Exercise 1. Show that $\mathbb{Q}$ is not a projective $\mathbb{Z}$-module.

Exercise 2. Show that every projective abelian group is free.

Exercise 3. Show that a direct product of R-modules $\prod_{i \in I} J_i$ is injective if and only if each J_i is injective.

Exercise 4. Let R be a commutative, unital ring. Show that the following are equivalent.

(i) Every R-module is projective.

(ii) Every R-module is injective.

(iii) Every short exact sequence of R-modules is split exact.