Multi-Criteria Optimization

Exercise 1: Concepts in Multi-Criteria Optimization

Analyse a Multi-Objective Optimization problem with the following six points and values in objective space:

$$\mathbf{x}^{(1)} \text{ with } \mathbf{f}^{(1)} = (10, 5)$$
$$\mathbf{x}^{(2)} \text{ with } \mathbf{f}^{(2)} = (7, 8)$$
$$\mathbf{x}^{(3)} \text{ with } \mathbf{f}^{(3)} = (4, 6)$$
$$\mathbf{x}^{(4)} \text{ with } \mathbf{f}^{(4)} = (6, 4)$$
$$\mathbf{x}^{(5)} \text{ with } \mathbf{f}^{(5)} = (9, 3)$$
$$\mathbf{x}^{(6)} \text{ with } \mathbf{f}^{(6)} = (3, 7)$$

(a) Determine which of these points are Pareto optimal (find $\mathcal{P}$).

(b) Sketch the objective space and indicate the Pareto front $f(\mathcal{P})$.

(c) Assume a reference point $R = (15, 15)$. Calculate the dominated hypervolume of the Pareto optimal points.

(d) Perform non-dominated sorting.

(e) Compute the crowding distance of the point $\mathbf{x}^{(3)}$ with the solution $\mathbf{f}^{(3)}$.

(f) Compute the hypervolume contribution of the point $\mathbf{x}^{(5)}$ with the solution $\mathbf{f}^{(5)}$. Again, assume a reference point $R = (15, 15)$.

Note: We want to minimize both objectives f_1, f_2.