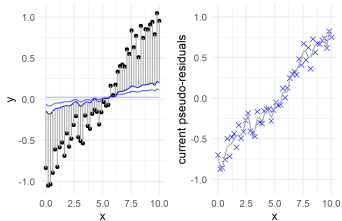
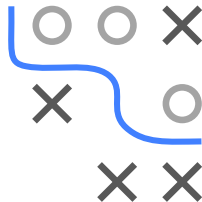


# Introduction to Machine Learning

## Boosting

### Gradient Boosting: Illustration



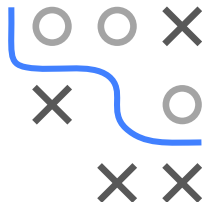
#### Learning goals

- See simple visualizations of boosting in regression
- Understand impact of different losses and base learners

## GRADIENT BOOSTING ILLUSTRATION - GAM

## GAM / Splines as BL and compare $L_2$ vs. $L_1$ loss.

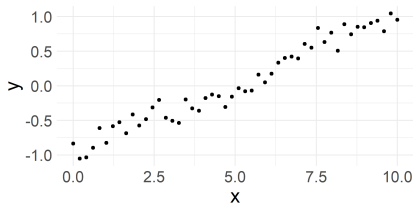
- L2: Init = optimal constant =  $\text{mean}(y)$ ; for L1 it's  $\text{median}(y)$
- BLs are cubic  $B$ -splines with 40 knots.
- PRs L2:  $\tilde{r}(f) = r(f) = y - f(\mathbf{x})$
- PRs L1:  $\tilde{r}(f) = \text{sign}(y - f(\mathbf{x}))$
- Constant learning rate 0.2



Univariate toy data:

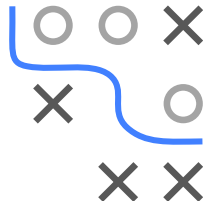
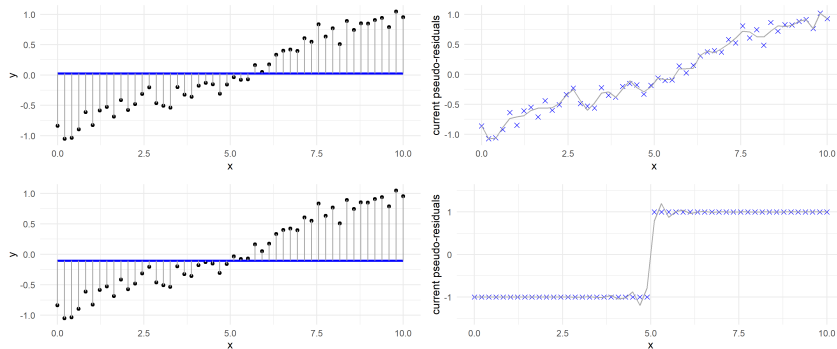
$$y^{(i)} = -1 + 0.2 \cdot x^{(i)} + 0.1 \cdot \sin(x^{(i)}) + \epsilon^{(i)}$$

$$n = 50 ; \epsilon^{(i)} \sim \mathcal{N}(0, 0.1)$$



# GAM WITH $L_2$ VS $L_1$ LOSS

Top:  $L_2$  loss, bottom:  $L_1$  loss

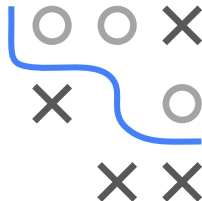
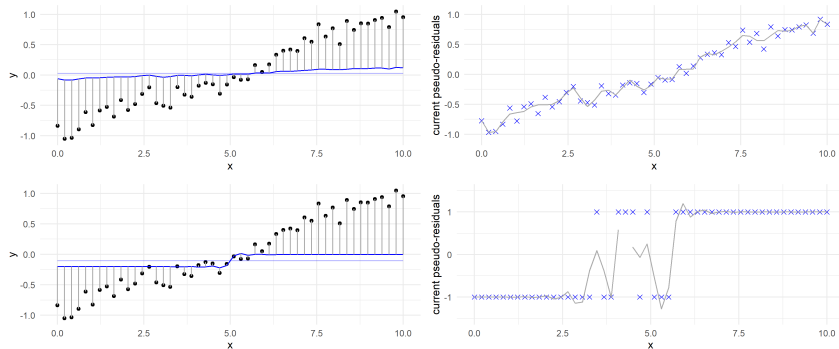


## Iteration 1

Shape of PRs affects gradual model fit:  $L_1$  only sees residuals' sign, BLs are not affected size of values as in  $L_2$  and hence lead to more moderate changes.

# GAM WITH $L_2$ VS $L_1$ LOSS

Top:  $L_2$  loss, bottom:  $L_1$  loss

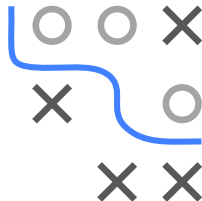
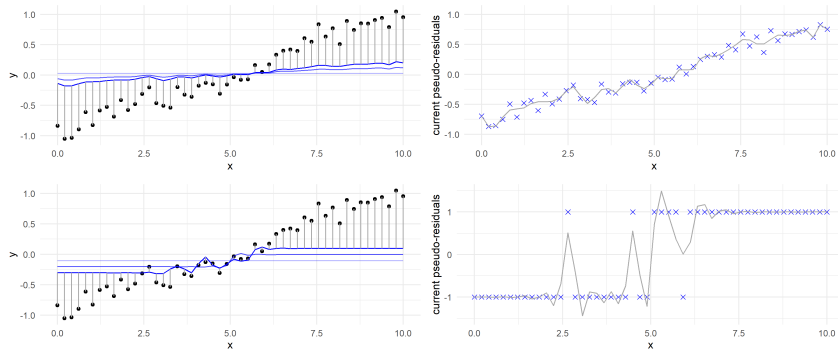


## Iteration 2

Shape of PRs affects gradual model fit:  $L_1$  only sees residuals' sign, BLs are not affected size of values as in  $L_2$  and hence lead to more moderate changes.

# GAM WITH $L_2$ VS $L_1$ LOSS

Top:  $L_2$  loss, bottom:  $L_1$  loss

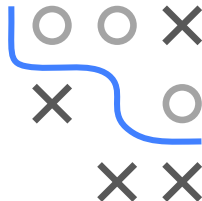
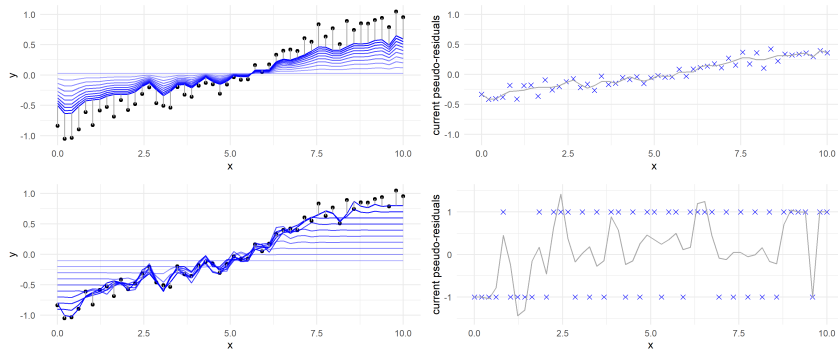


## Iteration 3

Shape of PRs affects gradual model fit:  $L_1$  only sees residuals' sign, BLs are not affected size of values as in  $L_2$  and hence lead to more moderate changes.

# GAM WITH $L_2$ VS $L_1$ LOSS

Top:  $L_2$  loss, bottom:  $L_1$  loss

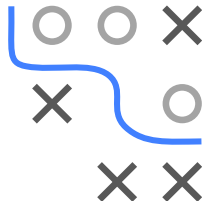
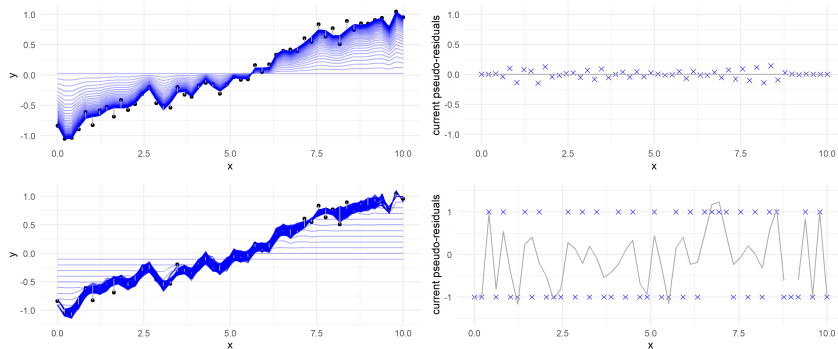


Iteration 10

Shape of PRs affects gradual model fit:  $L_1$  only sees residuals' sign, BLs are not affected size of values as in  $L_2$  and hence lead to more moderate changes.

# GAM WITH $L_2$ VS $L_1$ LOSS

Top:  $L_2$  loss, bottom:  $L_1$  loss

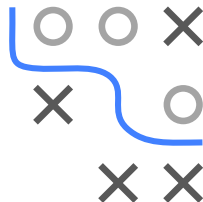
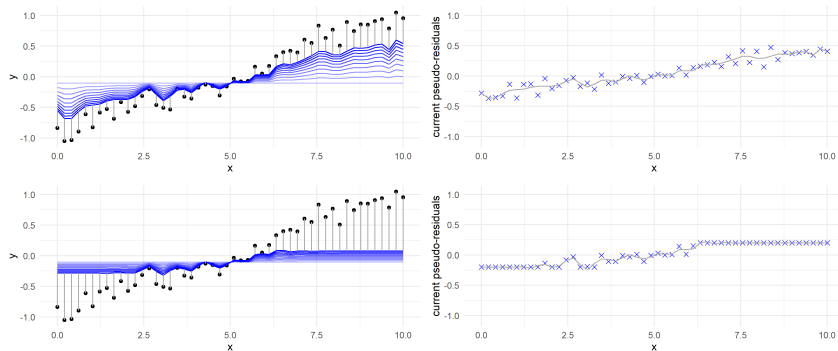


Iteration 100

Shape of PRs affects gradual model fit:  $L_1$  only sees residuals' sign, BLs are not affected size of values as in  $L_2$  and hence lead to more moderate changes.

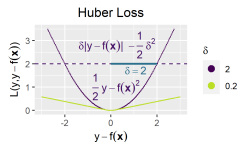
# GAM WITH HUBER LOSS

Top:  $\delta = 2$ , bottom:  $\delta = 0.2$ .



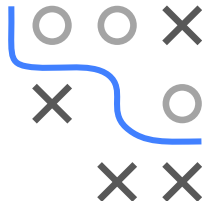
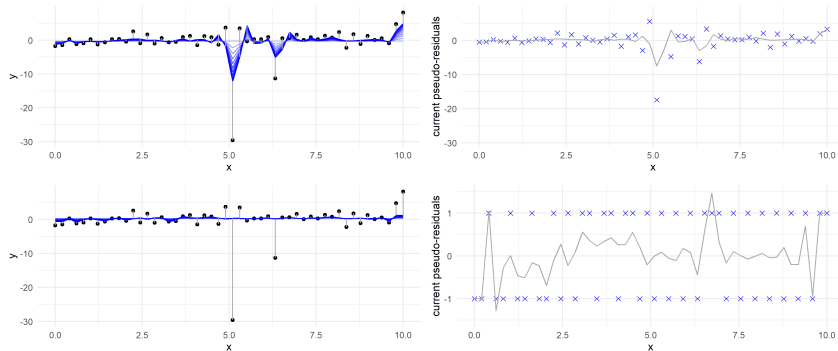
Iteration 10

For small  $\delta$ , PRs are often bounded, resulting in  $L1$ -like behavior, while the upper plot more closely resembles  $L2$  loss.



# GAM WITH OUTLIERS

Instead of Gaussian noise, let's use  $t$ -distrib, that leads to outliers in  $y$ .  
Top:  $L2$ , bottom:  $L1$ .

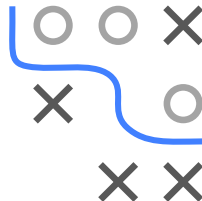
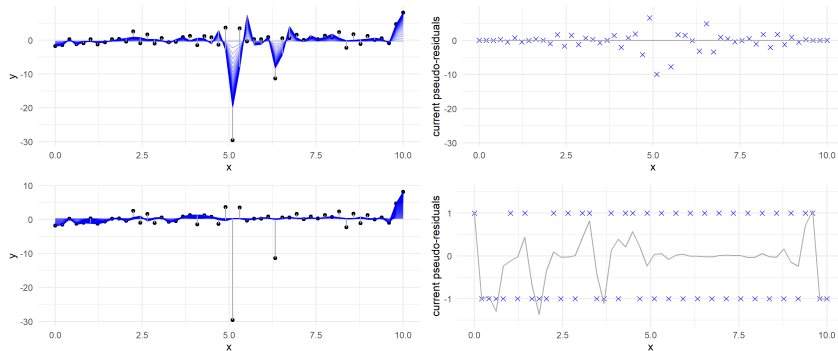


## Iteration 10

$L2$  loss is affected by outliers rather strongly, whereas  $L1$  solely considers residuals' sign and not their magnitude, resulting in a more robust model.

# GAM WITH OUTLIERS

Instead of Gaussian noise, let's use  $t$ -distrib, that leads to outliers in  $y$ .  
Top:  $L2$ , bottom:  $L1$ .

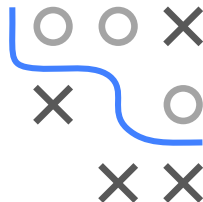
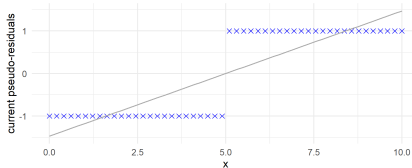
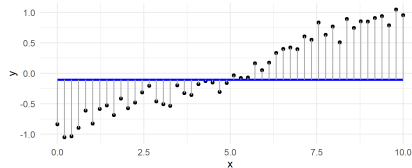
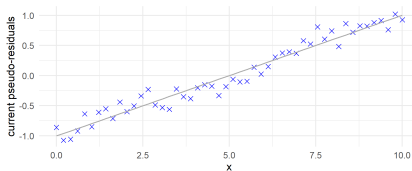
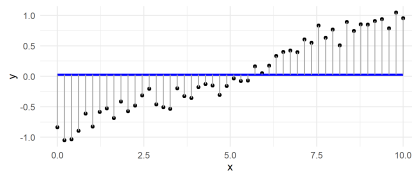


Iteration 100

$L2$  loss is affected by outliers rather strongly, whereas  $L1$  solely considers residuals' sign and not their magnitude, resulting in a more robust model.

# LM WITH $L_2$ VS $L_1$ LOSS

Top:  $L_2$ , bottom:  $L_1$ .

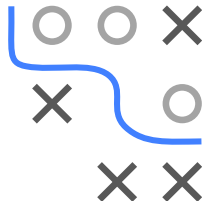


Iteration 1

$L_2$ : as  $\tilde{r}(f) = r(f)$ , BL of 1st iter already optimal; but learn rate slows us down.

Top:  $L2$ , bottom:  $L1$ .

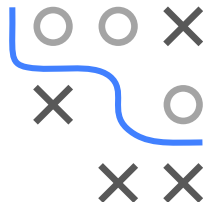
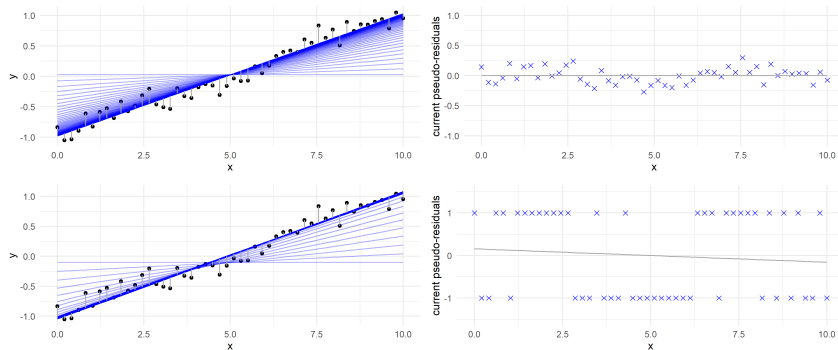
Top:  $L2$ , bottom:  $L1$ .



L2: as  $\tilde{r}(f) = r(f)$ , BL of 1st iter already optimal; but learn rate slows us down.

# LM WITH $L_2$ VS $L_1$ LOSS

Top:  $L_2$ , bottom:  $L_1$ .



Iteration 100

$L_2$ : as  $\tilde{r}(f) = r(f)$ , BL of 1st iter already optimal; but learn rate slows us down.