

Algorithms for linear programs



- Definition
- Max. Likelihood
- Normal regression
- Risk Minimization



SIMPLEX ALGORITHM

The Simplex algorithm is the most important method for solving Linear programming. It was published in 1947 by Georg Dantzig.

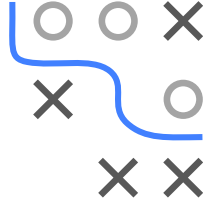
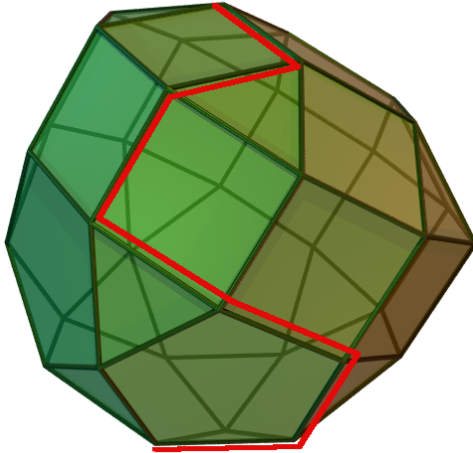
Basic idea: start from an arbitrary corner of the polytope. Run along this edge as long as the solution improves. Find a new edge, ...

Output: a path along the corners of the polytope that ends at the optimal point of the polytope.

Since linear programming is a **convex** optimization problem, the optimal corner found in this way is also a global optimum.



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The simplex algorithm can be divided into two steps:

- **Phase I:** determination of a **starting point**
- **Phase II:** determination of the **optimal solution**

To be able to start, a starting point must first be found in **Phase I**, i.e. a feasible corner \mathbf{x}_0 .

In **phase II** this solution is iteratively improved by searching for an edge that improves the solution and running along it to the next corner.



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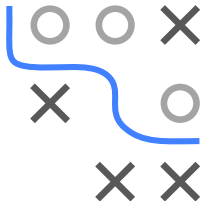
Phase I:

One way to find a starting point \mathbf{x}_0 is to solve a auxiliary linear problem with artificial variables ϵ :

$$\begin{aligned} \min_{\epsilon_1, \dots, \epsilon_m} \quad & \sum_{i=1}^m \epsilon_i \\ \text{s.t.} \quad & \mathbf{Ax} + \epsilon \geq \mathbf{b} \\ & \epsilon_1, \dots, \epsilon_m \geq 0 \\ & \mathbf{x} \geq 0 \end{aligned}$$

- A feasible starting point for the auxiliary problem is $\mathbf{x} = \mathbf{0}$ and

$$\epsilon_i = \begin{cases} 0 & \text{if } b_i < 0 \\ b_i & \text{if } b_i \geq 0 \end{cases}$$

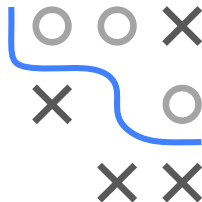


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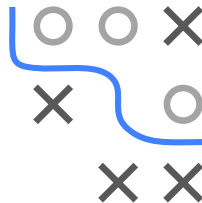
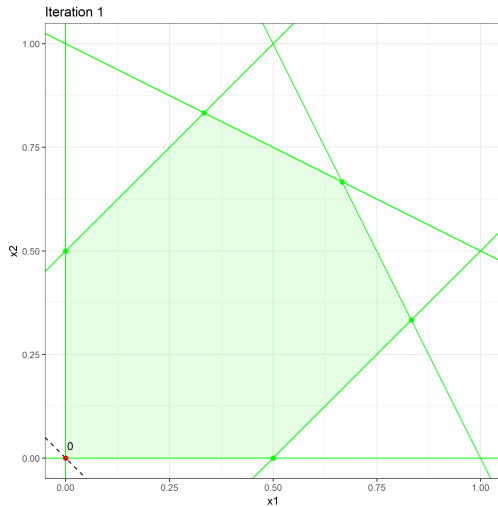
Example:

$$\begin{array}{ll}\min_{\mathbf{x} \in \mathbb{R}^2} & -x_1 - x_2 \\ \text{s.t.} & x_1 - x_2 \geq -0.5 \\ & -x_1 - 2x_2 \geq -2 \\ & -2x_1 - x_2 \geq -2 \\ & -x_1 + x_2 \geq -0.5 \\ & \mathbf{x} \geq 0\end{array}$$

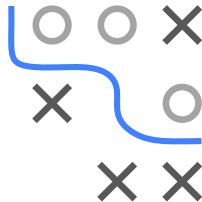
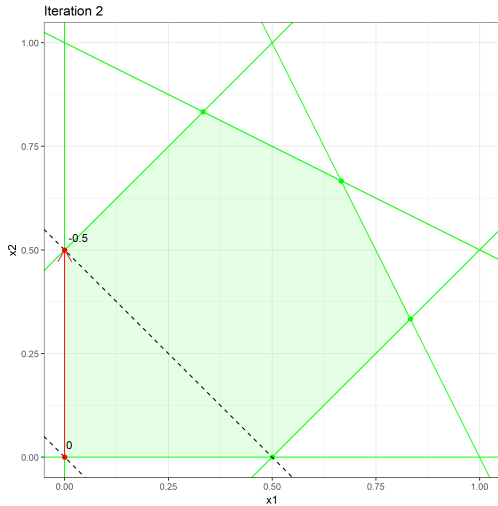
A starting point is the corner $(0, 0)$.



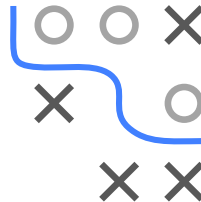
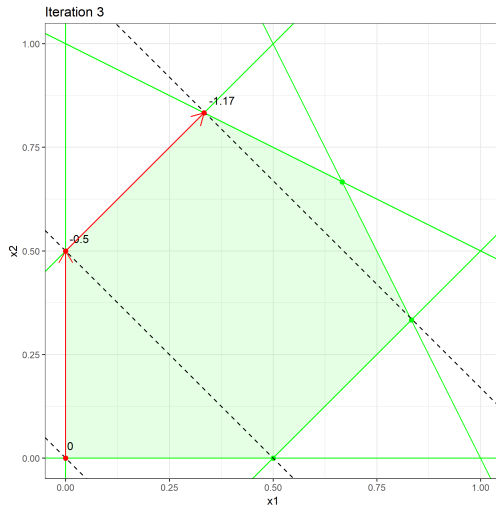
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