Exercise 1: Kullback-Leibler Divergence and model misspecification

Consider a double-exponential distributed random variable X with unknown parameters $\mu_0 \in \mathbb{R}$ and $\sigma_0 > 0$. In other words: $X \sim \text{DE}(\mu_0, \sigma_0)$ with the following density function:

$$g(x) = \frac{1}{2\sigma_0} \exp\left(-\frac{|x - \mu_0|}{\sigma_0}\right)$$

Unfortunately, the model is misspecified and X is assumed to be normally distributed with a set of parameters $\theta = (\mu, \sigma^2)$, meaning that $X \sim \mathcal{N}(\mu, \sigma^2)$

$$f_\theta(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x - \mu}{\sigma}\right)^2\right)$$

(a) Calculate the set of parameters θ that minimizes the Kullback-Leibler Divergence $D_{KL}(g \| f_\theta)$

Hint: Use the fact that for $X \sim \text{DE}(\mu_0, \sigma_0)$, the following properties apply: $\mathbb{E}(X) = \mu_0$ and $\mathsf{Var}(X) = 2\sigma_0^2$.