

Exercise 1. Let R be a PID, and $I \subset R$ an ideal. Show that R/I is both Noetherian and Artinian.

Exercise 2. Let R be Noetherian, and $P \subset R$ a prime ideal. Show that R_P is Noetherian.

Exercise 3. Let R be an Artinian ring. Show that every prime ideal of R is maximal.

Exercise 4. Let R be a ring, $S \subset R$ a multiplicative set, and $I \subset R$ an ideal. Show that $S^{-1}(\text{rad } I) = \text{rad}(S^{-1}I)$.

Exercise 5. Let R be Noetherian, and $I, J \subset R$ ideals with $J \subset \text{rad } I$. Show that there exists $n \in \mathbb{N}$ with $J^n \subset I$.