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MODULE THEORY

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Dedicated to my family and my best friend

Neeraj K. Gaud !

Introduction

This sample book discusses the course Module Theory of pure mathematics being taught to post-graduate students in University of Delhi.

Introduction to Modules

Defination of Module

Left Module:

Let R be a ring with identity and M be an abelian group with addition. We say M is a left R -module if there exists a mapping¹

¹ often called as scaler multiplication.

$$R \times M \rightarrow M$$

defined by

$$(a, x) \rightarrow ax$$

for each $a \in R$ and $x \in M$ satisfying following properties :

$$\forall a, b \in R \text{ and } x, y \in M$$

$$\begin{aligned}(a + b)x &= ax + bx \\ a(x + y) &= ax + ay \\ (ab)x &= a(bx) \\ 1x &= x\end{aligned}$$

and denoted by ${}_R M$

Right Module:

Let R be a ring with identity and M be an abelian group with addition. We say M is a right R -module if there exists a mapping

$$M \times R \rightarrow M$$

defined by

$$(x, a) \rightarrow xa$$

for each $a \in R$ and $x \in M$ satisfying following properties :

$$\forall a, b \in R \text{ and } x, y \in M$$

$$\begin{aligned}x(a + b) &= xa + xb \\ (x + y)a &= xa + ya \\ x(ab) &= (xa)b \\ x1 &= x\end{aligned}$$

and denoted by M_R

Examples :

1. Let V be a vector space over a field F then V is a left as well as right F -Module.
2. Let G be any abelian group under addition , then G is a \mathbb{Z} -Module where \mathbb{Z} is set of integers.
3. Let R be ring and $M = R[x]$ where $R[x]$ is a group of all polynomials with coefficients in R then M is a left as well as a right R -Module with scalar multiplication being usual multiplication.
4. Let M be collection of all $m \times n$ matrices over ring R , then M is left R -Module where scalar multiplication being usual multiplication of a scalar to a matrix like

Suppose ring R is a field then R -Module $R[x]$ is a vector space over field R .

$$\begin{aligned} A &= [a_{ij}] \mid i = 1, 2, 3, \dots, m \ j = 1, 2, 3, \dots, n \\ c \times A &= [c \times a_{ij}] \mid i = 1, 2, 3, \dots, m \ j = 1, 2, 3, \dots, n \end{aligned}$$