Multi-Criteria Optimization

Exercise 1: Concepts in Multi-Criteria Optimization

We are given a multi-objective optimization problem where we want to minimize both objectives f_1 and f_2:

$$f_1 : \mathcal{X} \to \mathbb{R}, f_2 : \mathcal{X} \to \mathbb{R},$$

where $\mathcal{X} = \{\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \mathbf{x}^{(3)}, \mathbf{x}^{(4)}, \mathbf{x}^{(5)}, \mathbf{x}^{(6)}\}$.

For each point in the domain, we know their objective function values:

$$\begin{aligned}
&\mathbf{x}^{(1)} \text{ with } \mathbf{f}^{(1)} = (10, 5)\\
&\mathbf{x}^{(2)} \text{ with } \mathbf{f}^{(2)} = (7, 8)\\
&\mathbf{x}^{(3)} \text{ with } \mathbf{f}^{(3)} = (4, 6)\\
&\mathbf{x}^{(4)} \text{ with } \mathbf{f}^{(4)} = (6, 4)\\
&\mathbf{x}^{(5)} \text{ with } \mathbf{f}^{(5)} = (9, 3)\\
&\mathbf{x}^{(6)} \text{ with } \mathbf{f}^{(6)} = (3, 7)
\end{aligned}$$

(a) Determine which of these six points are Pareto optimal (find $\mathcal{P}$).

(b) Sketch the objective space and visualize the Pareto front $\mathbf{f}(\mathcal{P})$.

(c) Assume we are given a reference point $R = (15, 15)$. Compute the dominated hypervolume of the Pareto optimal points.

(d) Perform non-dominated sorting.

(e) Compute the crowding distance of the point $\mathbf{x}^{(3)}$ with $\mathbf{f}^{(3)}$.

(f) Compute the hypervolume contribution of the point $\mathbf{x}^{(5)}$ with $\mathbf{f}^{(5)}$. Again, assume a reference point $R = (15, 15)$.