Exercise 1. Let R be an integral domain, and let T be an integral domain such that $R \subset T \subset \operatorname{Frac} R$. Show that $\operatorname{Frac} R = \operatorname{Frac} T$.

Exercise 2. Let R be an integral domain, and $S \subset R$ a multiplicative subset that does not contain 0. Show that if R is a PID, then so is $S^{-1}R$.

Exercise 3. Let R be a commutative unital ring, $S \subset R$ a multiplicative subset, and $I \subset R$ an ideal. Show that $S^{-1}\sqrt{I} = \sqrt{S^{-1}I}$ (Recall that $\sqrt{I} = \{x \in R \mid x^n \in I \text{ for some } n \in \mathbb{N}\}$).

Exercise 4. Let R be a commutative unital ring. Show that R is local if and only if whenever $r + s = 1$, then either $r \in R^*$ or $s \in R^*$

Exercise 5. Show that every nonzero homomorphic image of a local ring is local.