Exercise 1. Let R be a ring in which every maximal ideal is of the form cR for some $c \in R$ satisfying $c^2 = c$. Show that R is Noetherian (**Hint:** Show that every prime ideal is maximal).

Exercise 2. Let $(R, \mathfrak{M})$ be a Noetherian local ring. Suppose that $\mathfrak{M}/\mathfrak{M}^2$ is generated by the set $\{a_1 + \mathfrak{M}^2, \dots, a_n + \mathfrak{M}^2\}$. Show that $\mathfrak{M} = a_1R + \cdots + a_nR$.

Exercise 3. Let $R \subset S$ be an integral extension, and suppose that R and S are both integral domains. Show that R is a field if and only if S is a field.

Exercise 4. Show that if $R \subset S$ is an integral extension, then $S[x_1, \dots, x_n]$ is integral over $R[x_1, \dots, x_n]$.

Exercise 5. Let R be an integral domain with fractional field k. Show that if R is integrally closed and t is transcendental over k, then $R[t]$ is integrally closed.