Introduction to Digital Signal Processing

Signals

Sivakumar Balasubramanian

Department of Bioengineering
Christian Medical College, Bagayam
Vellore 632002

What is signal processing?

"Signal processing is an enabling technology that encompasses the fundamental theory, applications, algorithms, and implementations of **processing or transferring information** *contained in many different physical, symbolic, or abstract formats broadly designated as signals and uses* **mathematical, statistical, computational, heuristic, and/or linguistic representations***, formalisms, and techniques for* **representation, modeling, analysis, synthesis, discovery, recovery, sensing, acquisition, extraction, learning, security, or forensics***."*[1]

[1] Moura, J.M.F. (2009). "What is signal processing?, President's Message". IEEE Signal Processing Magazine 26 (6). doi:10.1109/MSP.2009.934636

What is a signal?

Any physical quantity carrying information that varies with one or more independent variables.

$$s\left(t\right) = 1.23t^2 - 5.11t + 41.5$$

$$s\left(x, y\right) = e^{-\left(x^2 + y^2 + 0.5xy\right)}$$

Mathematical representation will not be possible (e.g. *physiological signals, either because the exact function is not known or is too complicated*.)

What is a signal? (Contd ...)

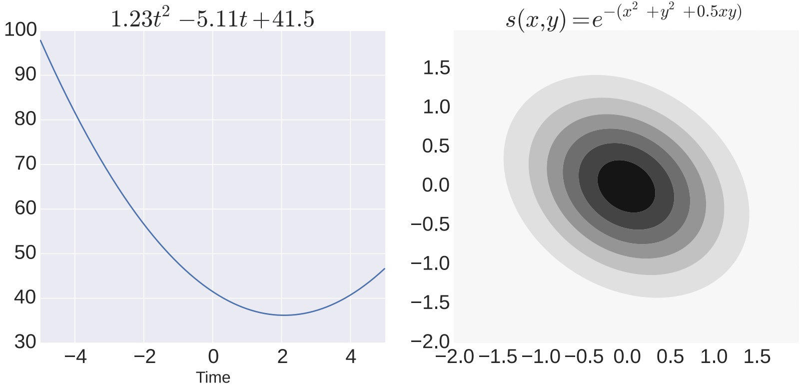


Figure: Example of 1D and 2D signals

Can you think of examples of 3D and 4D signals?

Classification of signals

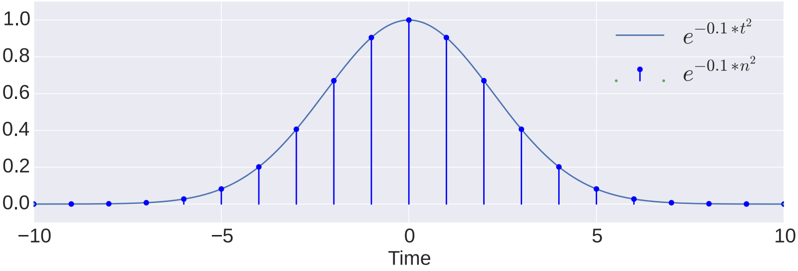
- Based on the signal dimensions. *e.g.* 1D, 2D ...
- **Scalar** vs. **Vector** signals: *e.g. gray scale versus RGB image*

$$I_g\left(x, y\right) \in \mathbb{R} \text{ and } I_{color}\left(x, y\right) \in \mathbb{R}^3$$

Classification of signals

- **Continuous-time** vs. **Discrete-time**: *based on the values assumed by the independent variable.*

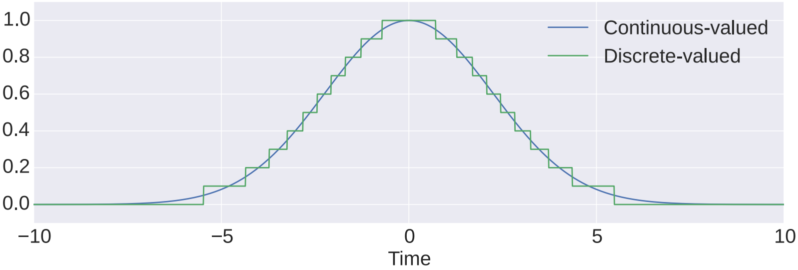
$$\begin{cases} x(t) = e^{-0.1t^2}, \ t \in \mathbb{R} & \text{Continuous-time} \\ x[n] = e^{-0.1n^2}, \ n \in \mathbb{Z} & \text{Discrete-time} \end{cases}$$



Classification of signals (Contd ...)

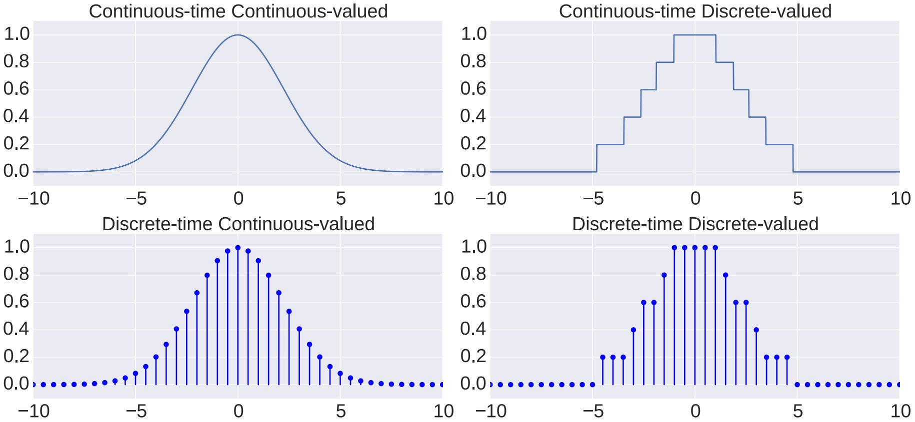
- **Continuous-valued** vs. **Discrete-valued**: *based on the values assumed by the dependent variable.*

$$\begin{cases} x(t) \in [a, b] & \text{Continuous-valued} \\ x(t) \in \{a_1, a_2, \cdots\} & \text{Discrete-valued} \end{cases}$$



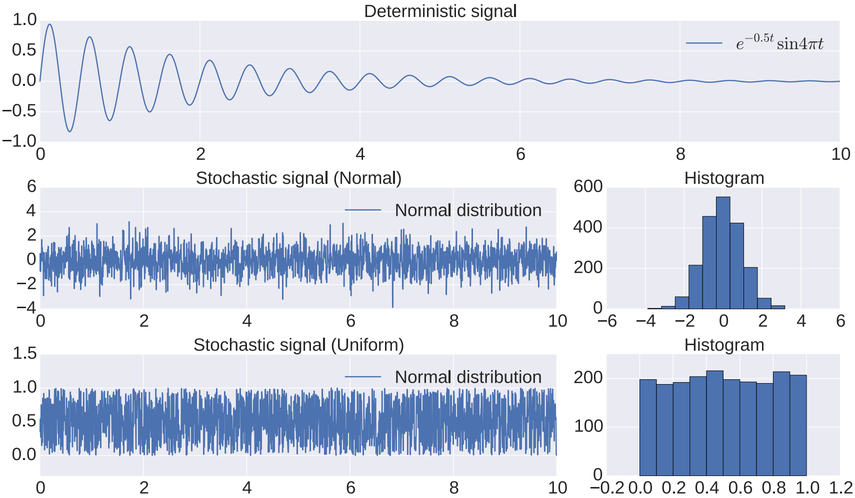
Classification of signals (Contd ...)

Four types of signals



Classification of signals (Contd ...)

- **Deterministic** vs. **Stochastic**: *e.g. EMG is an example of a stochastic signal.*



Classification of signals (Contd ...)

- **Even** vs. **Odd**: *based on the symmetry about the $t = 0$ axis.*

$$\begin{cases} x(t) = x(-t), & \text{Even signal} \\ x(t) = -x(-t), & \text{Odd signal} \end{cases}$$

Can there be signals that are neither even nor odd?

Theorem

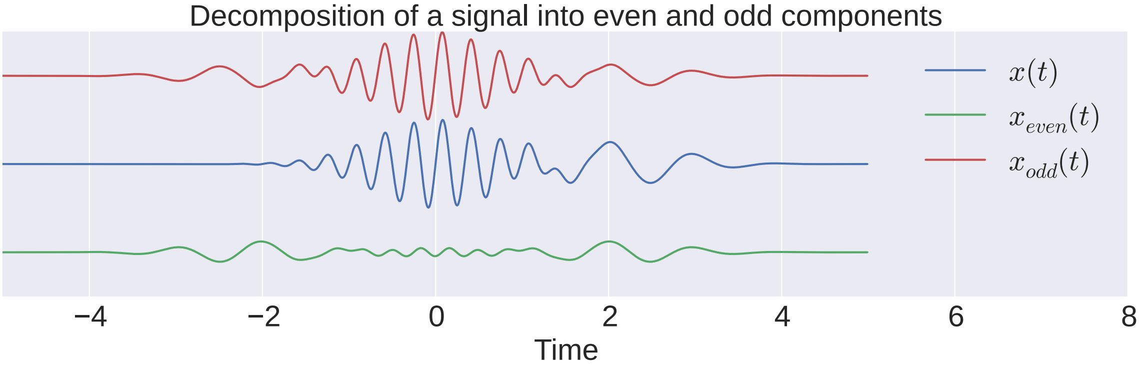
Any arbitrary function can be represented as a sum of an odd and even function.

$$x(t) = x_{even}(t) + x_{odd}(t)$$

where, $x_{even}(t) = \frac{x(t)+x(-t)}{2}$ and $x_{odd}(t) = \frac{x(t)-x(-t)}{2}$.

Classification of signals (Contd ...)

Decomposition of an arbitrary signal into even and odd components



Classification of signals (Contd ...)

- **Periodic** vs. **Non-periodic**: *a signal is periodic, if and only if*

$$x(t) = x(t+T), \forall t$$

where, T is the fundamental period.

- **Energy** vs. **Power**: *indicates if a signal is short-lived.*

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \qquad P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

$$E = \sum_{-\infty}^{\infty} |x(t)|^2 \qquad P = \frac{1}{2N+1}\sum_{-N}^{N} |x(t)|^2$$

A signal is an **energy** *signal, if* $0 < E < \infty$.
A signal is an **power** *signal, if* $0 < P < \infty$.

Useful signals in continuous and discrete-time

We will look at some important signals, that we will often come across and are useful in the analysis of signals and systems.

- Exponential signals
- (Complex) Sinusoids
- Exponential sinusoids
- Impulse/Dirac delta function
- Step function

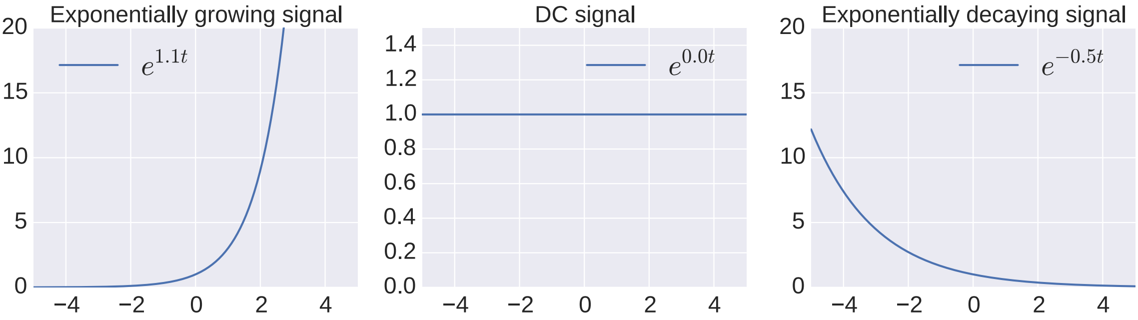
There are some important differences between the corresponding continuous and discrete-time signals.

Real Exponentials

Continuous-time version

$$x(t) = be^{at}$$

where, $a, b, t \in \mathbb{R}$. b is the amplitude and a is the exponential growth or decay rate.

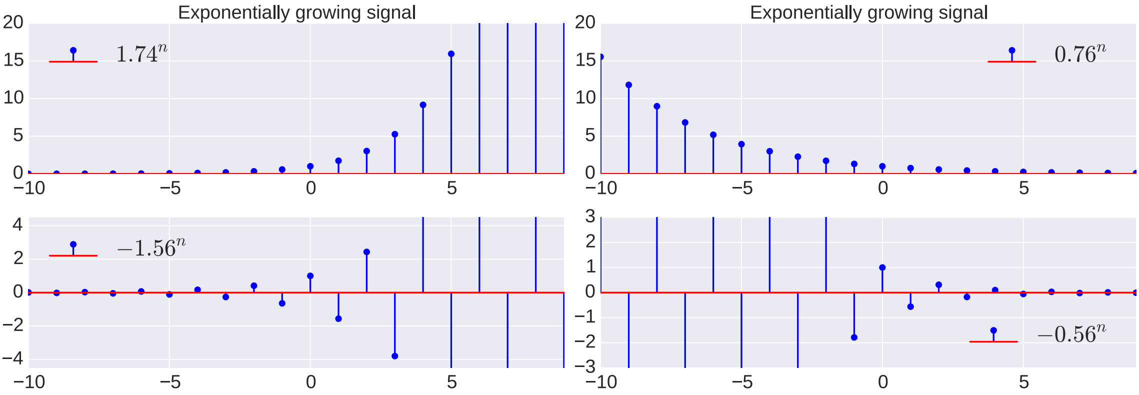


Real Exponentials (Contd ...)

Discrete-time version

$$x[n] = b\,(a)^n$$

where, $a, b \in \mathbb{R}$ and $n \in \mathbb{Z}$. b is the amplitude and a is the exponential growth or decay rate.



Real Exponentials (Contd ...)

These are encountered as solution to first order differential and difference equations.

$$\frac{d}{dt}x(t) = kx(t) \implies x(t) = Ce^{kt}$$

$$x[n] = kx[n-1] \implies x(t) = C(k)^n$$

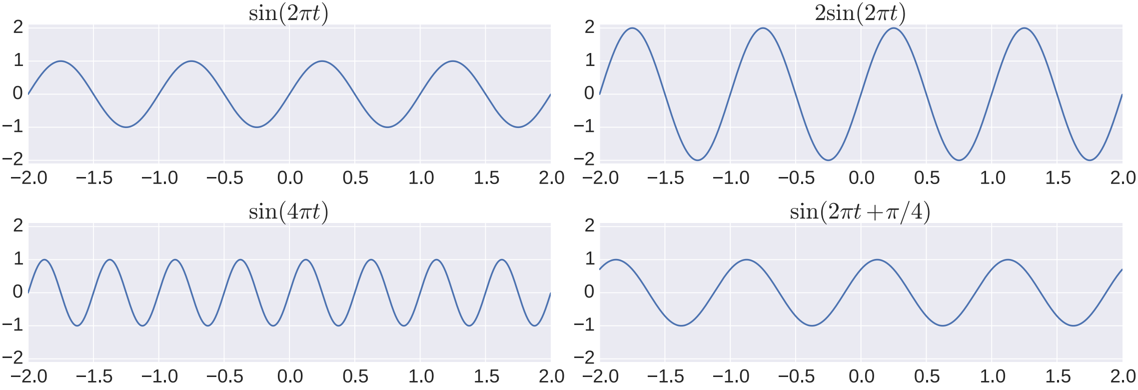
Can you think of practical examples of systems that result in such signals?

Sinusoidal signals

Continuous-time version

$$x(t) = A\sin(\omega t + \phi)$$

where, A is the amplitude, ω is the angular frequency $\left(\mathrm{rad.sec}^{-1}\right)$, and ϕ is the phase angle.



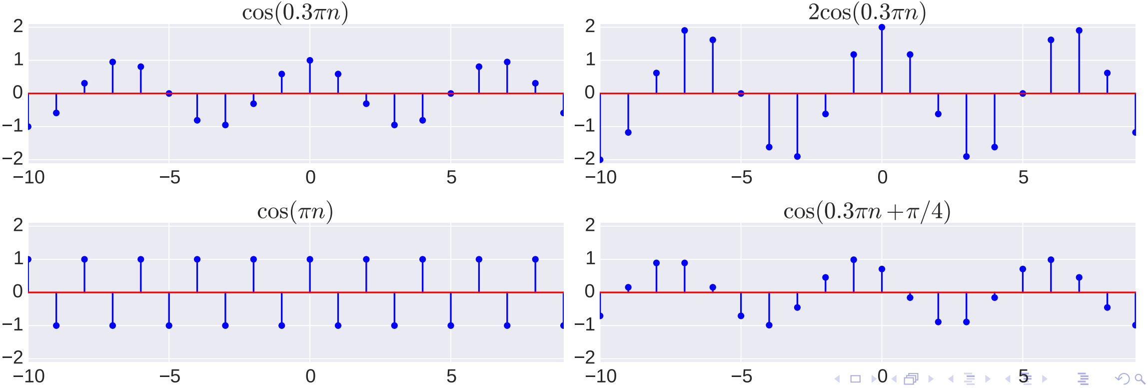
What is the fundamental period of sinusoid?

Sinusoidal signals (Contd ...)

Discrete-time version

$$x[n] = A\sin\left(\Omega n + \phi\right)$$

where, A is the amplitude, Ω is the digital frequency $\left(\text{rad.sample}^{-1}\right)$, and ϕ is the phase angle.



Sinusoidal signals (Contd ...)

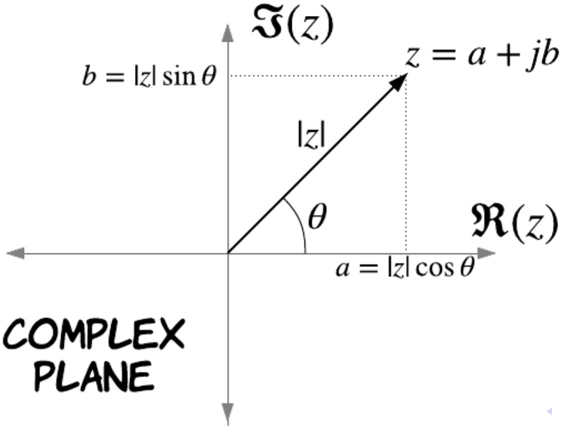
There are some peculiarities to the discrete sinusoid:

- Not all sinusoids are periodic! e.g. $\sin(n)$
- There is a maximum frequency for discrete sinusoids. What is it?
- Two sinusoids that differ by a discrete frequency of 2π are the same sinusoids.

Sinusoidal signals (Contd ...)

Complex exponential representation of sinusoids

$$z = a + jb = |z|\, e^{j\theta} = |z| \cos\theta + j\, |z| \sin\theta$$

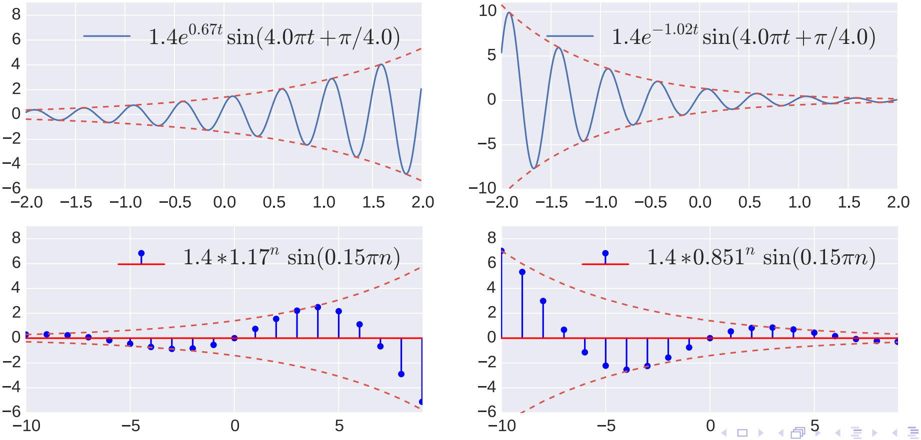


Exponential sinusoids

Continuous-time version

Amplitude modulated sinusoids

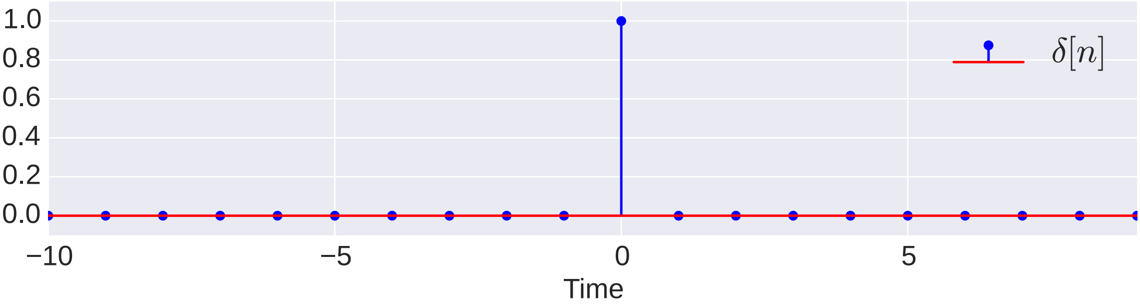
$$x(t) = ae^{bt} \sin\left(\omega t + \phi\right), \quad a, b, \omega, \phi \in \mathbb{R}$$



Impulse function $\delta[n]$ (Contd ...)

Kronecker delta function or sequence $\delta[n]$

- $$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{Otherwise} \end{cases}$$



Step function $u(t), u[n]$

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$