5.7 (a)

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$$d=1: \quad \hat{f}(z) := \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} f(x) \cdot e^{izx} \, dx \quad \wedge \quad \mathcal{F}_N(f_j) := \sum_{k=1}^{N} f_k \cdot e^{-i\omega_{j-1}(k-1)}$$

$$= \sum_{k=0}^{N-1} f_k \cdot e^{-i2\pi jk\frac{1}{N}}, \; j \in \{0, \dots, N-1\}$$

$$d=2: \hat{f}(\vec{z}) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}^2} f(\vec{z}) \cdot e^{i\langle \vec{z}, \vec{x} \rangle} \, d\vec{x}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(z_1, z_2) e^{iz_1x_1} \cdot e^{iz_2x_2} \, dz_2 \, dz_1$$

$$\Downarrow$$

$$\Rightarrow \mathcal{F}_{N,M}(f_{j_1,j_2}) = \sum_{l=0}^{M-1} \sum_{k=0}^{N-1} f_{k,l} \cdot e^{-i2\pi j_1 k\frac{1}{N}} \cdot e^{-i2\pi j_2 l\frac{1}{M}},$$

$$j_1 \in \{0, \dots, N-1\} \wedge j_2 \in \{0, \dots, M-1\}$$