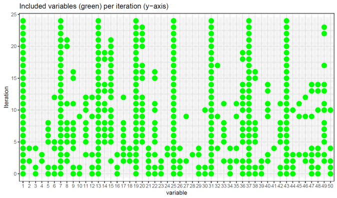
Optimization in Machine Learning

Evolutionary Algorithms

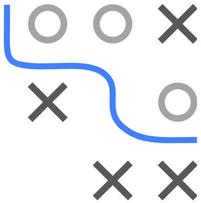
GA / Bit Strings



Learning goals

- Recombination
- Mutation
- Simple examples

BINARY ENCODING



- In theory: Each problem can be encoded binary
- In practice: Binary not always best representation (e.g., if values are numeric, trees or programs)

We typically encode problems with **binary decision variables** in binary representation.

Examples:

- Scheduling problems
- Integer / binary linear programming
- Feature selection
- ...

RECOMBINATION FOR BIT STRINGS

Two individuals $\mathbf{x}, \tilde{\mathbf{x}} \in \{0, 1\}^d$ encoded as bit strings can be recombined as follows:

- **1-point crossover:** Select crossover $k \in \{1, ..., d-1\}$ randomly. Take first k bits from parent 1 and last $d-k$ bits from parent 2.

$$\begin{array}{cccc} 1 & 1 & & 1 \\ 0 & 0 & & 0 \\ \hline 0 & 1 & \Rightarrow & 1 \\ 1 & 1 & & 1 \\ 1 & 0 & & 0 \end{array}$$

- **Uniform crossover:** Select bit j with probability p from parent 1 and $1-p$ from parent 2.

$$\begin{array}{cccc} 1 & 0 & & 1 \\ 0 & 0 & & 0 \\ 0 & 1 & \Rightarrow & 1 \\ 0 & 1 & & 1 \\ 1 & 0 & & 1 \end{array}$$

MUTATION FOR BIT STRINGS

Offspring $\mathbf{x} \in \{0,1\}^d$ encoded as a bit string can be mutated as follows:

- **Bitflip:** Each bit j is flipped with probability $p \in (0,1)$.

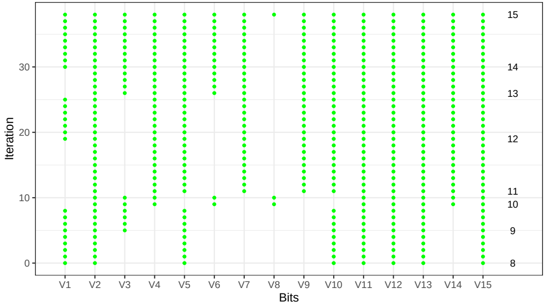
$$\begin{array}{ccc} 1 & & 0 \\ 0 & & 0 \\ 0 & \Rightarrow & 0 \\ 0 & & 1 \\ 1 & & 1 \end{array}$$

EXAMPLE 1: ONE-MAX EXAMPLE

$\mathbf{x} \in \{0, 1\}^d$, $d = 15$ bit vector representation.

Goal: Find the vector with the maximum number of 1's.

- Fitness: $f(\mathbf{x}) = \sum_{i=1}^{d} x_i$
- $\mu = 15$, $\lambda = 5$, $(\mu + \lambda)$-strategy, bitflip mutation, no recombination



Green: Representation of best individual per iteration. Right scale shows fitness.

EXAMPLE 2: FEATURE SELECTION

We consider the following toy setting:

- Generate design matrix $\mathbf{X} \in \mathbb{R}^{n \times p}$ by drawing $n = 1000$ samples of $p = 50$ independent normally distributed features with $\mu_j = 0$ and $\sigma_j^2 > 0$ varying between 1 and 5 for $j = 1, \ldots, p$.
- Linear regression problem with dependent variable $\mathbf{y}$:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\theta} + \epsilon$$

with $\epsilon \sim \mathcal{N}(0, 1)$.

Parameter $\boldsymbol{\theta}$:

$$\theta_0 = -1.2$$

$$\theta_j = \begin{cases} 1 & \text{for } j \in 1, 7, 13, 19, 25, 31, 37, 43 \\ 0 & \text{otherwise} \end{cases}$$

$\Rightarrow$ Only 8 out of 50 equally influential features