Examination

Mathematical Foundations for Software Engineering

Course codes: DIT022 / DIT023

Date:	2022-01-05
Time:	14:00-18:00
Place:	Johanneberg/Lindholmen
Teacher:	Christian Berger Alexander Stotsky
Visit to e*xam hall:*	14:30 and 16:30
Questions:	6
Results:	Will be posted by 2022-01-27.
Grade Limits (DIT022):	Pass (3) 50% Pass with honors (VG) 90%
Grade Limits (DIT023):	Pass (3): 50% Pass with credit (4): 70% Pass with distinction (5): 90%
Allowed aids:	Calculators: Casio FX-82..., Texas TI-30... and Sharp EL-W531... Attached appendix with formulas and tables.

Please observe the following:

- <u>DO NOT</u> write your name on any answer sheet or exam sheet – write the anonymized code instead.
- Write in legible English (unreadable responses mean no points!).
- Motivate your answers and clearly state any assumptions made.
- Start each part of the exam on a new sheet!
- Write only on one side of the paper!
- Only answers written on the answer sheets will be graded, do not write on the exam sheets!
- Before handing in your exam, number and sort the sheets in task order!

NOTE:

Not following these instructions may result in the deduction of points!

Question 1 (1 + 1 + 1 + 3 = 6 pt)
"Languages and Grammars"

1.1 What language is generated by the given grammar G1? (1pt)
*Note that we are **not** asking for partially correct solution(s) but for the fully correct one(s). Also, "choosing multiple solutions when only one is correct" will not be assessed as correct. Mark your choice(s) on your answer sheets, not this exam sheet.*

G1 = (V, T, S, P) and V = {S, X, Y, Z, d, p, λ}, where S is the start variable, T = {d, p, λ} set of terminals and rules:

S → dXp | pYd
X→ pYd | λ
Y→ dXp | λ
Z→ pXp

a) $\mathcal{L}(G1)$ ={w∈ {d,p}* | the length of w is even and the middle symbol is p}
b) $\mathcal{L}(G1)$ ={w∈ {d,p}* | w is a random series of p's and d's}
c) $\mathcal{L}(G1)$ ={w∈ {d,p}* | w contains d's followed by the same amount of p's, or vice-versa} (Example: ddpp, dddppp, …)
d) $\mathcal{L}(G1)$ ={w∈ {d,p}* | w contains the same amount of d's and p's in alternating order} (Example: dpdpdp, pdpdpd, …)
e) $\mathcal{L}(G1)$ ={w∈ {d,p}* | the length of w is odd and the middle symbol is d}

1.2 What language is generated by the given grammar G2? (1pt)
*Note that we are **not** asking for partially correct solution(s) but for the fully correct one(s). Also, "choosing multiple solutions when only one is correct" will not be assessed as correct. Mark your choice(s) on your answer sheets, not this exam sheet.*

G2 = (V, T, S, P) and V = {S, X, 0, 1}, where S is the start variable, T = {0,1} set of terminals and rules:

S → 0X
X→ 1 | 1X

a) $\mathcal{L}(G2)$ ={w∈ {0,1} * | w starts with a 1 or ends with a 0}
b) $\mathcal{L}(G2)$ ={w∈ {0,1} * | w contains at least one 1 and at least one 0}
c) $\mathcal{L}(G2)$ ={w∈ {0,1} * | w contains 1s and 0s}
d) $\mathcal{L}(G2)$ ={w∈ {0,1} * | w contains only 1s except for the first symbol which is a 0}
e) $\mathcal{L}(G2)$ ={w∈ {0,1} * | w contains a 0 as the first symbol followed by a random series of 1s and 0s}

1.3 What language is generated by the given grammar G3? (1pt)
*Note that we are **not** asking for partially correct solution(s) but for the fully correct one(s). Also, "choosing multiple solutions when only one is correct" will not be assessed as correct. Mark your choice(s) on your answer sheets, not this exam sheet.*

G3 = (V, T, S, P) and V = {S, B, a, b, c, λ}, where S is the start variable, T = {a, b, c, λ} set of terminals and rules P:

S → aSBc | λ
aB → ab
bB → bb
cB → Bc

Please note that the empty word (λ) is allowed.

a) $\mathcal{L}(G3) = \{a^m b^n c^n$ | where $m, n \geq 1\}$
b) $\mathcal{L}(G3) = \{a^n b^{2n} c^n c^m$ | where $n \geq 0, m = 0\}$
c) $\mathcal{L}(G3) = \{a^n b^n c^n$ | where $n \geq 1\}$
d) $\mathcal{L}(G3) = \{a^n b^n c^n$ | where $n \geq 0\}$
e) $\mathcal{L}(G3) = \{a^n b^n c^{2n}$ | where $n \geq 1\}$

1.4 Provide **three** different non-empty words that are generated by the given grammar G4. (3pt)

G4 = (V, T, S, P) and V = {postalcode, forward_or_stationarea, space, localdeliveryunit, provarea, loctype, letter, digit, rural, urban, A, B, C, E, G, H, J, K, L, M, N, P, R, S, T, V, X, Y, W, Z, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, where postalcode is the start variable, T ={”A”, “B”, “C”, “E”, “G”, “H”, “J”, “K”, “L”, “M”, “N”, “P”, “R”, “S”, “T”, “V”, “X”, “Y”, “W”, “Z”, “0”, “1”, “2”, “3”, “4”, “5”, “6”, “7”, “8”, “9”, “_”} set of terminals and rules P:

<postalcode> ::= <forward_or_stationarea> < underscore > <localdeliveryunit>
<forward_or_stationarea> ::= <provarea> <loctype> <letter>
<localdeliveryunit> ::= <digit> <letter> <digit>
<provarea> ::= A | B | C | E | G | H | J | K | L | M | N | P | R | S | T | V | X | Y
<loctype> ::= <rural> | <urban>
<rural> ::= 0
<urban> ::= 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
<letter> ::= A | B | C | E | G | H | J | K | L | M | N | P | R | S | T | V | W | X | Y | Z
<digit> ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
<underscore> ::= _

Question 2 (1 + 6 + 15 = 22 pt)
"Automata"

2.1 What does automaton A1 do? (1pt)
*Note that we are **not** asking for partially correct solution(s) but for the fully correct one(s). Also, "choosing multiple solutions when only one is correct" will not be assessed as correct. Mark your choice(s) on your answer sheets, not this exam sheet.*

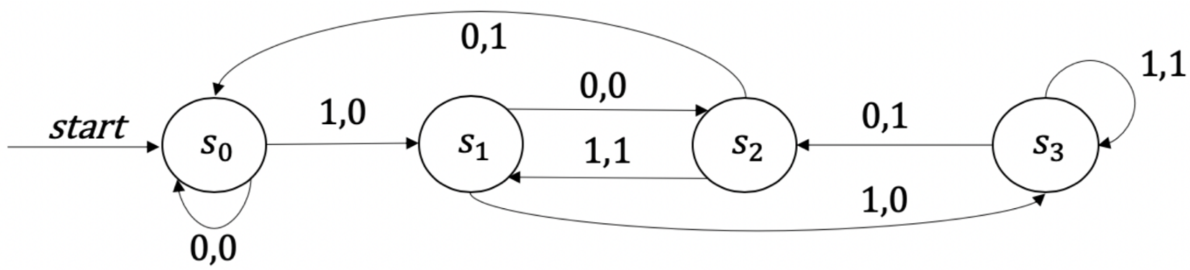


Figure 1: A1

a) It recognizes all possible input bit strings of length 2 (e.g., '00' is a bit string of length 2).
b) It recognizes when the bit string "110000" is on the input.
c) It recognizes bit strings with an even number of '1's on the input.
d) It recognizes when the output equals the input delayed by two 0's.
e) It recognizes all possible input bit strings of length 4 (e.g., '0000' is a bit string of length 3).

2.2 Repair the deterministic finite-state automaton A3 with no output so that it only accepts strings with an odd number of '1's **and** an odd number of '0's. (6pt)

Note that we are not asking for partially correct solution(s) but for the fully correct one(s). Unreadable drawings will be awarded with 0 points.

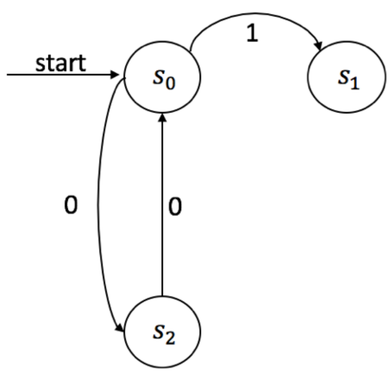


Figure 2: A3

2.3 Draw a deterministic finite-state machine with no output that accepts the set of bit strings $\{1^n | \; n = 2,3,4, \dots\}$. (15pt)
Note that unreadable drawings will be awarded with 0 points.

Question 3 (8 + 1 + 1 + 9 = 19 pt)
"Logic"

3.1 Provide the complete truth table for the given compound proposition. (8pt)

$$((p \to q) \leftrightarrow ((r \vee q) \wedge ((r \to q) \wedge p)))$$

3.2 To what logical expression can the compound proposition from 3.1 be simplified to (ie., is the compound proposition logically equivalent)? (1pt)

3.3 Express this statement using predicates and quantifiers. (1pt)
Note that we are not asking for partially correct solution(s) but for the fully correct one(s). Also, choosing multiple solutions, when only one is correct will not be assessed as correct. Mark clearly the correct solution.

Let $Q(x, y)$ be the statement "student x has been a contestant on quiz show y." Express S_1 in terms of $Q(x, y)$, quantifiers, and logical connectives, where the domain for x consists of all students at your school and for y consists of all quiz shows on television.

S_1 = "At least two students from your school have been contestants on Jeopardy."

a) $S_1 = \exists x_1 \exists x_2 (Q(x_1, Jeopardy) \wedge Q(x_2, Jeopardy))$
b) $S_1 = \exists x_1 \exists x_2 (Q(x_1 \wedge x_2, Jeopardy))$
c) $S_1 = \exists x_1 \exists x_2 \exists y_1 \exists y_2 (Q(x_1, y_1) \wedge Q(x_2, y_2) \wedge y_1 \neq y_2)$
d) $S_1 = \exists x_1 \exists x_2 (Q(x_1, Jeopardy) \wedge Q(x_2, Jeopardy) \wedge x_1 \neq x_2)$
e) $S_1 = \exists x_1 \exists x_2 \exists y_1 (Q(x_1, y_1) \wedge Q(x_2, y_1) \wedge x_1 \neq x_2)$

3.4 Look at the following compound proposition and answer the questions below. (9pt in total)

$$((p \to q) \to ((r \vee p) \vee (\neg p \to \neg q)))$$

i) When is a compound proposition "satisfiable"? (1pt)
ii) When is a compound proposition "not a tautology"? (1pt)
iii) When is a compound proposition a contingency? (1pt)
iv) Why is the given compound proposition not a tautology? (6pt)

Question 4 (4 + 4 = 8 pt)
"Proofs"

4.1 Prove that $2 + 4 + 6 + \cdots + 2n = n + n^2$ for all $n > 0$. (4pt)

4.2 Prove that $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$ for all $n > 0$. (4pt)

Question 5 (1 + 1 + 1 + 8 + 4 + 5 = 20 pt)
"Complexity and Graphs"

5.1 What is the complexity of the following code snippet (in terms of Big-O notation)? (1pt)

```
function(int n){
   int result = 0;
   for (int i=1; i<=n; i++){
      for (int j=1; j<=n; j++){
         if (j < 2){
            for (int k=1; k<=j; k++) {
              result++;
            }
         }
      }
   }
}
```

5.2 What is the complexity of the following code snippet (in terms of Big-O notation)? (1pt)

```
function(int n){
   for (int i=1; i<n; i= i*2){
      for (int j=1; j<n; j++){
         System.out.println("Hej!"); // This is O(1)
      }
   }
}
```

5.3 What is the complexity of the following code snippet (in terms of Big-O notation)? (1pt)

```
function(int n){
  // a[..] is an array defined outside this function with at least n elements
  for(int i = 1; i < n; i++) {
    bool not_found = true;
    for(int j = 1; j < n; j++) {
        if (a[i] == a[j]) {
             not_found = false;
             break; // break means leaving the current loop
        }
    }
    if (not_found) {
        System.out.println("not found");
    }
  }
  return true;
```

5.4 Trace the execution of the **bubble sort** algorithm over the array below; please note that this question consists of the two sub-questions 5.4.1) and 5.4.2) and hence, please read the entire question before you start. (8pt in total)

Array: 12 3 5 13 17 11 9 10 14

5.4.1) Show **only** the state of the array **after** every single iteration of the algorithm did complete and before the next iteration is starting (ie., **one** single line per iteration!) until the array is sorted. Include in your trace also the state of the array at the beginning before applying your sorting algorithm. Assume that the bubble sort implementation is using an internal variable `swapped` to keep track whether two elements have been swapped in a single iteration; if no swapping did occur for a complete iteration, the algorithm will end.

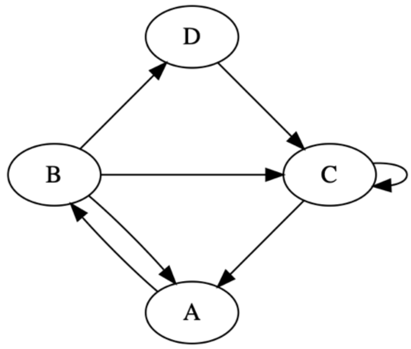
5.4.2) What is the best-case and worst-case complexity of bubble sort?

5.5 Assume you have three algorithms A, B, C with the respective complexities O(log(n)), O(n^2), and O(n^3). Each algorithm spends 5s to process 20,000 items.

5.5.1) What are the processing times for each algorithm for 30,000 items? Please provide the results of your calculations with an accuracy of two decimal places (the decimal place accuracy of a number is the number of digits to the right of the decimal point). (3pt)

5.5.2) What algorithm would be the fastest to process 10,000 items? (1pt)

5.6 Look at the directed graph G5 = (V, E) below, where V is the set of vertices V={A,B,C,D} and E is the set of edges, and answer the questions below.



5.6.1) Determine $all = \sum_{v \in V} deg^{-}(v) + \sum_{v \in V} deg^{+}(v)$ (2pt)

5.6.2) How many different Euler paths do exist in G5? (1pt)

5.6.3) How many different Hamilton paths do exist in G5? (1pt)

5.6.4) Is the following statement "true" or "false":

$\nexists v \in V$: there is a Hamilton circuit in G5. (1pt)

Question 6 (5 + 6 + 9 + 5 = 25 pt)
"Statistics"

6.1 For the following data:

x	y
0	1
2	3
3	9
7	11

(a) Please calculate the following sums:

(1) $S_{xx} = \sum_{i=1}^{n}(x_i - \overline{x})^2$, 0.5 pt

(2) $S_{yy} = \sum_{i=1}^{n}(y_i - \overline{y})^2$, 0.5 pt

(3) $S_{xy} = \sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})$, 0.5 pt

where $\overline{x}$ and $\overline{y}$ are the mean values and $n = 4$.

(b) Please calculate the correlation coefficient using the sums above, 0.5 pt.

(c) Please find the parameters α and β of the linear regression $y = \alpha + \beta x$, using the sums above, 1 pt .

(d) Please report the standard error of the estimate **without explicit calculation of the sum of squares** $\sum(y - \hat{y})^2$: (2 pt)

Hint: please use the following relation :

$$\sum_{i=1}^{n}(y_i - \overline{y} - r\frac{s_y}{s_x}(x - \overline{x}))^2 = (n-1)s_y^2(1 - r^2) = \frac{S_{xx}S_{yy} - S_{xy}^2}{S_{xx}} = SS_R{}^1$$

The total cost of the problem is 5 pt.

[1] SS_R is the the sum of squares of the residuals or residual sum of squares

6.2 In each of 4 elections, the Democrats have a 51% chance of winning.

We assume that the elections are independent of each other.

(a) What is the probability that the Democrats will win
0 elections, 1 election, 2 elections, 3 elections or all 4 elections ?
Please report five probabilities with four decimal place accuracy[2], (5 pt).

(b) Calculate the sum of probabilities, (1 pt).

Total number of points is 6 pt.
Please, apply the formula for the binomial distribution.

[2]The decimal place accuracy of a number is the number of digits to the right of the decimal point

6.3 For the following three groups of data :

Group 1	Group 2	Group 3
7	4	6
9	3	8
5	6	5
8	2	4

test whether the mean values are equal for the groups, please use $\alpha = 5\%$ (totally 9 points, see the distribution below:)

1) State the null and alternative hypothesis. (2 pt)

2) Calculate the appropriate test statistic. (4 pt)
Please report the test statistic with three decimal place accuracy.[3]

3) Find the critical value. (2 pt)
The table for critical values is enclosed.

4) What is the decision rule? (0.5 pt)

5) What is your interpretation of the findings? (0.5 pt)

[3]The decimal place accuracy of a number is the number of digits to the right of the decimal point

6.4 Suppose that we have two populations of Type A and Type B which follow normal distributions with variances σ_1^2 and σ_2^2 respectively. Suppose also that we calculated sample variances for 21 samples of Type A and Type B. The results are summarized in the Table.

(a) Test, at the $\alpha = 10\%$ level of significance, whether the data given in the Table provide sufficient evidence to conclude that Type B population has lower variance than Type A. (in total 5 pt, see the distribution below).
1) State the null and alternative hypothesis. (2 pt)
2) Calculate the appropriate test statistic. (1 pt)
3) Find the critical value. (1 pt)
The table for critical values is enclosed.

(b) What is your decision and interpretation of the findings? (1 pt).

The table for critical values is enclosed.

Population	Sample Size	Sample Variance
A	$n_1 = 21$	$s_1^2 = 1.78$
B	$n_2 = 21$	$s_2^2 = 0.98$

Formulas for Examen: Statistical Part

1 Linear Regression

1.1 Pearson Correlation Coefficient and Variances:

$$r = \frac{1}{s_x s_y}\frac{1}{(n-1)}\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})$$

$$= \frac{\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\overline{x})^2\sum_{i=1}^{n}(y_i-\overline{y})^2}} = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$$

$$s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\overline{x})^2$$

$$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i-\overline{y})^2$$

$$S_{xx} = (n-1)s_x^2 = \sum_{i=1}^{n}(x_i-\overline{x})^2$$

$$S_{yy} = (n-1)s_y^2 = \sum_{i=1}^{n}(y_i-\overline{y})^2$$

$$S_{xy} = \sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})$$

$$SS_R = \frac{S_{xx}S_{yy}-S_{xy}^2}{S_{xx}}$$

1.2 Equation for Linear Regression:

$$
\begin{aligned}
y - \overline{y} &= r\frac{s_y}{s_x}(x - \overline{x}) \\
y - \overline{y} &= b(x - \overline{x}) = a + bx \\
b &= \frac{\sum_{i=1}^{n} x_i y_i - n\ \overline{x}\ \overline{y}}{\sum_{i=1}^{n} x_i^2 - n(\overline{x})^2} = r\frac{s_y}{s_x} = \frac{S_{xy}}{S_{xx}} \\
a &= \bar{y} - b\bar{x}
\end{aligned}
$$

1.3 Regression Error Estimation:

$$
\epsilon = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-2}} = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \overline{y} - r\frac{s_y}{s_x}(x - \overline{x}))^2}{n-2}} = \sqrt{\frac{(n-1)s_y^2(1-r^2)}{n-2}} = \sqrt{\frac{S_{xx}S_{yy} - S_{xy}^2}{(n-2)S_{xx}}}
$$

2 Probability and Distributions

2.1 Bayes Theorem and Total Probablity

$$
P(B|A) = \frac{P(B)P(A|B)}{P(A)}
$$

$$
P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i)
$$

2.2 Poisson Distribution

$$
P\{X = i\} = e^{-\lambda}\frac{\lambda^i}{i!}
$$

$$
E(x) = \lambda, \quad Var(X) = \lambda
$$

2.3 Binomial Distribution

$$
P_{i,n} = \binom{n}{i} p^i q^{n-i}, \ \binom{n}{i} = \frac{n!}{i!(n-i)!}, \ q = 1 - p, \ i = 0, 1, ..., n
$$

3 ANOVA

$$
\begin{aligned}
MSTR &= \frac{n\displaystyle\sum_{i=1}^{k}(\bar{x}_i - \bar{X})^2}{(k-1)} \\
MSE &= \frac{(n-1)\displaystyle\sum_{i=1}^{k} s_i^2}{(n_t - k)}, \quad n_t = k\ n \\
F &= \frac{MSTR}{MSE} = \frac{\frac{SSB}{df_{SSB}}}{\frac{SSW}{df_{SSW}}} \\
SSB &= n\sum_{i=1}^{k}(\overline{x}_i - \overline{X})^2 \\
SSW &= \sum\sum x_{ij}^2 - SSB - n\ k\ \overline{X}^2
\end{aligned}
$$

$$\text{critical value,} \quad F_{k-1,\ n_t - k,\ \alpha}$$

4 Hypothesis Testing

4.1 One Sample χ^2 test (Test on Variance)

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

$$\text{critical value,} \quad \chi^2_{\alpha,n-1}$$

4.2 F-Test

$$
\begin{aligned}
F &= \frac{s_1^2}{s_2^2} \\
df_1 &= n_1 - 1 \\
df_2 &= n_2 - 1
\end{aligned}
$$

4.3 Two Sample t-Test

$$
\begin{aligned}
t &= \frac{\overline{x} - \overline{y}}{S\sqrt{1/n + 1/m}} \\
S &= \sqrt{\frac{(n-1)S_x^2 + (m-1)S_y^2}{(n-1) + (m-1)}} \\
df &= n + m - 2
\end{aligned}
$$

4.4 Paired t-Test

$$t = \frac{\overline{d} - \mu_0}{S/\sqrt{n}}$$

critical value, $t_{\alpha,n-1}$

Table of critical values for the F distribution (for use with ANOVA):

How to use this table:

There are two tables here. The first one gives critical values of F at the p = 0.05 level of significance.
The second table gives critical values of F at the p = 0.01 level of significance.
1. Obtain your F-ratio. This has (x,y) degrees of freedom associated with it.
2. Go along x columns, and down y rows. The point of intersection is your critical F-ratio.
3. If your obtained value of F is equal to or larger than this critical F-value, then your result is significant at that level of probability.
An example: I obtain an F ratio of 3.96 with (2, 24) degrees of freedom.
I go along 2 columns and down 24 rows. The critical value of F is 3.40. My obtained F-ratio is larger than this, and so I conclude that my obtained F-ratio is likely to occur by chance with a p<.05.

Critical values of F for the 0.05 significance level:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.39	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.97	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.10	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.97	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.56	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.33	3.47	3.07	2.84	2.69	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.38	2.32	2.28
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.26
25	4.24	3.39	2.99	2.76	2.60	2.49	2.41	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.17
31	4.16	3.31	2.91	2.68	2.52	2.41	2.32	2.26	2.20	2.15
32	4.15	3.30	2.90	2.67	2.51	2.40	2.31	2.24	2.19	2.14
33	4.14	3.29	2.89	2.66	2.50	2.39	2.30	2.24	2.18	2.13
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17	2.12
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11

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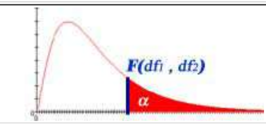
F Distribution Tables

The F distribution is a right-skewed distribution used most commonly in Analysis of Variance. When referencing the F distribution, the **numerator degrees of freedom are always given first**, as switching the order of degrees of freedom changes the distribution (e.g., $F_{(10,12)}$ does not equal $F_{(12,10)}$). For the four F tables below, the rows represent denominator degrees of freedom and the columns represent numerator degrees of freedom. The right tail area is given in the name of the table. For example, to determine the .05 critical value for an F distribution with 10 and 12 degrees of freedom, look in the 10 column (numerator) and 12 row (denominator) of the F Table for alpha=.05. $F_{(.05, 10, 12)} = 2.7534$. You can use the interactive F-Distribution Applet to obtain more accurate measures.

F Table for α = 0.10



\	df_1=1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
df_2=1	39.86346	49.50000	53.59324	55.83296	57.24008	58.20442	58.90595	59.43898	59.85759	60.19498	60.70521	61.22034	61.74029	62.00205	62.26497	62.52905	62.79428	63.06064	63.32812
2	8.52632	9.00000	9.16179	9.24342	9.29263	9.32553	9.34908	9.36677	9.38054	9.39157	9.40813	9.42471	9.44131	9.44962	9.45793	9.46624	9.47456	9.48289	9.49122
3	5.53832	5.46238	5.39077	5.34264	5.30916	5.28473	5.26619	5.25167	5.24000	5.23041	5.21562	5.20031	5.18448	5.17636	5.16811	5.15972	5.15119	5.14251	5.13370
4	4.54477	4.32456	4.19086	4.10725	4.05058	4.00975	3.97897	3.95494	3.93567	3.91988	3.89553	3.87036	3.84434	3.83099	3.81742	3.80361	3.78957	3.77527	3.76073
5	4.06042	3.77972	3.61948	3.52020	3.45298	3.40451	3.36790	3.33928	3.31628	3.29740	3.26824	3.23801	3.20665	3.19052	3.17408	3.15732	3.14023	3.12279	3.10500
6	3.77595	3.46330	3.28876	3.18076	3.10751	3.05455	3.01446	2.98304	2.95774	2.93693	2.90472	2.87122	2.83634	2.81834	2.79996	2.78117	2.76195	2.74229	2.72216
7	3.58943	3.25744	3.07407	2.96053	2.88334	2.82739	2.78493	2.75158	2.72468	2.70251	2.66811	2.63223	2.59473	2.57533	2.55546	2.53510	2.51422	2.49279	2.47079
8	3.45792	3.11312	2.92380	2.80643	2.72645	2.66833	2.62413	2.58935	2.56124	2.53804	2.50196	2.46422	2.42464	2.40410	2.38302	2.36136	2.33910	2.31618	2.29257
9	3.36030	3.00645	2.81286	2.69268	2.61061	2.55086	2.50531	2.46941	2.44034	2.41632	2.37888	2.33962	2.29832	2.27683	2.25472	2.23196	2.20849	2.18427	2.15923
10	3.28502	2.92447	2.72767	2.60534	2.52164	2.46058	2.41397	2.37715	2.34731	2.32260	2.28405	2.24351	2.20074	2.17843	2.15543	2.13169	2.10716	2.08176	2.05542
11	3.22520	2.85951	2.66023	2.53619	2.45118	2.38907	2.34157	2.30400	2.27350	2.24823	2.20873	2.16709	2.12305	2.10001	2.07621	2.05161	2.02612	1.99965	1.97211
12	3.17655	2.80680	2.60552	2.48010	2.39402	2.33102	2.28278	2.24457	2.21352	2.18776	2.14744	2.10485	2.05968	2.03599	2.01149	1.98610	1.95973	1.93228	1.90361
13	3.13621	2.76317	2.56027	2.43371	2.34672	2.28298	2.23410	2.19535	2.16382	2.13763	2.09659	2.05316	2.00698	1.98272	1.95757	1.93147	1.90429	1.87591	1.84620
14	3.10221	2.72647	2.52222	2.39469	2.30694	2.24256	2.19313	2.15390	2.12195	2.09540	2.05371	2.00953	1.96245	1.93766	1.91193	1.88516	1.85723	1.82800	1.79728
15	3.07319	2.69517	2.48979	2.36143	2.27302	2.20808	2.15818	2.11853	2.08621	2.05932	2.01707	1.97222	1.92431	1.89904	1.87277	1.84539	1.81676	1.78672	1.75505
16	3.04811	2.66817	2.46181	2.33274	2.24376	2.17833	2.12800	2.08798	2.05533	2.02815	1.98539	1.93992	1.89127	1.86556	1.83879	1.81084	1.78156	1.75075	1.71817
17	3.02623	2.64464	2.43743	2.30775	2.21825	2.15239	2.10169	2.06134	2.02839	2.00094	1.95772	1.91169	1.86236	1.83624	1.80901	1.78053	1.75063	1.71909	1.68564
18	3.00698	2.62395	2.41601	2.28577	2.19583	2.12958	2.07854	2.03789	2.00467	1.97698	1.93334	1.88681	1.83685	1.81035	1.78269	1.75371	1.72322	1.69099	1.65671
19	2.98990	2.60561	2.39702	2.26630	2.17596	2.10936	2.05802	2.01710	1.98364	1.95573	1.91170	1.86471	1.81416	1.78731	1.75924	1.72979	1.69876	1.66587	1.63077
20	2.97465	2.58925	2.38009	2.24893	2.15823	2.09132	2.03970	1.99853	1.96485	1.93674	1.89236	1.84494	1.79384	1.76667	1.73822	1.70833	1.67678	1.64326	1.60738
21	2.96096	2.57457	2.36489	2.23334	2.14231	2.07512	2.02325	1.98186	1.94797	1.91967	1.87497	1.82715	1.77555	1.74807	1.71927	1.68896	1.65691	1.62278	1.58615
22	2.94858	2.56131	2.35117	2.21927	2.12794	2.06050	2.00840	1.96680	1.93273	1.90425	1.85925	1.81106	1.75899	1.73122	1.70208	1.67138	1.63885	1.60415	1.56678
23	2.93736	2.54929	2.33873	2.20651	2.11491	2.04723	1.99492	1.95312	1.91888	1.89025	1.84497	1.79643	1.74392	1.71588	1.68643	1.65535	1.62237	1.58711	1.54903
24	2.92712	2.53833	2.32739	2.19488	2.10303	2.03513	1.98263	1.94066	1.90625	1.87748	1.83194	1.78308	1.73015	1.70185	1.67210	1.64067	1.60726	1.57146	1.53270
25	2.91774	2.52831	2.31702	2.18424	2.09216	2.02406	1.97138	1.92925	1.89469	1.86578	1.82000	1.77083	1.71752	1.68898	1.65895	1.62718	1.59335	1.55703	1.51760
26	2.90913	2.51910	2.30749	2.17447	2.08218	2.01389	1.96104	1.91876	1.88407	1.85503	1.80902	1.75957	1.70589	1.67712	1.64682	1.61472	1.58050	1.54368	1.50360
27	2.90119	2.51061	2.29871	2.16546	2.07298	2.00452	1.95151	1.90909	1.87427	1.84511	1.79889	1.74917	1.69514	1.66616	1.63560	1.60320	1.56859	1.53129	1.49057
28	2.89385	2.50276	2.29060	2.15714	2.06447	1.99585	1.94270	1.90014	1.86520	1.83593	1.78951	1.73954	1.68519	1.65600	1.62519	1.59250	1.55753	1.51976	1.47841
29	2.88703	2.49548	2.28307	2.14941	2.05658	1.98781	1.93452	1.89184	1.85679	1.82741	1.78081	1.73060	1.67593	1.64655	1.61551	1.58253	1.54721	1.50899	1.46704
30	2.88069	2.48872	2.27607	2.14223	2.04925	1.98033	1.92692	1.88412	1.84896	1.81949	1.77270	1.72227	1.66731	1.63774	1.60648	1.57323	1.53757	1.49891	1.45636
40	2.83535	2.44037	2.22609	2.09095	1.99682	1.92688	1.87252	1.82886	1.79290	1.76269	1.71456	1.66241	1.60515	1.57411	1.54108	1.50562	1.46716	1.42476	1.37691
60	2.79107	2.39325	2.17741	2.04099	1.94571	1.87472	1.81939	1.77483	1.73802	1.70701	1.65743	1.60337	1.54349	1.51072	1.47554	1.43734	1.39520	1.34757	1.29146