3.4 (d)

Parseval's theorem $\langle f, g \rangle_2 = \langle \hat{f}, \hat{g} \rangle$

$\iff$ $\| \hat{f} \|_2 = \| f \|_2$

$$\int_0^\infty \frac{\sin^2(\alpha w)}{w^2} dw = \int_0^\infty \left| \frac{\sin(aw)}{w} \right|^2 dw \quad \begin{array}{l} \text{Substitute} \\ w = \varphi/a \\ x = \frac{1}{a} \cdot \varphi \end{array}$$

$$= \int_{a \cdot 0}^{a \cdot \infty} \left| \frac{\sin(\varphi)}{\varphi/a} \right|^2 \frac{1}{a} \cdot d\varphi = \int_0^\infty a^2 \cdot \left| \frac{\sin(\varphi)}{\varphi} \right|^2 \frac{1}{a} d\varphi$$

$$= \int_0^\infty a \cdot \left| \frac{\sin(\varphi)}{\varphi} \right|^2 d\varphi = \int_0^\infty \left| \sqrt{a} \cdot \frac{\sin(\varphi)}{\varphi} \right|^2 d\varphi$$

$f(\varphi) = \sqrt{a} \cdot \mathrm{sinc}\left(\frac{\varphi}{\pi} \right)$ is symmetric

$f(\varphi) = f(-\varphi) \quad \wedge \quad \| f \|_2^2 = \int_{\mathbb{R}} \left(\sqrt{a} \cdot \mathrm{sinc}(\frac{\varphi}{\pi}) \right)^2 d\varphi$

$$= \int_{\mathbb{R}} \frac{\sin^2(aw)}{w^2} dw$$

$\Rightarrow \int_{\mathbb{R}} \frac{\sin^2(aw)}{w^2} = 2 \cdot \int_0^\infty \frac{\sin^2(aw)}{w^2} dw$

$f(x) = \sqrt{a} \cdot \mathrm{sinc}\left(\frac{x}{\pi} \right):$

$\hat{f}(z) = \frac{1}{(2\pi)^{\frac{1}{2}}} \cdot \int_{\mathbb{R}} \sqrt{a} \cdot \mathrm{sinc}\left(\frac{x}{\pi} \right) \cdot e^{-izx} dx$

$\hat{f}(z) = \sqrt{a} \cdot \frac{1}{(2\pi)^{\frac{1}{2}}} \int_{\mathbb{R}} \mathrm{sinc}\left(\frac{x}{\pi} \right) \cdot e^{-izx} dx$

$$\overset{r=1}{=} \sqrt{a} \cdot \frac{1}{\sqrt{\frac{2}{\pi}}} \cdot \frac{1}{(2\pi)^{\frac{1}{2}}} \cdot \int_{\mathbb{R}} \sqrt{\frac{2}{\pi}} \cdot \operatorname{sinc}\left(\frac{x}{\pi}\right) e^{-izx} dx$$

$$= \sqrt{a} \cdot \frac{1}{\sqrt{\frac{2}{\pi}}} \cdot \chi_{C_1}(0)$$

$$\| \hat{f} \|_2^2 :$$

$$\| \hat{f} \|_2^2 = \int_{\mathbb{R}} \left(\sqrt{a} \cdot \sqrt{\frac{\pi}{2}} \, \chi_{C_1(0)}(x) \right)^2 dx$$

$$= \int_{\mathbb{R}} a \cdot \frac{\pi}{2} \left(\chi_{C_1(0)}(x) \right)^2 dx$$

$$= a \cdot \frac{\pi}{2} \int_{\mathbb{R}} \left(\chi_{C_1(0)}(x) \right)^2 dx \qquad \overset{\text{here}}{\chi_{C_1(\omega)}(x)}$$
$$\in \{0, 1\}$$

$$= a \cdot \frac{\pi}{2} \int \chi_{C_1(\omega)}(x) \, dx \quad \rightsquigarrow$$

$$= a \cdot \frac{\pi}{2} \cdot 2 = \pi \cdot a$$

appling theorem : $\| f \|_2^2 = \| \hat{f} \|_2^2$

$$\Longrightarrow \int_{\mathbb{R}} \left(\sqrt{a} \cdot \operatorname{sinc}\left(\frac{x}{a}\right) \right)^2 dx = \int_{\mathbb{R}} \left(\sqrt{a} \cdot \sqrt{\frac{\pi}{2}} \cdot \chi_{C_1(0)}(x) \right)^2 dx$$

$$\int_{\mathbb{R}} \frac{\sin^2(a\omega)}{\omega^2} d\omega = \pi \cdot a \qquad \int_0^\infty \frac{\sin^2(a\omega)}{\omega^2} d\omega = a \cdot \frac{\pi}{2} \; /\!/$$

$$2 \cdot \int_0^\infty \frac{\sin^2(a\omega)}{\omega^2} d\omega = \pi \cdot a$$