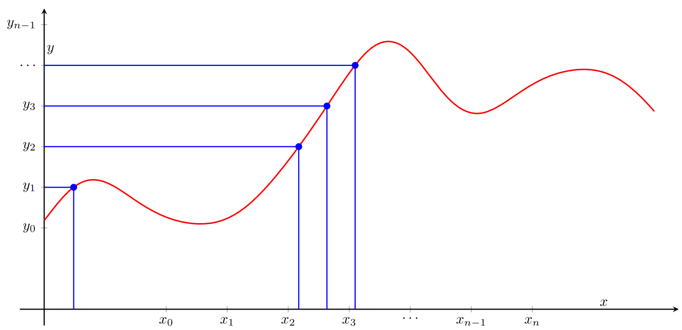

Figure 2. Lebesgue Sampling.

retained while maintaining lower computational expense. However, the system trajectories tend to move away from the sliding manifold till the control is updated again but remain bounded within a band. Detailed discussions have been carried out in later sections.

3. Dynamics of the MAS and problem formulation

Consider first order heterogeneous MAS with a virtual leader and a finite number of followers that are interconnected in a well defined directed topology. Under this interconnection, information of the leader's state is not available globally. However, local information is obtained by communication between follower agents. The dynamics of the leader and followers in such a system are described by nonlinear differential equations as

$$\dot{x}_0 = f(x_0(t)) + u_0(t) + \varsigma_0(t) \tag{6}$$

$$\dot{x}_i = f(x_i(t)) + u_i(t) + \varsigma_i(t); \quad i \in \mathbb{N} \tag{7}$$

where $f(\cdot) : \mathbb{R}^+ \times X \to \mathbb{R}^m$ denotes the nonlinear dynamics of each agent in the MAS. Here, the map $f(\cdot)$ is taken to be continuous in t. Also $X \subset \mathbb{R}^m$ is a domain in which origin is contained. $x_0(t)$ is the state of the virtual leader and $u_0(t)$ is the control associated with it such that $\|u_0(t)\|_\infty \le \Lambda$ for some $\Lambda \in \mathbb{R}^+$. This makes quite a practical case when upper limits on hardware constraints are known but the information on real time control effort is not concrete. Quite similarly$x_i(t)$ and $u_i(t)$ are the state of ith follower and the associated control, respectively. $\varsigma_0(t)$ and $\varsigma_i(t)$ are exogenous bounded disturbances that can be measured such thatmax$\{\varsigma_0(t), \varsigma_i(t)\} \le \varsigma_{\max}$.

Assumption 3.1: *The function $f(\cdot)$ described in (6) satisfies a Lipschitz condition over some fairly large domain $\mathbb{D}_{\mathbb{L}}$ with Lipschitz constant $\bar{L}$ such that*

$$\|f(z_1) - f(z_2)\| \le \bar{L}\|z_1 - z_2\| \tag{8}$$

$\forall t \in \mathbb{R}^+ \cup \{0\}$; $z_1, z_2 \in X$ and $\bar{L} \in \mathbb{R}^+$. Thus, the function $f(\cdot)$ is locally Lipschitz with respect to its arguments.

Assumption 3.2: *We assume that the function $f_i(\cdot)$ is input-to-state stable (ISS) in the sense of*Sontag (2008). *Recall that a system $\dot{z} = g(z, t, u)$ is said to be ISS if there exist functions $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$ such that for any initial condition $z(t_0)$ and any bounded input $u(t)$, the solution $z(t)$ exists for all time $t \ge t_0$ and the following criterion is satisfied*:

$$\|z(t)\| \le \beta(z(t_0), t) + \gamma(\|u(t)\|_\infty). \tag{9}$$

It is also worthy to note that whenever a feedback law stabilises the system, there also exists a (possibly different) feedback such that the system with external input is ISS (Sontag, 2008).

Remark 3.1: Note that a function $\alpha : \mathbb{R}^+ \cup \{0\} \to \mathbb{R}^+ \cup \{0\}$ is of class $\mathcal{K}_\infty$ if it is continuous, strictly increasing, unbounded and satisfies $\alpha(0) = 0$. A function $\beta : \mathbb{R}^+ \cup \{0\} \times \mathbb{R}^+ \cup \{0\} \to \mathbb{R}^+ \cup \{0\}$ is said to be of class $\mathcal{KL}$ if $\beta(\cdot, t) \in \mathcal{K}_\infty$ $\forall t$ and $\beta(\cdot, t) \to 0$ as $t \to \infty$.

Remark 3.2: For brevity, we shall carry out the discussion in $\mathbb{R}^1$. However, the same can be extended to higher dimensions by the use of Kronecker products.

The essential focus of this study is to ensure that the trajectory of the leader agent gets tracked accurately by other agents in finite time with minimum computational expense. The consensus tracking aims to maintain follower's state in consistence with leader's state in finite time by local communication. However, the leader moves independently of followers.

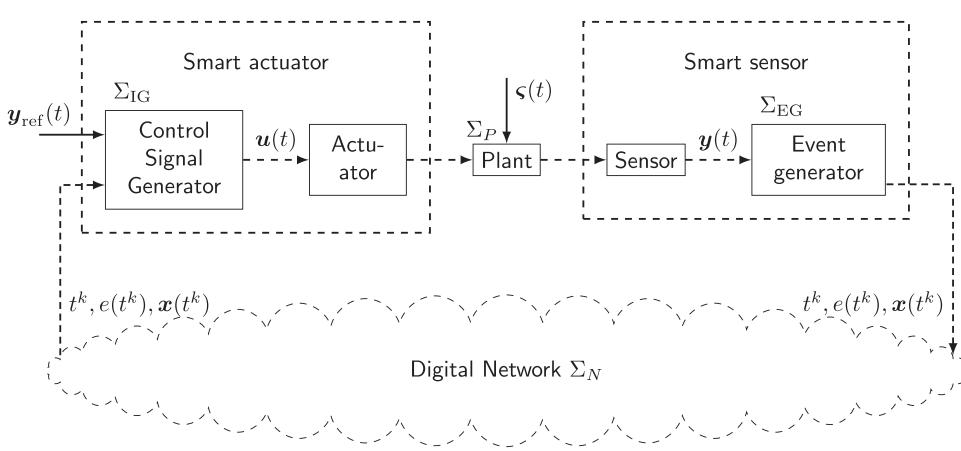

Figure 3. An illustration of event-based control.

4. Synthesis of the proposed controller

Let us define the tracking error for ith agent as

$$e_i(t) = x_i(t) - x_0(t)\mathbf{1}_N. \tag{10}$$

$e_i(t)$ is a vector of appropriate dimension such that $e_i(t) = [e_1(t)\ e_2(t)\ \dots e_r(t)]^{\mathrm{T}}$ for some $r \in \mathbb{N}$, where r is the number of followers in the MAS. In the coming discussions, $x_0(t)\mathbf{1}_N$ is simply denoted as $x_0(t)$. In terms of graph theory, the error variable (10) modifies to Liu, Ling, Huang, and Zhu (2015)

$$\begin{aligned}\bar{e}_i(t) &= (\mathcal{L}+\mathcal{B})e_i(t) = (\mathcal{L}+\mathcal{B})(x_i(t) - x_0(t)) \\ &= \mathcal{H}(x_i(t) - x_0(t)).\end{aligned} \tag{11}$$

Remark 4.1: From this point onwards, we shall refer to $\mathcal{L}+\mathcal{B}$ as $\mathcal{H}$ if no confusion arises.

Similarly, $\bar{e}_i(t)$ is also a vector of appropriate dimension. It is required that the error variable (11) vanishes quickly to establish a common agreement among agents and they attain accurate tracking. The controller that guarantees finite-time consensus is synthesised on archetype of variable structure control techniques with a slight modification. For any sliding mode controller design (David Young et al., 1999; Zak, 2003) is a two-step process: the design of a sliding surface where trajectories are required to be confined in finite time, and a control to force the trajectories onto this surface.

Theorem 4.1: *Given plant dynamics* (6) *and the tracking error* (11), *the control protocol due to traditional sliding mode controller that stabilises the MAS is given by*

$$\begin{aligned}u_i(t) = &-\mathcal{H}^{-1}(K|\sigma_i(t)|^{\tau}\operatorname{sign}(\sigma_i(t)) + f(x_i(t)) - f(x_0(t)) \\ &- u_0(t) + \varsigma_i(t) - \varsigma_0(t))\end{aligned}$$

where $\sigma_i(t)$ is the sliding manifold, $K|\sigma_i(t)|^{\tau}\operatorname{sign}(\sigma_i(t))$ is the discontinuous forcing function and $K > \sup\{\varsigma_{\max}\}$ is adjustable gain which can be tuned as per design needs. The exponent $\tau \in (0,1)$ decides the convergence speed to the sliding manifold.

Proof: In the notions of sliding modes, the sliding manifold $\sigma(t)$ is expressed in terms of error $e(t)$ as

$$\sigma(t) = \left(\frac{\mathrm{d}}{\mathrm{d}t} + \lambda\right)^{n-1} e(t), \tag{12}$$

where λ is a scalar weighting parameter and n is the order of the system. Thus, for the first order system given by (6), the sliding manifold takes the form

$$\begin{aligned}&\sigma_i(t) = \bar{e}_i(t) = \mathcal{H}e_i(t) = \mathcal{H}(x_i(t) - x_0(t)). \\ &\Rightarrow \dot{\sigma}_i(t) = \dot{\bar{e}}_i(t) = \mathcal{H}\dot{e}_i(t) = \mathcal{H}(\dot{x}_i(t) - \dot{x}_0(t)) \\ &\Rightarrow \dot{\sigma}_i(t) = \mathcal{H}(f(x_i(t)) + u_i(t) + \varsigma_i(t) \\ &\qquad - f(x_0(t)) - u_0(t) - \varsigma_0(t)) \\ &\Rightarrow u_i(t) = -\mathcal{H}^{-1}(K|\sigma_i(t)|^{\tau}\operatorname{sign}(\sigma_i(t)) + f(x_i(t)) \\ &\qquad - f(x_0(t)) - u_0(t)).\end{aligned} \tag{13}$$

The control law described by Theorem 4.1 is same as that in (13). This concludes the proof. ■

There are several advantages of using this law such as faster convergence and chattering free operation due to the presence of power rate term in the law. By suitably tailoring the power rate term, one can vary the speed of response. However, an optimal choice of parameters is necessary to establish a compromise between chattering and speed of response. When power rate reaching law is enforced in the system, higher control magnitudes are invoked initially but the sliding motion is short lived due to the presence of power rate term. Chattering phenomenon, which is a consequence of sliding motion in the vicinity of the sliding manifold, is thus avoided due to reduction in sliding motion.

The sudden increase of interest in the event-driven design of circuits and systems is due to enhanced performance in applications where resources are constrained. In networked control system like MAS connected over shared network consisting of rapid information exchange between nodes, resources such as bandwidth and processor time are always constrained. In such scenario, event-based control is expected to yield better results. The control gets updated only when an event (noticeable change) occurs, thereby significantly minimising computational requirement and power consumption. The event-based control is a good candidate if the requirement is to execute different tasks intime-shared manner and also where control is expensive. It is also advantageous in situations when steady state needs to be upper bounded at start regardless of any initial condition and the manner in which the states of the system evolve.

It has often been described as an alternative to periodic sampling owing to its nature. Next sample instant is dependent on the triggering of an *event*. Hence, our control law (13) is modified for $\forall t \in [t^k, t^{k+1})$ as

$$\begin{aligned}u_i(t) = &-\mathcal{H}^{-1}(K|\sigma_i(t^k)|^{\tau}\operatorname{sign}(\sigma_i(t^k)) + f(x_i(t^k)) \\ &- f(x_0(t^k)) - u_0(t^k)).\end{aligned} \tag{14}$$

The error introduced due to discretisation of the control is given by

$$\bar{\epsilon}(t) = x(t^k) - x(t), \tag{15}$$

such that at t^k, $\bar{\epsilon}(t) = 0$. It should be noted that t_i^k denotes the triggering instant for i^{th} agent. Henceforth, $\bar{\epsilon}_0(t)$ and $\bar{\epsilon}_i(t)$ shall denote the discretisation errors corresponding to leader and follower agents, respectively.

From (10), $e_i(t) = x_i(t) - x_0(t)$, so $e_i(t^k) = x_i(t^k) - x_0(t^k)$.

Theorem 4.2: *Consider the system described by* (6), *error variables* (10), (11), (15), *sliding manifold (hyperplane) $\sigma_i(t)$ in the notions of sliding mode and the control law* (14).

(i) *Sliding mode is said to exist in vicinity of the sliding surface if the surface is an essential attractor. In other words, reachability to the surface is ascertained for some reachability constant $\eta > 0$.*

(ii) *The event-driven sliding mode control law* (14) *provides stability to the system in the sense of Lyapunov if gain K is* *selected as*

$$K > \sup\Big\{\digamma + \bar{L}\|\bar{\epsilon}_i(t)\| - \bar{L}\|\bar{\epsilon}_0(t)\| + \bar{L}\|e_i(t^k)\| - \bar{L}\|e_i(t^k)\|\Big\}, \tag{16}$$

where $\digamma = \mathcal{H}\varsigma_{\max} - \mathcal{H}u_0(t) + u_0(t^k)$.

Proof: (i) Let us consider a Lyapunov candidate V such that

$$V = \frac{1}{2}\sigma_i^{\mathrm{T}}(t)\sigma_i(t). \tag{17}$$

Time derivative of (17) along the state trajectories for $t \in [t^k, t^{k+1})$ yields

$$\begin{aligned}\dot{V} &= \sigma_i^{\mathrm{T}}(t)\dot{\sigma}_i(t) = \sigma_i^{\mathrm{T}}(t)\dot{\bar{e}}_i(t) = \sigma_i^{\mathrm{T}}(t)\mathcal{H}\dot{e}_i(t)\\ &= \mathcal{H}(\dot{x}_i(t) - \dot{x}_0(t))\\ &= \sigma_i^{\mathrm{T}}(t)\mathcal{H}\Big(f(x_i(t)) + u_i(t) + \varsigma_i(t) - f(x_0(t))\\ &\quad - u_0(t) - \varsigma_0(t)\Big).\end{aligned} \tag{18}$$

We can substitute (14) into (18) to simplify the expression as

$$\begin{aligned}\dot{V} &= \sigma_i^{\mathrm{T}}(t)\Big\{\mathcal{H}f(x_i(t)) - \mathcal{H}f(x_0(t)) + \mathcal{H}\varsigma_i(t) - \mathcal{H}\varsigma_0(t) - \mathcal{H}u_0(t)\\ &\quad - K|\sigma_i(t^k)|^\tau \operatorname{sign}(\sigma_i(t^k)) - f(x_i(t^k)) + f(x_0(t^k)) + u_0(t^k)\Big\}\\ &\le \sigma_i^{\mathrm{T}}(t)\Big\{\mathcal{H}f(x_i(t)) - \mathcal{H}f(x_0(t)) + \mathcal{H}\varsigma_{\max} - \mathcal{H}u_0(t)\\ &\quad - K|\sigma_i(t^k)|^\tau \operatorname{sign}(\sigma_i(t^k)) - f(x_i(t^k)) + f(x_0(t^k)) + u_0(t^k)\Big\}\\ &\le \sigma_i^{\mathrm{T}}(t)\Big\{\mathcal{H}f(x_i(t)) - \mathcal{H}f(x_i(t^k)) + \mathcal{H}f(x_i(t^k))\\ &\quad - \mathcal{H}f(x_0(t)) + \mathcal{H}f(x_0(t^k))\\ &\quad - \mathcal{H}f(x_0(t^k)) + \digamma - K|\sigma_i(t^k)|^\tau \operatorname{sign}(\sigma_i(t^k))\\ &\quad - f(x_i(t^k)) + f(x_0(t^k))\Big\}\\ &\le \sigma_i^{\mathrm{T}}(t)\Big\{\digamma - K|\sigma_i(t^k)|^\tau \operatorname{sign}(\sigma_i(t^k)) + \mathcal{H}\Big(f(x_i(t))\\ &\quad - f(x_i(t^k))\Big) + \mathcal{H}f(x_i(t^k))\\ &\quad - \mathcal{H}\Big(f(x_0(t)) - f(x_0(t^k))\Big) - \mathcal{H}f(x_0(t^k))\\ &\quad - \Big(f(x_i(t^k)) - f(x_0(t^k))\Big)\Big\}\\ &\le \sigma_i^{\mathrm{T}}(t)\Big\{\digamma - K|\sigma_i(t^k)|^\tau \operatorname{sign}(\sigma_i(t^k))\\ &\quad + \mathcal{H}\Big(\bar{L}\|x_i(t) - x_i(t^k)\|\Big)\\ &\quad + \mathcal{H}\Big(\bar{L}\|x_i(t^k) - x_0(t^k)\|\Big) - \mathcal{H}\Big(\bar{L}\|x_0(t) - x_0(t^k)\|\Big)\\ &\quad - \Big(\bar{L}\|x_i(t^k) - x_0(t^k)\|\Big)\Big\}\\ &\le \sigma_i^{\mathrm{T}}(t)\Big\{\digamma - K|\sigma_i(t^k)|^\tau \operatorname{sign}(\sigma_i(t^k)) + \mathcal{H}\Big(\bar{L}\|\bar{\epsilon}_i(t)\|\Big)\\ &\quad - \mathcal{H}\Big(\bar{L}\|\bar{\epsilon}_0(t)\|\Big) + \mathcal{H}\Big(\bar{L}\|e_i(t^k)\|\Big) - \Big(\bar{L}\|e_i(t^k)\|\Big)\Big\}\\ &\le \sigma_i^{\mathrm{T}}(t)\Big\{\digamma - K|\sigma_i(t^k)|^\tau \operatorname{sign}(\sigma_i(t^k)) + \|\mathcal{H}\|\bar{L}\|\bar{\epsilon}_i(t)\|\\ &\quad - \|\mathcal{H}\|\bar{L}\|\bar{\epsilon}_0(t)\| + \|\mathcal{H}\|\bar{L}\|e_i(t^k)\| - \bar{L}\|e_i(t^k)\|\Big\}\\ &\le \|\sigma_i^{\mathrm{T}}(t)\|\digamma - K\sigma_i^{\mathrm{T}}(t)|\sigma_i(t^k)|^\tau \operatorname{sign}(\sigma_i(t^k)) + \|\sigma_i^{\mathrm{T}}(t)\|\\ &\quad \|\mathcal{H}\|\bar{L}\|\bar{\epsilon}_i(t)\| - \|\sigma_i^{\mathrm{T}}(t)\|\|\mathcal{H}\|\bar{L}\|\bar{\epsilon}_0(t)\| + \|\sigma_i^{\mathrm{T}}(t)\|\\ &\quad \|\mathcal{H}\|\bar{L}\|e_i(t^k)\| - \|\sigma_i^{\mathrm{T}}(t)\|\bar{L}\|e_i(t^k)\|.\end{aligned} \tag{19}$$

As long as $\sigma_i(t) > 0$ or $\sigma_i(t) < 0$, the condition $\operatorname{sign}(\sigma_i(t)) = \operatorname{sign}(\sigma_i(t^k))$ is strictly met $\forall t \in [t^k, t^{k+1})$. Hence, when trajectories are just outside the sliding surface, (19) can be represented as

$$\begin{aligned}\dot{V} &\le \|\sigma_i^{\mathrm{T}}(t)\|\digamma - K\|\sigma_i^{\mathrm{T}}(t)\||\sigma_i(t^k)|^\tau\\ &\quad + \|\sigma_i^{\mathrm{T}}(t)\|\|\mathcal{H}\|\bar{L}\|\bar{\epsilon}_i(t)\| - \|\sigma_i^{\mathrm{T}}(t)\|\|\mathcal{H}\|\bar{L}\|\bar{\epsilon}_0(t)\|\\ &\quad + \|\sigma_i^{\mathrm{T}}(t)\|\|\mathcal{H}\|\bar{L}\|e_i(t^k)\| - \|\sigma_i^{\mathrm{T}}(t)\|\bar{L}\|e_i(t^k)\|\\ \Rightarrow \dot{V} &\le \|\sigma_i^{\mathrm{T}}(t)\|\Big\{\digamma - K|\sigma_i(t^k)|^\tau + \|\mathcal{H}\|\bar{L}\|\bar{\epsilon}_i(t)\|\\ &\quad - \|\mathcal{H}\|\bar{L}\|\bar{\epsilon}_0(t)\| + \|\mathcal{H}\|\bar{L}\|e_i(t^k)\| - \bar{L}\|e_i(t^k)\|\Big\}.\\ &\therefore\ \dot{V} \le -\eta\|\sigma_i^{\mathrm{T}}(t)\| = -\eta\|\sigma_i(t)\|,\end{aligned} \tag{20}$$

where $\eta > 0$ and $K > \sup\{\digamma + \bar{L}\|\bar{\epsilon}_i(t)\| - \bar{L}\|\bar{\epsilon}_0(t)\| + \bar{L}\|e_i(t^k)\| - \bar{L}\|e_i(t^k)\|\}$. This implies that the sliding manifold is an attractor and state trajectories continuously decrease towards it $\forall t \in [t^k, t^{k+1})$. This completes the proof of reachability. ■

Proof: (ii) Now, it requires to be shown that if negative definiteness of time derivative of the Lyapunov candidate (17) be ascertained, stability in the sense of Lyapunov can be guaranteed. At instant t^k, the control signal gets updated, thereby nullifying the discretisation errors. Hence, $\|\bar{\epsilon}_i(t^k)\| \to 0$ and $\|\bar{\epsilon}_0(t^k)\| \to 0$. When $\sigma_i(t) = 0$ and $t = t^k$, it follows that

$$\sigma_i(t) = \bar{e}_i(t) = \mathcal{H}e_i(t) = \mathcal{H}(x_i(t) - x_0(t)) = x_i(t) - x_0(t) = 0. \tag{21}$$

(21) is a direct consequence of Lemma 2.2, wherein $\mathcal{H}$ is of full rank and thus, is invertible. Hence, at $t = t^k$,

$$x_i(t^k) - x_0(t^k) = e_i(t^k) = 0. \tag{22}$$

$\because$ $\|\bar{\epsilon}_i(t^k)\| \to 0$, $\|\bar{\epsilon}_0(t^k)\| \to 0$ and from (22), we can conclude that (20) can be written as

$$\dot{V} \le \|\sigma_i^{\mathrm{T}}(t)\|(-K|\sigma_i(t^k)|^\tau + \digamma). \tag{23}$$

From the results of part (i), it is clear that $\dot{V} < 0$, concluding that stability in the sense of Lyapunov can be ensured. This concludes the proof of stability. ■

Remark 4.2: For time instants between $[t^k, t^{k+1})$ the states show a tendency to deviate from the sliding manifold but remain bounded within a band near the manifold. In other words, practical sliding mode occurs.

The triggering instant t^k is completely characterised by a triggering rule. Next sampling instant is by virtue of this criterion.

As long as this criterion is respected, next clock pulse is not called upon and the control signal is maintained constant at the previous value. A novel triggering rule has been used in this study and is given by

$$\delta = \|\nu_1 e_i + \nu_2 \dot{e}_i\| - (c_0 + c_1 e^{-\varpi t}) \tag{24}$$

such that $\nu_1 > 0$, $\nu_2 > 0$, $c_0 \geq 0$, $c_1 \geq 0$, $c_0 + c_1 > 0$ and $\varpi \in (0, \lambda_2(\mathcal{L}))$. Here $\lambda_2(\mathcal{L})$ is the second eigenvalue if all the eigenvalues of $\mathcal{L}$ are arranged in ascending order. This means $\lambda_1(\mathcal{L}) < \lambda_2(\mathcal{L}) < \lambda_3(\mathcal{L}) < \ldots < \lambda_n(\mathcal{L})$. The triggering rule is such that error and rate of change of change of error, both, are taken into consideration to ascertain desired closed-loop performance. The second term $(c_0 + c_1 e^{-\varpi t})$ is a time-varying but state independent threshold, ensuring a finite positive lower bound on inter-event execution time, thereby excluding *Zeno* behaviour, i.e. no two consecutive events occur at same time. Another advantage of using this threshold is that communication with neighbour agents can be avoided. The purpose of reducing the communication burden in a hybrid system is defeated if this phenomenon occurs in the system. This can lead to prevention of existence of global solutions to be defined for all time. In order to prevent the accumulation of samples at the same time, it is necessary to ensure that adjacent samples are separated in time by a small finite quantity, thereby ensuring a positive lower bound on inter-event execution time. The following theorem establishes the existence of a finite lower bound for inter-event execution time for each agent triggered separately.

Theorem 4.3: *Consider the system described by* (6), *the event-based control protocol given in* (14) *and the discretisation error as defined in* (15). *The sequence of triggering instants* $\{t_i^k\}_{k=0}^{\infty}$ *for each agent respects the triggering rule given in* (24). *Consequently, Zeno phenomenon is not exhibited and the inter-event execution time for each agent* $T_i^k = t_i^{k+1} - t_i^k$ *is bounded below by a finite positive quantity* ϕ.

Proof: Before proceeding to a formal proof, we make the following assumption. ■

Assumption 4.1: *A finite but not necessarily constant delay* Δ *might occur during sampling and is unavoidable due to hardware characteristics. In such cases the control is maintained constant* $\forall\, t_i \in [t_i^k + \Delta, t_i^{k+1} + \Delta)$. *It has been assumed that* Δ *is negligible and has been neglected innocuously. Hence for our case, the control is constant in the interval* $[t_i^k, t_i^{k+1})$.

Between kth and $(k+1)$th sampling instant in the execution of control, the discretisation error is non zero. T_i^k is the time it takes the discretisation error to rise from 0 to $\|\bar{\epsilon}(t)\|_\infty$. Thus,

$$\begin{aligned} \frac{d}{dt}\left\|\bar{\epsilon}_i(t)\right\| &\leq \left\|\frac{d}{dt}\bar{\epsilon}_i(t)\right\| \leq \left\|\frac{d}{dt}x_i(t)\right\| \\ \Rightarrow \left\|\frac{d}{dt}\bar{\epsilon}_i(t)\right\| &\leq \left\|f(x_i(t)) + u_i(t) + \varsigma_i(t)\right\| \end{aligned} \tag{25}$$

Substituting the control input (14) in the above inequality (25), we get

$$\begin{aligned} \left\|\frac{d}{dt}\bar{\epsilon}_i(t)\right\| \leq &\Big\| f(x_i(t)) - \mathcal{H}^{-1}\Big(K|\sigma_i(t^k)|^\tau \operatorname{sign}(\sigma_i(t^k)) \\ &+ f(x_i(t^k)) - f(x_0(t^k)) - u_0(t^k)\Big) + \varsigma_i\Big\| \\ \leq &\bar{L}\|x_i(t)\| + \|\mathcal{H}^{-1}\|K|\sigma_i(t^k)|^\tau + \|\mathcal{H}^{-1}\|\bar{L}\|x_i(t^k)\| \\ &+ \|\mathcal{H}^{-1}\|\bar{L}\|x_0(t^k)\| + \|\mathcal{H}^{-1}\|\|u_0(t^k)\| + \|\varsigma_i\| \\ \leq &\bar{L}(\|x_i(t^k)\| + \|\bar{\epsilon}_i(t)\|) + \|\mathcal{H}^{-1}\|K|\sigma_i(t^k)|^\tau \\ &+ \|\mathcal{H}^{-1}\|\bar{L}\|x_i(t^k)\| + \mho \\ \leq &\bar{L}\|\bar{\epsilon}_i(t)\| + (\mathbf{1}_N + \|\mathcal{H}^{-1}\|)\bar{L}\|x_i(t^k)\| + \mho \\ \leq &\bar{L}\|\bar{\epsilon}_i(t)\| + \Omega\|x_i(t^k)\| + \mho \end{aligned} \tag{26}$$

where $\Omega = (\mathbf{1}_N + \|\mathcal{H}^{-1}\|)\bar{L}$ and $\mho = \|\mathcal{H}^{-1}\|K|\sigma_i(t^k)|^\tau + \|\mathcal{H}^{-1}\|\bar{L}\|x_0(t^k)\| + \|\mathcal{H}^{-1}\|\|u_0(t^k)\| + \|\varsigma_i\|$.

For $t \in [t^k, t^{k+1})$, the solution to the differential inequality (26) can be understood by using Comparison Lemma (Khalil, 2002) with initial condition $\|\bar{\epsilon}_i(t)\| = 0$. Comparison Lemma (Khalil, 2002; Ramm & Hoang, 2011) is particularly useful when information on bounds on the solution is more important than the solution itself.

Since Ω and $\mho$ are always positive, the solution to the differential inequality of (26) qualifies to be a finite positive value (Girard, 2013). Thus,

$$\|\bar{\epsilon}_i(t)\| \leq \phi; \quad \phi \in \mathbb{R}^+. \tag{27}$$

This implies that T_i^k is always lower bounded by some positive quantity, and hence this concludes the proof.

Remark 4.3: The results established in the proof of Theorem 4.3 hold locally over some fairly large compact domain $\mathbb{D}_{\mathbb{L}}$. Under varied settings, the design parameters can be tuned selectively to attain desired closed-loop performance. From (24), as long as $\delta < 0$, next sample is not taken and the system is said to deliver acceptable closed-loop performance.

The triggering rule (24) can be presented in an iterative way as

$$t_i^{k+1} = \inf\{t_i \in [t_i^k, \infty) : \delta \geq 0\}. \tag{28}$$

5. Numerical simulations

The objective of consensus is to bring a set of agents move towards a common state-dependent quantity of interest by exploiting only local interactions. The communication topology that is used for information exchange among agents in this study is shown in Figure 4 and is directed in nature. In this particular topology, agents 1 and 2 receive information directly from the leader. Agent 2 passes its information to agent 3, and agent 3 passes its information to agents 1 and 4. In this way, there happens local interactions, and the information of the leader is acquired by all the agents. Agents 1 and 2 are the informed agents, and agents 3 and 4 are uninformed agents in this topology. Using spectral graph theory, we can compute the graph

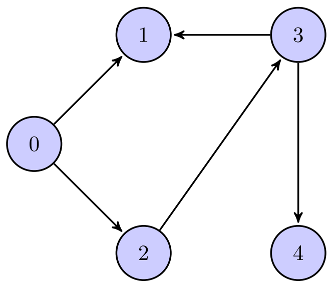

Figure 4. Topology of MAS– a digraph with leader indexed with 0 and followers indexed with 1, 2, 3 and 4.

matrices associated with the topology under consideration as follows.

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{D} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{29}$$

$$\mathcal{L} = \mathcal{D} - \mathcal{A} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix},$$

$$\mathcal{L} + \mathcal{B} = \mathcal{H} = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}. \tag{30}$$

The eigenvalues of $\mathcal{H}$ are 2,1,1 and 1, i.e. there is a simple eigenvalue 2 and an eigenvalue 1 of multiplicity 3. Note that none of the eigenvalue is 0, implying that $\mathcal{H}$ has full rank. This is a direct consequence of Lemma 2.2. Thus, there exists at least one spanning tree in the topology of Figure 4. The eigenvalues of the Laplacian $\mathcal{L}$ are 1, 1, 1 and 0, which are all real. It is worthy to note that the same discussion applies and can be extended to Laplacian with complex eigenvalues too.

The dynamics of the leader has been taken as

$$\dot{x}_0(t) = u_0(t)\cos(t) + 0.2\sin(x_0(t)) + \varsigma_0(t) \tag{31}$$

where $u_0(t) = \frac{2\cos(0.1\pi t)}{1+e^{-t}}$ is the control input of the leader. Dynamics of the follower agents are non-identical, and are described as

$$\dot{x}_1(t) = 0.1\sin(x_1(t))^{1/3} + \cos^2(2\pi t) + e^{-t} + u_1(t) + \varsigma_1(t), \tag{32}$$

$$\dot{x}_2(t) = 0.1\sin(x_2(t)) + \cos(2\pi t) + u_2(t) + \varsigma_2(t), \tag{33}$$

$$\dot{x}_3(t) = -x_3(t)\cos(t) - \sin(x_3(t)) - \cos(x_3(t)) + u_3(t) + \varsigma_3(t), \tag{34}$$

$$\dot{x}_4(t) = \sin(x_4(t)) + cos(e^{-t\,x_4(t)}) + u_4(t) + \varsigma_4(t). \tag{35}$$

These dynamics are locally Lipschitz over some fairly large domain $\mathbb{D}_{\mathbb{L}}$. The parametric values used in simulation are tabulated below in Table 1. Simulations have been performed for two cases – behaviour of the MAS in the absence of perturbations, and MAS operating under the influence of an unknown but bounded disturbance. To further aid the proposition, formation keeping by agents has also been illustrated. Setting $\varsigma_i = 0$ in (32)–(35), implies that the system is operating in the absence of disturbance. Figures 5 and 6 illustrate agents coming to consensus in finite time, and the tracking error during consensus. In Figure 5, the agents start from far away initial points (initial conditions) and track the trajectory of the leader in finite time. There is no erroneous behaviour during the short transient phase. By adjusting the parameters τ and K, the speed of

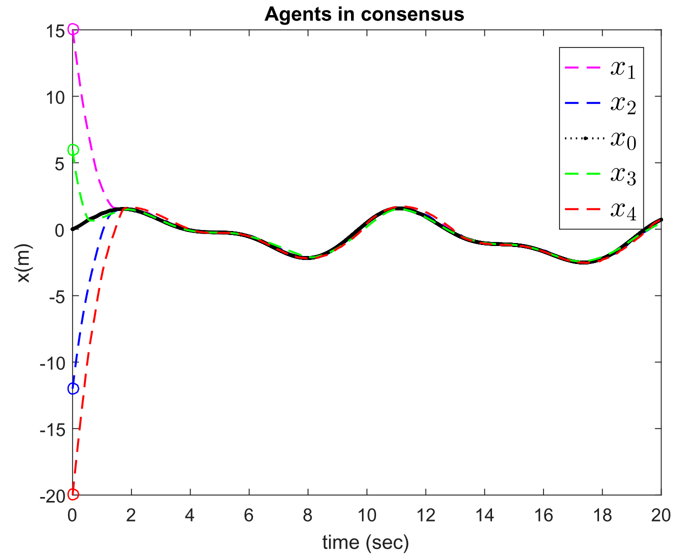

Figure 5. Leader–follower consensus in absence of disturbances.

Table 1. Numerical values of different parameters used in simulation.

c_0	c_1	ν_1	ν_2	K	ϖ	τ	$x_0(0)$	$x_1(0)$	$x_2(0)$	$x_3(0)$	$x_4(0)$
10^{-4}	0.2499	0.8	0.6	10	0.75	0.5	0	15	−12	6	−20

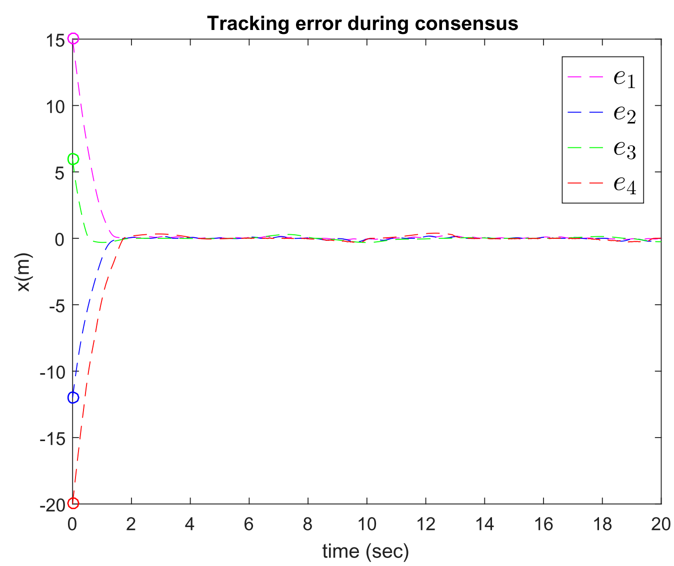

Figure 6. Tracking error profile of follower agents in absence of disturbances.

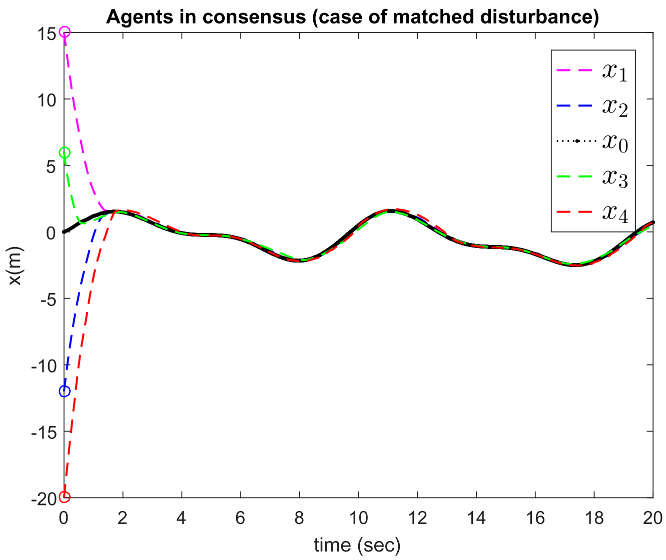

Figure 7. Consensus tracking in presence of disturbance $0.3\sin(\pi^2t^2)$.

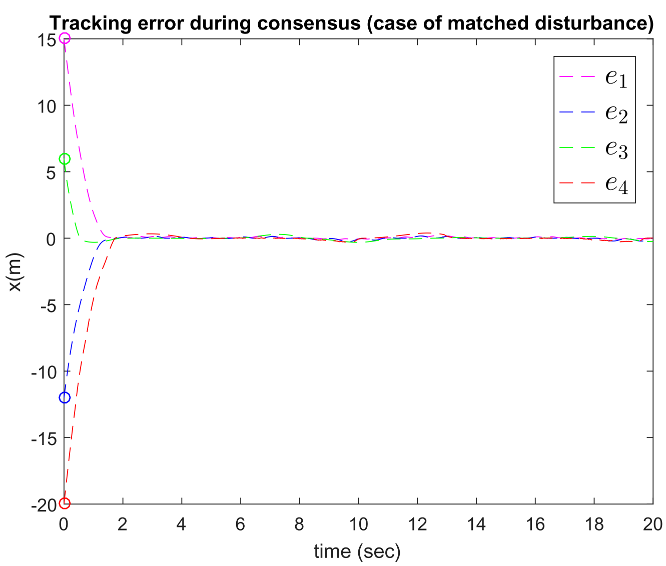

Figure 8. Tracking error profile of follower agents in presence of disturbance $0.3\sin(\pi^2t^2)$.

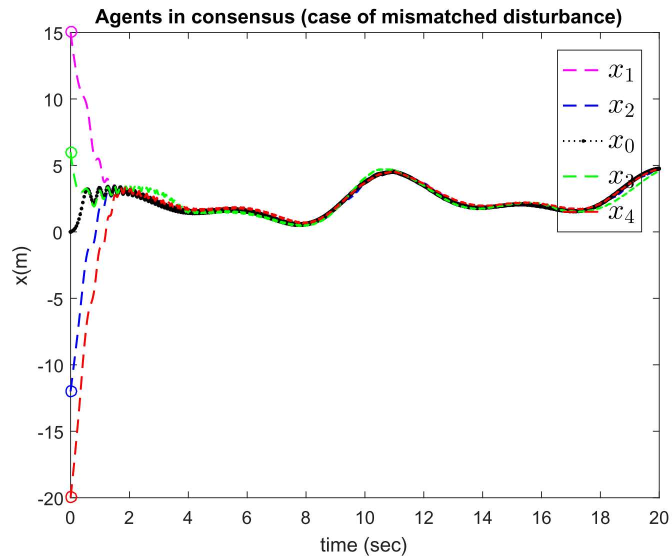

Figure 9. Consensus tracking in presence of disturbance $9\sin(\pi^2t^2)$.

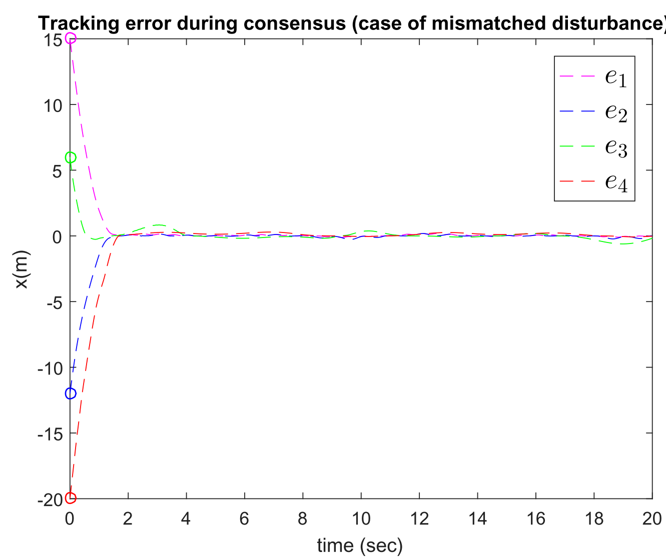

Figure 10. Tracking error profile of follower agents in presence of disturbance $9\sin(\pi^2t^2)$.

convergence to the leader's trajectory can be tailored. Figure 6 is the error profile of the follower agents. Since the convergence is quite fast, the error variables die out rapidly. This faster convergence of error variable to zero shows desirable closed-loop dynamics of the system and proves the effectiveness of the controller. It is also worthy to investigate the behaviour of the MAS under bounded and matched perturbations. We designate the term matched perturbations to exogenous disturbances entering the system through input channel.

Sliding mode is known for its inherent robustness, and is known to reject any disturbance that is matched. Consider a time-varying matched disturbance $\varsigma_i = 0.3\sin(\pi^2t^2)$. Figures 7 and 8 depict consensus tracking and the tracking errors of the follower agents respectively under the influence of disturbance $0.3\sin(\pi^2t^2)$. Since, the magnitude of this disturbance is very small, trajectory of the agents are unchanged (Figure 7). The tracking errors, depicted in Figure 8, once again converge to zero as soon as consensus is established. However, if the magnitude of the disturbance is large, e.g. $9\sin(\pi^2t^2)$, the trajectories are slightly changed from those when there was small or no disturbance acting on the system. In spite of corruption in trajectories, consensus to the leader is achieved in finite time. This behaviour has been illustrated in Figure 9. Tracking error profile for this case has been shown in Figure 10.

In sampled data control systems, the sampling paradigm is very crucial. Under the explicit assumption of infinite computational power and communication bandwidth, many time-based controllers have been designed. In practice, bandwidth and processor time are always constrained. In addition, the central processing unit may be required to perform several tasks in a time-shared manner. Under such applications, event-based control is expected to yield better results. Having used a novel

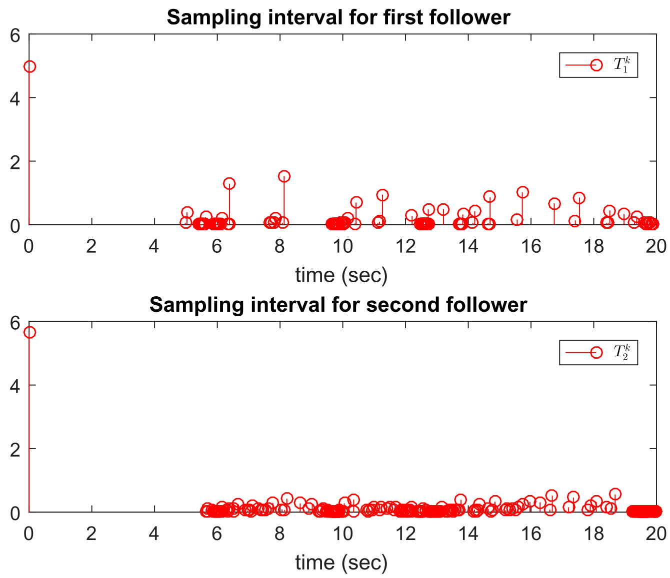

Figure 11. Sampling intervals for followers 1 and 2.

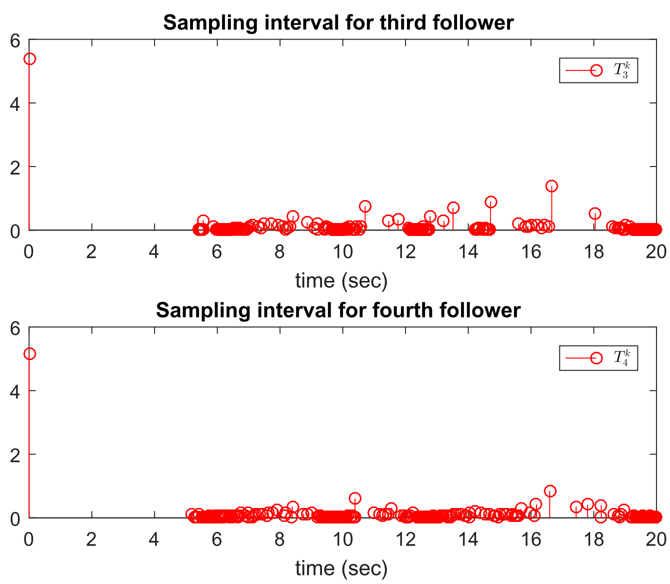

Figure 12. Sampling intervals for followers 3 and 4.

Table 2. Controller updates for the system under control law (14).

Agent	Follower 1	Follower 2	Follower 3	Follower 4
No. of updates	365	725	887	751
Total number of updates				2728

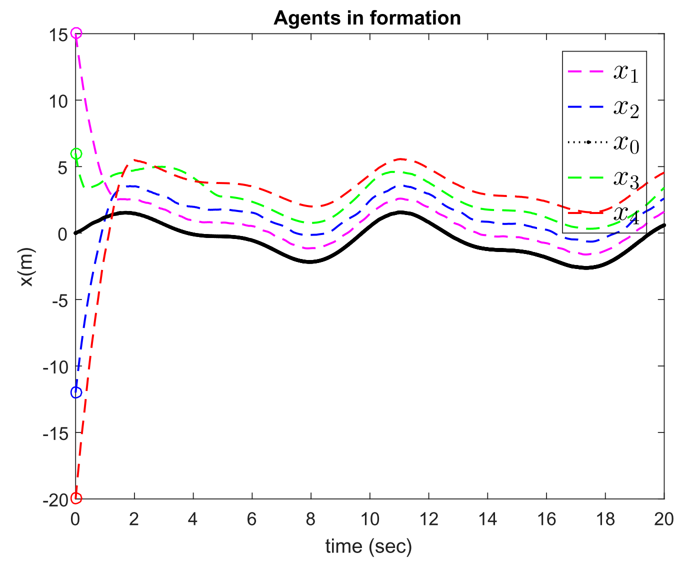

Figure 13. Agents in parallel formation.

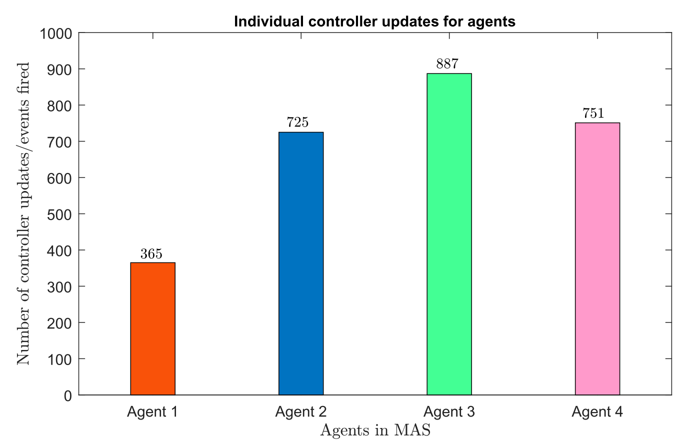

Figure 14. Individual controller updates of agents in the MAS.

triggering rule (24), the controller updates are minimal. Control effort is required only when necessary. Accuracy adjustment and the decision to take a sample is attributed to the threshold parameter in a triggering rule. In many practical applications, it is unreasonable to permanently fix the threshold. Suppose there are multiple agents tasked to maintain a prescribed geometrical formation. The inter-agent communication should be maintained even after the geometrical formation is achieved, but if the threshold remains fixed at some constant value, some of the data packets containing important information on maintaining the formation may not be triggered. Another instance is of some widely used practical communication networks such as IEEE 802.11 WLAN. This network has a time-varying data transmission rate owing to variable interference and random wireless fading. It is, thus, required that the accuracy adjustment parameter (threshold) should be altered in a dynamic fashion to schedule agents' communication while ensuring resouce efficiency and desired closed-loop performance.

Figures 11 and 12 are presented here to illustrate the inter-event execution time of follower agents. It is clearly established from these plots that the sampling is non-uniform on the horizontal axis. In Figures 11 and 12, the vertical axis represents the sampling interval $T^k = t^{k+1} - t^k$, and the horizontal axis is the time axis. The height of the bar in Figures 11 and 12 represents the sampling interval at corresponding time instants. It is also easy to conclude that *Zeno* behaviour has been excluded. The number of controller updates of the follower agents is tabulated in Table 2 and also shown as a graph in Figure 13.

Maintaining formation in certain scenarios is one of the most important practical applications of MAS operation. Formation keeping of agents is demonstrated here to further aid the propositions made in this study. Figure 14 depict agents

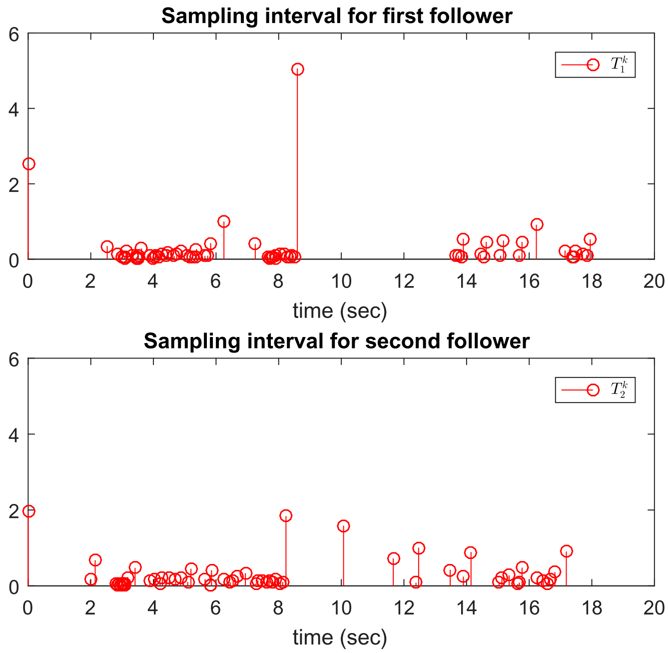

Figure 15. Sampling intervals for followers 1 and 2 during parallel formation.

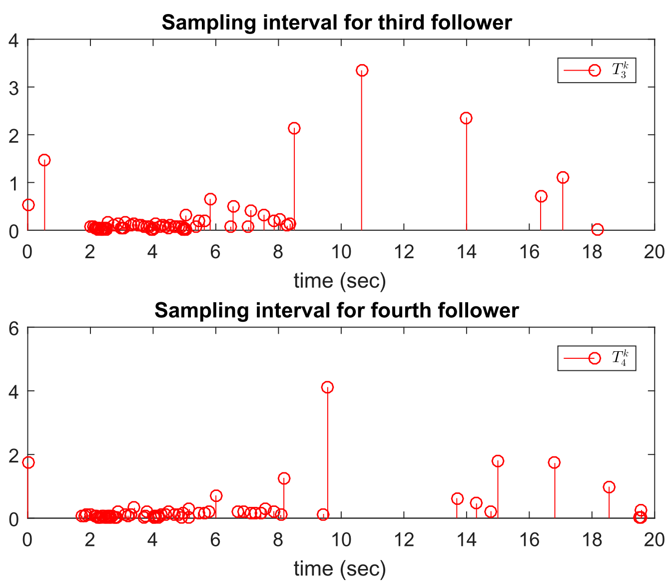

Figure 16. Sampling intervals for followers 3 and 4 during parallel formation.

in parallel formation, keeping a constant separation between themselves throughout the experiment. The corresponding triggering intervals when agents make parallel formation are shown in Figures 15 and 16. It is evident from the plots that formation is also achieved in finite time and therefore the control is robust.

6. Concluding remarks and outlook

In this study, we have addressed the consensus problem in heterogeneous MAS with inherent nonlinear dynamics via event-based sliding mode control. A novel dynamicevent-triggering rule has been proposed to ensure that the closed-loop performance does not deteriorate when lesser number of samples are taken to update the controller. Incorporating sliding mode controller with paradigms of event-based scheme helped achieve low computational power and control by exception. Mathematical foundations ascertain stable behaviour and exclusion of Zeno phenomenon. Furthermore, numerical simulations bolster the findings surfaced using the proposed control. The advantage of using this scheme is to retain the robustness offered by sliding mode control with minimum effort possible. We look forward to extend the study to arbitrary order agents under communication delay and packet loss.

Disclosure statement

No potential conflict of interest was reported by the authors.

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