5.4 (d)

Parseval's theorem $\langle f, g\rangle_2 = \langle \hat{f}, \hat{g}\rangle$

$\Leftrightarrow \quad \|\hat{f}\|_2 = \|f\|_2$

$$\int_0^\infty \frac{\sin^2(\alpha\omega)}{\omega^2}\,d\omega = \int_0^\infty \left|\frac{\sin(a\omega)}{\omega}\right|^2 d\omega$$

Substitute
$\omega = \varphi/a$
$x = \frac{1}{a}\cdot\varphi$

$$= \int_{a\cdot 0}^{a\cdot\infty} \left|\frac{\sin(\varphi)}{\varphi/a}\right|^2 \frac{1}{a}\cdot d\varphi = \int_0^\infty a^2\cdot\left|\frac{\sin(\varphi)}{\varphi}\right|^2 \frac{1}{a}\,d\varphi$$

$$= \int_0^\infty a\cdot\left|\frac{\sin(\varphi)}{\varphi}\right|^2 d\varphi = \int_0^\infty \left|\sqrt{a}\cdot\frac{\sin(\varphi)}{\varphi}\right|^2 d\varphi$$

$\Rightarrow \quad f(\varphi) = \sqrt{a}\cdot \operatorname{sinc}\left(\frac{\varphi}{\pi}\right)$ is symmetric

$\Rightarrow \quad f(\varphi) = f(-\varphi) \ \wedge\ \|f\|_2^2 = \int_{\mathbb{R}} \left(\sqrt{a}\cdot\operatorname{sinc}\left(\frac{\varphi}{\pi}\right)\right)^2 d\varphi$

$= \int_{\mathbb{R}} \frac{\sin^2(a\omega)}{\omega^2}\,d\omega$

$$\Rightarrow \int_{\mathbb{R}} \frac{\sin^2(a\omega)}{\omega^2} = 2\cdot\int_0^\infty \frac{\sin^2(a\omega)}{\omega^2}\,d\omega$$

$f(x) = \sqrt{a}\cdot\operatorname{sinc}\left(\frac{x}{\pi}\right)$:

$$\hat{f}(z) = \frac{1}{(2\pi)^{\frac{1}{2}}}\cdot\int_{\mathbb{R}} \sqrt{a}\cdot\operatorname{sinc}\left(\frac{x}{\pi}\right)\cdot e^{-izx}\,dx$$

$$\hat{f}(z) = \sqrt{a}\cdot\frac{1}{(2\pi)^{\frac{1}{2}}}\int_{\mathbb{R}} \operatorname{sinc}\left(\frac{x}{\pi}\right)e^{-izx}\,dx$$

$$\overset{r=1}{=} \sqrt{a}\cdot\frac{1}{\sqrt{\frac{2}{\pi}}}\cdot\frac{1}{(2\pi)^{\frac{1}{2}}}\cdot\int_{\mathbb{R}}\sqrt{\frac{2}{\pi}}\cdot \operatorname{sinc}\left(\frac{x}{\pi}\right) e^{-izx}\,dx$$

$$= \sqrt{a}\cdot\frac{1}{\sqrt{\frac{2}{\pi}}}\cdot\chi_{C_1}(0)$$

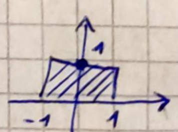
$\|\hat{f}\|_2^2$:

$$\|\hat{f}\|_2^2 = \int_{\mathbb{R}}\left(\sqrt{a}\cdot\sqrt{\frac{\pi}{2}}\,\chi_{C_1(0)}(x)\right)^2dx$$

$$= \int_{\mathbb{R}} a\cdot\frac{\pi}{2}\left(\chi_{C_1(0)}(x)\right)^2dx$$

$$= a\cdot\frac{\pi}{2}\int_{\mathbb{R}}\left(\chi_{C_1(0)}(x)\right)^2dx \quad \Big| \quad \text{here } \chi_{C_1(0)}(x) \in \{0,1\}$$

$$= a\cdot\frac{\pi}{2}\int\chi_{C_1(0)}(x)\,dx \rightsquigarrow$$



$$= a\cdot\frac{\pi}{2}\cdot 2 = \pi\cdot a$$

appling theorem: $\|f\|_2^2 = \|\hat{f}\|_2^2$

$$\Leftrightarrow \int_{\mathbb{R}}\left(\sqrt{a}\cdot\operatorname{sinc}\left(\frac{x}{a}\right)\right)^2dx = \int_{\mathbb{R}}\left(\sqrt{a}\cdot\sqrt{\frac{\pi}{2}}\cdot\chi_{C_1(0)}(x)\right)^2dx$$

$$\int_{\mathbb{R}}\frac{\sin^2(a\omega)}{\omega^2}\,d\omega = \pi\cdot a \quad\Big|\quad \int_0^\infty\frac{\sin^2(a\omega)}{\omega^2}\,d\omega = a\cdot\frac{\pi}{2}$$

$$2\cdot\int_0^\infty\frac{\sin^2(a\omega)}{\omega^2}\,d\omega = \pi\cdot a$$