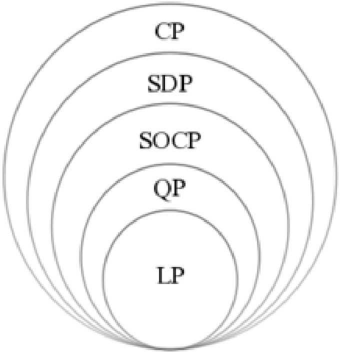
Optimization in Machine Learning

Linear Programming



Learning goals

- Instances of LPs underlying statistical estimation
- Definition of an LP
- Geometric intuition of LPs

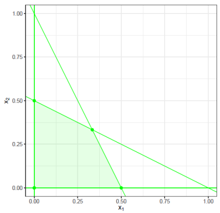
LINEAR PROGRAMMING

Linear problems (LP):

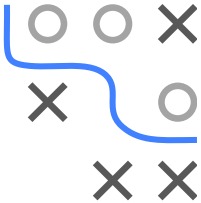
linear objective function + **linear** constraints

Example:

$$\begin{aligned} \min \quad & -x_1 - x_2 \\ \text{s.t.} \quad & x_1 + 2x_2 \leq 1 \\ & 2x_1 + x_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{aligned}$$



GEOMETRIC INTERPRETATION



Linear programming can be interpreted geometrically.

Feasible set:

- *i*-th inequality constraint: $\mathbf{a}_i^\top \mathbf{x} \leq b_i$
- Points $\{\mathbf{x} : \mathbf{a}_i^\top \mathbf{x} = b_i\}$ form a hyperplane in $\mathbb{R}^n$
  ($\mathbf{a}_i$ is perpendicular to the hyperplane and called **normal vector**)
- Points $\{\mathbf{x} : \mathbf{a}_i^\top \mathbf{x} \geq b_i\}$ lie on the side of the hyperplane into which the normal vector points ("half-space")

SOLUTIONS TO LP

There are 3 ways to solve linear programming:

1. Feasible set is **empty** $\Rightarrow$ LP is infeasible
2. Feasible set is **"unbounded"**
3. Feasible set is **"bounded"** $\Rightarrow$ LP has at least one solution