5.5 (g)

Mittwoch, 17. November 2021 20:04

$$\left(\mathcal{F}_8(f)_j\right)_{j=1}^{8} = (0,0,-4i,0,0,0,4i,0)$$

$$\mathcal{F}_8^{-1}\left(\quad\right)_j = \frac{1}{8}\sum_{k=1}^{8} \mathcal{F}_k \cdot e^{i\omega_{k-1}(j-1)}$$

$$\overset{\text{peri.}}{=} \frac{1}{8}\sum_{k=-3}^{4} \mathcal{F}_k \cdot e^{i\omega_{k-1}(j-1)}$$

$$= \frac{1}{8}\left[-4i\cdot e^{i\omega_{(-2)}(j-1)} + 4i e^{i\omega_2(j-1)}\right]$$

$$= \frac{1}{2}\left[i e^{\frac{4}{8}\pi i(j-1)} - i e^{-\frac{4}{8}\pi i(j-1)}\right]$$

$$= \frac{1}{2}\left[i\cos\left(\frac{4}{8}\pi(j-1)\right) - \sin\left(\frac{4}{8}\pi(j-1)\right) - i\cos\left(-\frac{4}{8}\pi(j-1)\right) + \sin\left(-\frac{4}{8}\pi(j-1)\right)\right]$$

$$= \sin\left(\frac{4}{8}\pi(j-1)\right) = \sin\left(4\pi\,\frac{(j-1)}{8}\right) \Rightarrow \underline{\underline{\omega = 4\pi}} \wedge \underline{\underline{N=8}}$$