Gravitational Lensing Expressions

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In the following I use O, S and L to denote the Observer, Source and Lens positions, respectively. P denotes any point in the ray of light we deal with. The lensing equation is

$$\frac{d^2u}{d\theta^2} + u = \frac{3}{2} r_g u^2 \tag{1}$$

where $u \equiv 1/r$, the radius $r \equiv LP$, and $\theta \equiv \measuredangle SLP$. The gravitational radius r_g is the Schwarzschild radius

$$r_g = \frac{2MG}{c^2} \tag{2}$$

where c is the speed of light and M the lens mass.

For thin lens approximation I use a couple of definitions: the "breaking point" (B) is a point on the incomming ray at which $\measuredangle SBL = \pi/2$, the incoming ray angle $\phi \equiv \measuredangle BSL$ and the impact parameter $b \equiv BL$. Given some ϕ, the breaking point B and the impact parameter b are calculated, the incoming ray is deflected at B (stays the same before B) such that $\phi_{afterB} = \phi - 2r_g/b$. This assumes that the impact parameter b is much larger than the gravitational radius r_g (this is the thins lens approximation assumption)

Calculating the time of travel is done as follows:

$$\Delta t_{ray} = \int_{Ray} \frac{dp}{c} \left(1 - \frac{r_g}{r(p)}\right)^{-\frac{1}{2}} \tag{3}$$

the square root term is the regular gravitational time dilation.

For extended masses I think changing r_g to depend on the inner mass at P, the numerics should work just fine for both thin and thick lenses. The metric outside a static spherical symmetric mass doesn't care of the exact inner structure.

Thoughts, comments?