Non-Linear Compton Scattering

$e \quad + \quad n\gamma \rightarrow e' \quad + \quad \gamma'$ in a strong laser field with ξ or a_0

$(m_e^*, 0) + n(\omega^*, \boldsymbol{k}) \rightarrow (m_e^*, \boldsymbol{p}'_n) + (\omega'_n, \boldsymbol{k}'_n)$, for $m_e^* = m_e\sqrt{1+\xi^2}$, $\omega^* = \omega/\sqrt{1+\xi^2}$

$(E_e, \boldsymbol{p}) + n(\omega_0, \boldsymbol{k}_0) \rightarrow (E'_c, \boldsymbol{p}'_{n,c}) + (\omega'_{n,c}, \boldsymbol{k}'_{n,c})$

$$\frac{d\sigma}{dy}(\zeta, \xi_2) = \frac{2\pi r_e^2}{x\xi^2} \sum_{n=1}^{\infty} (F_{1n} + \zeta\xi_2 F_{2n}) \qquad \frac{d\sigma}{d\bar{z}_n}(\zeta, \xi_2) = \frac{2\pi r_e^2}{x\xi^2} \frac{L_{u_n}\omega'_n\sqrt{1+\xi^2}}{n\omega} (F_{1n} + \zeta\xi_2 F_{2n})$$

$$\equiv \pi r_e^2 \frac{4L_{u_n}}{u_n} F(\bar{z}_n)$$

$$\boxed{\begin{aligned} F_{1n} &= -4J_n^2 + \xi^2\left(1 - y + \frac{1}{1-y}\right)(J_{n-1}^2 + J_{n+1}^2 - 2J_n^2) \\ F_{2n} &= \xi^2\left(-1 + y + \frac{1}{1-y}\right)\left(1 - 2\frac{y}{y_n}\frac{(1-y_n)}{(1-y)}\right)(J_{n-1}^2 - J_{n+1}^2) \end{aligned}}$$

Simple transformations : Linear $\rightarrow$ Non-Linear Compton scattering

Linear	Non-Linear
x	$u_n = nx/(1+\xi^2)$
ω	$n\omega/\sqrt{1+\xi^2}$
$\omega' = \dfrac{m_e\omega}{\omega(1-\cos\theta) + m_e}$, $d\omega' = \omega dy$	$\omega'_n = \dfrac{nm_e\omega}{\omega(1-\cos\theta_n)/\sqrt{1+\xi^2} + m_e\sqrt{1+\xi^2}}$, $d\omega'_n = \dfrac{n\omega}{\sqrt{1+\xi^2}}dy$
$\omega'_{min} = \dfrac{\omega}{1+x}$	$\omega'_{n,min} = \dfrac{n\omega/\sqrt{1+\xi^2}}{1+u_n}$
$y_{max} = \dfrac{x}{1+x}$	$y_n = \dfrac{u_n}{1+u_n}$, $u_n = \dfrac{y_n}{1-y_n}$, and $y = \dfrac{\omega'_{n,c}}{E_e}$
$\bar{z} = \dfrac{\ln(\omega/\omega')}{\ln(1+x)}$, $d\bar{z} = \dfrac{\omega}{L_x\omega'}dy$	$\bar{z}_n = \dfrac{\ln\left(n\omega/\left(\omega'_n\sqrt{1+\xi^2}\right)\right)}{\ln(1+u_n)}$, $d\bar{z}_n = \dfrac{n\omega/\sqrt{1+\xi^2}}{L_{u_n}\omega'_n}dy$
$\gamma = \dfrac{E_e}{m_e}$	$\dfrac{\gamma}{\sqrt{1+\xi^2}}$

Variables used for simulation in CAIN

$$u \equiv \frac{y}{1-y} \qquad \therefore y = \frac{u}{1+u}$$

$$v_n \equiv \frac{u}{u_n} \quad \text{for } 0 \geq v_n \leq 1 \quad \text{and} \quad dv_n = \frac{du}{u_n}$$

$$dy = \frac{u_n}{(1+u)^2} dv_n$$

$$\therefore \frac{d\sigma}{dv_n}(\zeta, \xi_2) = \frac{2\pi r_e^2}{x\xi^2} \left(F_{1n} + \zeta\xi_2 F_{2n}\right) \frac{u_n}{(1+u)^2}$$

Variables suggested in CAIN

$$\bar{z}_n = \ln\left(n\omega / \left(\omega_n' \sqrt{1+\xi^2}\right)\right) / L_{u_n}$$

$$y \approx 1 - \frac{1+\xi^2}{n} \frac{\omega_n'}{\omega} = 1 - e^{-L_{u_n} \bar{z}_n}$$

$$\text{with} \quad \gamma^2 \gg 1, \quad \beta \approx 1$$

$$\frac{d\sigma}{d\bar{z}_n}(\zeta, \xi_2) = \frac{2\pi r_e^2}{x\xi^2} \left(F_{1n} + \zeta\xi_2 F_{2n}\right) L_{u_n} e^{-L_{u_n} \bar{z}_n}$$

Transformation to variables in CAIN

$$x \to \lambda$$

$$u \to v$$

$$u_n \to v_n$$

$$v_n = u/u_n \to y = v/v_n$$

Compare with following cross section formulas,

$$\frac{d\sigma}{dy}(\zeta, \xi_2) = \frac{2\pi r_e^2}{x\xi^2} \sum_{n=1}^{\infty} \left(F_{1n} + \zeta\xi_2 F_{2n}\right)$$

Event generation in the non-linear Compton scattering

1. Uniform random number: $0 \leq \bar{z}_n \leq 1$ with $\bar{z}_n \equiv \dfrac{\log\left(n\omega / \left(w'_n \sqrt{1+\xi^2}\right)\right)}{L_{u_n}}$

2. $\omega'_n = \dfrac{n\omega}{\sqrt{1+\xi^2}}\, e^{-L_{u_n}\bar{z}_n}$

3. $\cos\theta_n = m\sqrt{1+\xi^2}\left(\dfrac{\sqrt{1+\xi^2}}{n\omega} - \dfrac{1}{\omega'_n}\right) + 1 = \dfrac{2}{u_n} - \dfrac{m\sqrt{1+\xi^2}}{\omega'_n} + 1$

$$\therefore 1 - \cos\theta_n = \frac{m\sqrt{1+\xi^2}}{\omega}\left(\frac{\omega}{\omega'_n} - \frac{\sqrt{1+\xi^2}}{n}\right) \quad \text{with } u_n = \frac{nx}{1+\xi^2},\ x = \frac{2\omega}{m_e}$$

4. Lorentz transformation from the electron rest frame to the laboratory frame :

$$\omega'_{c,n} = \frac{\gamma}{\sqrt{1+\xi^2}}\omega'_n\left(1 - \sqrt{1 - \frac{1+\xi^2}{\gamma^2}}\cos\theta_n\right)$$

$$y = \frac{\omega'_{c,n}}{E_e} = \frac{\omega'_n}{m_e\sqrt{1+\xi^2}}\left(1 - \sqrt{1 - \frac{1+\xi^2}{\gamma^2}}\left(1 + \frac{m_e\sqrt{1+\xi^2}}{\omega}\left(\frac{\sqrt{1+\xi^2}}{n} - \frac{\omega}{\omega'_n}\right)\right)\right)$$

$$y = -\sqrt{1 - \frac{1+\xi^2}{\gamma^2}}\frac{\omega'_n}{\omega}\left(\frac{\sqrt{1+\xi^2}}{n} - \frac{\omega}{\omega'_n}\right) + \frac{\omega'_n}{m_e\sqrt{1+\xi^2}}\left(1 - \sqrt{1 - \frac{1+\xi^2}{\gamma^2}}\right)$$

$$y \approx \frac{\omega'_n}{\omega}\left(\frac{\omega}{\omega'_n} - \frac{\sqrt{1+\xi^2}}{n}\right) = 1 - \frac{\omega'_n}{\omega}\frac{\sqrt{1+\xi^2}}{n} = 1 - e^{-L_{u_n}\bar{z}_n} \text{ for } 1 \gg \frac{1+\xi^2}{\gamma^2}$$

Compton Process in a Strong Laser Field

$$\text{Laser strength}: \ \xi = \frac{e\sqrt{-a^2}}{m} = \frac{e\sqrt{-<A^\mu A_\mu>}}{m} = \frac{\lambda\!\!\!^{-}_L}{m}\sqrt{\mu_0 cP}$$

$$\text{with } A^\mu = (\Phi, \boldsymbol{A}) \text{ and } -a^2 = \frac{1}{2\omega_L V} \times 2 = \frac{1}{\omega_L V}$$

$$S_{fi} = \frac{1}{(2\omega' \cdot 2q_0 \cdot 2q_0')^{1/2}} \sum_n M_{fi}^{(n)} (2\pi)^4 i\delta^{(4)}(nk + q - q' - k')$$

$$q^\mu = p^\mu - e^2 \frac{a^2}{2(kp)} k^\mu, \ \ q'^\mu = p'^\mu - e^2 \frac{a^2}{2(kp')} k^\mu, \ \ k = (\omega, \boldsymbol{k}), \ \ k' = (\omega', \boldsymbol{k}')$$

$$q^2 = q'^2 = m_e^2(1+\xi^2) = m_e^{*2}$$

$$nk + q = q' + k' \quad \therefore kq = kq' + kk', \ \ kp = kp' + kk' \text{ with } k^2 = 0$$

$$u = \frac{kk'}{kp'} = \frac{k(p-p')}{kp'} = \frac{kp}{kp'} - 1 = \frac{kq}{kq'} - 1 = \frac{k(q'+k')}{kq'} - 1$$

In the CM system, $n\boldsymbol{k} + \boldsymbol{q} = \boldsymbol{q}' + \boldsymbol{k}' = 0$ and $k(q'+k') = \omega(q_0' + \omega')$

$$\therefore \ u = \frac{E_n}{q_0' - |\boldsymbol{q}'|\cos\theta} - 1 \ \text{ with } E_n = q_0' + \omega' = n\omega + q_0$$

$$u_n = \frac{E_n}{q_0' - |\boldsymbol{q}'|} - 1 = \frac{E_n(q_0' + |\boldsymbol{q}'|)}{q'^2} - 1 = \frac{E_n^2}{m_e^{*2}} - 1 = \frac{2n(kp)}{m_e^{*2}} \ \text{ with } kq = kp$$

$$\because E_n^2 = (n\omega + q_0)^2 = n^2\omega^2 + 2n\omega q_0 + q_0^2 = 2n\omega|\boldsymbol{q}| + 2n\omega q_0 + q_0^2 - |\boldsymbol{q}|^2 \text{ with } n\boldsymbol{k} + \boldsymbol{q} = 0$$

$$u_n = \frac{nx}{1+\xi^2} \text{ with } x = \frac{2kp}{m_e^2} = \frac{4\omega_0 E_e}{m_e^2} \text{ for headon collision} \qquad 0 \le u \le u_n$$

Probability of n-th harmonic emission per unit volume and unit time:

$$dW_n = |M_{fi}^{(n)}|^2 \frac{d^3k' d^3q'}{(2\pi)^6 \cdot 2\omega' \cdot 2q_0 \cdot 2q_0'} (2\pi)^4 i\delta^{(4)}(nk + q - q' - k')$$

In the CM system, $i.e.$ $n\boldsymbol{k} + \boldsymbol{q} = \boldsymbol{q}' + \boldsymbol{k}' = 0$

$$I = \delta^{(4)}(nk + q - q' - k') \frac{d^3q' d^3k'}{q_0'\omega'} = \delta^{(3)}(\boldsymbol{q}' + \boldsymbol{k}')\delta(q_0' + \omega' - E_n) \frac{d^3q' d^3k'}{q_0'\omega'}$$

integrating over d^3k', $\quad I = \delta(q_0' + \omega' - E_n)\dfrac{d^3q'}{q_0'\omega'}, \quad \boldsymbol{k}' = -\boldsymbol{q}'$

$d^3q' = |\boldsymbol{q}'|^2 d|\boldsymbol{q}'| d\Omega$ and $q_0' = \sqrt{m^{*2} + |\boldsymbol{q}'|^2}$, $\omega' = |\boldsymbol{q}'|$

$$\therefore \frac{\partial(q_0' + \omega')}{\partial|\boldsymbol{q}'|} = |\boldsymbol{q}'| \left(\frac{1}{q_0'} + \frac{1}{|\boldsymbol{q}'|} \right) = |\boldsymbol{q}'| \frac{E_n}{q_0'\omega'} \qquad \text{,so} \quad d|\boldsymbol{q}'| = \frac{q_0'\omega'}{|\boldsymbol{q}'| E_n} d(q_0' + \omega')$$

$$\therefore d^3q' = \frac{|\boldsymbol{q}'| q_0'\omega'}{E_n} d(q_0' + \omega') d\Omega$$

$$I = \delta(q_0' + \omega' - E_n) \frac{1}{q_0'\omega'} \frac{|\boldsymbol{q}'| q_0'\omega'}{E_n} d(q_0' + \omega') d\Omega$$

integrating over $\delta(q_0' + \omega' - E_n)$, $\quad I = \dfrac{|\boldsymbol{q}'|}{E_n} d\Omega = \dfrac{2\pi|\boldsymbol{q}'|}{E_n} d\cos\theta$

$$\frac{1}{u} = \frac{kp'}{kk'} = \frac{k(p - k')}{kk'} = \frac{kp}{kk'} - 1 = \frac{1}{y} - 1 \quad \therefore u = \frac{y}{1 - y}, \; y = \frac{u}{1 + u} = \frac{kk'}{kp} = \frac{\omega'}{E_e}$$

$$u = \frac{E_n}{q_0' - |\boldsymbol{q}'|\cos\theta} - 1 \quad , \text{so} \quad \cos\theta = \frac{q_0'}{|\boldsymbol{q}'|} - \frac{E_n}{|\boldsymbol{q}'|(1+u)}$$

$$\therefore d\cos\theta = \frac{E_n}{|\boldsymbol{q}'|(1+u)^2} du$$

$$\therefore I = \frac{2\pi du}{(1+u)^2}$$

$$dW_n = \frac{\left|M_{fi}^{(n)}\right|^2}{4\pi} \frac{1}{4q_0} \frac{du}{(1+u)^2}$$

The total propability of emission from unit volume in unit time is

$$\boxed{W = \sum_{n=1}^{\infty} W_n = \frac{\alpha m^2}{4q_0} \sum_{n=1}^{\infty} \int_0^{u_n} \frac{du}{(1+u)^2} \left[F_{1n} + \zeta\xi_2 F_{2n}\right]}$$

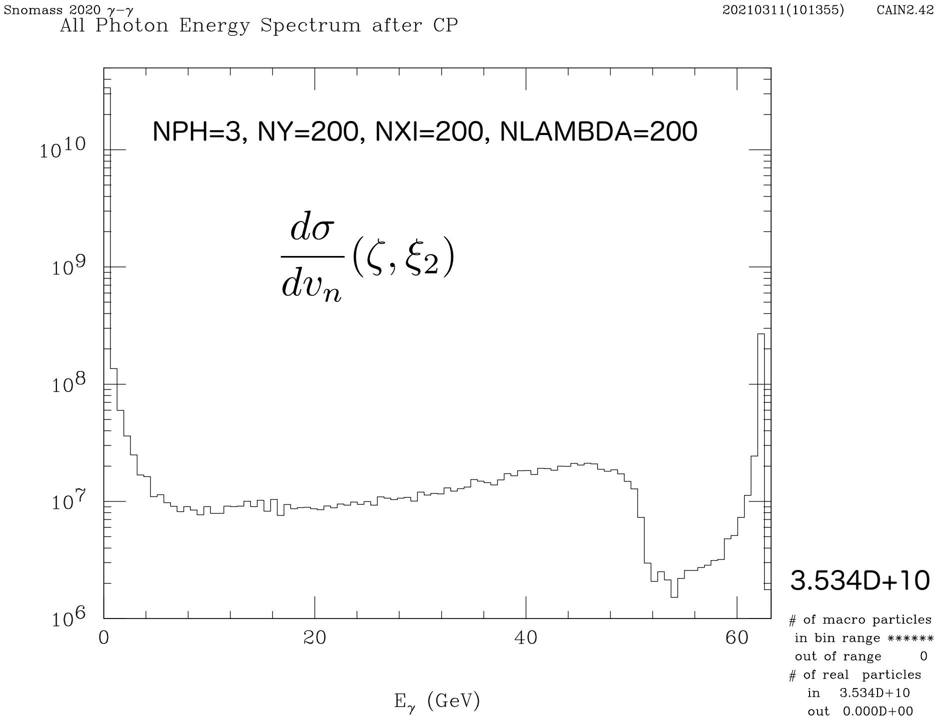
Dividing by the flux of incident particles, $\boldsymbol{J}_{inc}$, the cross section is $d\sigma = dW/|\boldsymbol{J}_{inc}|$

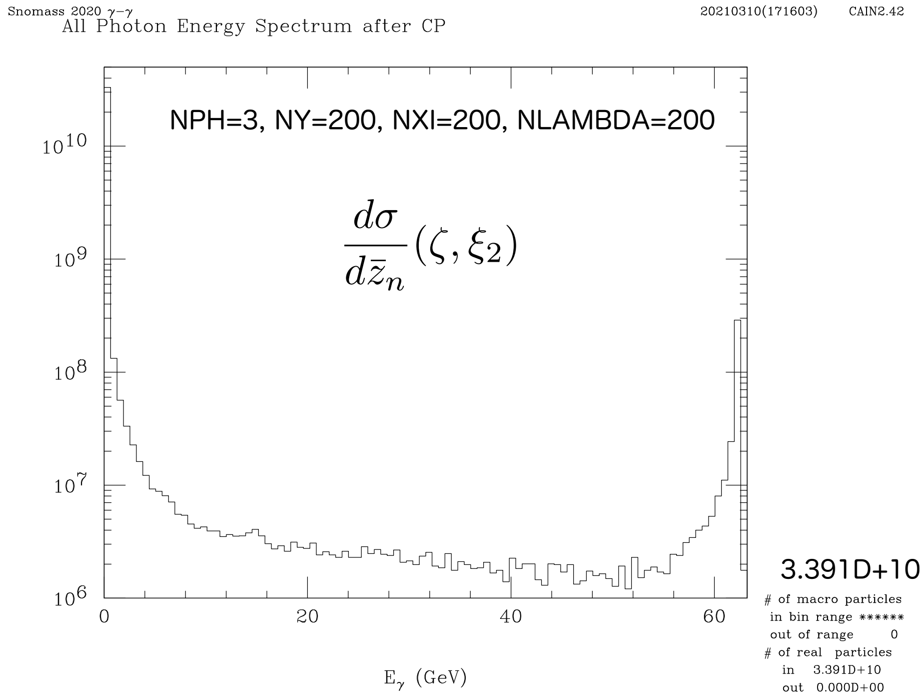
$$|\boldsymbol{J}_{inc}| = \frac{|\boldsymbol{v}_1 - \boldsymbol{v}_2|}{V} = \frac{\sqrt{(p_1 p_2)^2 - m_1^2 m_2^2}}{E_1 E_2 V} = \frac{n(kq)}{n\omega q_0 V} = \frac{pk}{\omega q_0 V} = \frac{2E_e\omega}{\omega q_0 V} = \frac{2E_e}{q_0 V} \quad \text{with } m_1^2 = 0$$

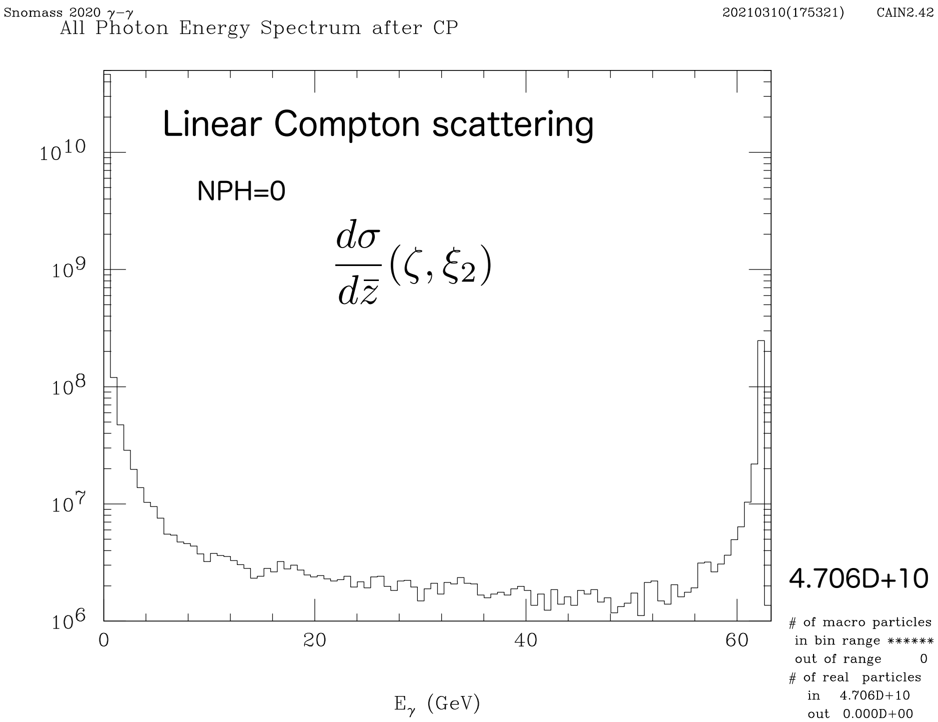
$$\frac{\alpha m^2}{4q_0|\boldsymbol{J}_{inc}|} = \frac{\alpha m^2 V}{8E_e} = \left(\frac{\alpha}{m}\right)^2 \left(\frac{m}{\alpha}\right)^2 \frac{\alpha m^2 V}{8E_e} \frac{1}{x} \frac{4\omega E_e}{m^2} = \frac{r_e^2}{x} \frac{4\pi m^2}{e^2} \frac{\omega}{2} \quad \text{with } r_e = \frac{\alpha}{m}$$

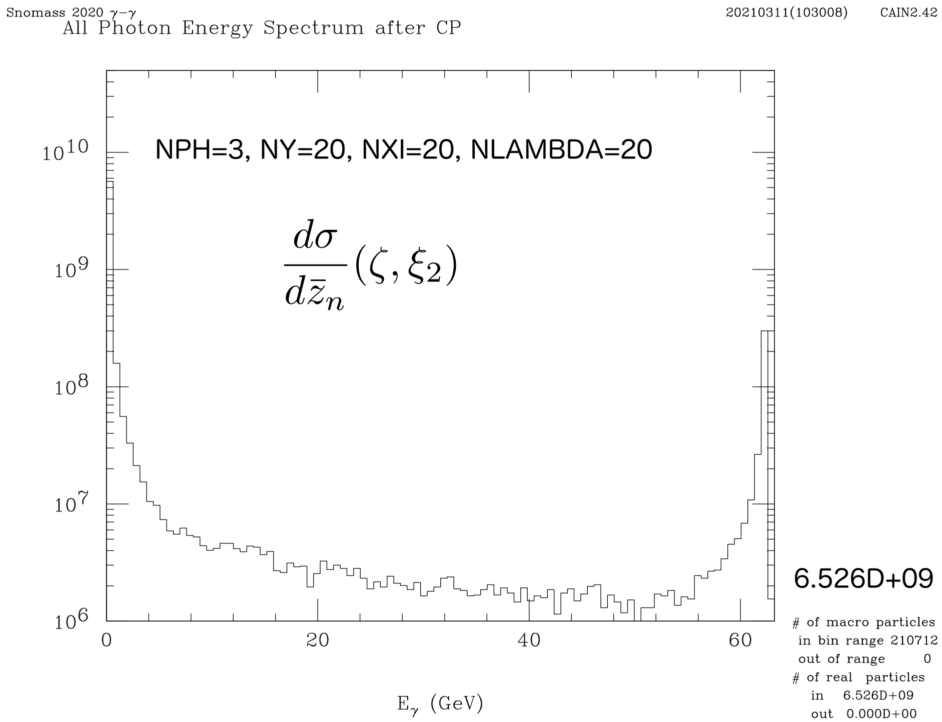
$$= \frac{r_e^2}{x} \frac{1}{\xi^2} \frac{e^2}{m^2\omega V} \frac{4\pi m^2 V}{e^2} \frac{\omega}{2} = \frac{2\pi r_e^2}{x\xi^2} \quad \text{with } \xi^2 = \frac{e^2(-a^2)}{m^2} = \frac{e^2}{m^2\omega V}, \quad -a^2 = \frac{1}{\omega V}$$

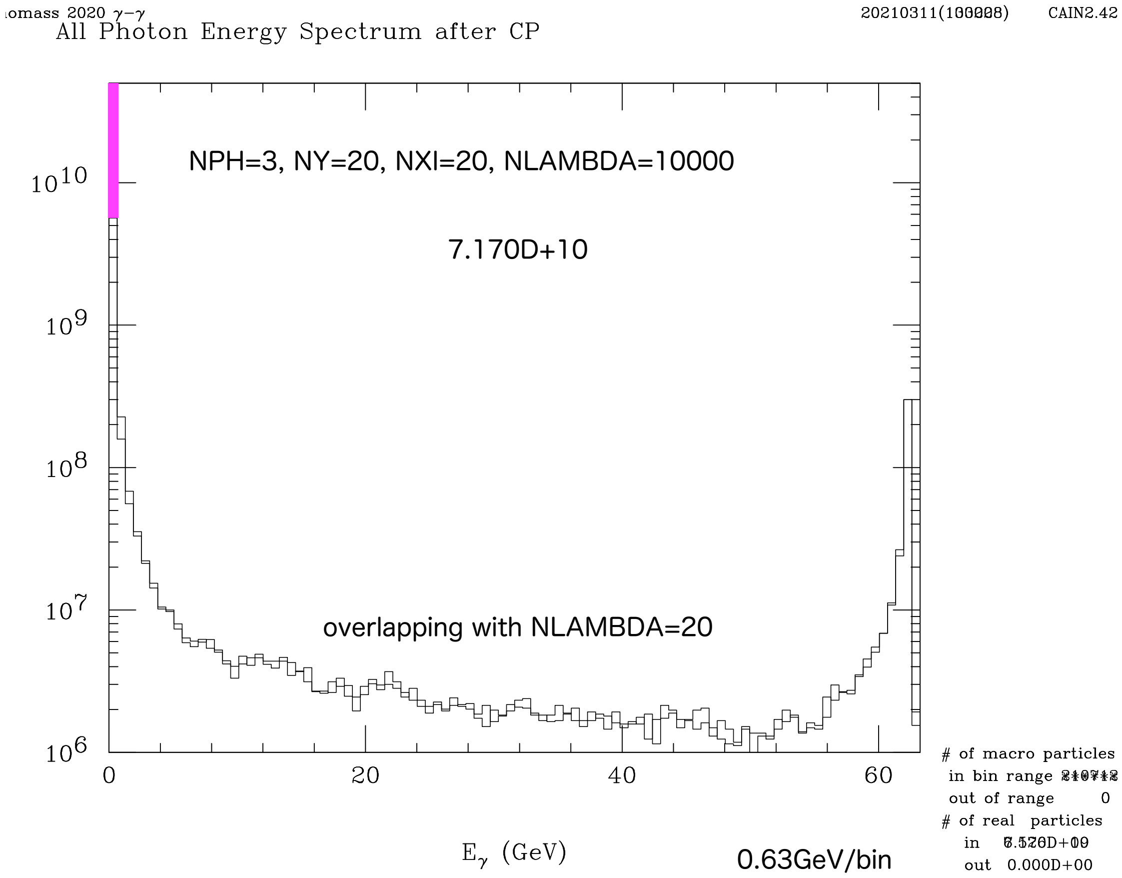
$$\boxed{\therefore \sigma = \frac{2\pi r_e^2}{x\xi^2} \sum_{n=1}^{\infty} \int_0^{u_n} \frac{du}{(1+u)^2} \left[F_{1n} + \zeta\xi_2 F_{2n}\right] = \frac{2\pi r_e^2}{x\xi^2} \sum_{n=1}^{\infty} \int_0^{y_n} dy \left[F_{1n} + \zeta\xi_2 F_{2n}\right] \quad \text{with } dy = \frac{du}{(1+u)^2}}$$

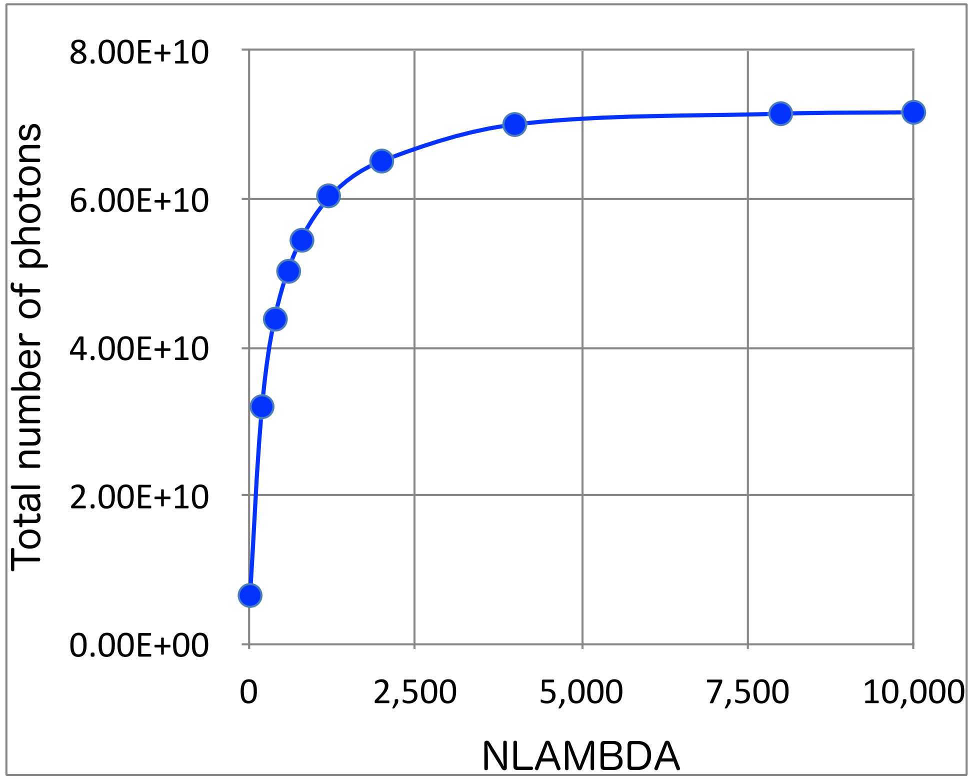

Snomass 2020 γ−γ
20210311(101355) CAIN2.42
All Photon Energy Spectrum after CP
NPH=3, NY=200, NXI=200, NLAMBDA=200
$\frac{d\sigma}{dv_n}(\zeta,\xi_2)$
10^{10}
10^9
10^8
10^7
10^6
0
20
40
60
E_γ (GeV)
3.534D+10
# of macro particles
in bin range ******
out of range 0
# of real particles
in 3.534D+10
out 0.000D+00


Snomass 2020 γ−γ
20210310(171603) CAIN2.42
All Photon Energy Spectrum after CP
NPH=3, NY=200, NXI=200, NLAMBDA=200
$\frac{d\sigma}{d\bar{z}_n}(\zeta,\xi_2)$
10^{10}
10^9
10^8
10^7
10^6
0
20
40
60
E_γ (GeV)
3.391D+10
# of macro particles
in bin range ******
out of range 0
# of real particles
in 3.391D+10
out 0.000D+00

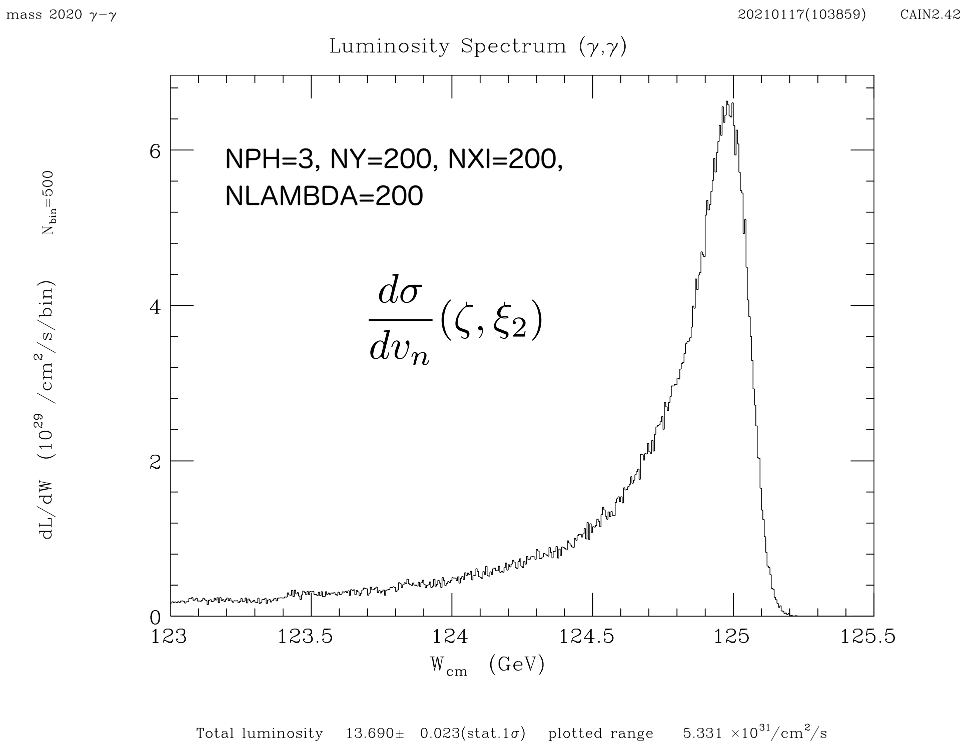

Snomass 2020 γ−γ
20210310(175321) CAIN2.42
All Photon Energy Spectrum after CP
Linear Compton scattering
NPH=0
$\frac{d\sigma}{d\bar{z}}(\zeta,\xi_2)$
10^{10}
10^9
10^8
10^7
10^6
0
20
40
60
E_γ (GeV)
4.706D+10
# of macro particles
in bin range ******
out of range 0
# of real particles
in 4.706D+10
out 0.000D+00

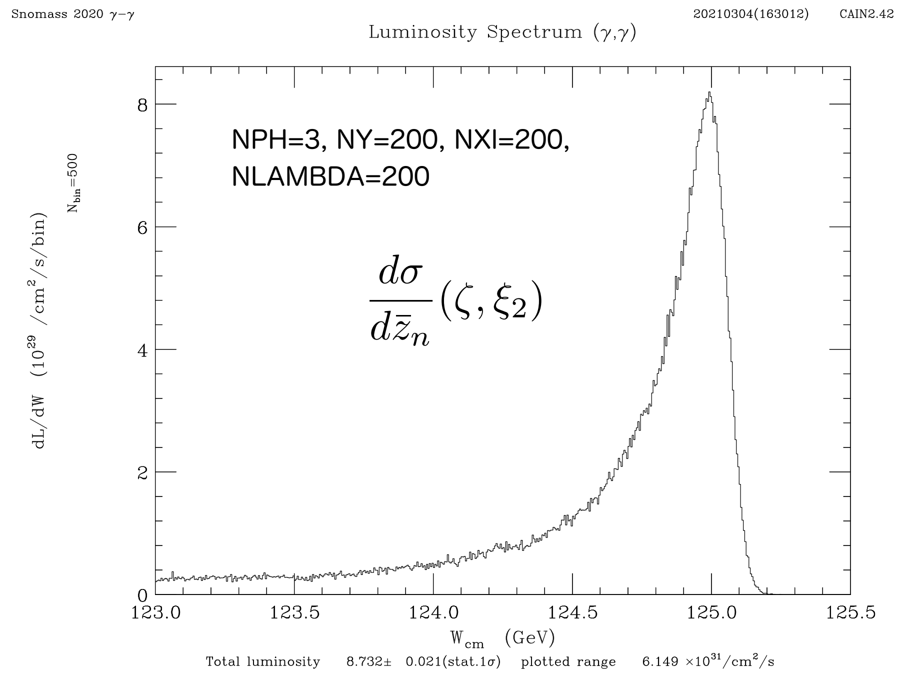

Snomass 2020 γ−γ
20210311(103008) CAIN2.42
All Photon Energy Spectrum after CP
NPH=3, NY=20, NXI=20, NLAMBDA=20
$\frac{d\sigma}{d\bar{z}_n}(\zeta,\xi_2)$
10^{10}
10^9
10^8
10^7
10^6
0
20
40
60
E_γ (GeV)
6.526D+09
# of macro particles
in bin range 210712
out of range 0
# of real particles
in 6.526D+09
out 0.000D+00

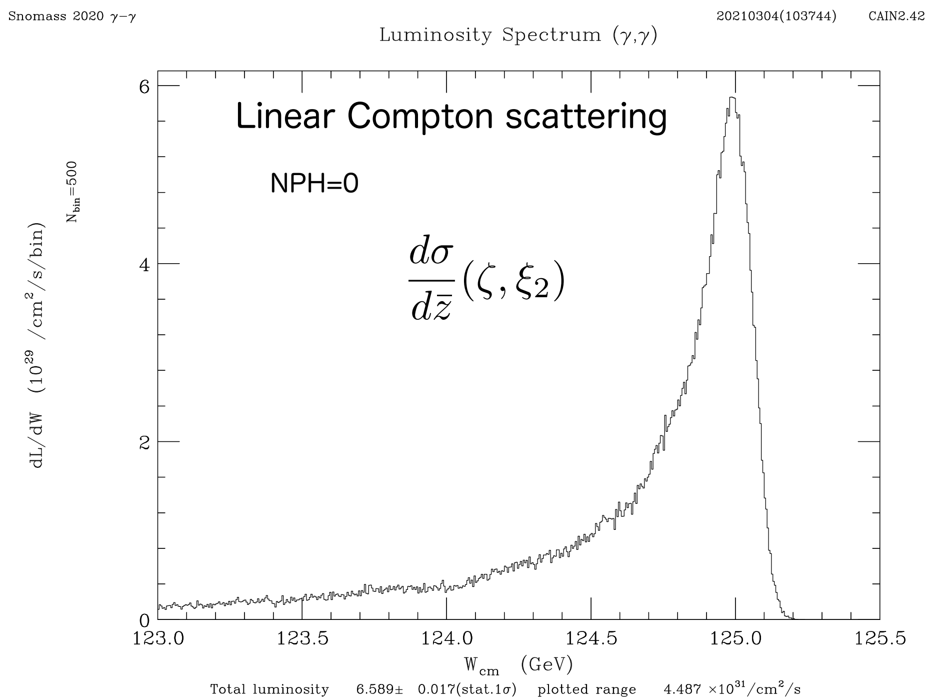


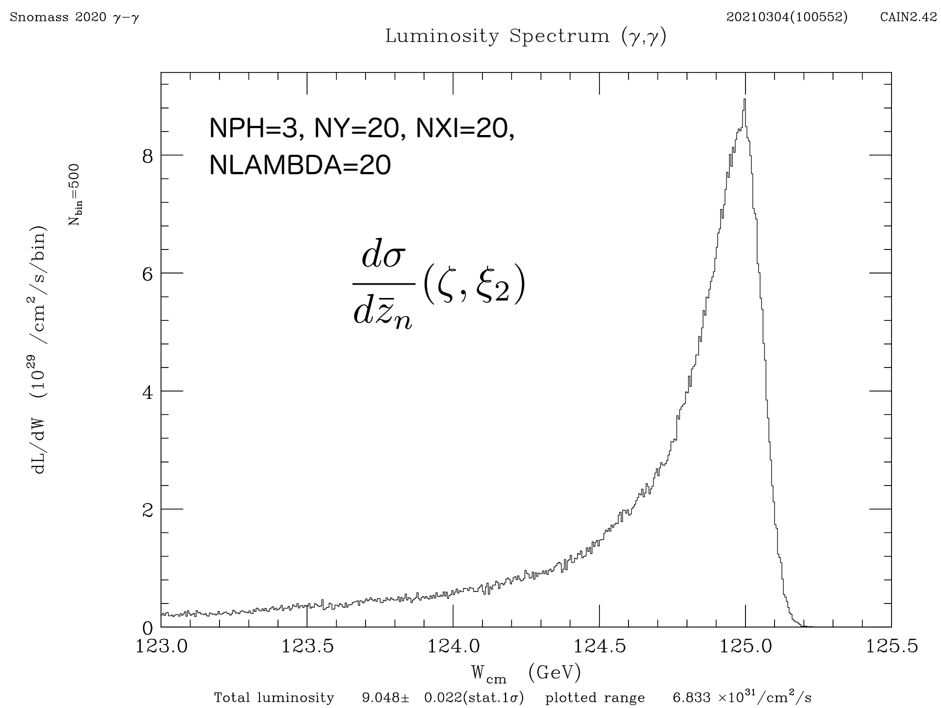


Difference of photons with NLAMBDA=10,000 and 20
= 6.517E+10 / 0.63GeV


mass 2020 γ−γ
20210117(103859) CAIN2.42
Luminosity Spectrum (γ,γ)
NPH=3, NY=200, NXI=200,
NLAMBDA=200
$\frac{d\sigma}{dv_n}(\zeta,\xi_2)$
dL/dW (10^{29} /cm^2/s/bin)
N_{bin}=500
W_{cm} (GeV)
Total luminosity 13.690± 0.023(stat.1σ) plotted range 5.331 ×10^{31}/cm^2/s


Snomass 2020 γ−γ
20210304(163012) CAIN2.42
Luminosity Spectrum (γ,γ)
NPH=3, NY=200, NXI=200,
NLAMBDA=200
$\frac{d\sigma}{d\bar{z}_n}(\zeta,\xi_2)$
dL/dW (10^{29} /cm^2/s/bin)
N_{bin}=500
W_{cm} (GeV)
Total luminosity 8.732± 0.021(stat.1σ) plotted range 6.149 ×10^{31}/cm^2/s


Snomass 2020 γ−γ
20210304(103744) CAIN2.42
Luminosity Spectrum (γ,γ)
Linear Compton scattering
NPH=0
$\frac{d\sigma}{d\bar{z}}(\zeta,\xi_2)$
dL/dW (10^{29} /cm^2/s/bin)
N_{bin}=500
W_{cm} (GeV)
Total luminosity 6.589± 0.017(stat.1σ) plotted range 4.487 ×10^{31}/cm^2/s


Snomass 2020 γ−γ
20210304(100552) CAIN2.42
Luminosity Spectrum (γ,γ)
NPH=3, NY=20, NXI=20,
NLAMBDA=20
$\frac{d\sigma}{d\bar{z}_n}(\zeta,\xi_2)$
dL/dW (10^{29} /cm^2/s/bin)
N_{bin}=500
W_{cm} (GeV)
Total luminosity 9.048± 0.022(stat.1σ) plotted range 6.833 ×10^{31}/cm^2/s