

## Univariate Optimization 1

### Exercise 1: Golden Ratio, Brent's Method

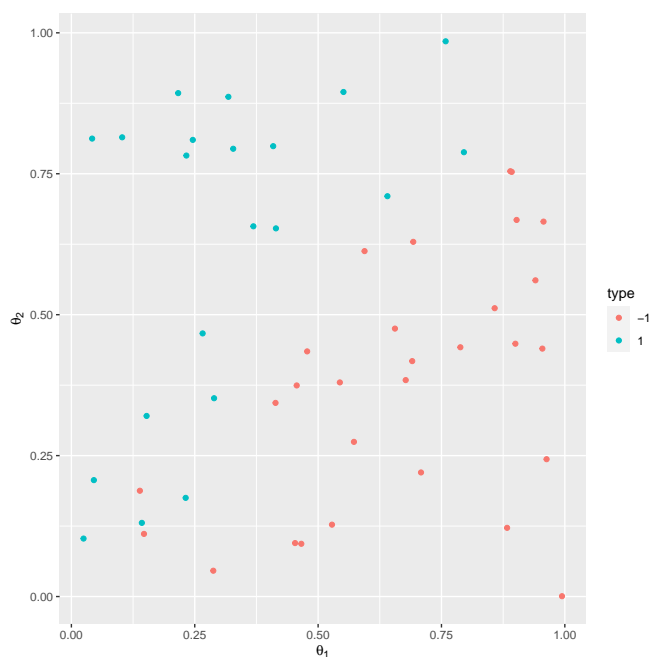
You are given the following data situation:

```
library(ggplot2)

set.seed(123)

X = matrix(runif(100), ncol = 2)
y = -((X %*% c(-1, 1) + rnorm(50, 0, 0.1) < 0) * 2 - 1)
df = as.data.frame(X)
df$type = as.character(y)

ggplot(df) +
  geom_point(aes(x = V1, y = V2, color=type)) +
  xlab(expression(theta[1])) +
  ylab(expression(theta[2]))
```



In the following we want to estimate a linear SVM without intercept and with  $\lambda = 1$ . We assume we know that  $\theta_2 = 2$ .

- Show that if  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  is convex then  $g_c : \mathbb{R} \rightarrow \mathbb{R}, x \mapsto f(x, c) \quad \forall c \in \mathbb{R}$  is convex.
- Explain why the non-geometric primal linear SVM formulation should be used rather than the geometric one if we want to find  $\theta_1$  via the golden ratio algorithm<sup>1</sup>.
- Find  $\theta_1$  via the golden ratio algorithm. Implement the algorithm in R. For the termination criterion, use an absolute error of 0.01. Use  $[-3, 3]$  as the starting interval.

<sup>1</sup>We choose this algorithm for educational purposes; in practice, we typically use more advanced algorithms.

- (d) Given the three points  $(x_1, y_1), (x_2, y_2), (x_3, y_3)$  show that the parameters  $a, b, c \in \mathbb{R}$  of the interpolating parabola can be found via  $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{pmatrix}^{-1} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$  when the parabola equation is given by  $p(x) = ax^2 + bx + c$ .
- (e) Find  $\theta_1$  via Brent's method<sup>2</sup>. Implement a simplified version<sup>3</sup> of the algorithm in R. For the termination criterion, use an absolute error of 0.01. Use  $[-3, 3]$  as the starting interval. For the first step, use a golden ratio step.
- (f) Now, assume we do not know  $\theta_2$ . Our initial guess is  $\theta_2 = 0$ . We now alternately minimize w.r.t. either  $\theta_1$  or  $\theta_2$  via the golden ratio method (the starting interval is always reset to  $[-3, 3]$ ) while the other parameter is held constant. We switch to minimizing the other parameter when the absolute error is smaller than 0.01. Repeat this procedure 10 times.
- (g) How does the optimization trace of f) look in parameter space?

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<sup>2</sup>We choose this algorithm for educational purposes; in practice, we typically use more advanced algorithms.

<sup>3</sup>Only check if the proposed point is in the current interval