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MODULE THEORY

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Contents

Dedicated to my family and my best friend

`Neeraj K. Gaud` *!*

Introduction

This sample book discusses the course `Module Theory` of `pure mathematics` being taught to `post-graduate` students in `University of Delhi`.

Introduction to Modules

Defination of Module

Left Module:

Let R be a ring with identity and M be an abelian group with addition. We say M is a left $R-$module if there exists a mapping[1]

[1] often called as scaler multiplication.

$$R \times M \to M$$

defined by

$$(a\ ,\ x) \to ax$$

for each $a \in R$ and $x \in M$ satisfying following properties :

$\forall\ a\ ,\ b \in R$ and $x\ ,y\ \in M$

$$\begin{aligned} (a+b)x &= ax + bx \\ a(x+y) &= ax + ay \\ (ab)x &= a(bx) \\ 1x &= x \end{aligned}$$

and denoted by ${}_RM$

Right Module:

Let R be a ring with identity and M be an abelian group with addition. We say M is a right $R-$module if there exists a mapping

$$M \times R \to M$$

defined by

$$(x\ ,\ a) \to xa$$

for each $a \in R$ and $x \in M$ satisfying following properties :

$\forall\ a\ ,\ b \in R$ and $x\ ,y\ \in M$

$$\begin{aligned} x(a+b) &= xa + xb \\ (x+y)a &= xa + ya \\ x(ab) &= (xa)b \\ x1 &= x \end{aligned}$$

and denoted by M_R

Examples :

1. Let V be a vector space over a field F then V is a left as well as right $F-$Module.

2. Let G be any abelian group under addition , then G is a $\mathbb{Z}-$Module where $\mathbb{Z}$ is set of integers.

3. Let R be ring and $M = R[x]$ where $R[x]$ is a group of all polynomials with coefficents in R then M is a left as well as a right $R-$Module with scaler multiplication being usual multiplication.

Suppose ring R is a field then $R-$Module $R[x]$ is a vector space over field R.

4. Let M be collection of all $m \times n$ matrices over ring R , then M is left $R-$Module where scaler multiplication being usual multiplication of a scaler to a matrix like

$$
\begin{aligned}
\mathtt{A} &= [\mathtt{a_{ij}}] \mid i = 1,2,3,\cdots,m \; j = 1,2,3,\cdots,n \\
c \times \mathtt{A} &= [c \times \mathtt{a_{ij}}] \mid i = 1,2,3,\cdots m \; j = 1,2,3,\cdots n
\end{aligned}
$$