

Exercise 1. If A is a finite abelian group, show that $A \otimes_{\mathbb{Z}} \mathbb{Q} = 0$.

Exercise 2. Show that $\mathbb{Z}_m \otimes_{\mathbb{Z}} \mathbb{Z}_n \cong \mathbb{Z}_d$, where $d = (m, n)$. **Hint:** Write $d = am + bn$ for some integers a, b .

Exercise 3. Let M be an R -module, and $I \subset R$ an ideal. Show that $R/I \otimes_R M \cong M/IM$.

Exercise 4. Let R be commutative, and $I, J \subset R$ ideals. Show that $R/I \otimes_R R/J \cong R/(I + J)$.

Exercise 5. Let R be commutative. An R -module is called *flat* if tensoring with that module is left exact. Show that every projective R -module is flat.