

Mathematical Concepts 2

**Exercise 1: Matrix Calculus**

Let  $\mathbf{x}, \mathbf{c} \in \mathbb{R}^d$ ,  $\mathbf{u}, \mathbf{v} : \mathbb{R}^d \rightarrow \mathbb{R}^d$ ,  $\mathbf{u}, \mathbf{v} \in \mathcal{C}^2$ ,  $\mathbf{Y} : \mathbb{R}^d \rightarrow \mathbb{R}^{d \times d}$

- (a) Compute  $\frac{\partial \|\mathbf{x} - \mathbf{c}\|_2^2}{\partial \mathbf{x}}$
- (b) Compute  $\frac{\partial \|\mathbf{x} - \mathbf{c}\|_2}{\partial \mathbf{x}}$
- (c) Compute  $\frac{\partial \mathbf{u}^\top \mathbf{v}}{\partial \mathbf{x}}$
- (d) Show that  $\frac{\partial \mathbf{Y}^\top \mathbf{u}}{\partial \mathbf{x}} = \begin{pmatrix} \mathbf{y}_1^\top \frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \mathbf{u}^\top \frac{\partial \mathbf{y}_1}{\partial \mathbf{x}} \\ \vdots \\ \mathbf{y}_d^\top \frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \mathbf{u}^\top \frac{\partial \mathbf{y}_d}{\partial \mathbf{x}} \end{pmatrix}$  where  $\mathbf{y}_i : \mathbb{R}^d \rightarrow \mathbb{R}^d, i = 1, \dots, d$  are the column vectors of  $\mathbf{Y}$ .
- (e) Compute  $\frac{\partial^2 \mathbf{u}^\top \mathbf{v}}{\partial \mathbf{x} \partial \mathbf{x}^\top}$   
*Hint: use c), d)*

**Exercise 2: Optimality in 1d**

Let  $f : [-1, 2] \rightarrow \mathbb{R}, x \mapsto \exp(x^3 - 2x^2)$

- (a) Compute  $f'$
- (b) Plot  $f$  and  $f'$  with R
- (c) Find all possible candidates  $x^*$  for maxima and minima.  
*Hint: exp is a strictly monotone function.*
- (d) Compute  $f''$
- (e) Determine if the candidates are local maxima, minima or neither.
- (f) Find the global maximum and global minimum of  $f$