Exercise 3.3 c.iv

Equations:

$$\sigma_B^2(s) = P_1(s)(\mu_1(s)-\mu_G)^2 + P_2(s)(\mu_2(s)-\mu_G)^2 \quad (1)$$

$$P_1(s)+P_2(s) = 1 \quad (2)$$

$$P_1(s)\mu_1(s) + P_2(s)\mu_2(s) = \mu_G \quad (3)$$

$$G(s) = P_1(s)\mu_1^2(s) + P_2(s)\mu_2^2(s) \quad (4)$$

(s) is dropped in the following

expand (1)

$$\sigma_B^2 = P_1\mu_1^2 - 2P_1\mu_1\mu_G + P_1\mu_G^2 + P_2\mu_2^2 - 2P_2\mu_2\mu_G + P_2\mu_G^2$$

$$= P_1\mu_1^2 + P_2\mu_2^2 - 2(P_1\mu_1 + P_2\mu_2)\mu_G + (P_1+P_2)\mu_G^2 =: (5)$$

(2) and (3) in (5)

$$\sigma_B^2 = P_1\mu_1^2 + P_2\mu_2^2 - 2\mu_G^2 + \mu_G^2$$

$$= P_1\mu_1^2 + P_2\mu_2^2 - \mu_G^2$$

$$\Rightarrow \sigma_B^2(s) = \underbrace{P_1(s)+\mu_1^2(s) + P_2(s)\mu_2^2(s)}_{} - \mu_G^2 \quad | \text{ with } (4)$$

$$\underline{\underline{= G(s) + C}} \quad , C \in \mathbb{R} \geq 0$$

$$\frac{\partial \sigma_B^2(s)}{\partial s} = \frac{\partial G(s)}{\partial s} \quad \square$$

Therefore max. $G(s)$ is equivalent to max. $\sigma_B^2(s)$, since they are only different in the const. term $-\mu_G^2$ (indep. of s).