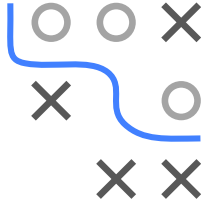
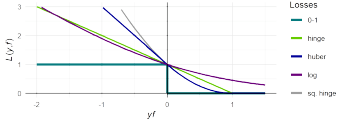
Introduction to Machine Learning



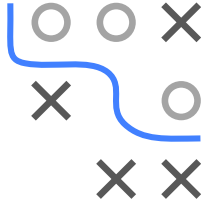
Linear Support Vector Machines
SVMs and Empirical Risk Minimization



Learning goals

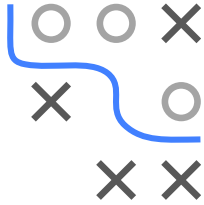
- Know why the SVM problem can be understood as (regularized) empirical risk minimization problem
- Know that the corresponding loss is the hinge loss

REGULARIZED EMPIRICAL RISK MINIMIZATION



- We motivated SVMs from a geometrical point of view: The margin is a distance to be maximized.
- This is not really true anymore under margin violations: The slack variables are not really distances. Instead, $\gamma \cdot \zeta^{(i)}$ is the distance by which an observation violates the margin.
- This already indicates that transferring the geometric intuition from hard-margin SVMs to the soft-margin case has its limits.
- There is an alternative approach to understanding soft-margin SVMs: They are **regularized empirical risk minimizers**.

SOFT-MARGIN SVM WITH ERM AND HINGE LOSS



We derived this QP for the soft-margin SVM:

$$\begin{aligned} \min_{\boldsymbol{\theta},\theta_0,\zeta^{(i)}} \quad & \frac{1}{2}\|\boldsymbol{\theta}\|^2 + C\sum_{i=1}^{n}\zeta^{(i)} \\ \text{s.t.} \quad & y^{(i)}\left(\left\langle \boldsymbol{\theta}, \mathbf{x}^{(i)} \right\rangle + \theta_0\right) \geq 1 - \zeta^{(i)} \quad \forall\, i \in \{1,\ldots,n\}, \\ \text{and} \quad & \zeta^{(i)} \geq 0 \quad \forall\, i \in \{1,\ldots,n\}. \end{aligned}$$

In the optimum, the inequalities will hold with equality (as we minimize the slacks), so $\zeta^{(i)} = 1 - y^{(i)} f\left(\mathbf{x}^{(i)}\right)$, but the lowest value $\zeta^{(i)}$ can take is 0 (we do no get a bonus for points beyond the margin on the correct side). So we can rewrite the above:

$$\frac{1}{2}\|\boldsymbol{\theta}\|^2 + C\sum_{i=1}^{n} L\left(y^{(i)}, f\left(\mathbf{x}^{(i)}\right)\right);\ L\left(y, f(\mathbf{x})\right) = \begin{cases} 1 - yf & \text{if } yf \leq 1 \\ 0 & \text{if } yf > 1 \end{cases}$$

We can also write $L\left(y, f(\mathbf{x})\right) = \max(1 - yf, 0)$.

OTHER LOSSES

SVMs can easily be generalized by changing the loss function.

- Squared hinge loss / Least Squares SVM:
  $L(y, f(\mathbf{x})) = \max(0, (1 - yf)^2)$
- Huber loss (smoothed hinge loss)
- Bernoulli/Log loss. This is L2-regularized logistic regression!
- NB: These other losses usually do not generate sparse solutions in terms of data weights and hence have no "support vectors".