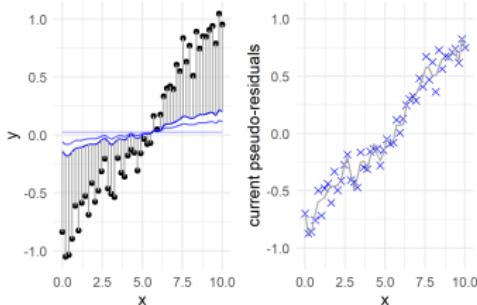


Introduction to Machine Learning

Boosting Gradient Boosting: Illustration



Learning goals

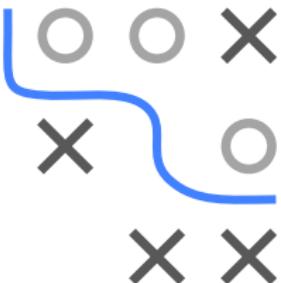
- See simple visualizations of boosting in regression
- Understand impact of different losses and base learners



GRADIENT BOOSTING ILLUSTRATION - GAM

GAM / Splines as BL and compare L_2 vs. L_1 loss.

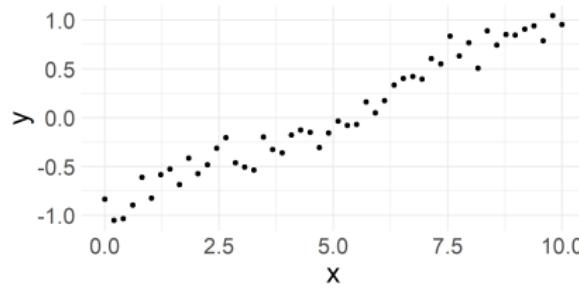
- L2: Init = optimal constant = $\text{mean}(y)$; for L1 it's $\text{median}(y)$
- BLs are cubic B -splines with 40 knots.
- PRs L_2 : $\tilde{r}(f) = r(f) = y - f(\mathbf{x})$
- PRs L_1 : $\tilde{r}(f) = \text{sign}(y - f(\mathbf{x}))$
- Constant learning rate 0.2



Univariate toy data:

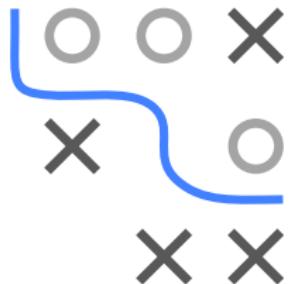
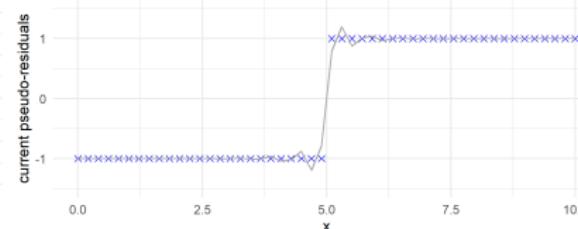
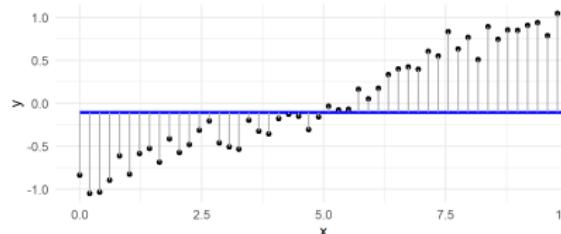
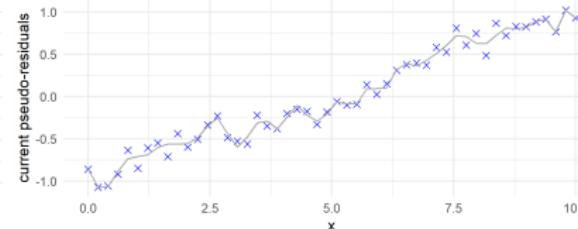
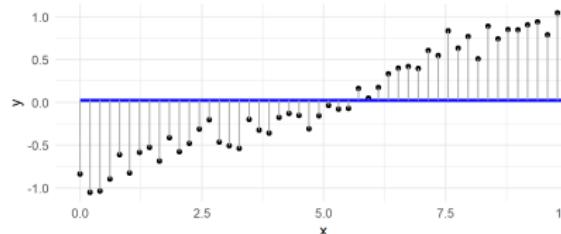
$$y^{(i)} = -1 + 0.2 \cdot x^{(i)} + 0.1 \cdot \sin(x^{(i)}) + \epsilon^{(i)}$$

$$n = 50 ; \epsilon^{(i)} \sim \mathcal{N}(0, 0.1)$$



GAM WITH L_2 VS L_1 LOSS

Top: L_2 loss, bottom: L_1 loss

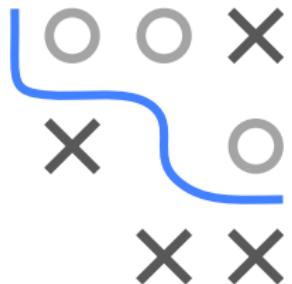
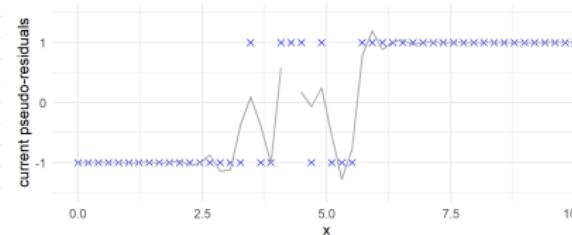
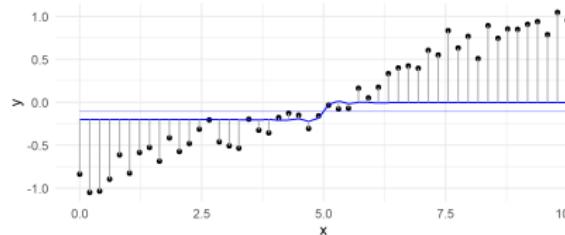
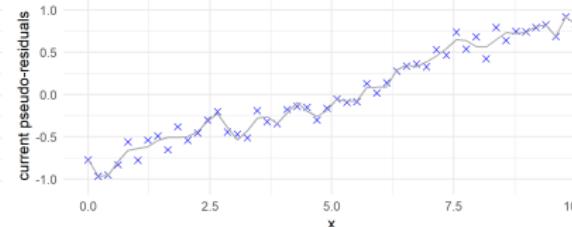
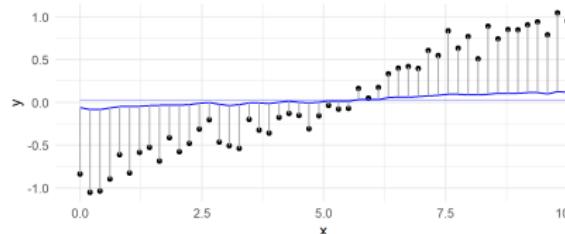


Iteration 1

Shape of PRs affects gradual model fit: L_1 only sees resids' sign, BLs are not affected size of values as in L_2 and hence lead to more moderate changes.

GAM WITH L_2 VS L_1 LOSS

Top: L_2 loss, bottom: L_1 loss

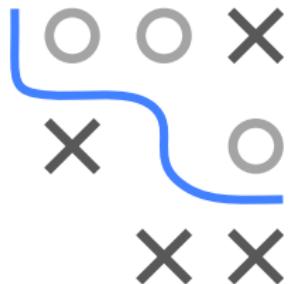
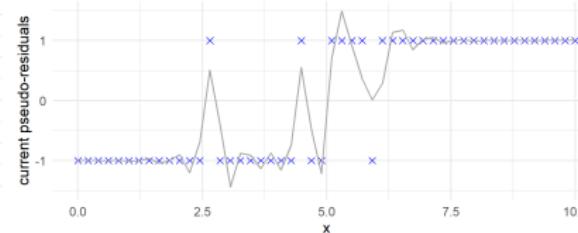
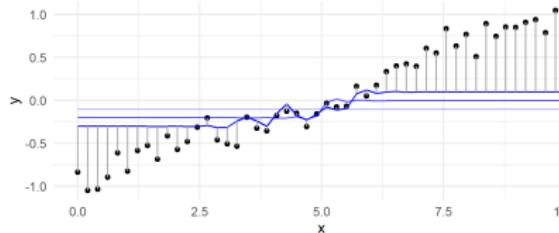
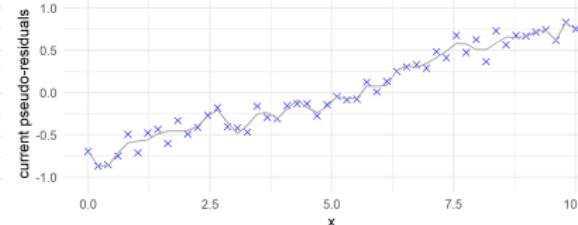
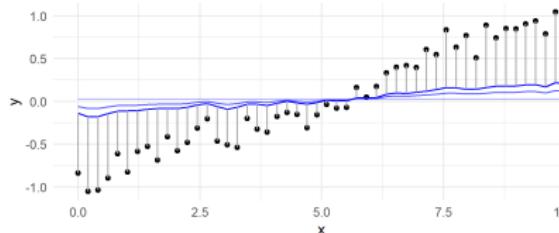


Iteration 2

Shape of PRs affects gradual model fit: L_1 only sees resids' sign, BLs are not affected size of values as in L_2 and hence lead to more moderate changes.

GAM WITH L_2 VS L_1 LOSS

Top: L_2 loss, bottom: L_1 loss

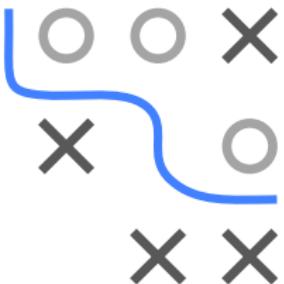
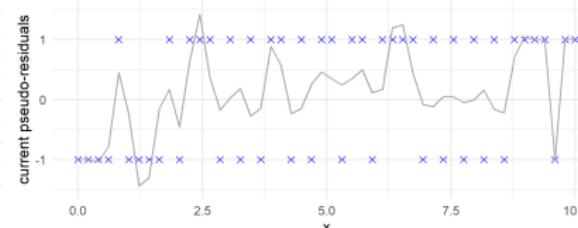
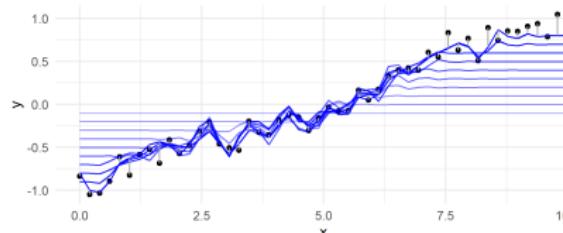
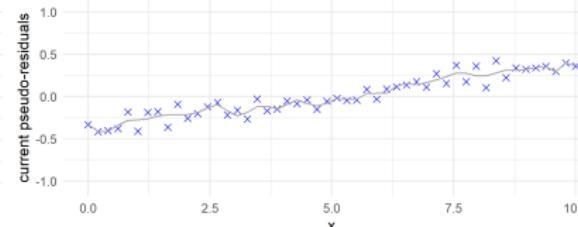
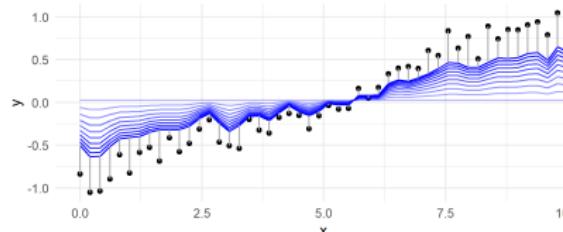


Iteration 3

Shape of PRs affects gradual model fit: L_1 only sees resids' sign, BLs are not affected size of values as in L_2 and hence lead to more moderate changes.

GAM WITH L_2 VS L_1 LOSS

Top: L_2 loss, bottom: L_1 loss

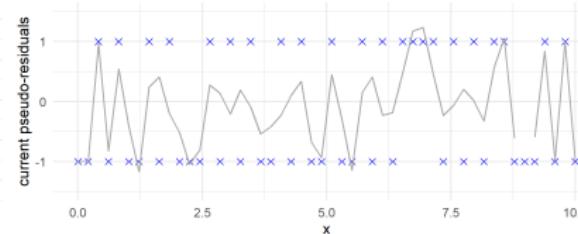
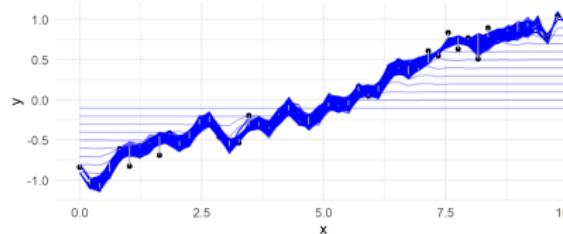
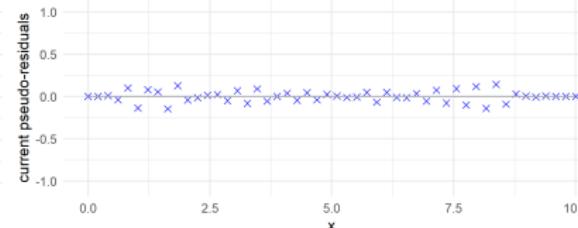
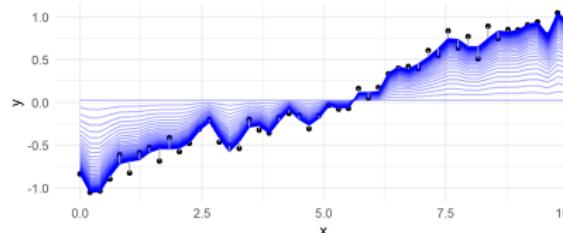


Iteration 10

Shape of PRs affects gradual model fit: L_1 only sees resid's sign, BLs are not affected size of values as in L_2 and hence lead to more moderate changes.

GAM WITH L_2 VS L_1 LOSS

Top: L_2 loss, bottom: L_1 loss

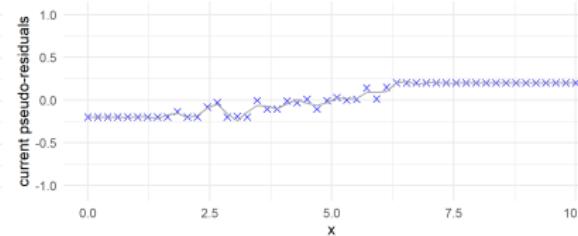
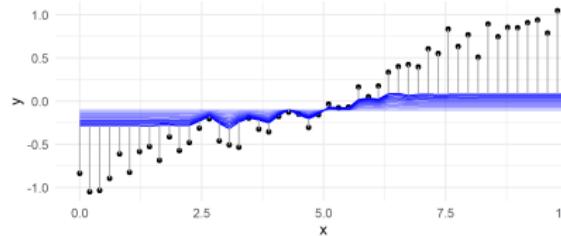
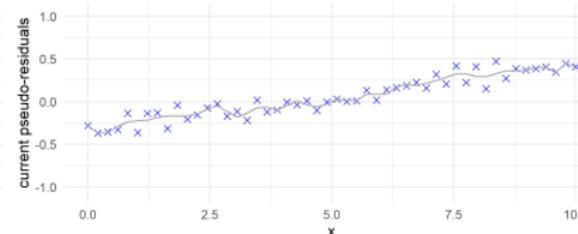
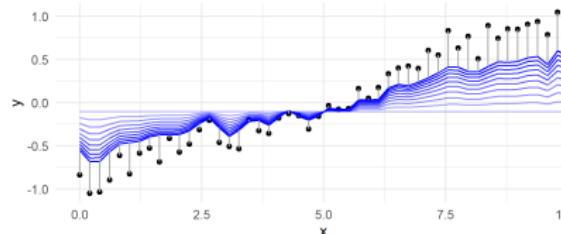


Iteration 100

Shape of PRs affects gradual model fit: L_1 only sees resids' sign, BLs are not affected
size of values as in L_2 and hence lead to more moderate changes.

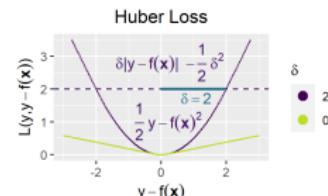
GAM WITH HUBER LOSS

Top: $\delta = 2$, bottom: $\delta = 0.2$.



Iteration 10

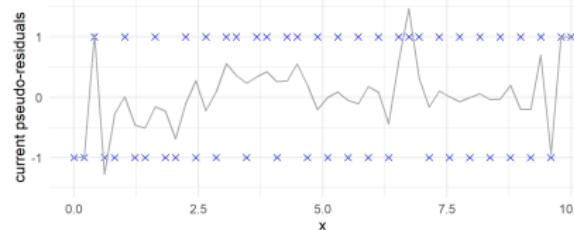
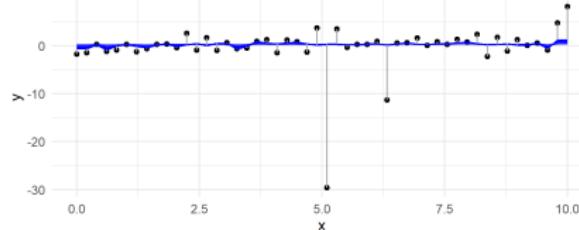
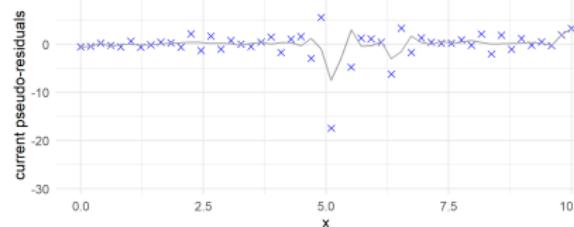
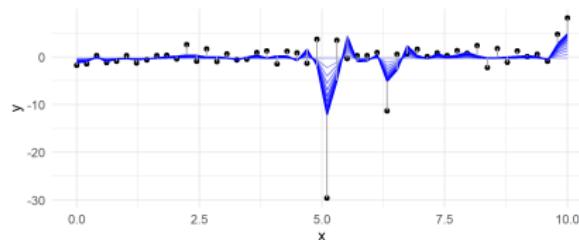
For small δ , PRs are often bounded, resulting in $L1$ -like behavior, while the upper plot more closely resembles $L2$ loss.



GAM WITH OUTLIERS

Instead of Gaussian noise, let's use t -distrib, that leads to outliers in y .

Top: L_2 , bottom: L_1 .



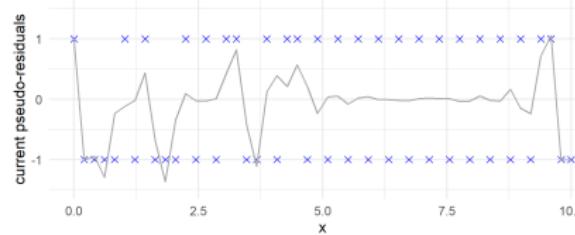
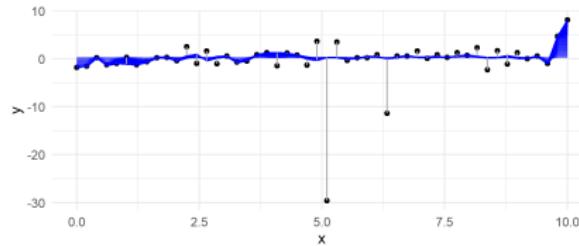
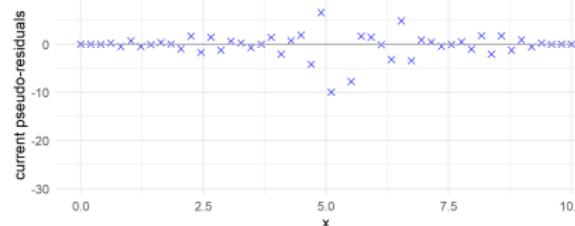
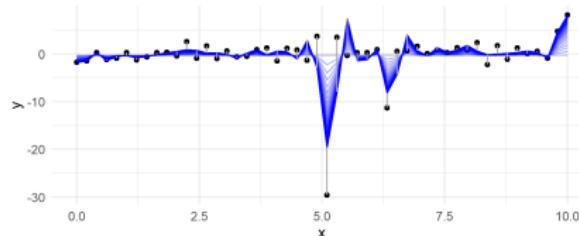
Iteration 10

L_2 loss is affected by outliers rather strongly, whereas L_1 solely considers residuals' sign and not their magnitude, resulting in a more robust model.

GAM WITH OUTLIERS

Instead of Gaussian noise, let's use t -distrib, that leads to outliers in y .

Top: L_2 , bottom: L_1 .

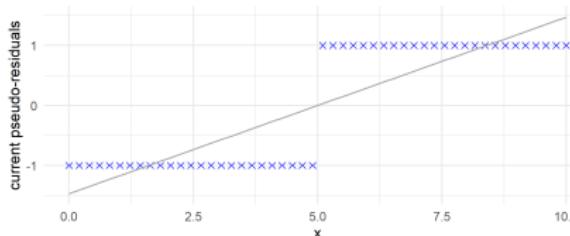
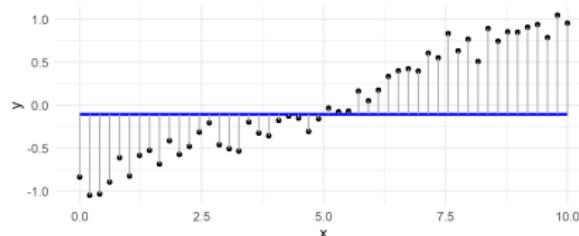
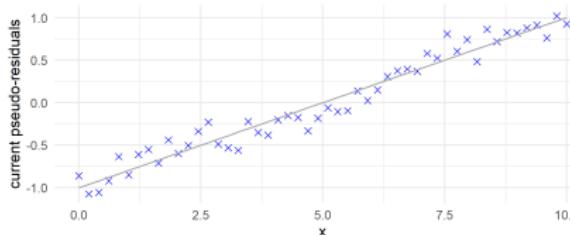
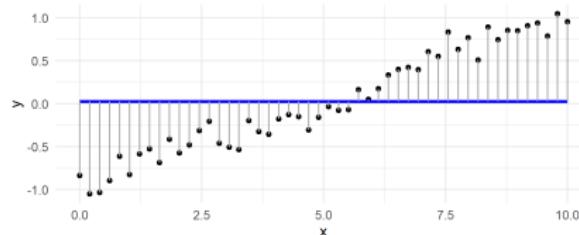


Iteration 100

L_2 loss is affected by outliers rather strongly, whereas L_1 solely considers residuals' sign and not their magnitude, resulting in a more robust model.

LM WITH $L2$ VS $L1$ LOSS

Top: $L2$, bottom: $L1$.

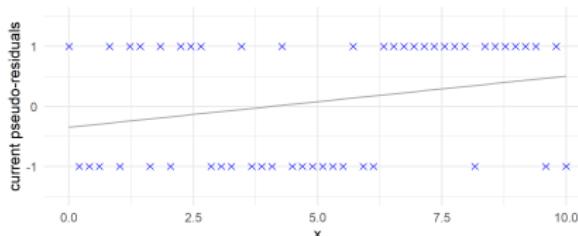
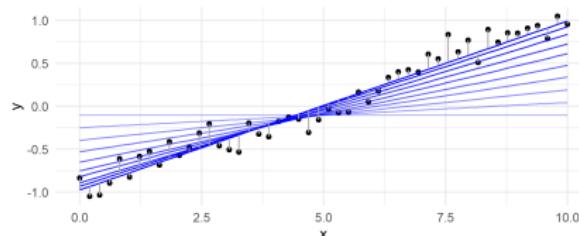
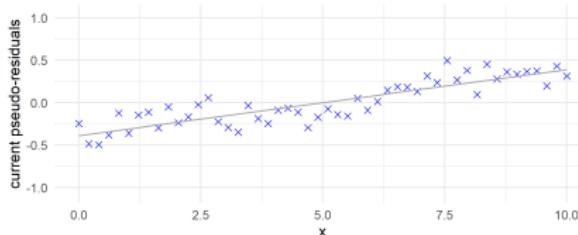
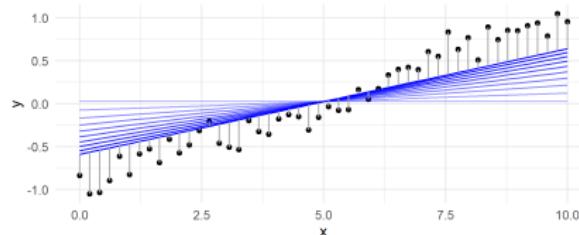


Iteration 1

$L2$: as $\tilde{r}(f) = r(f)$, BL of 1st iter already optimal; but learn rate slows us down.

LM WITH $L2$ VS $L1$ LOSS

Top: $L2$, bottom: $L1$.

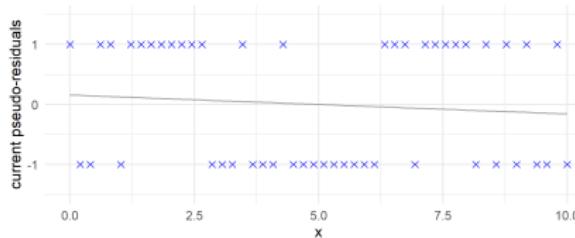
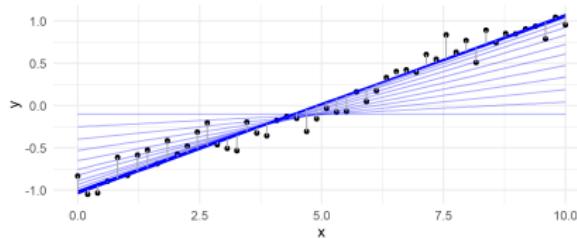
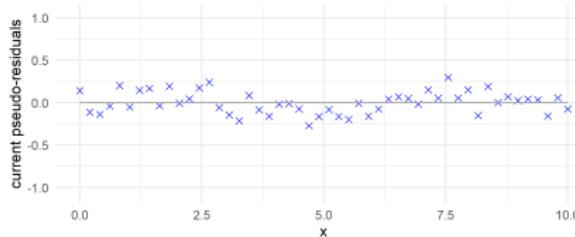
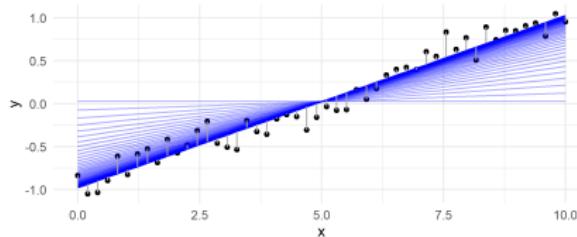


Iteration 10

$L2$: as $\tilde{r}(f) = r(f)$, BL of 1st iter already optimal; but learn rate slows us down.

LM WITH $L2$ VS $L1$ LOSS

Top: $L2$, bottom: $L1$.



Iteration 100

$L2$: as $\tilde{r}(f) = r(f)$, BL of 1st iter already optimal; but learn rate slows us down.