Problem 5.8 a.)

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$$G(x) = \frac{e^{-\left(\frac{x^2}{2s^2}\right)}}{(2\pi s^2)^{\left(\frac{d}{2}\right)}}$$

1. calculate Div(G(x))

$$\sum_{i=1}^{d} \frac{\partial^2 G(x)}{\partial^2 x_i} = \frac{1}{(2\pi s^2)^{\frac{d}{2}}} \cdot \left(\sum_{i=1}^{d} \left(\frac{\partial}{\partial x} \left(\underbrace{\frac{\partial}{\partial x} \left(e^{-\left(\frac{x^2}{2s^2}\right)} \right)}_{=} \right) \right) \right.$$

$$\frac{\partial}{\partial x} \left(\underbrace{\frac{\partial}{\partial x} \left(-\frac{x^2}{2s^2} \right) \cdot e^{-\left(\frac{x^2}{2s^2}\right)}}_{=} \right)$$

$$\frac{\partial}{\partial x_i} \left(-\frac{x_i}{s^2} \cdot e^{-\left(\frac{x^2}{2s^2}\right)} \right)$$

$$\left(\frac{-1}{s^2} \cdot e^{-\left(\frac{x^2}{2s^2}\right)} + \frac{-x_i}{s^2} \cdot -\frac{x_i}{s^2} \cdot e^{-\left(\frac{x^2}{2s^2}\right)} \right)$$

$$\left(-\frac{1}{s^2} + \frac{x_i^2}{s^4} \right) e^{-\left(\frac{x^2}{2s^2}\right)}$$

$$= \frac{1}{(2\pi s^2)^{\frac{d}{2}}} \cdot \sum_{i=1}^{d} \left(-\frac{1}{s^2} + \frac{x_i^2}{s^4} \right) e^{-\left(\frac{x^2}{2s^2}\right)}$$

$$= \frac{1}{(2\pi s^2)^{\frac{d}{2}}} \cdot \left(\frac{-d}{s^2} + \frac{x^2}{s^4} \right) \cdot e^{-\left(\frac{x^2}{2s^2}\right)} \quad = \square$$

2. calculat part. time derivative of G(x)

$$\frac{\partial G}{\partial t} = \frac{\partial G}{\partial s} \cdot \frac{\partial s}{\partial t} \quad (*)$$

$$= \frac{-(ds^2 - x^2) e^{-\left(\frac{x^2}{2s^2}\right)}}{(2\pi s^2)^{\frac{d}{2}} \cdot s^3} \cdot \frac{1}{s}$$

$$= \frac{1}{(2\pi s^2)^{\frac{d}{2}}} \cdot \frac{1}{s^4} \cdot -(ds^2 - x^2) e^{-\left(\frac{x^2}{2s^2}\right)}$$

$$= \frac{1}{(2\pi s^2)^{\frac{d}{2}}} \left(\frac{d}{s^2} + \frac{x^2}{s^4} \right) \cdot e^{-\left(\frac{x^2}{2s^2}\right)}$$

(*) with:

$$s = \sqrt{2t} = (2t)^{\frac{1}{2}}$$

$$\frac{\partial s}{\partial t} = \frac{\partial}{\partial t} (2t)^{\frac{1}{2}} = 2 \cdot \frac{1}{2} (2t)^{-\frac{1}{2}} = \frac{1}{s}$$

$$G(x) = \frac{e^{-\left(\frac{x^2}{2s^2}\right)}}{(2\pi s^2)^{\frac{d}{2}}}$$

$$\frac{\partial G}{\partial s} = \frac{-(ds^2 - x^2) e^{-\left(\frac{x^2}{2s^2}\right)}}{(2\pi s^2)^{\frac{d}{2}} \cdot s^3}$$