



THE UNIVERSITY OF
MELBOURNE

Generative models

COMP90051 Statistical Machine Learning

“What I Cannot Create, I Do Not Understand” —— Richard Feynman

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So far..

Classifier:

- SVM
- Perceptron
- Multi-layer perceptron
- CNN

Feature extraction:

- Pretrained CNN model

Using pretrained model in Keras

Feature extraction before the last classifier

```
from tensorflow.keras.applications.resnet50 import ResNet50
from tensorflow.keras.preprocessing import image
from tensorflow.keras.applications.resnet50 import preprocess_input
import numpy as np

model = ResNet50(weights='imagenet', include_top=False)

img_path = '1.jpeg'
img = image.load_img(img_path, target_size=(224, 224))
x = image.img_to_array(img)
x = np.expand_dims(x, axis=0)
x = preprocess_input(x)

features = model.predict(x)
```

Using pretrained model in Keras

Feature extraction from arbitrary layer

```
from tensorflow.keras.applications.resnet50 import ResNet50
from tensorflow.keras.preprocessing import image
from tensorflow.keras.applications.resnet50 import preprocess_input
import numpy as np
from tensorflow.keras.models import Model

model = ResNet50(weights='imagenet')
layers=model.layers #get name of the layers
model = Model(inputs=model.input, outputs=model.get_layer('conv5_block3_out').output)

img_path = '1.jpeg'
img = image.load_img(img_path, target_size=(224, 224))
x = image.img_to_array(img)
x = np.expand_dims(x, axis=0)
x = preprocess_input(x)

features = model.predict(x)
```

Using pretrained model in Keras

Perform classification

```
from tensorflow.keras.applications.resnet50 import ResNet50
from tensorflow.keras.preprocessing import image
from tensorflow.keras.applications.resnet50 import preprocess_input, decode_predictions
import numpy as np

model = ResNet50(weights='imagenet')

img_path = '1.png'
img = image.load_img(img_path, target_size=(224, 224))
x = image.img_to_array(img)
x = np.expand_dims(x, axis=0)
x = preprocess_input(x)

preds = model.predict(x)

print('Predicted:', decode_predictions(preds, top=3)[0])
```

Predict class using a pretrained ResNet 50

Pretrained on ImageNet (1000 classes)



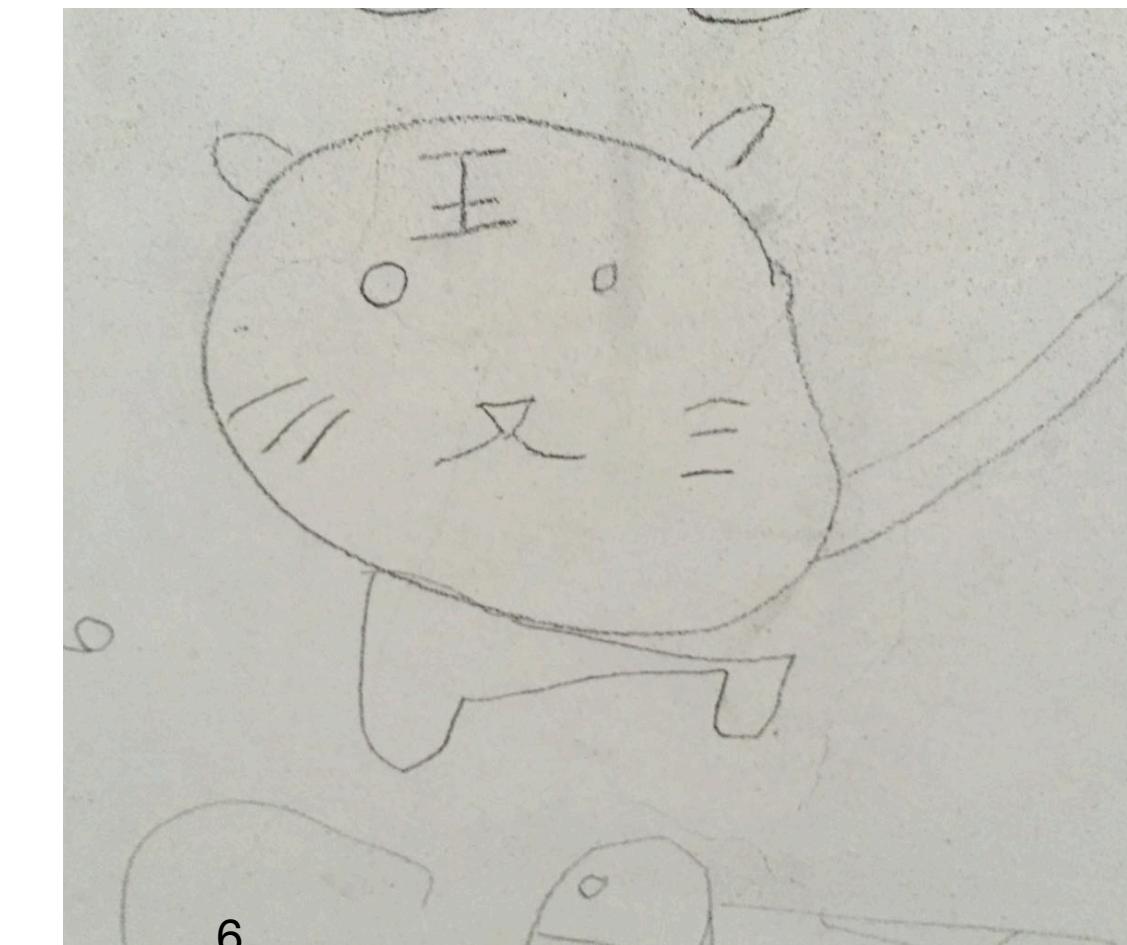
class	probability
espresso	0.9995716
cup	0.00022115342
coffee_mug	0.00020262193



class	probability
daisy	0.79781777
pot	0.10065505
picket_fence	0.05316206



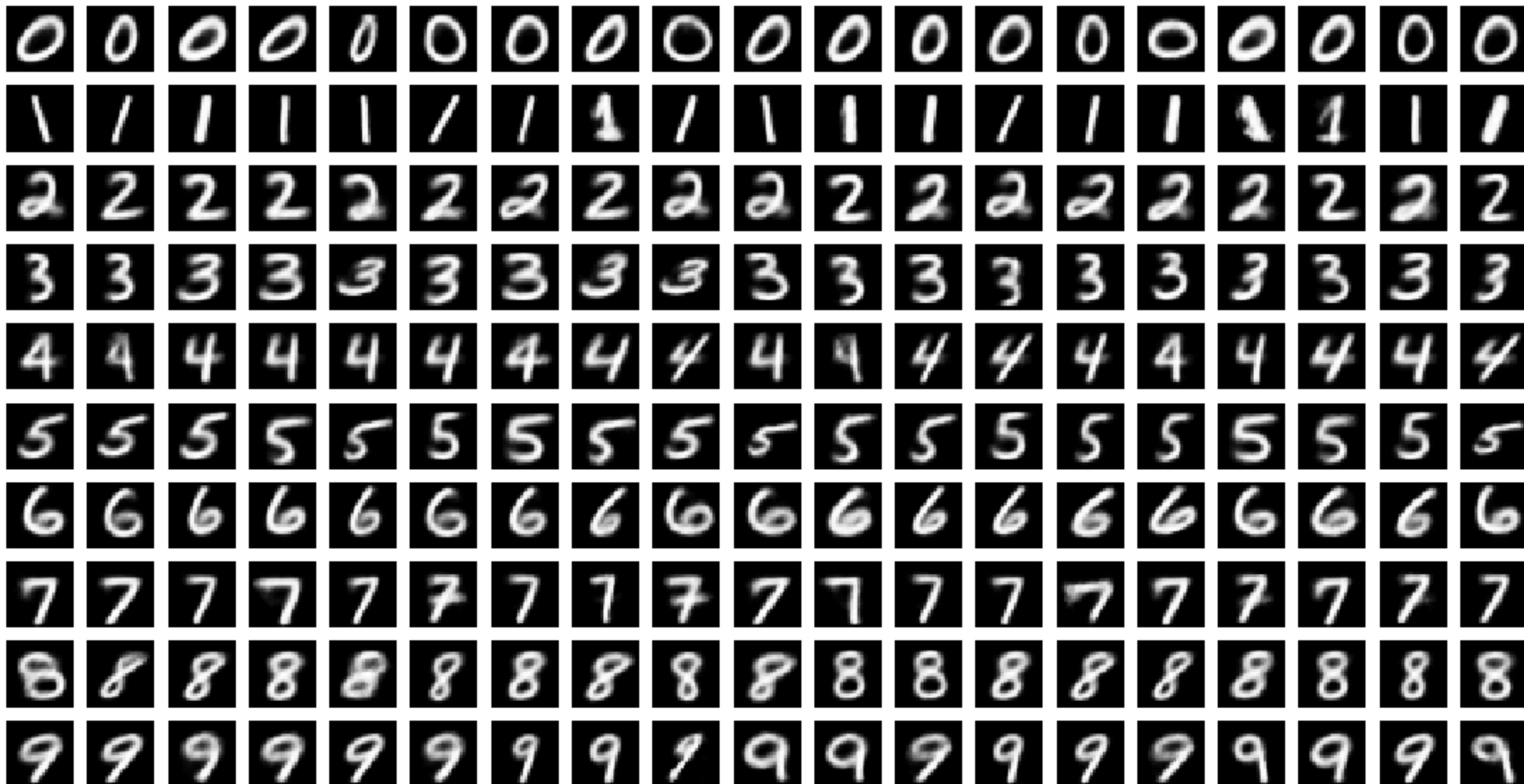
class	probability
alp (mountain)	0.7066206
geyser	0.106300876
valley	0.056517705



class	probability
rubber_eraser	0.53692865
envelope	0.1995637
paper_towel	0.040610846

Beyond classification

Image generation



Beyond classification

Image generation

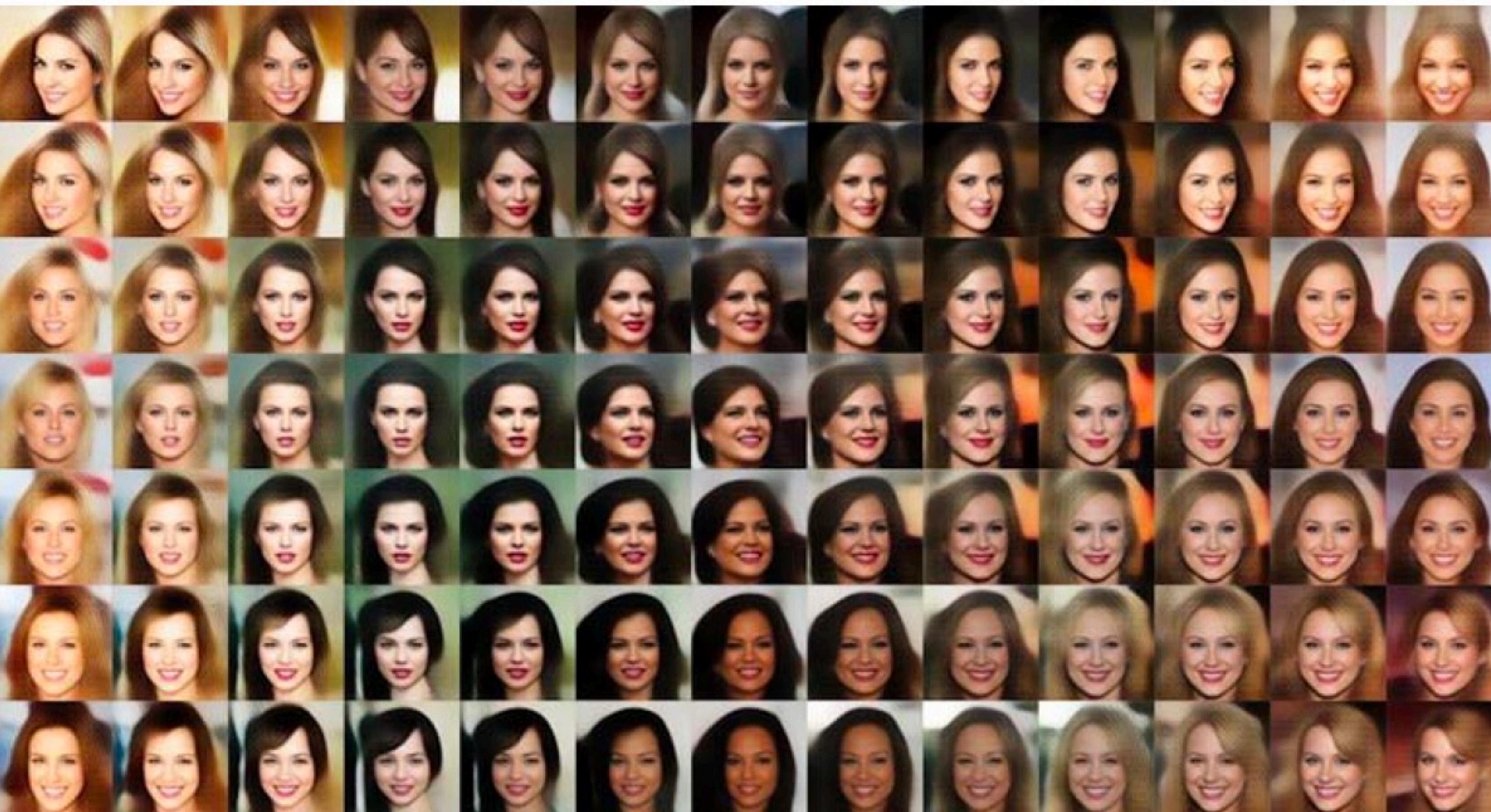


Figure 8.10 in Deep learning with python⁸ by Francois Chollet

Beyond classification

Image editing



(a) *Glasses*: remove and add the glasses



(b) *Mouth_open*: close and open the mouth



(c) *No_beard*: add and remove the beard

Generative Models

Cope with all of above tasks

- Variational Autoencoder (VAE)
- Generative Adversarial Network (GAN)

Outline

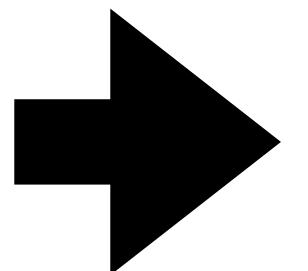
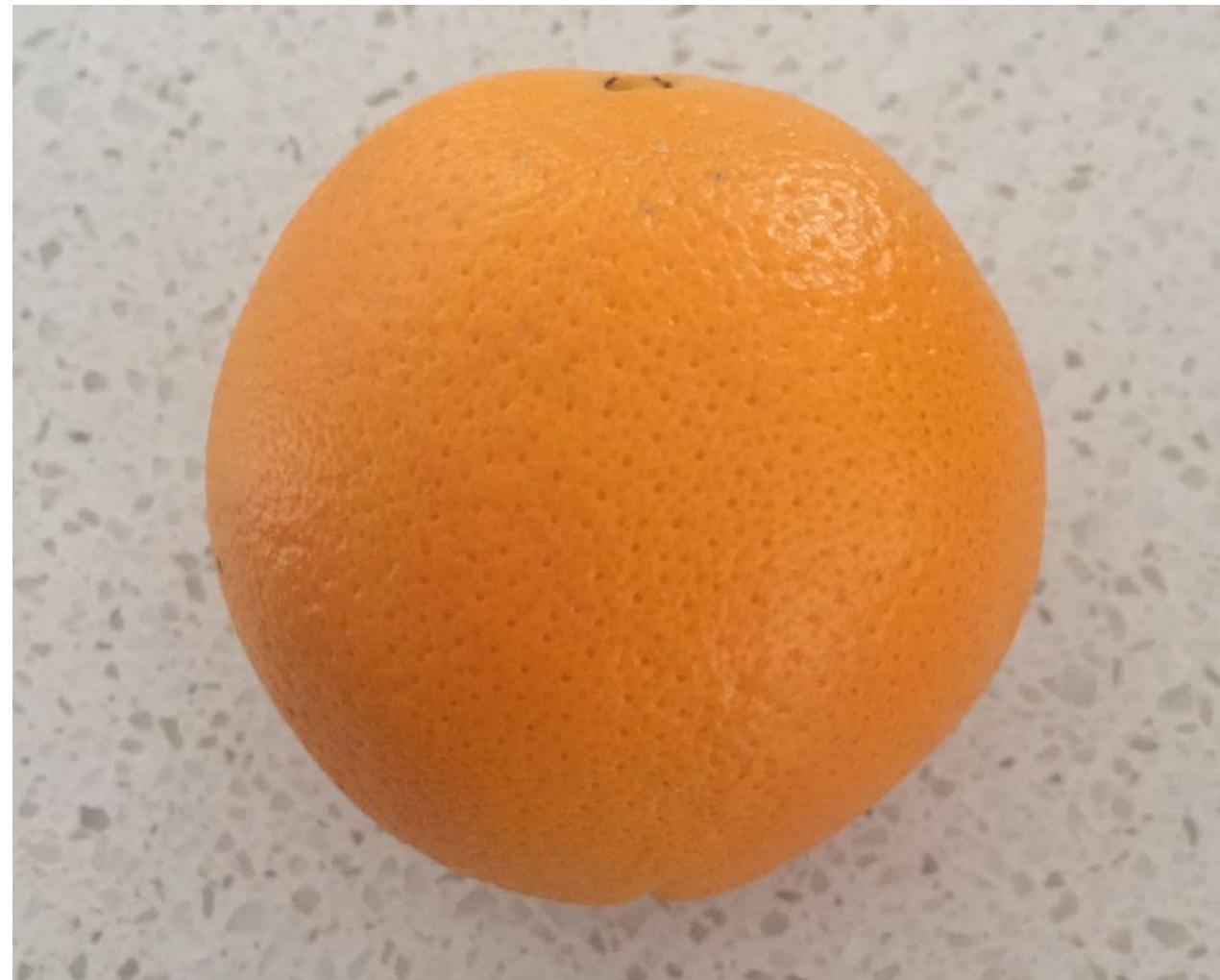
- Autoencoder (AE)
- Variational Autoencoder (VAE)
- Generative Adversarial Model (GAN)

AE

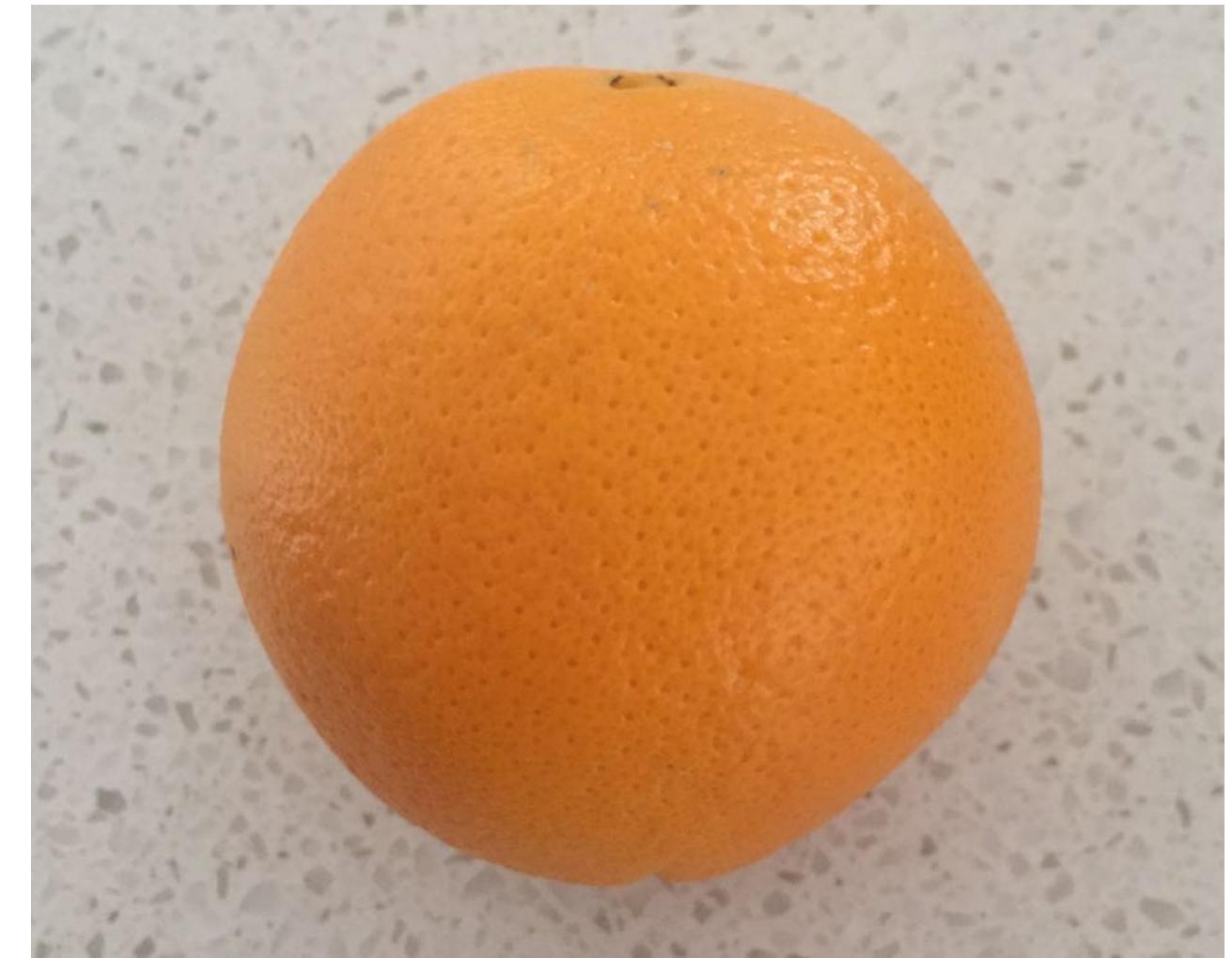
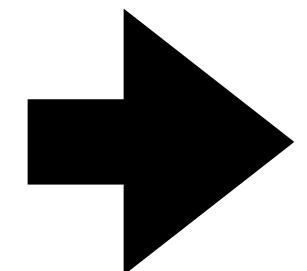
VAE

GAN

Meaningful representations can reconstruct the input data



Orange
Round



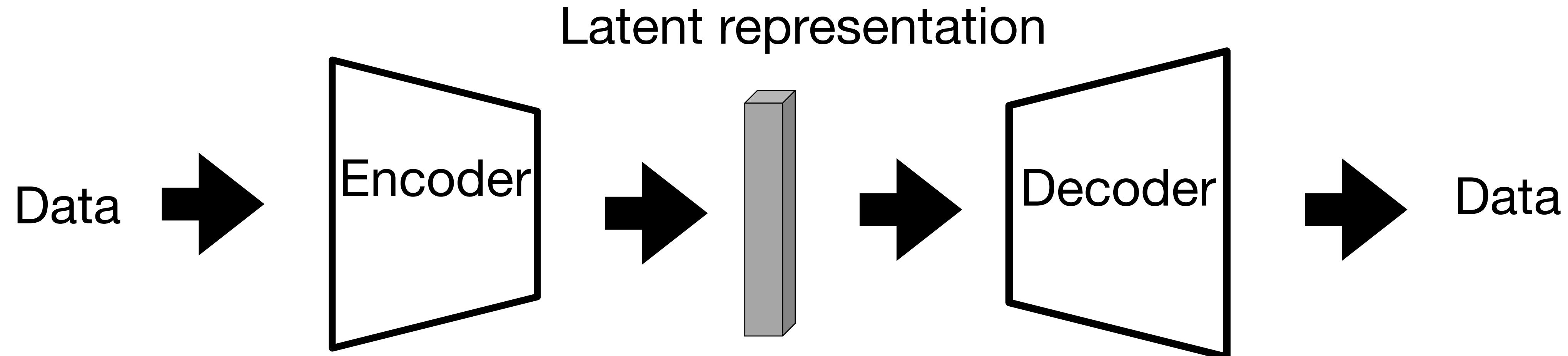
AE

VAE

GAN

Encoder - decoder

Extract meaningful representation to reconstruct the input data

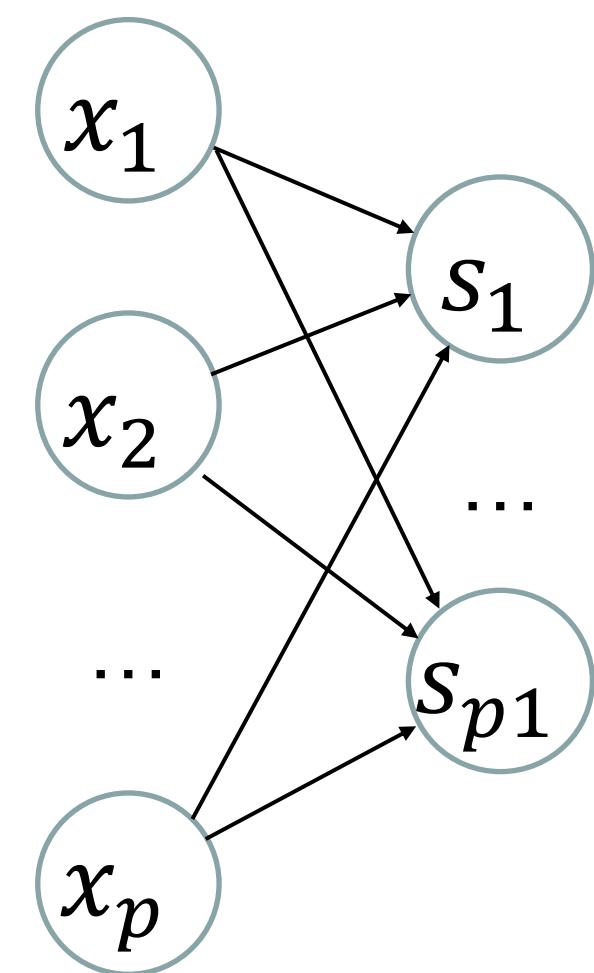
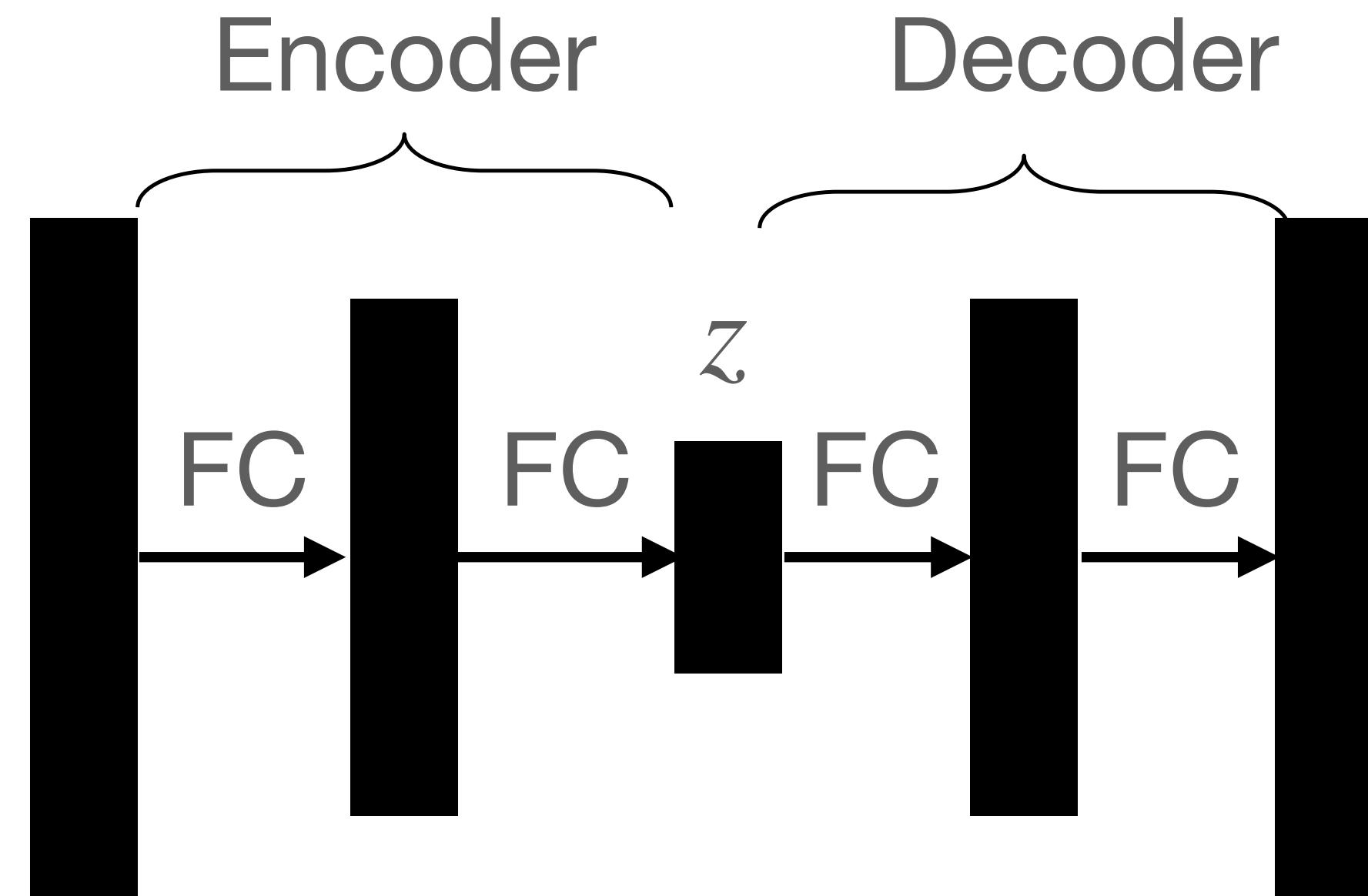


AE

VAE

GAN

Network architecture

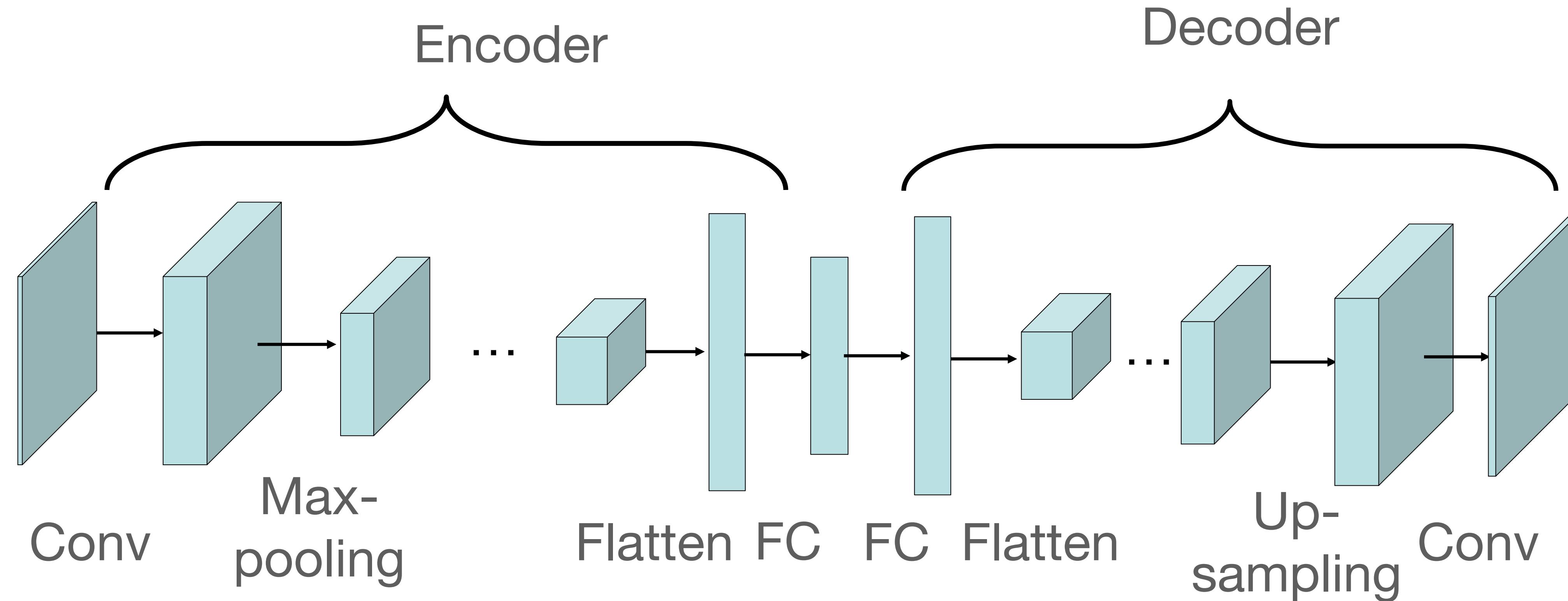


AE

VAE

GAN

Network architecture



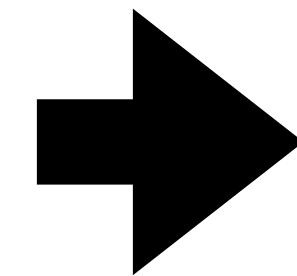
Upsampling

```
y = tf.keras.layers.UpSampling2D(size=(2, 2))(x)
```

Size: The upsampling factors for rows and columns

1	2
3	4

Upsampling



1	1	2	2
1	1	2	2
3	3	4	4
3	3	4	4

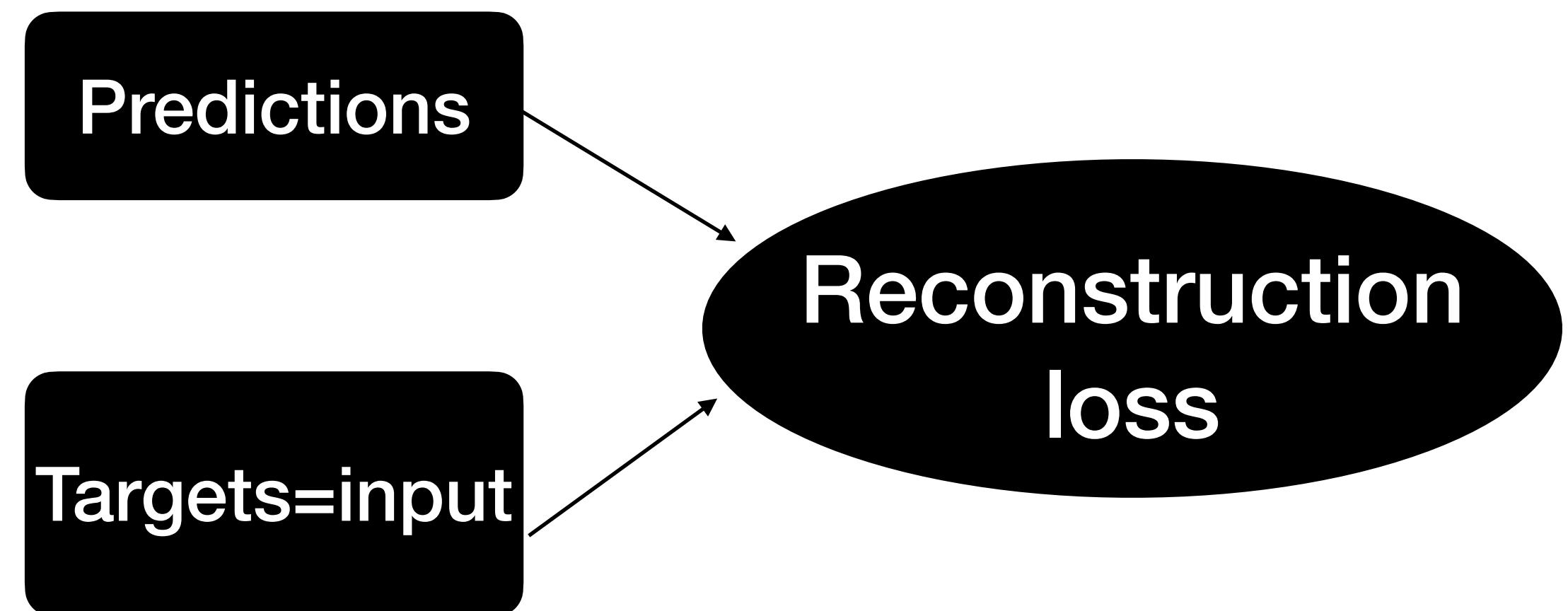
AE

VAE

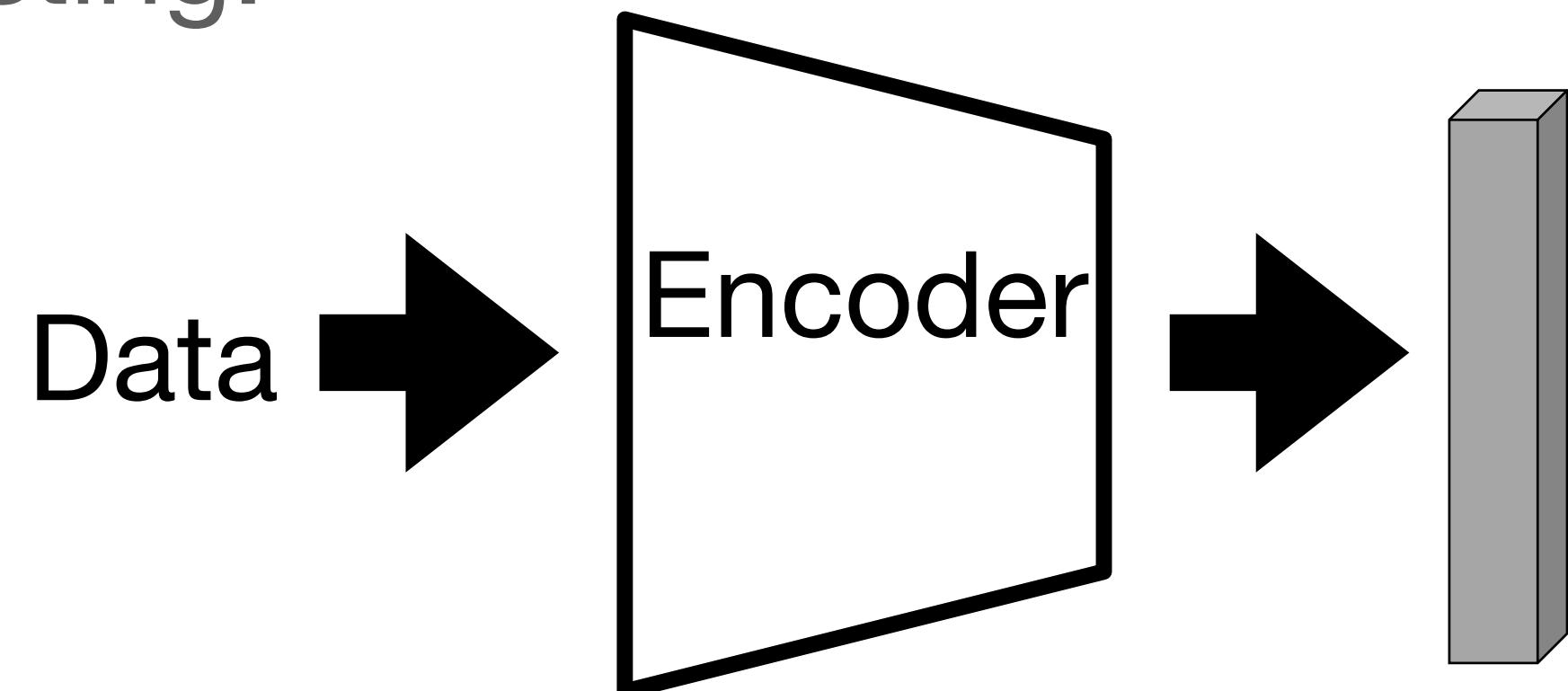
GAN

Training and testing

Training:



Testing:



Loss:

$$\text{'mse': } L = E[(X - y_{pre})^2]$$

'binary_crossentropy': Input is between 0~1

AE

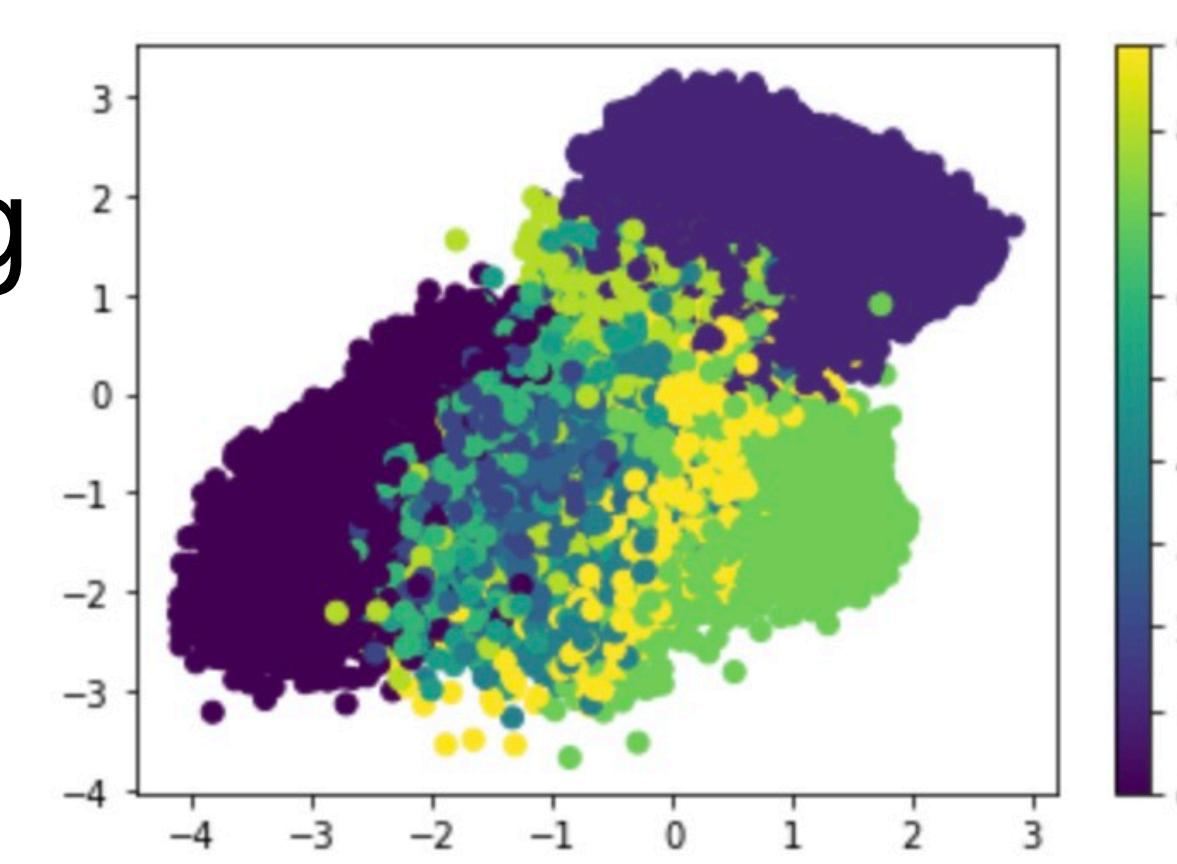
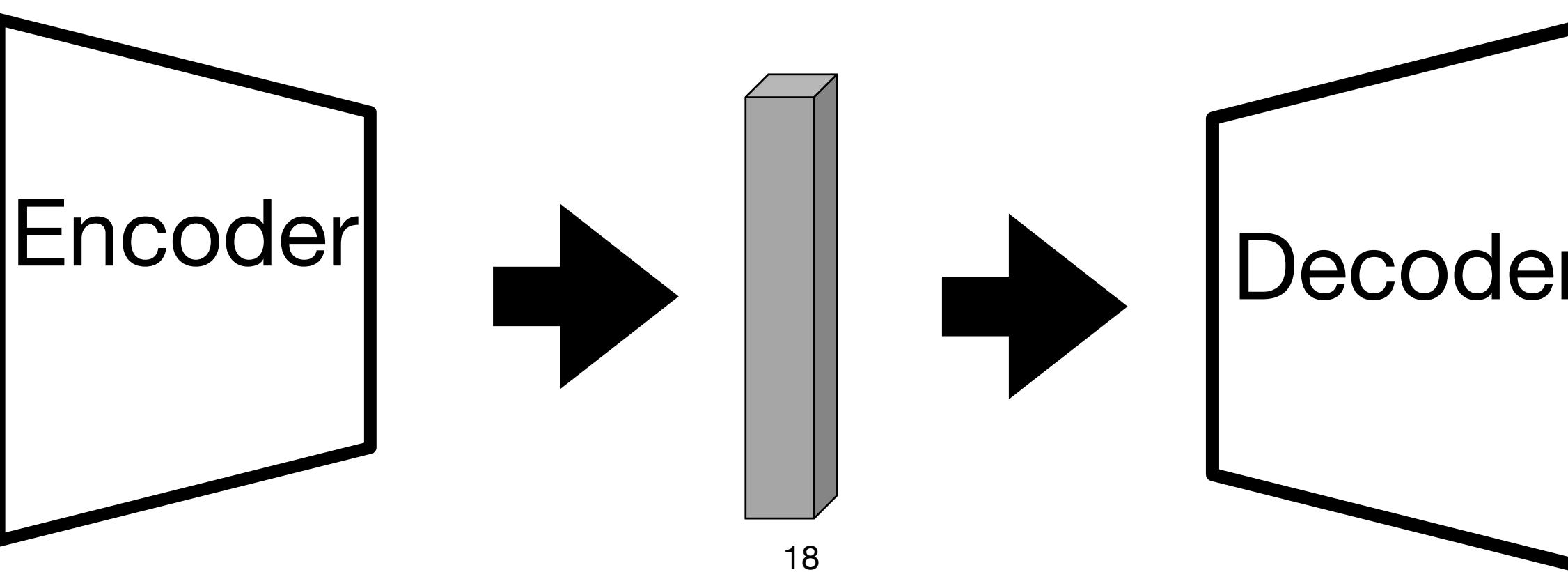
VAE

GAN

Dimension reduction

7	2	1	0	4	1	4	9	5	9
9	0	2	5	1	9	7	8	1	0
6	5	6	5	8	4	6	4	3	9
6	9	8	1	2	9	9	5	9	7
9	4	3	6	3	1	1	7	6	9
3	9	9	8	4	1	0	6	0	9
9	8	6	0	8	1	1	7	1	3
1	2	2	5	8	1	3	2	9	4
4	9	9	7	1	1	9	0	7	8
7	6	1	1	0	1	2	3	4	2

encoding



decoding

7	2	1	0	4	1	4	9	5	9
9	0	2	5	1	9	7	8	1	0
6	5	6	5	8	4	6	4	3	9
6	9	8	1	2	9	9	5	9	7
9	4	3	6	3	1	1	7	6	9
3	9	9	8	4	1	0	6	0	9
9	8	6	0	8	1	1	7	1	3
1	2	2	5	8	1	3	2	9	4
4	9	9	7	1	1	9	0	7	8
7	6	1	1	0	1	2	3	4	2

AE

VAE

GAN

Image denoising

Noisy



Clean



Predicted

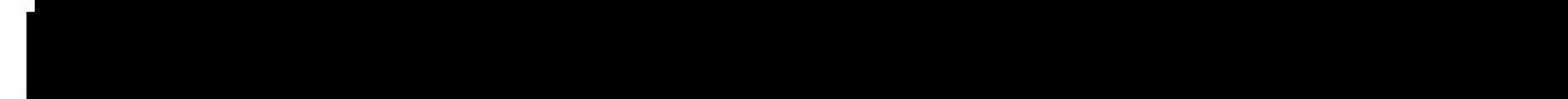


Ground-truth

Noisy



Clean



Predicted



Ground-truth

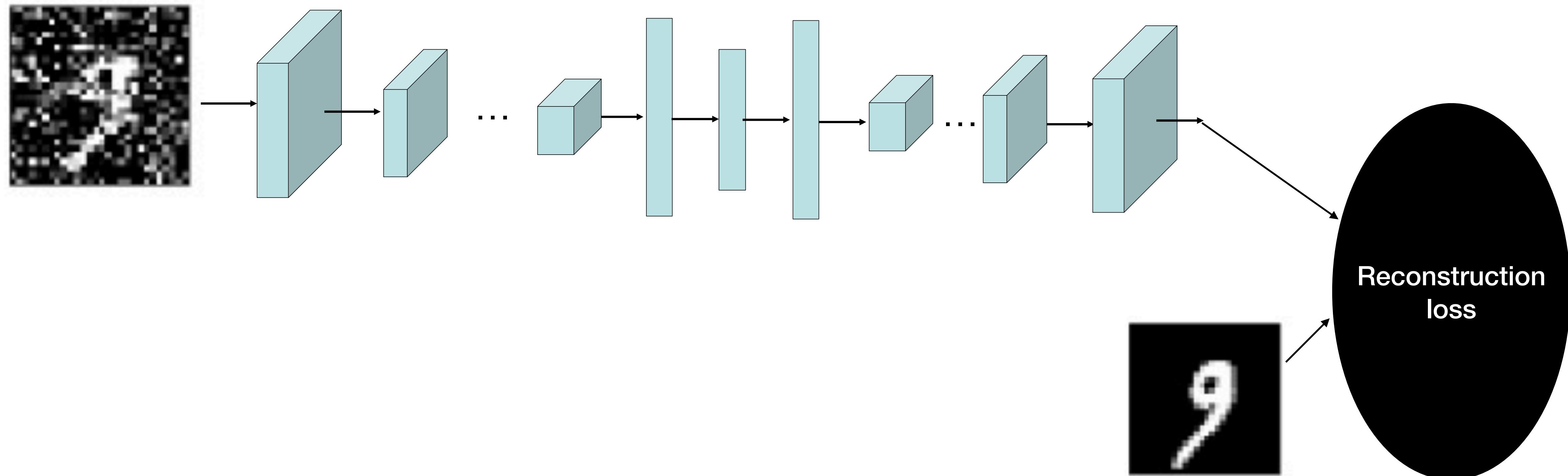
AE

VAE

GAN

Denoising Autoencoder

Training (have access to clean data):



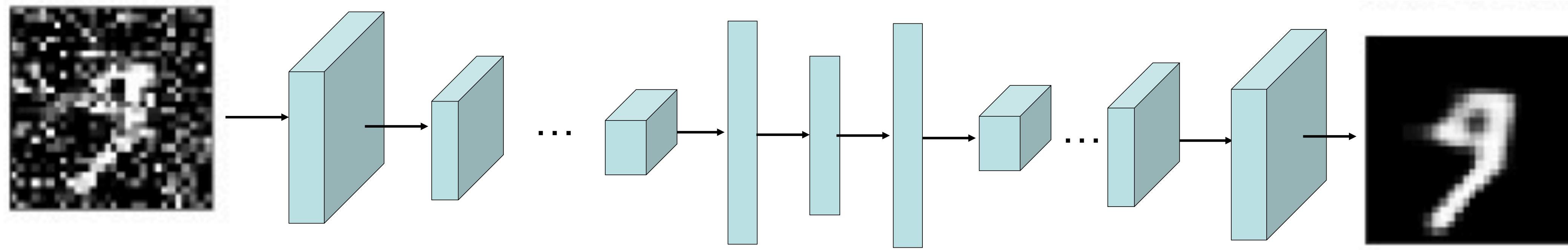
AE

VAE

GAN

Denoising Autoencoder

Testing:

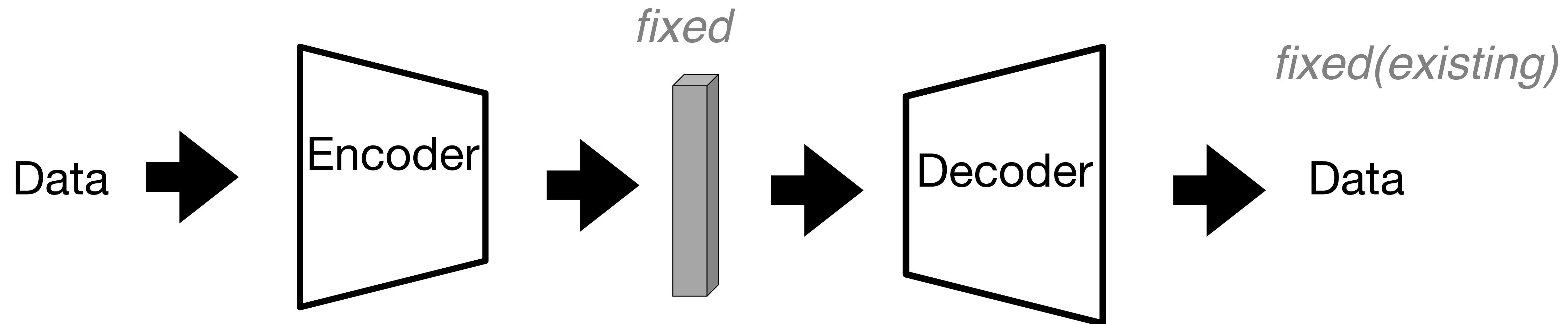


AE

VAE

GAN

Generate new data? NO

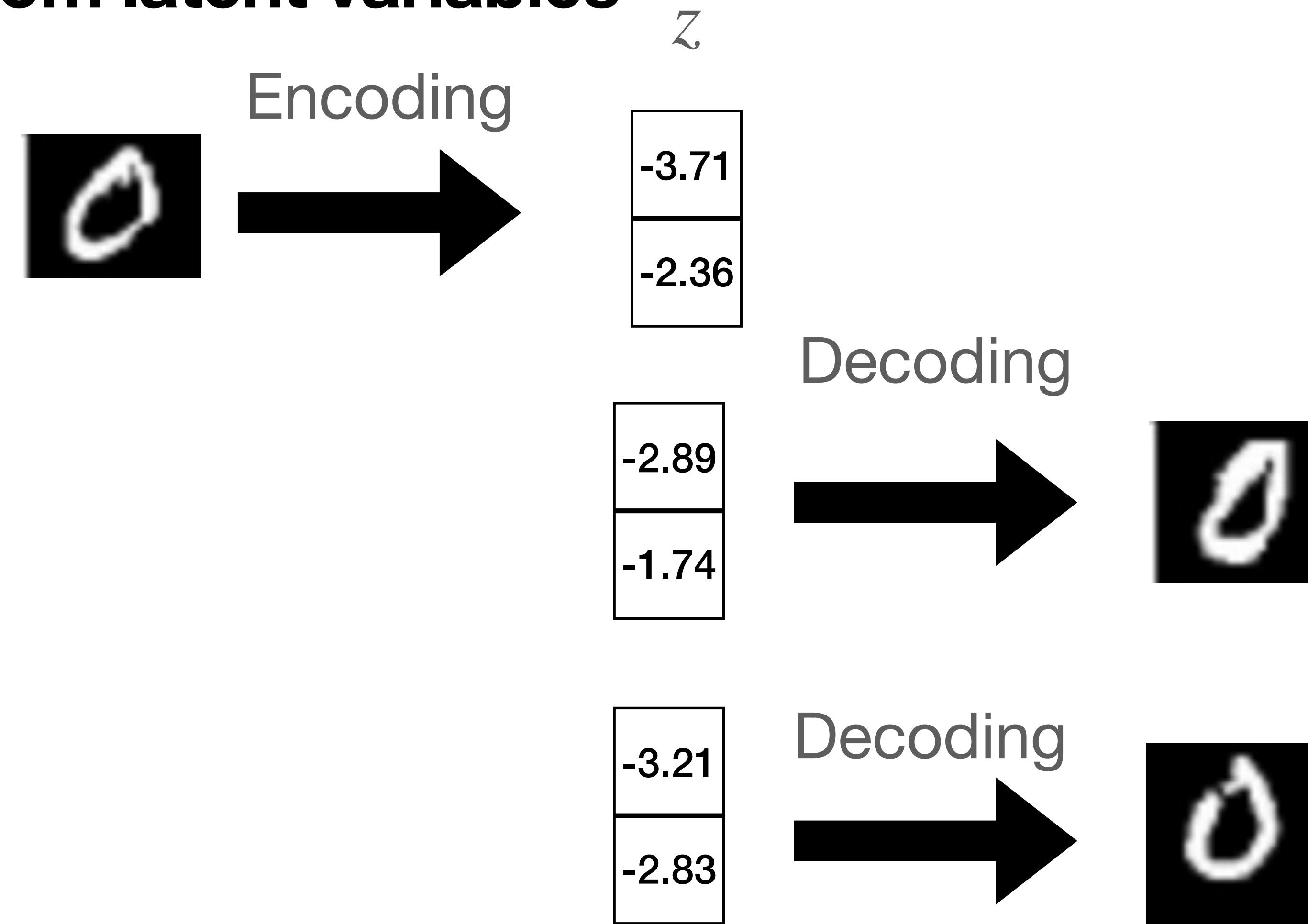


AE

VAE

GAN

Data generation from latent variables



AE

VAE

GAN

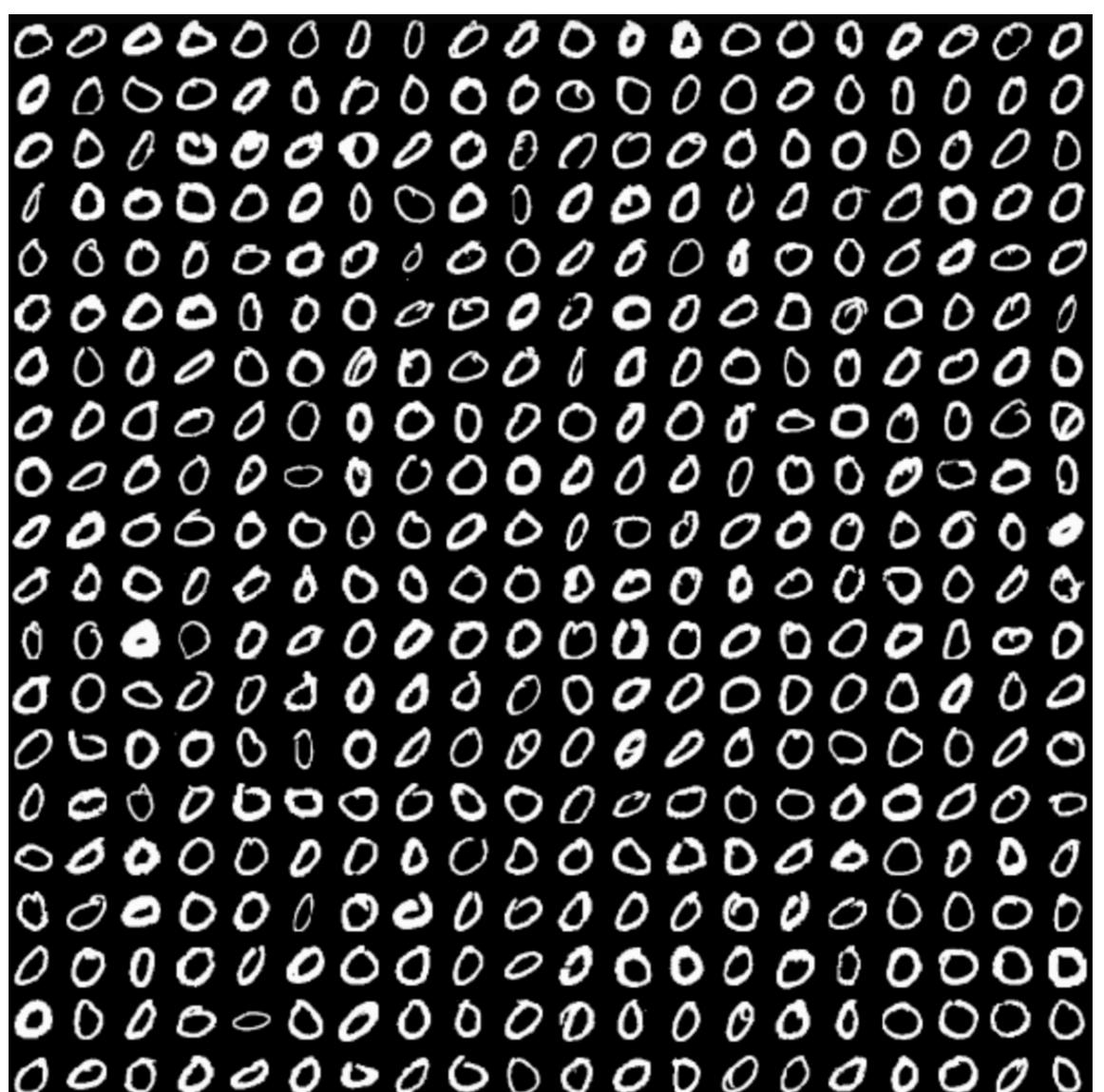
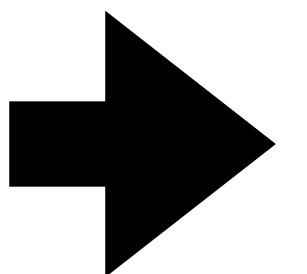
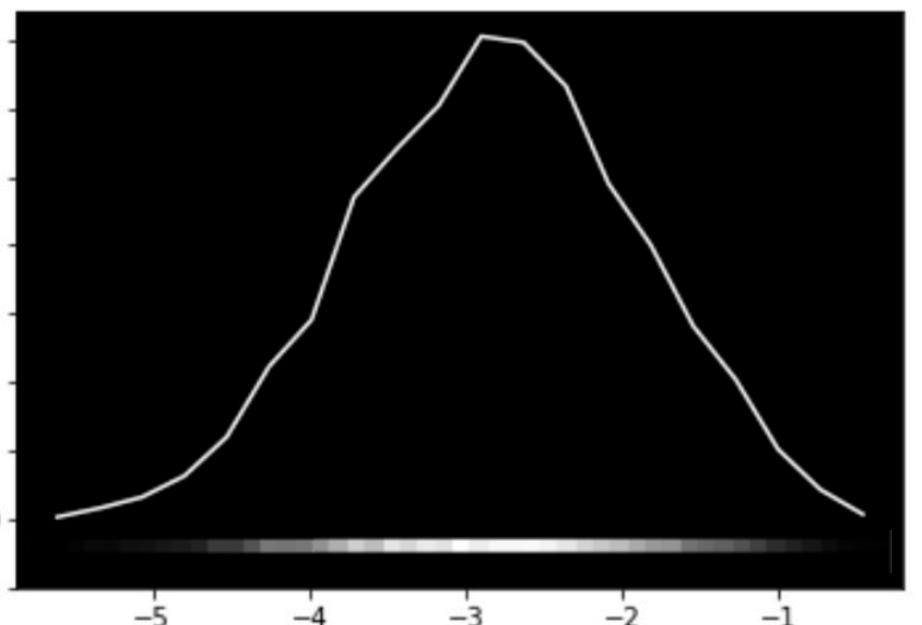
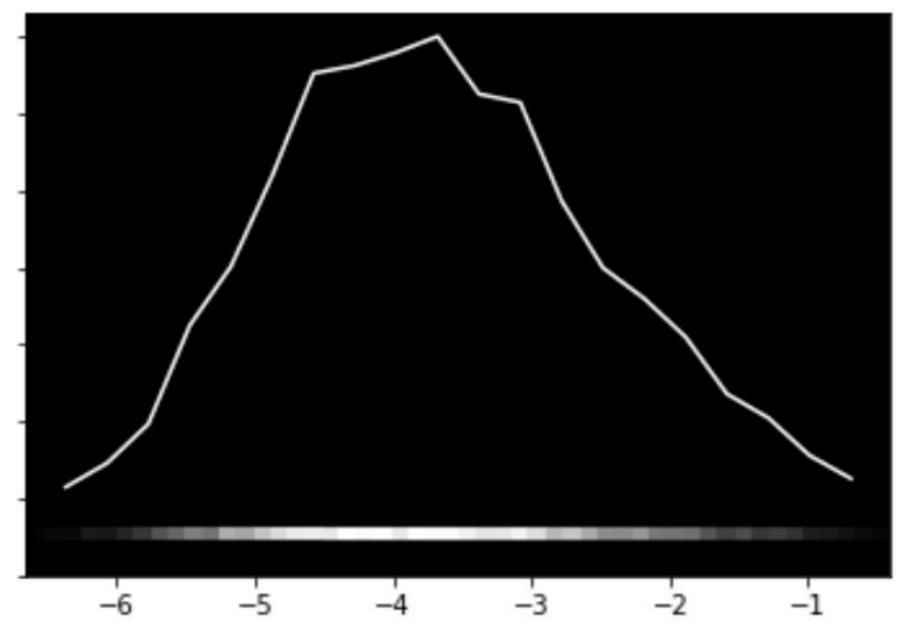
Data generation from latent variable

$$p(x) = \int p(x | z)p(z)dz$$

$p(x)$: probability of the data x

$p(z)$: probability of the latent variable z

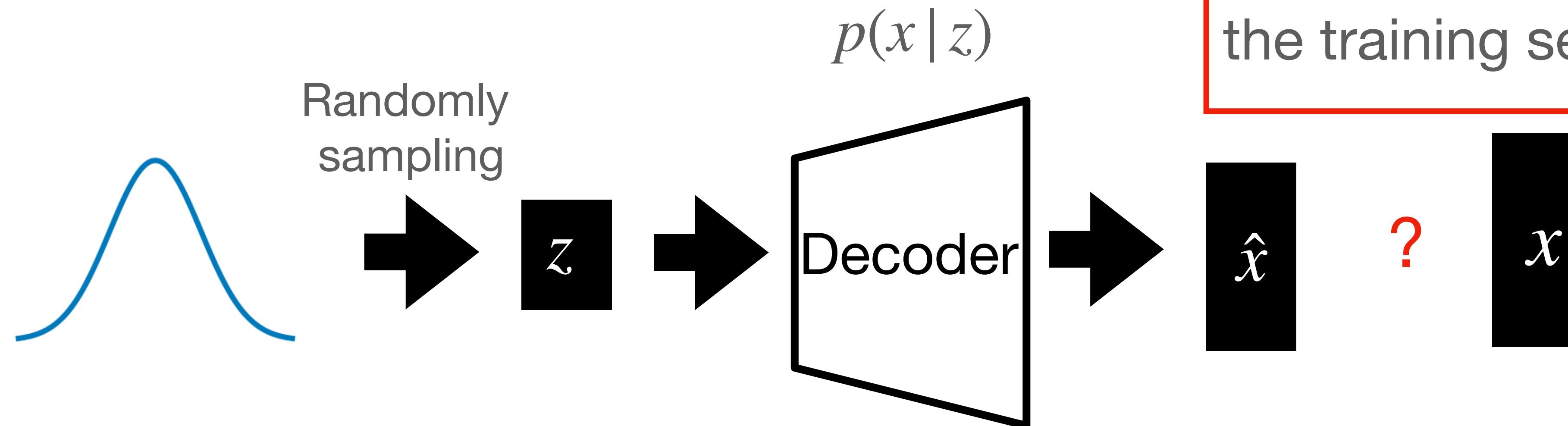
$p(x|z)$: probability of x given z



Turn latent variable into data using a decoder (network)

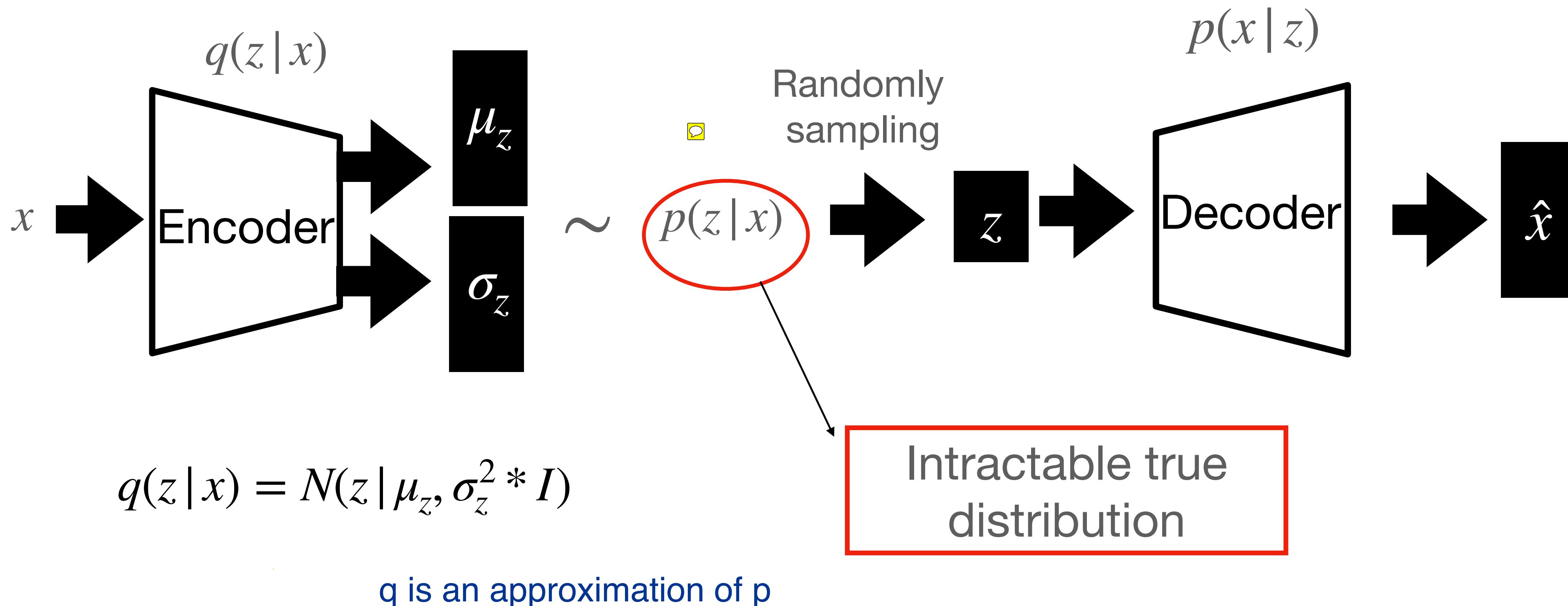
Assume: $p(z) = N(z | 0, I)$

I: identity matrix



Encoder: Reduce sampling space

Use posterior distribution to sample z is more likely to produce x

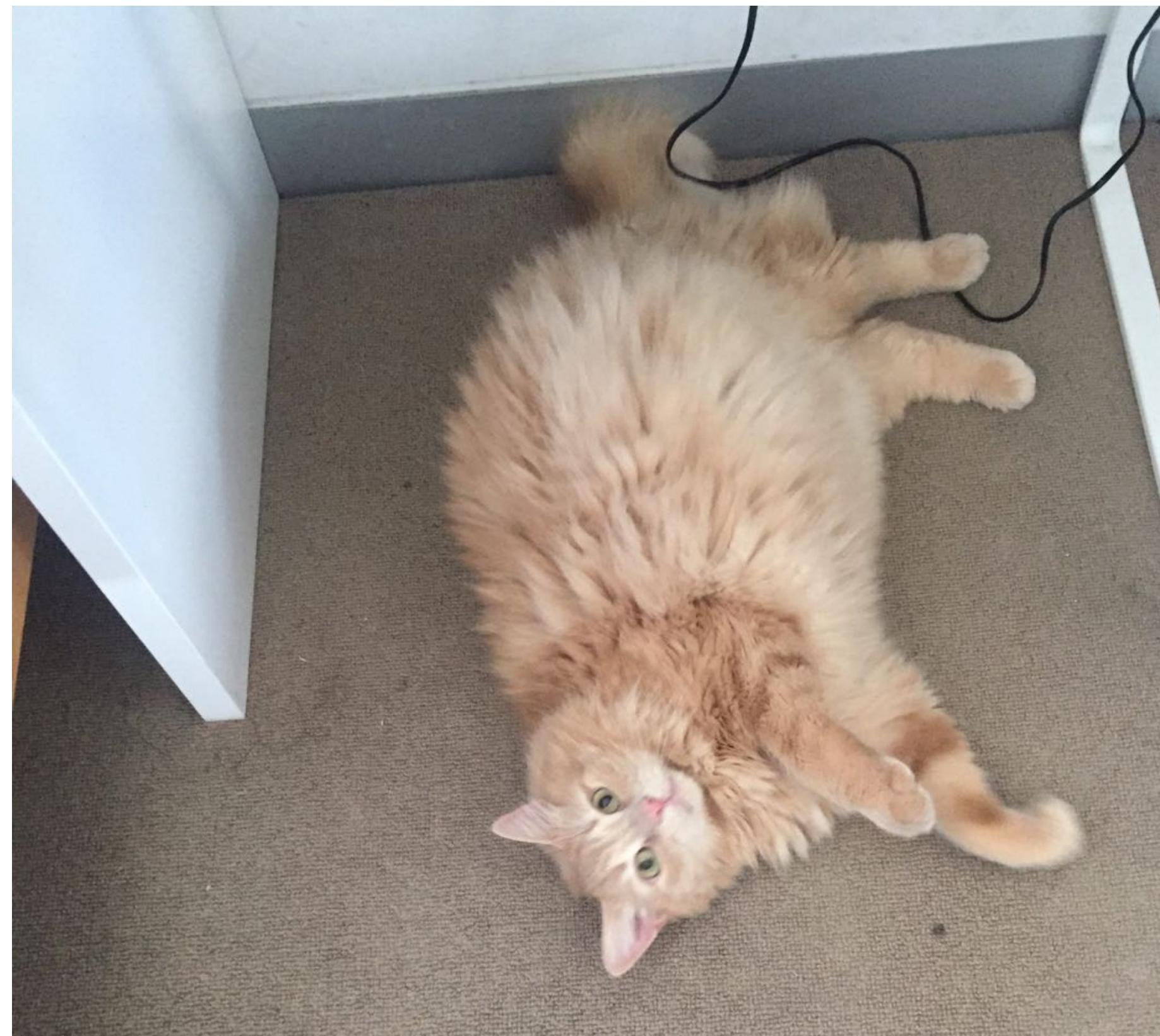


AE

VAE

GAN

Math is coming...



Network optimisation: Maximize log likelihood and gradient ascend

$$\begin{aligned}
 \log p(x^{(i)}) &= E_{z \sim q(z|x)} [\log p(x^{(i)})] \\
 &= E_{z \sim q(z|x)} \left[\log \frac{p(x^{(i)}|z)p(z)}{p(z|x^{(i)})} \right] \quad \xleftarrow{\text{Reduction}} \quad = E_{z \sim q(z|x)} \left[\log \frac{p(x^{(i)}|z)p(z)}{p(z|x^{(i)})} \frac{q(z|x^{(i)})}{q(z|x^{(i)})} \right] \\
 &= E_{z \sim q(z|x)} [\log p(x^{(i)}|z)] - \boxed{E_{z \sim q(z|x)} \left[\log \frac{q(z|x^{(i)})}{p(z)} \right]} + \boxed{E_{z \sim q(z|x)} \left[\log \frac{q(z|x^{(i)})}{p(z|x^{(i)})} \right]} \\
 &= E_{z \sim q(z|x)} [\log p(x^{(i)}|z)] - D_{KL} [q(z|x^{(i)}) || p(z)] + D_{KL} [q(z|x^{(i)}) || p(z|x^{(i)})]
 \end{aligned}$$

$\begin{array}{l} p(x, z) \\ = p(x|z)p(z) \\ = p(z|x)p(x) \end{array}$

KL
Divergence

Maximize lower bound

Unknown true posterior

$$\begin{aligned} \log p(x^{(i)}) &= E_{z \sim q(z|x)} [\log p(x^{(i)}|z)] - D_{KL} [q(z|x^{(i)}) || p(z)] + D_{KL} [q(z|x^{(i)}) || p(z|x^{(i)})] \\ &\geq E_{z \sim q(z|x)} [\log p(x^{(i)}|z)] - D_{KL} [q(z|x^{(i)}) || p(z)] \end{aligned}$$

lower bound

$\max E_{z \sim q(z|x)} [\log p(x^{(i)}|z)]$: Minimize reconstruction loss between output and input

$q(z|x)$: Output of encoder $q(z) = N(z|\mu, \sigma^2 I)$

$p(z)$: Prior distribution, assumed to be standard Gaussian distribution $p(z) = N(z|0, I)$

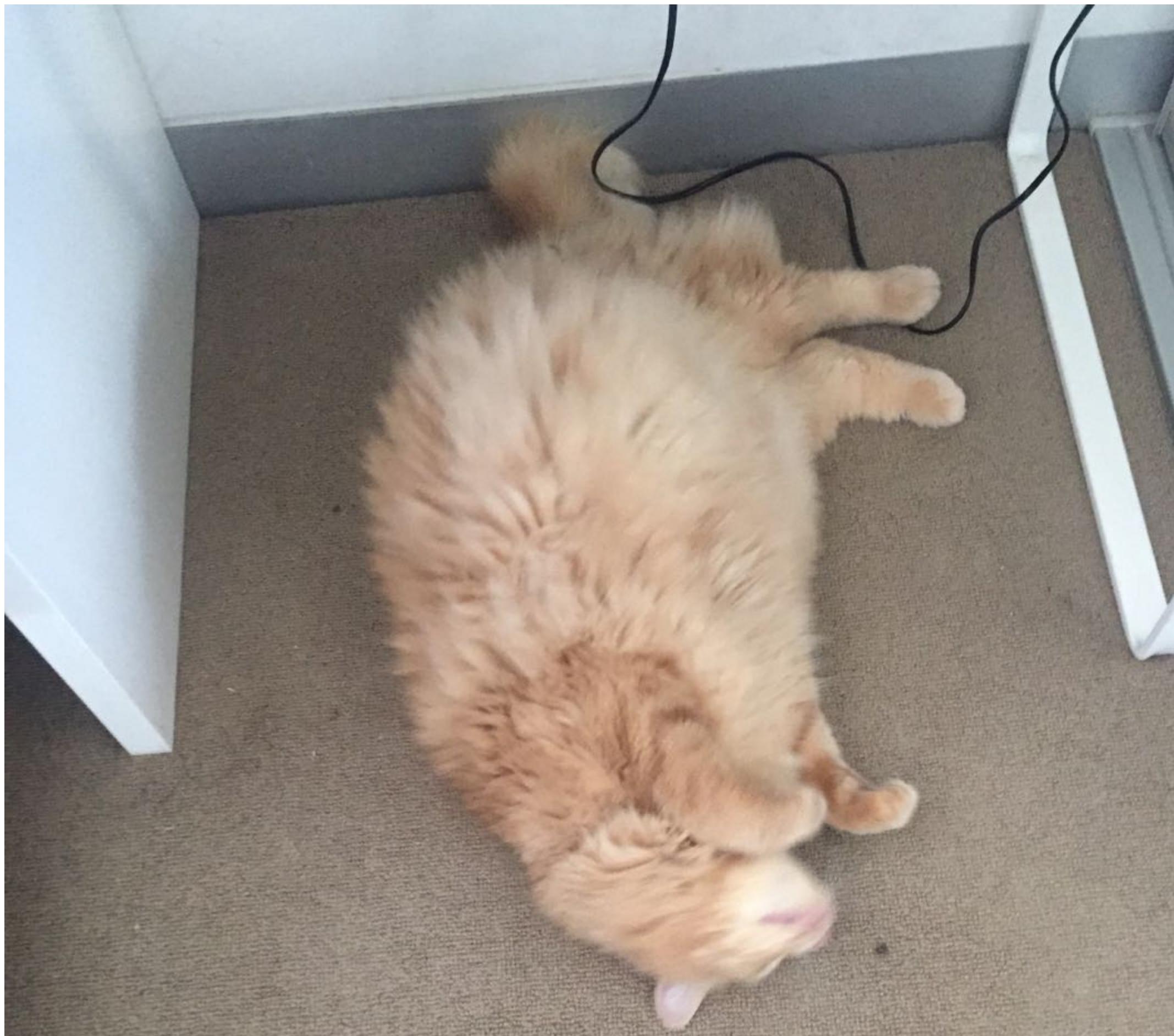
$D_{KL} [q(z|x^{(i)}) || p(z)]$: KL divergence of two gaussian distributions

AE

VAE

GAN

Math is coming AGAIN...



AE

VAE

GAN

Warning of math: KL divergence

$$KL(p_1 || p_2) = \int p_1(x) \log \frac{p_1(x)}{p_2(x)} dx \quad p_1(x) = \frac{1}{\sigma_1 \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x - \mu_1}{\sigma_1} \right)^2} \quad p_2(x) = \frac{1}{\sigma_2 \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x - \mu_2}{\sigma_2} \right)^2}$$

$$= \int [-\log \sigma_1 - \frac{1}{2}(\log(2\pi)) - \frac{1}{2} \left(\frac{x - \mu_1}{\sigma_1} \right)^2 - (-\log \sigma_2 - \frac{1}{2}(\log(2\pi)) - \frac{1}{2} \left(\frac{x - \mu_2}{\sigma_2} \right)^2] p_1(x) dx$$

$$= -\log \sigma_1 + \log \sigma_2 - \frac{1}{2\sigma_1^2} E_1[(x - \mu_1)^2] + \frac{1}{2\sigma_2^2} E_1[(x - \mu_2)^2]$$

$$= -\log \sigma_1 + \log \sigma_2 - \frac{1}{2} + \frac{1}{2\sigma_2^2} E_1[(x - \mu_2)^2]$$

AE

VAE

GAN

Warning of math: KL divergence

$$\begin{aligned}
 KL(p_1 || p_2) &= \int p_1(x) \log \frac{p_1(x)}{p_2(x)} dx = -\log \sigma + \log \sigma_2 - \frac{1}{2} + \frac{1}{2\sigma_2^2} E_1[(x - \mu_2)^2] \\
 p_1(x) \sim N(\mu, \sigma^2) \quad p_2(x) \sim N(0, 1) &\quad = -\log \sigma + 0 - \frac{1}{2} + \frac{1}{2} E_1(x^2) \\
 KL(p_1, p_2) &= -\log \sigma - \frac{1}{2} + \frac{1}{2} (\sigma^2 + \mu^2) \quad \left. \begin{array}{l} \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \quad \left. \begin{array}{l} \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \\
 &= \frac{1}{2} (-\log \sigma^2 + \sigma^2 + \mu^2 - 1)
 \end{aligned}$$

$x^2 = (x - \mu)^2 + 2\mu x - \mu^2$
 $E(x^2) = E(x^2)$
 $E(x^2) = \sigma^2 + \mu^2$

$$= \frac{1}{2} (-\log \sigma^2 + \sigma^2 + \mu^2 - 1)$$

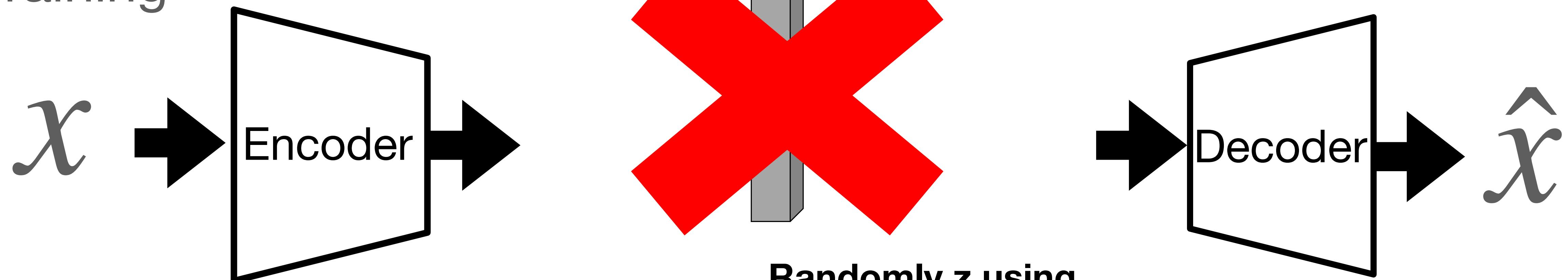
AE

VAE

GAN

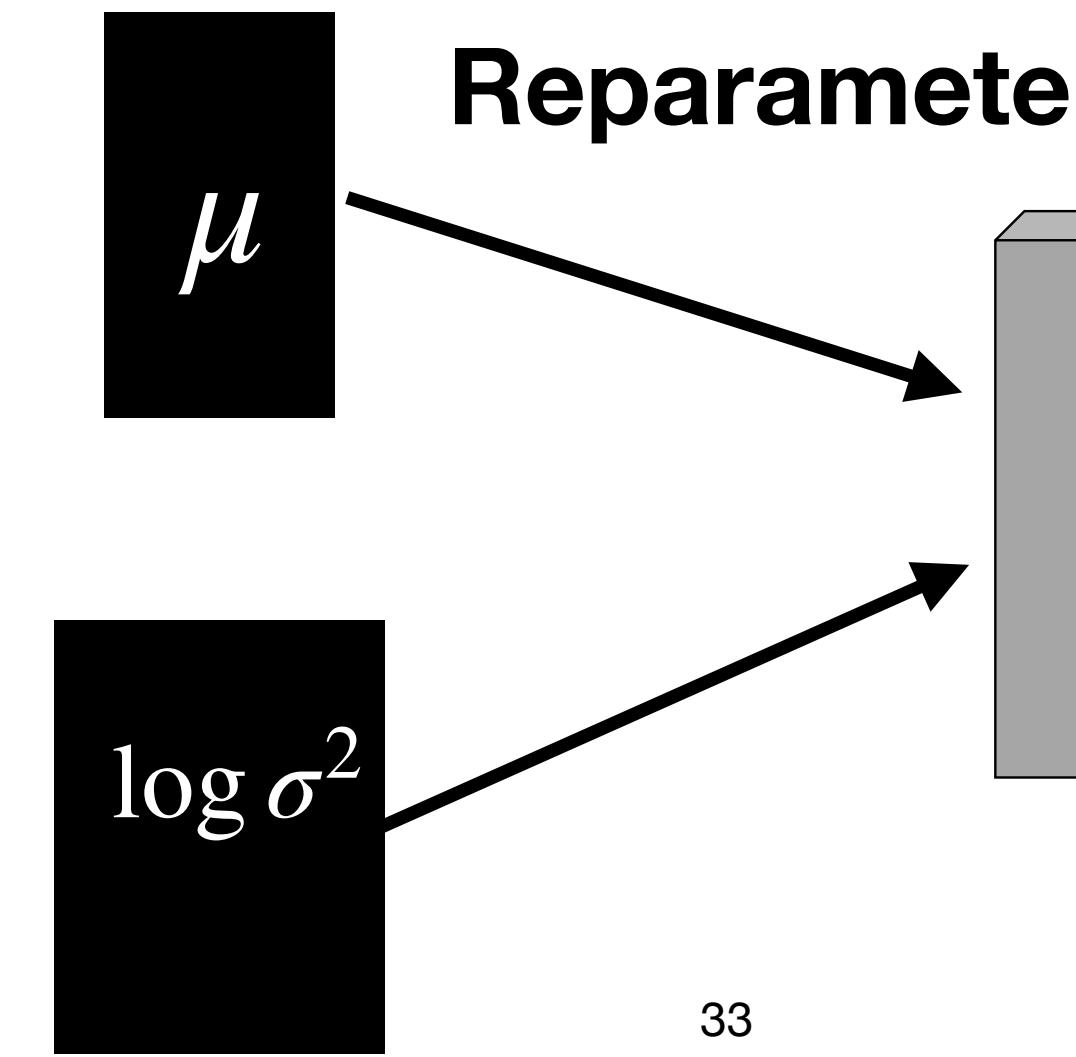
(Summary) Different from AE: estimate the distribution of z

Training



Randomly z using
Reparameterization trick

**For easy and stable
training: output $\log \sigma^2$**

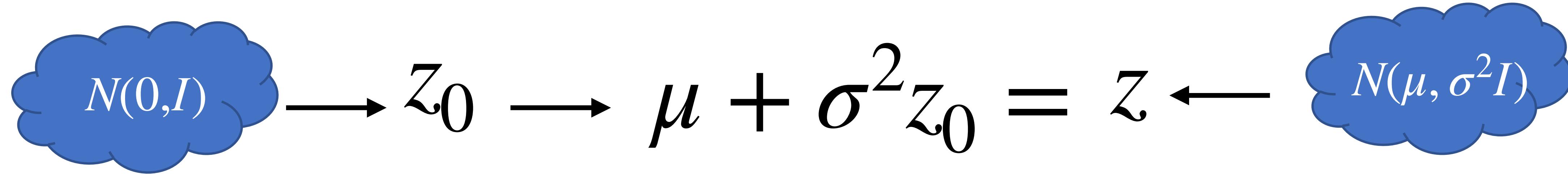


AE

VAE

GAN

Sample z: Reparameterization trick to make network differentiable



```
z0 = K.random_normal(shape=(batch_size, z_dim), mean=0., stddev=1.0)
z=mu + K.exp(log_var/2) * z0
```

AE

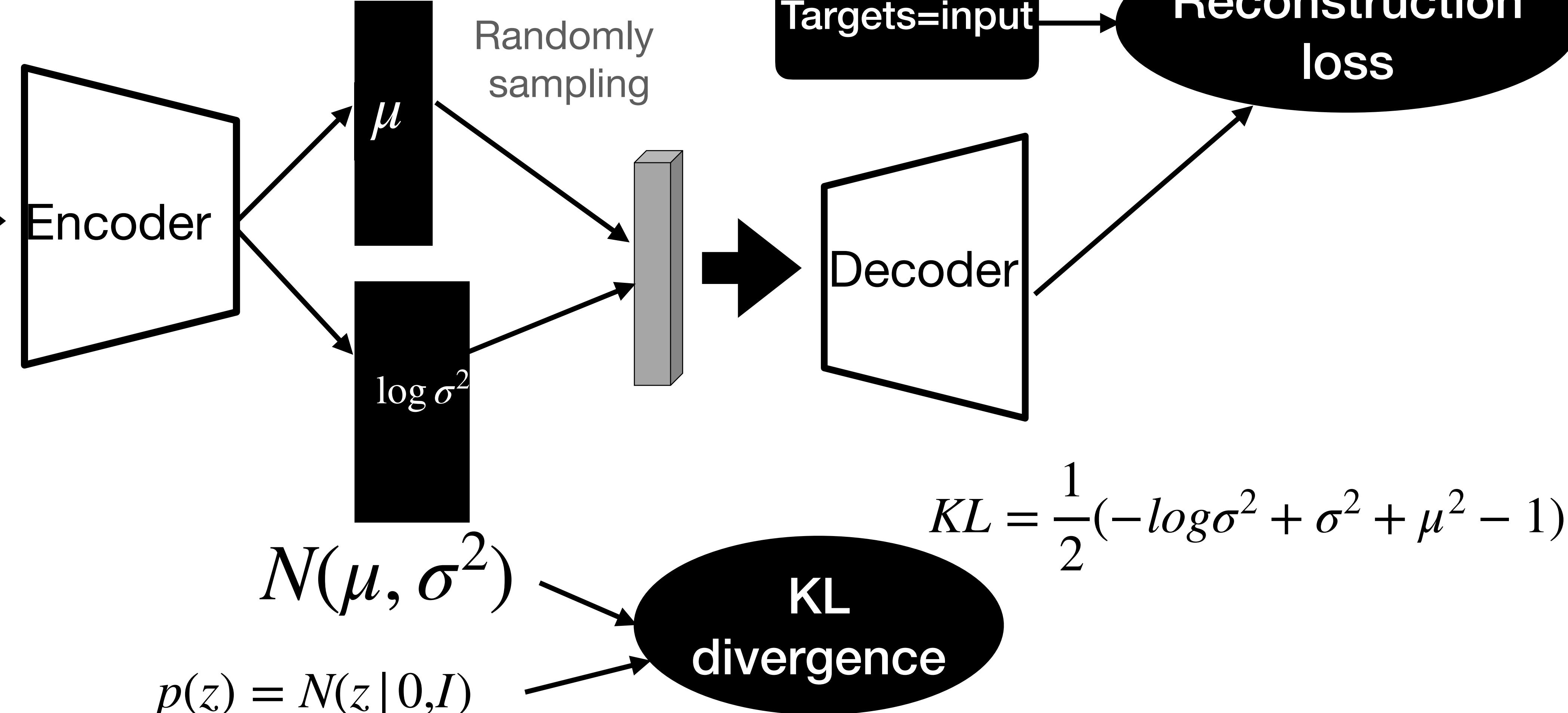
VAE

GAN

(Summary) Different from AE: Two losses

Training

Data

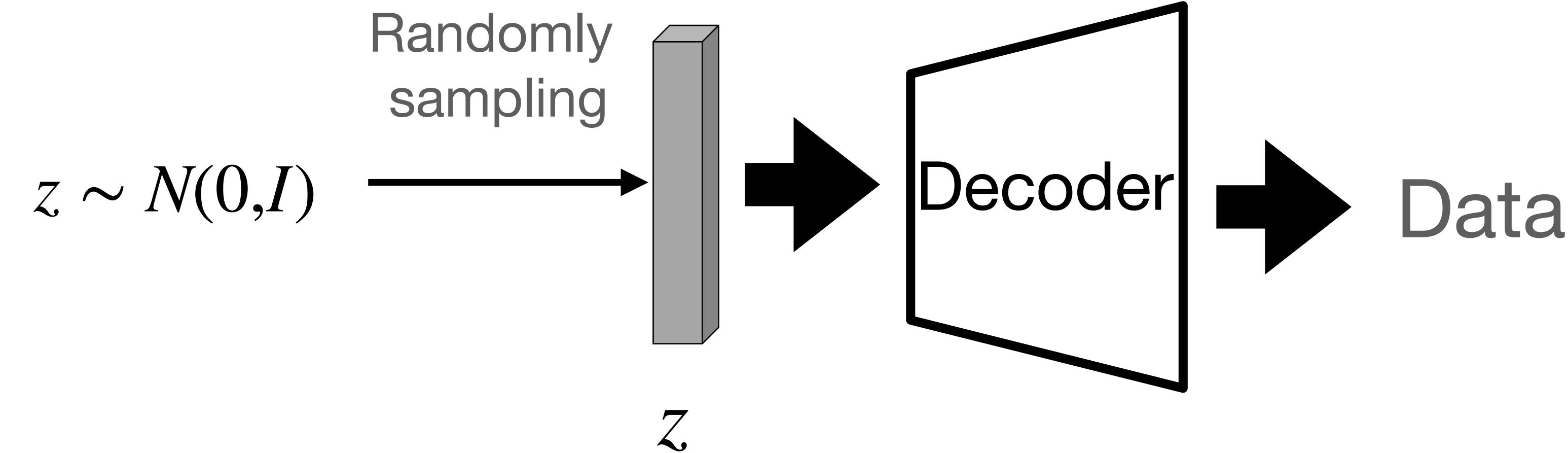


AE

VAE

GAN

Testing



AE

VAE

GAN

Testing

Random z

$$z \sim N(0, I)$$



Random digit

3 7 6 7 / 9 1 9 6 5 0 8 1 9 / 9 8 7 3 / 1 3 9 8 5 9 6 2 5 1
7 5 7 6 9 2 6 9 / 8 0 9 0 7 9 5 6 0 9 7 6 7 0 9 5 1 2 0 6 0
9 0 9 8 5 6 1 8 1 3 6 9 2 6 5 5 3 2 9 1 9 9 9 2 1 1 3 1 5 8
9 7 2 8 2 8 6 0 6 2 / 6 9 6 9 2 2 0 7 3 9 9 5 6 3 7 7 8 2 8
5 6 9 4 9 4 8 6 9 2 5 5 9 7 5 9 2 2 3 7 1 9 6 9 7 8 9 5 9
1 0 1 0 2 1 3 6 7 1 7 6 3 5 8 0 9 0 7 9 8 2 1 7 7 2 8 3 1 9 0
7 9 5 0 3 1 9 0 5 7 6 9 2 6 7 9 2 8 4 1 6 6 7 6 0 0 1 1 5 2
9 9 6 2 0 5 5 0 6 3 9 6 6 5 7 6 9 8 7 0 8 0 1 2 1 3 9 1 5 1
1 9 5 7 2 9 5 3 2 1 9 9 9 5 9 9 9 6 5 6 8 5 5 8 9 9 9 7 7 0
7 6 9 3 9 9 2 5 6 6 0 9 5 0 1 9 0 9 3 2 9 2 4 9 4 8 9 0 6 1
0 4 1 3 6 9 6 8 6 5 9 4 9 8 7 5 7 6 9 7 5 2 2 1 1 5 2 3 8 9
1 5 4 5 9 1 7 5 3 1 0 3 9 2 1 8 6 5 6 2 8 8 7 5 6 9 8 9 8 9 3
8 4 9 9 2 5 9 7 7 9 0 7 6 9 9 3 7 8 7 5 6 7 6 4 6 5 2 0 1 9
2 3 9 3 0 9 9 0 9 2 9 1 5 8 1 8 5 0 7 7 2 9 6 0 9 5 5 0 0 7
8 5 7 0 3 0 2 6 9 2 1 6 9 6 6 2 3 9 6 2 1 3 2 7 5 3 7 8 7 4
9 5 2 3 8 8 8 0 0 5 3 1 9 7 8 3 9 5 0 1 5 2 0 2 8 9 0 6 5 9
9 1 9 4 6 5 1 5 7 5 8 3 7 5 0 0 9 6 9 6 1 5 2 9 0 2 3 9 1 9
5 8 9 6 9 9 6 9 8 5 2 1 2 8 3 3 2 2 1 0 4 7 9 8 7 9 3 3 2 9
8 3 9 6 7 0 2 5 7 5 5 7 9 5 2 2 0 6 7 9 5 3 2 5 0 9 6 0 4 6
5 7 2 6 2 9 6 1 9 2 1 7 0 4 9 7 8 7 1 1 0 3 1 0 2 4 8 6 5 6
8 7 6 0 9 7 3 9 8 2 6 3 5 9 2 7 1 7 3 4 7 0 8 7 1 6 7 6 7 8
0 9 1 0 6 2 8 9 5 9 4 9 3 5 1 4 5 8 2 6 6 0 7 5 0 1 3 3 1 9
2 9 0 8 6 9 9 1 9 9 6 7 6 0 9 8 8 5 5 9 6 4 7 0 5 1 8 7 6
0 2 9 0 7 3 7 2 5 5 6 2 3 4 6 9 5 9 2 9 1 4 1 9 0 8 0 5 3
0 8 1 8 7 9 6 9 6 0 8 6 1 9 0 8 1 0 1 9 3 6 3 2 9 6 2 6 0 3
7 7 8 9 7 8 9 5 2 8 9 5 7 0 7 2 8 1 2 2 9 9 7 9 7 0 2 9 9 0
3 2 1 9 0 6 3 0 8 9 9 1 8 5 6 9 8 9 2 1 9 6 6 7 8 9 0 5 6 5
7 2 8 1 7 8 1 1 6 9 8 7 1 8 4 4 9 7 0 1 4 7 0 6 3 2 9 0 0
3 0 2 2 1 9 3 6 8 0 5 5 9 1 1 0 9 9 8 0 0 3 0 9 9 5 9 2 2
1 6 9 9 7 7 9 9 9 1 0 1 6 3 9 2 7 9 9 7 5 5 9 9 4 1 9 5 1 8

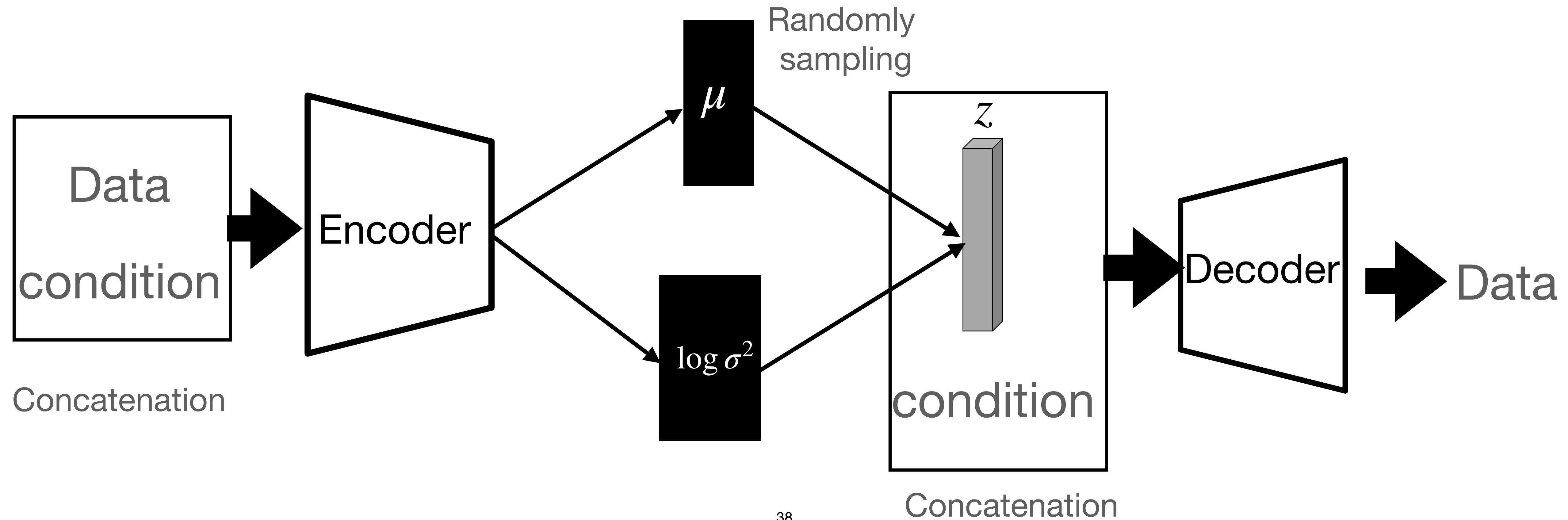
AE

VAE

GAN

Conditional VAE

condition: $c = [1, 0, 0, \dots, 0]^T$



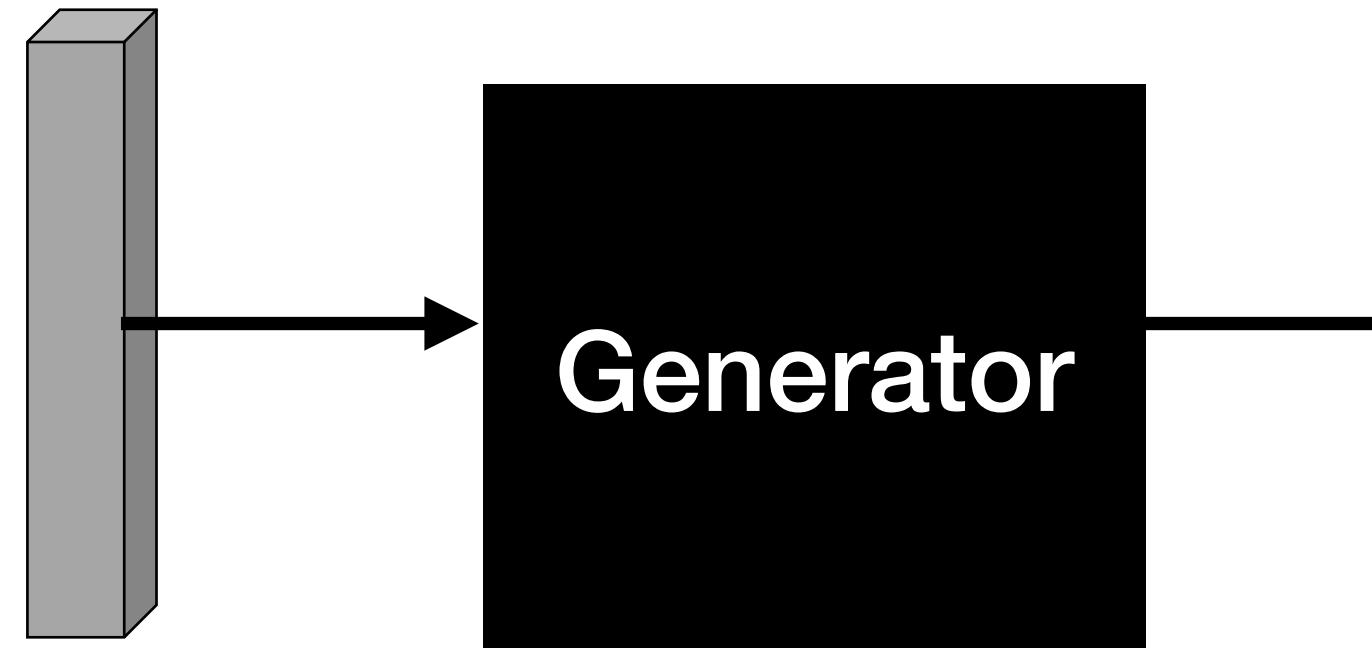
AE

VAE

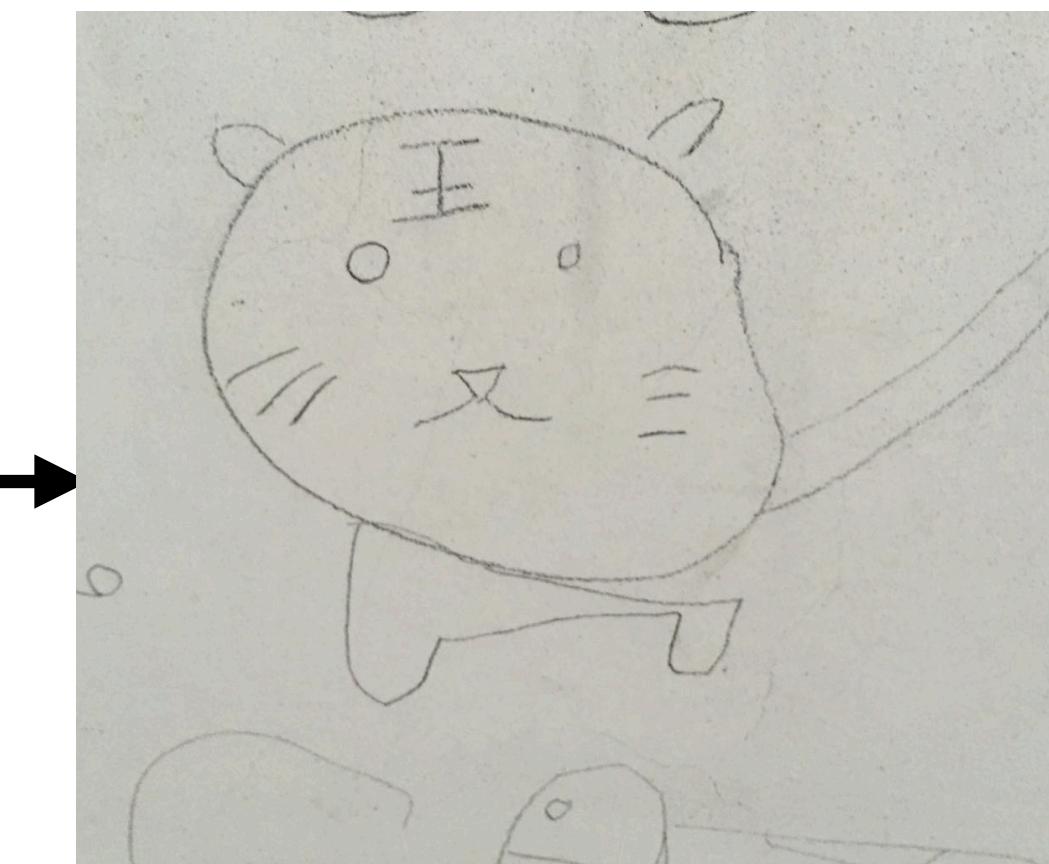
GAN

Two-player game

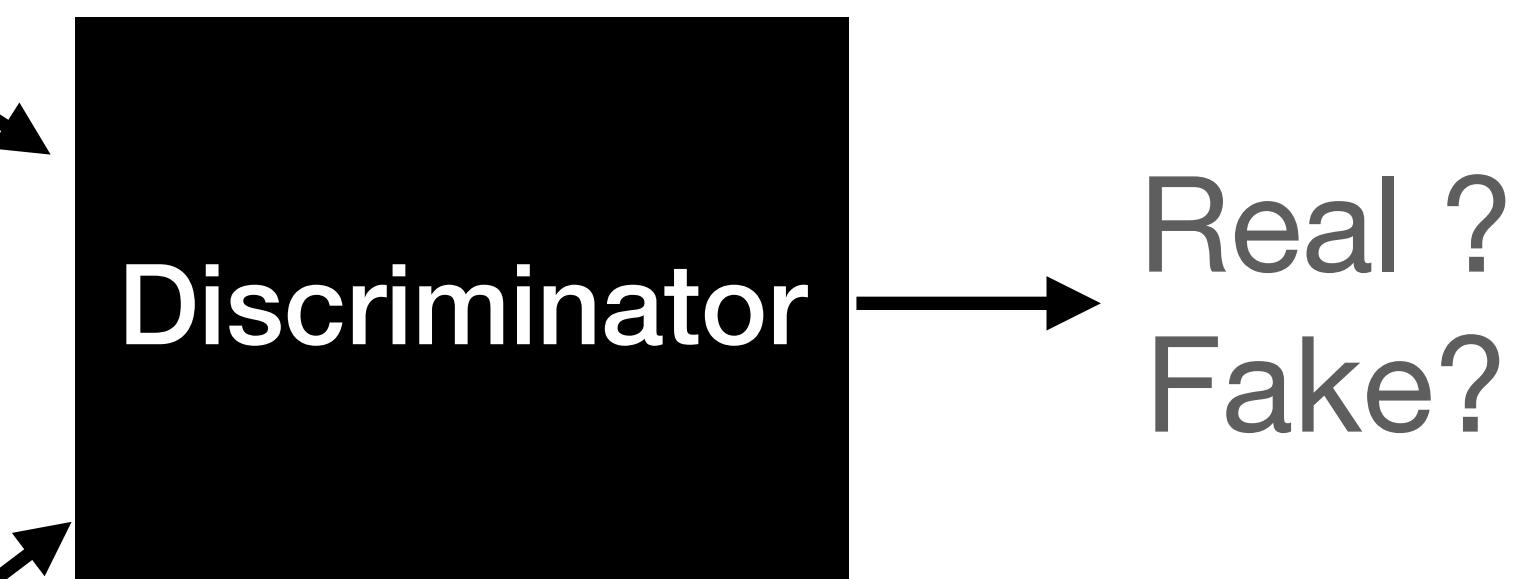
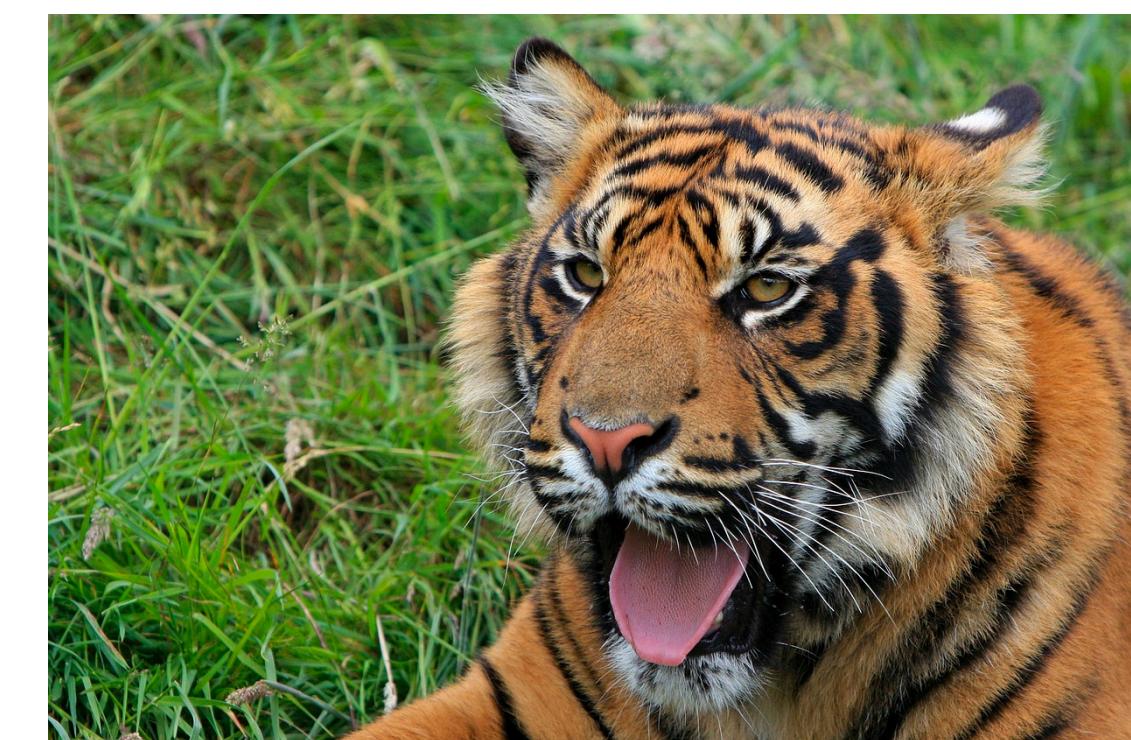
$$z \sim N(0, I)$$



$$\min_G \max_D V(D, G) = \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p_{\mathbf{z}}(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))].$$



Aims to tell whether the image is generated or not



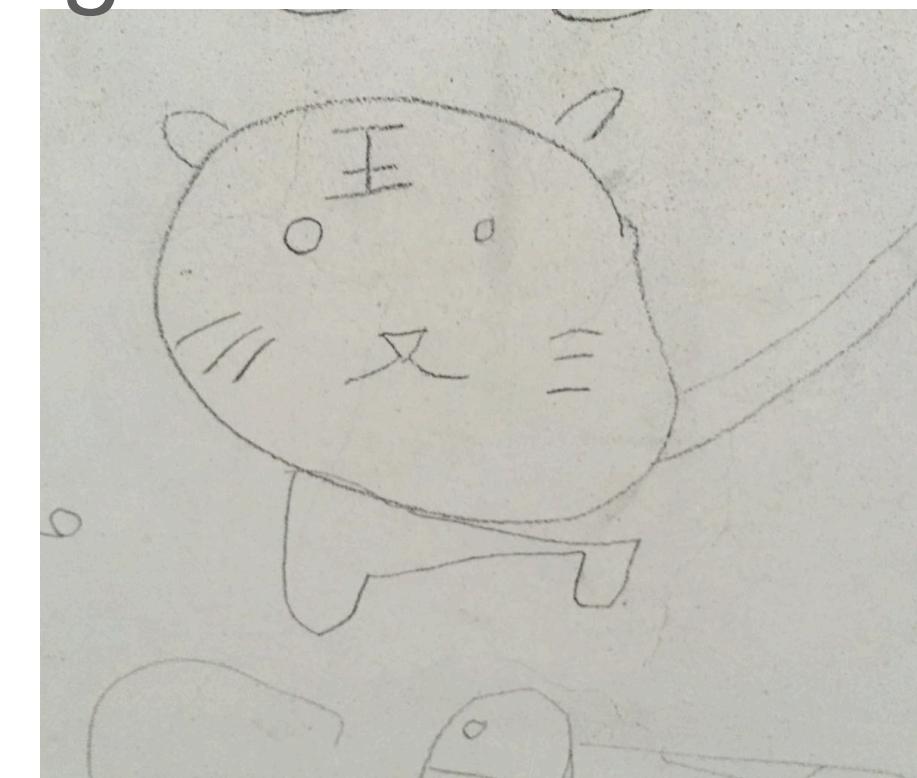
AE

VAE

GAN

Discriminator: binary cross-entropy

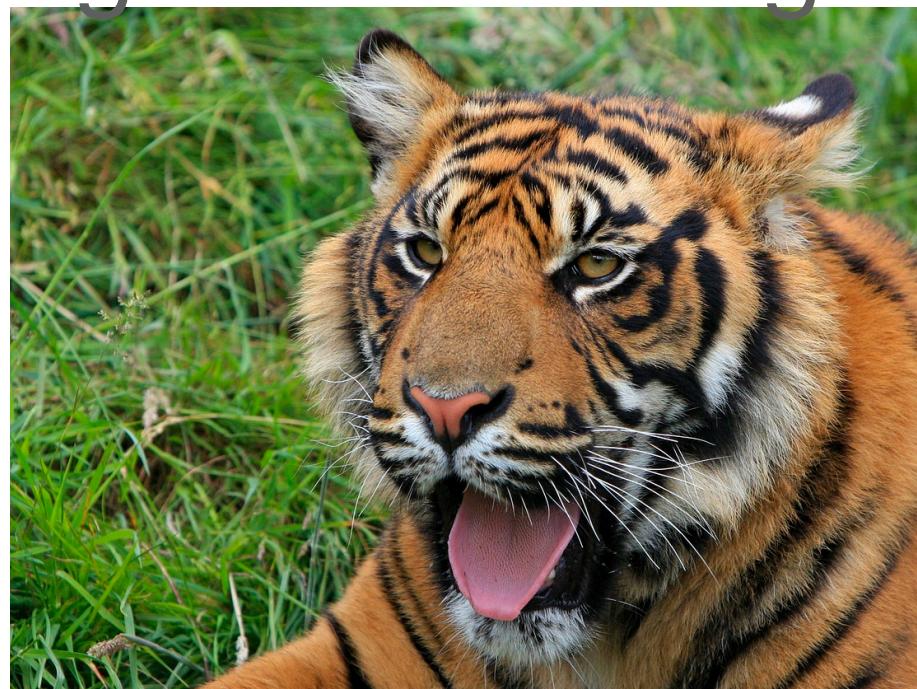
Generated image from noise with generator



$y=0$

binary cross-
entropy

Real image from training dataset



$y=1$

binary cross-
entropy

Keras code for discriminator

```
optimizer = Adam(0.001)

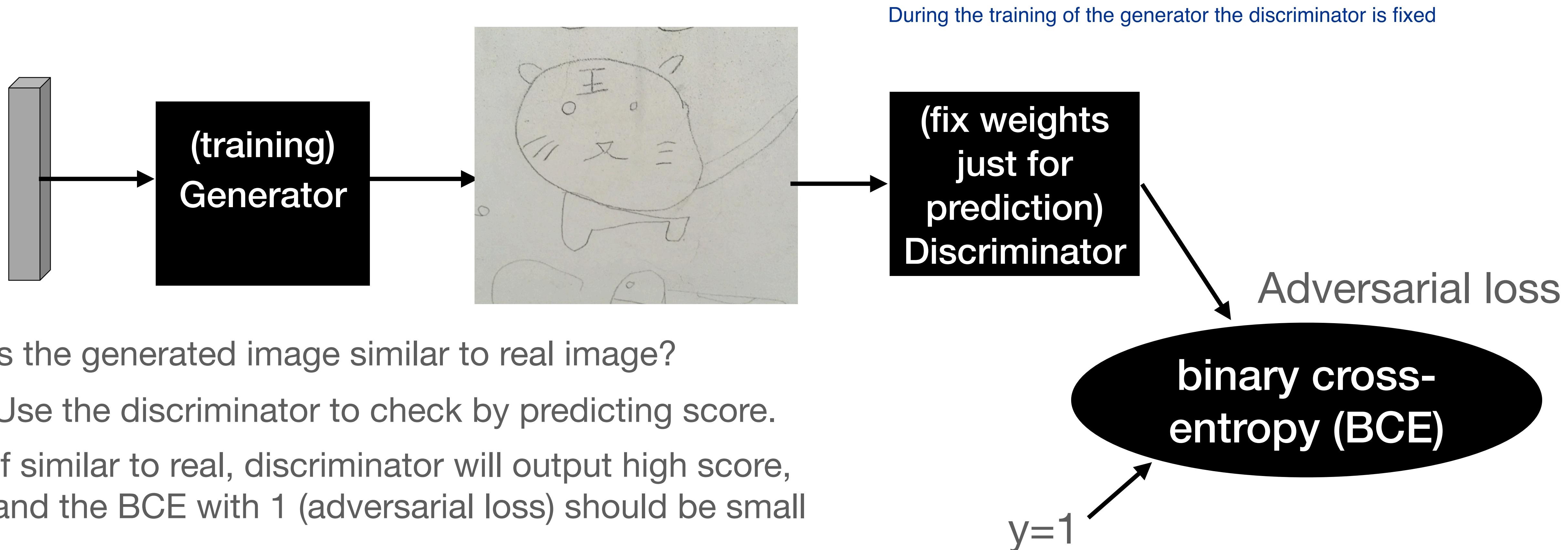
discriminator.compile(loss='binary_crossentropy',
                      optimizer=optimizer,
                      metrics=['accuracy'])
```

```
valid = np.ones((batch_size, 1))
fake = np.zeros((batch_size, 1))

# Generate a batch of new images
noise = np.random.normal(0, 1, (batch_size, self.latent_dim))
gen_imgs = generator.predict(noise)

d_loss_real = discriminator.train_on_batch(real_imgs, valid)
d_loss_fake = discriminator.train_on_batch(gen_imgs, fake)
```

Generator: Fool discriminator by adversarial loss



So the generator update weights to
minimise the adversarial loss

Keras code for generator

```
# generate image
z = Input(shape=(self.latent_dim,))
gen_img = generator(z)

# fix weights of discriminator for prediction only
discriminator.trainable = False

# output score for generated image
validity = discriminator(gen_img)

# The combined model: input noise, output prediction of discriminator to calculate loss
combined_model = Model(z, validity)
combined_model.compile(loss='binary_crossentropy', optimizer=optimizer)

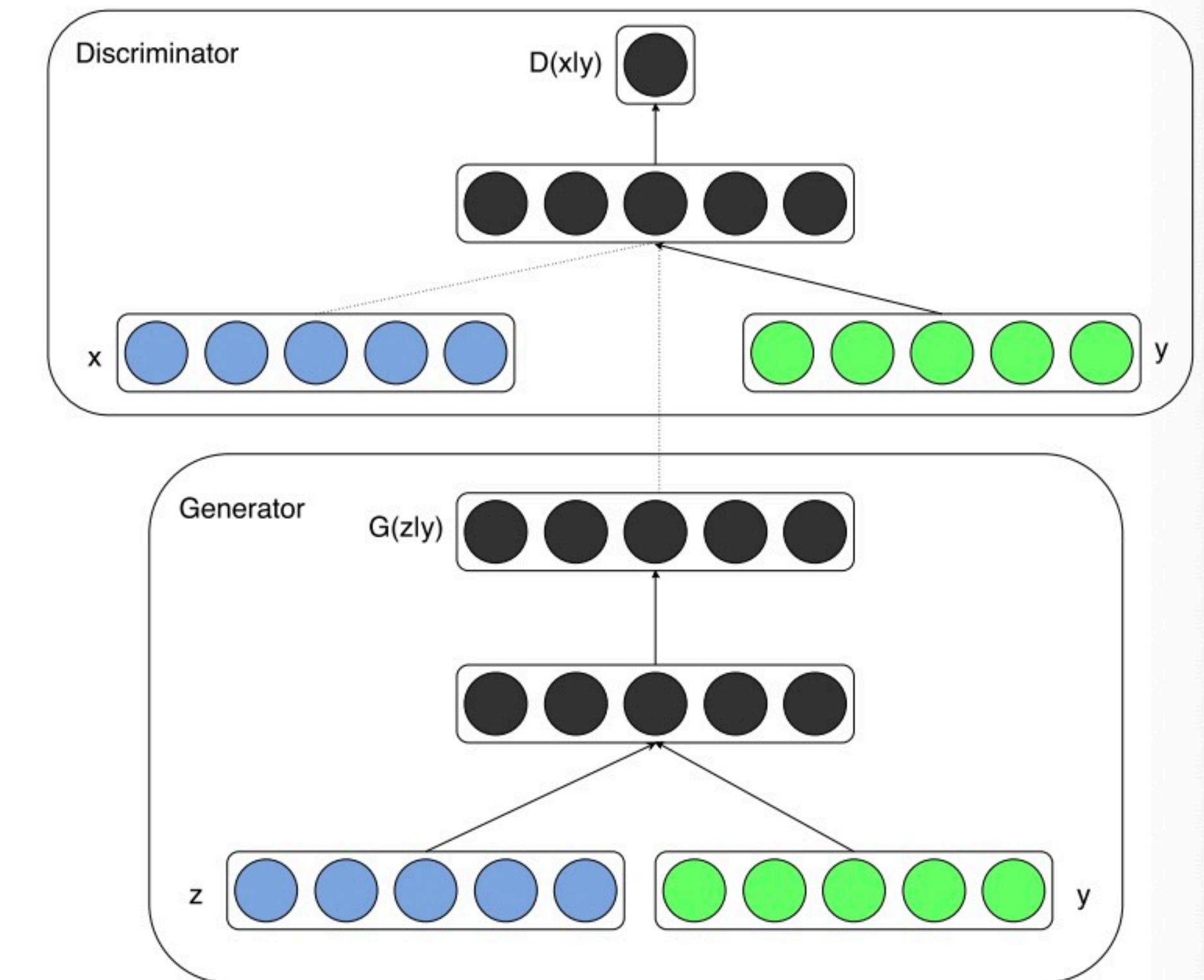
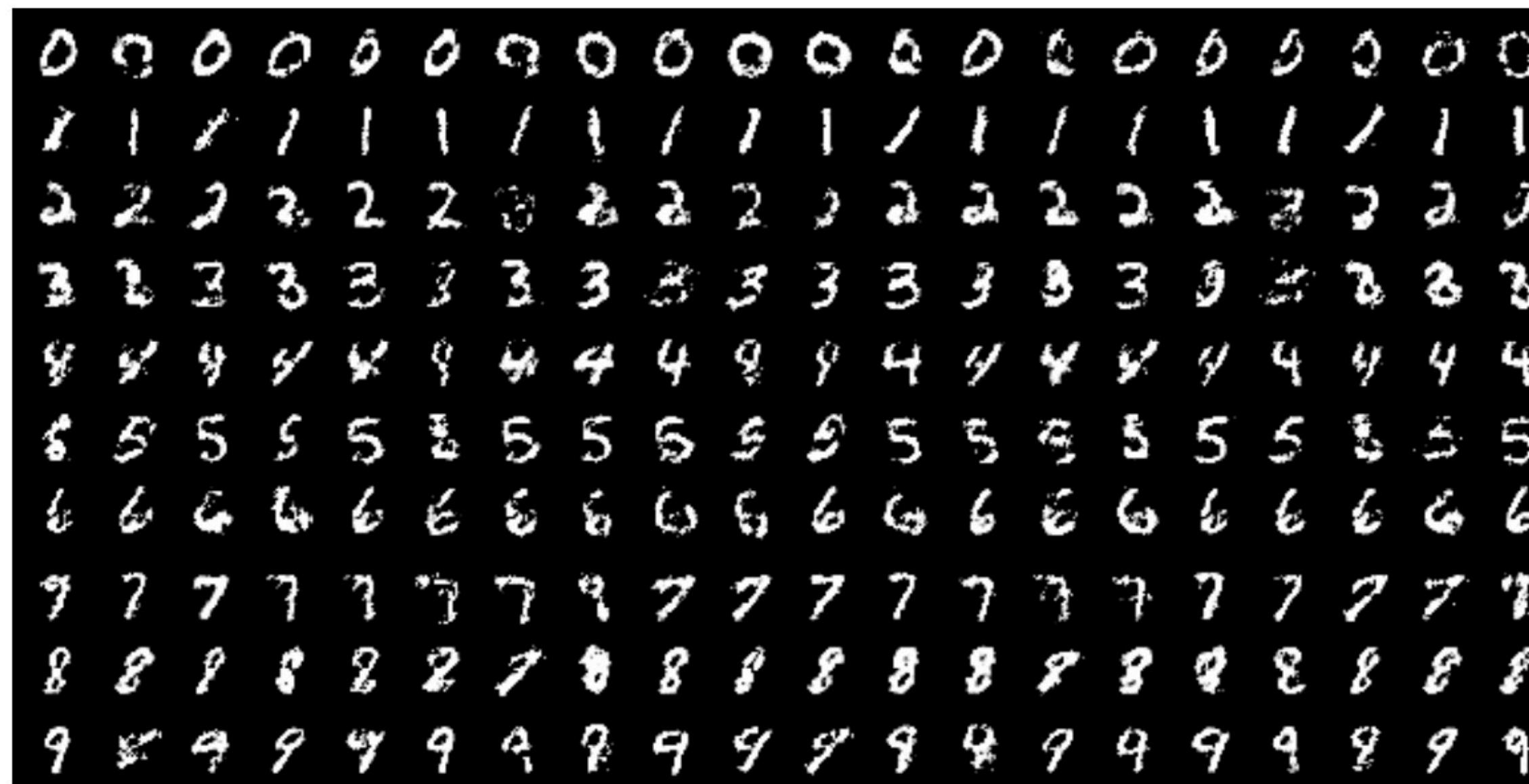
valid = np.ones((batch_size, 1))
noise = np.random.normal(0, 1, (batch_size, self.latent_dim))
# update generator to make the output of discriminator to be similar to 1
g_loss = combined_model.train_on_batch(noise, valid)
```

AE

VAE

GAN

Conditional generative adversarial nets



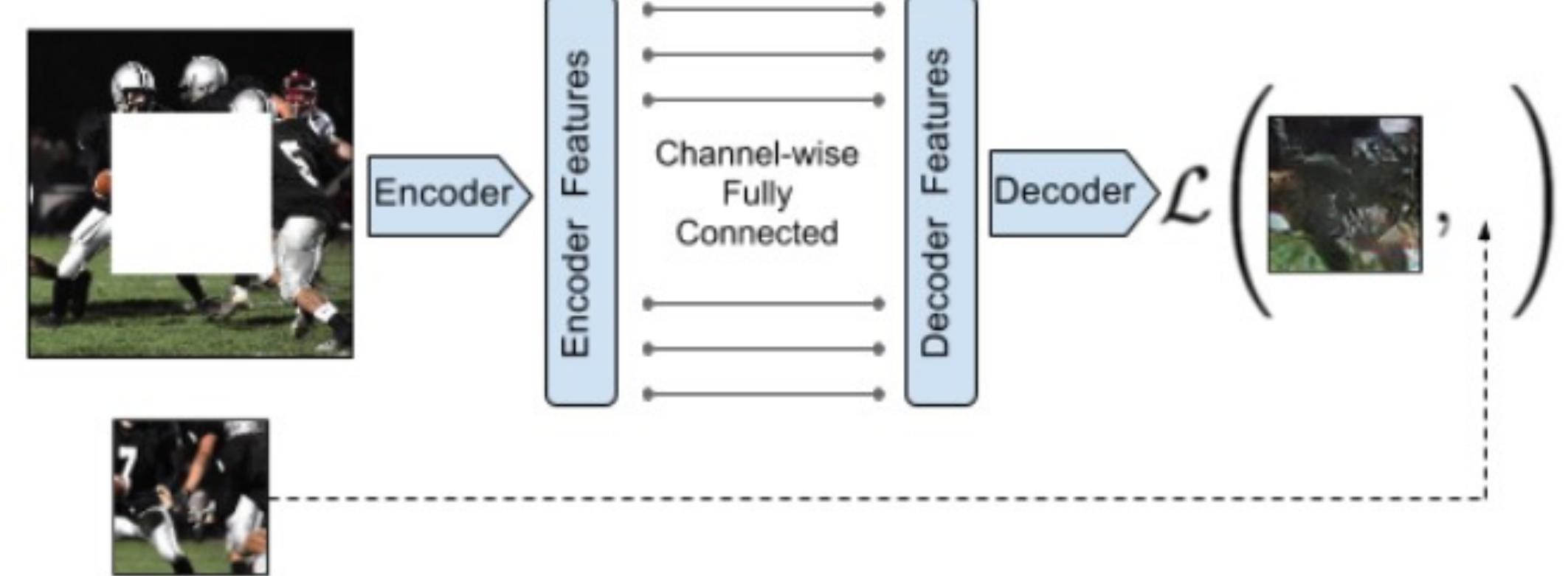
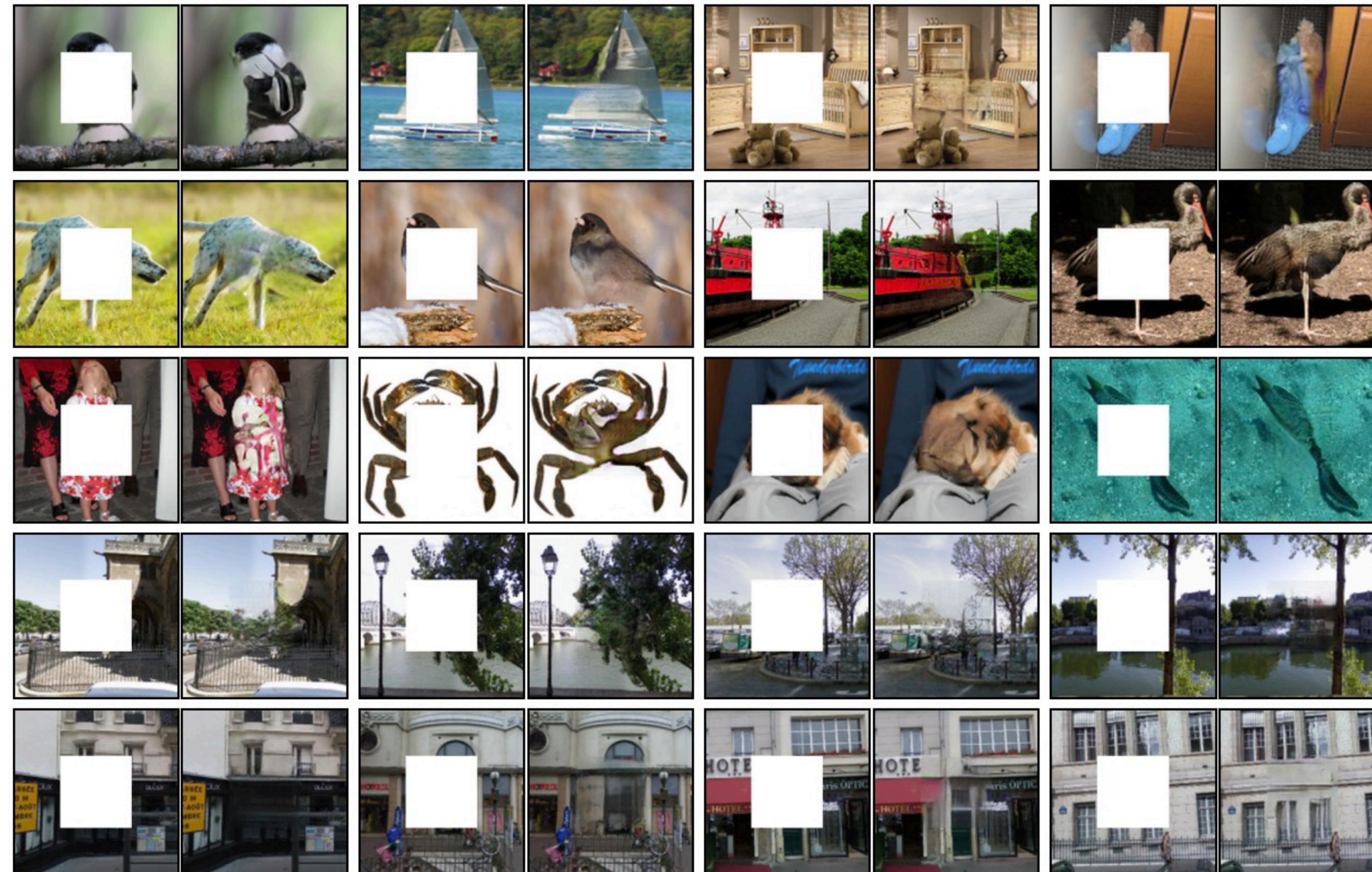
Combine the class vector with the noise

AE

VAE

GAN

Context encoder



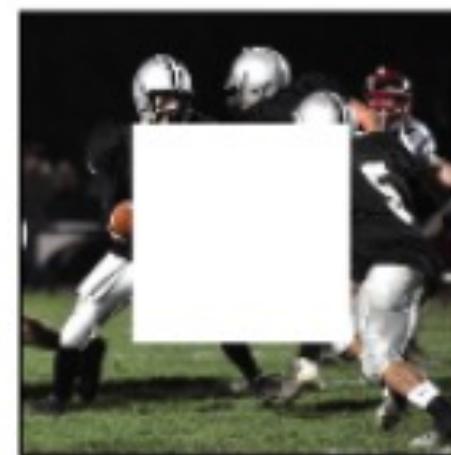
AE

VAE

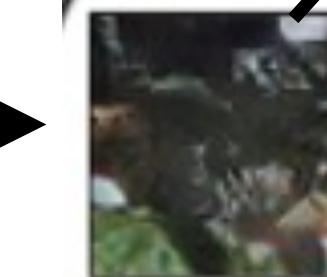
GAN

Context encoder

Generator



(training)
Generator



Reconstruction
loss (L2)

(Frozen)
Discriminator

y=1

Adversarial loss

binary cross-
entropy

Discriminator is the same as the original gan

AE

VAE

GAN

Other applications



Other applications

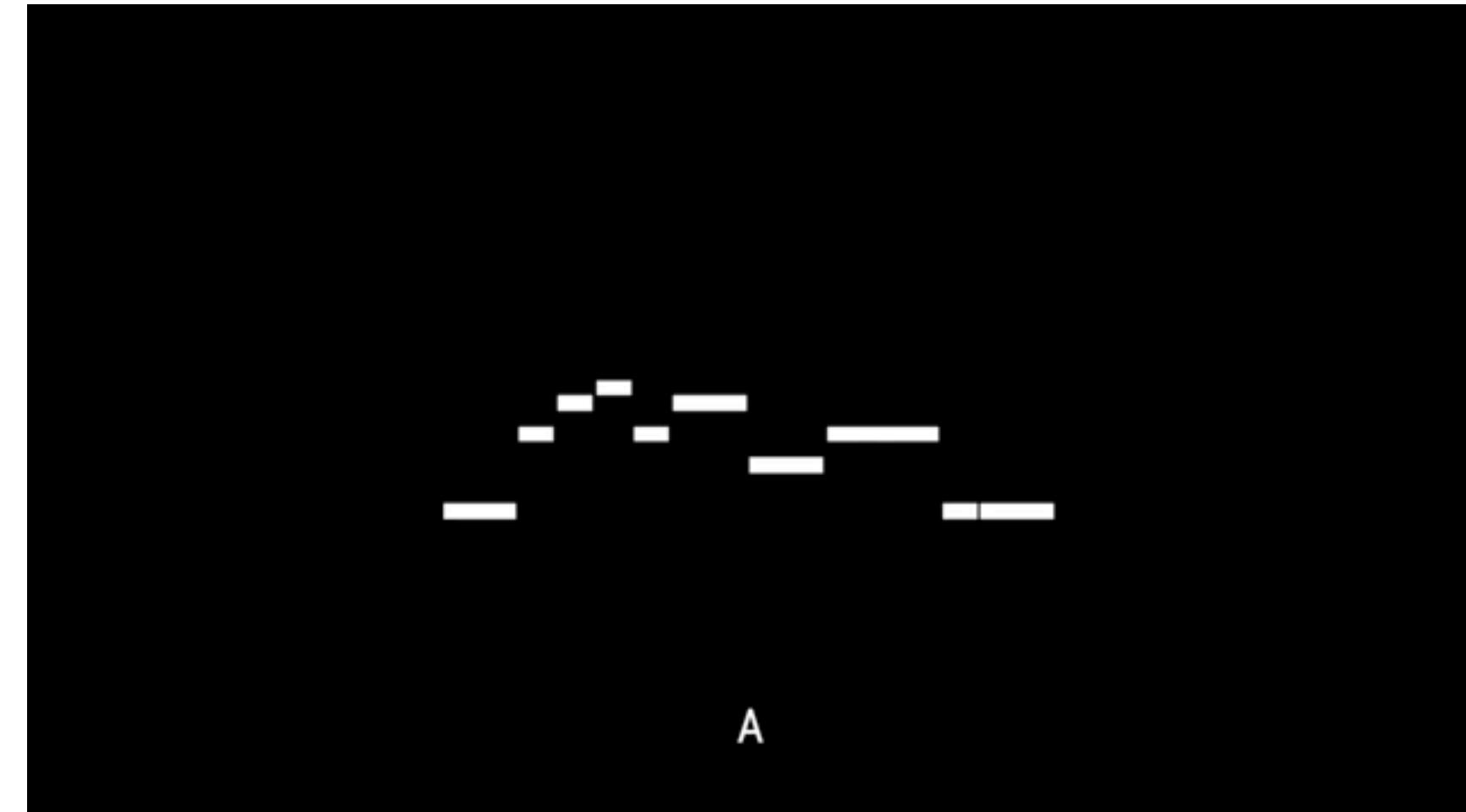
Turning a horse video into a zebra video (by CycleGAN)



https://www.youtube.com/watch?time_continue=1&v=9reHvktoLY&feature=emb_logo

Summary

- How to train AE to learn meaningful representation and denoisy image?
- How to train VAE?
- How to train GAN?



https://www.youtube.com/watch?time_continue=87&v=G5JT16flZwM&feature=emb_logo