5.2 (b) (i)

Mittwoch, 10. November 2021 20:23

$$\|(x * y)\|_1 = \sqrt[1]{\sum_{k \in \Omega} |(x*y)(k)|}$$

$$\overset{\Omega = \mathbb{Z}}{=} \sum_{k \in \mathbb{Z}} \left| \sum_{\ell \in \mathbb{Z}} x(\ell) \cdot y(k-\ell) \right|$$

$$\leq \sum_{k \in \mathbb{Z}} \sum_{\ell \in \mathbb{Z}} |x(\ell) \cdot y(k-\ell)|$$

$$\overset{\text{commutative}}{=} \sum_{\ell \in \mathbb{Z}} \sum_{k \in \mathbb{Z}} |x(\ell) \cdot y(k-\ell)|$$

$$\overset{\text{sum over } \mathbb{Z}}{=} \sum_{\ell \in \mathbb{Z}} \sum_{k \in \mathbb{Z}} |x(\ell) \cdot y(k)|$$

$$= \sum_{\ell \in \mathbb{Z}} \sum_{k \in \mathbb{Z}} |x(\ell)| \cdot |y(k)|$$

$$= \sum_{\ell \in \mathbb{Z}} |x(\ell)| \cdot \sum_{k \in \mathbb{Z}} |y(k)|$$

$$= \|x\| \cdot \|y\|$$