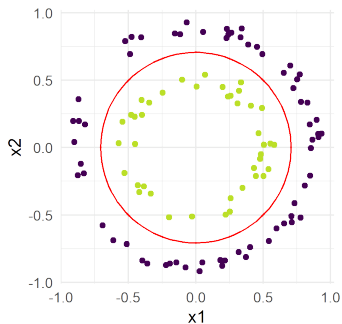
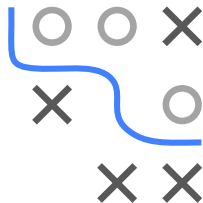


# Introduction to Machine Learning

## Nonlinear Support Vector Machines Feature Generation for Nonlinear Separation

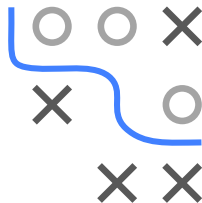
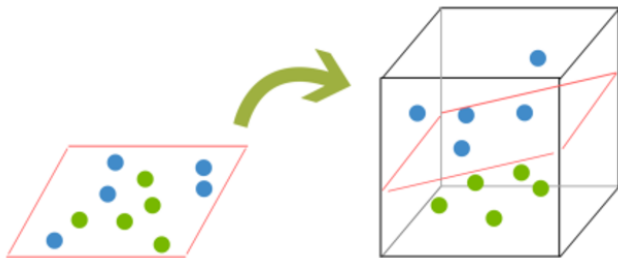


### Learning goals

- Understand how nonlinearity can be introduced via feature maps in SVMs
- Know the limitation of feature maps

# NONLINEARITY VIA FEATURE MAPS

- How to extend a linear classifier, e.g. the SVM, to nonlinear separation between classes?
- We could project the data from 2D into a richer 3D feature space!



# FEATURE MAPS: COMPUTATIONAL LIMITATIONS

Let us have a look at a similar nonlinear feature map  $\phi : \mathbb{R}^2 \rightarrow \mathbb{R}^5$ , where we collect all monomial feature extractors up to degree 2 (pairwise interactions and quadratic effects):

$$\phi(x_1, x_2) = (x_1^2, x_2^2, x_1 x_2, x_1, x_2).$$

For  $p$  features vectors, there are  $k_1$  different monomials where the degree is exactly  $d$ , and  $k_2$  different monomials up to degree  $d$ .

$$k_1 = \binom{d+p-1}{d} \quad k_2 = \binom{d+p}{d} - 1$$

Which is quite a lot, if  $p$  is large.

