Problem Set 2: Collusion and Entry

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In this problem set, you will investigate the behavior of a cartel and the conditions under which a competing firm might decide to enter the market. The data you will use comes from Igami and Sugaya (2021, REStud).

Collaboration among groups of 2-3 students is encouraged, though I will leave it up to you to determine the optimal group size. Each group should submit a single set of solutions. Groups also should feel free to communicate with each other or with me regarding any difficult parts of the problem set. If you are stuck on a question then move on, as the exercises are designed for you to learn the material, and once you are no longer learning for a question, you should stop spending time on it. That said, you should plan to spend a substantial amount of time on these exercises.

Data Description

The data contains an annual time series of the following variables from 1980-1998: a year identifier, an indicator for whether the cartel was active in that year (I_cartel), price (P), total quantity (Q), quantity produced by each of the four cartel firms (q_basf, q_roche, etc), and the quantity produced by the fringe firms (q_fri).

1. Replicate figure 3. That is, graph output, price and cost over time.

Calibration

Assume the firms Roche, Takeda, E. Merck, and BASF compete in Cournot (quantity) competition. There is also a competitive fringe that produces an exogenous quantity that does not depend on price. The market demand curve is given by:

$$Q_t^D = \alpha_1 P_t + \epsilon_t$$

where P_t is price and ϵ_t will be referred to as the *effective demand shifter*. The demand faced by the non-fringe firms, $Q_{C,t}$, is market demand less fringe production, $Q_{F,t}$. In other words, the demand faced by the non-fringe firms is:

$$Q_{C,t} = Q_t^D - Q_{F,t}$$

2. Write down the maximization problem that the firms face, and derive their first order conditions.

Assume the marginal cost of each firm is constant over time and quantity. Further, assume the marginal cost of Roche, $c_{roche} = 5.9$.

3. Use Roche's first order condition (FOC) to calibrate α_1. To do so, first calculate the value of α_1 that satisfies the FOC in each pre-cartel period. Then average over these values to obtain your estimate. Periods during which collusion occurred should be ignored in this calibration because the FOCs do not apply. Report your estimate.

4. Now use your $\hat{\alpha}_1$ and calibrate the other firms' marginal costs. To do so, obtain the costs that satisfy their FOCs in each pre-cartel period, and average over the periods to obtain your estimates. Again, only use time periods when the cartel was not active. Report your estimates.

5. Using your $\hat{\alpha}_1$, calculate the effective demand shifter, ϵ_t, in every period. Graph your result.

6. Suppose the four non-fringe firms were to merge. Using your calibrated effective demand shifter, determine what the new firm's optimal quantity would be in each time period and calculate the market clearing price. Create a graph comparing this monopoly price to the price in the data. Do the prices in the data approach the monopoly prices during the cartel periods?

Collusion

Suppose the four non-fringe firms agree to collude with each other starting in 1991. More specifically, suppose that Roche is the cartel's leader, and they choose the cartel's production target in every year to maximize their own profit. Each firm in the cartel then produces their share of this target, and these shares are set at the market shares in the last pre-cartel period (1990). Assume that if any firm deviates from its production quota, the cartel reverts to Cournot competition in the next period and forevermore.

7. Write down Roche's maximization problem as the cartel leader.

8. In each period from 1991 to 1998, compute:

   (a) The cartel level of output, the market clearing price, and the profit of each firm.

   (b) The profit each firm would earn if they were to deviate from their prescribed output. Assume that the deviating firm produces the quantity that maximizes its profit, conditional on the its competitors producing according to the collusive scheme, and also on the fringe production.

   (c) The profit each firm would earn under Cournot competition.

In order for collusion to be sustainable, it must be true that each firm is better off continuing to cooperate with the cartel than they are deviating. This means that for each firm the net present value of profits from continued cooperation must exceed the net present value of profits from deviation. We call this the incentive compatibility constraint (ICC).

9. Write down the ICC faced by the firms in the coalition.

10. Assume firms have a discount factor of 0.8, and that they assume the effective demand shifter and fringe production will not change in future periods ($\epsilon_{t+1} = \epsilon_t$, $Q_{F,t+1} = Q_{F,t}$). Calculate the ICC for each firm at each time period in 1991-1998. Graph your results.

Entry

Suppose it is 1995 and a new firm is considering entering the Vitamin C market. If it enters, it would have a marginal cost equal to that of Roche.

11. In a static entry game, under what fixed cost would the new firm be willing to enter the market? Consider two scenarios:

    (a) The incumbent firms continue to operate as a cartel, in the sense that they set quantities taking as given the production of the entrant and the fringe. In this scenario, the entrant enters without joining the coalition. To evaluate the scenario, search for the quantity that maximizes the entrant's profit; for each candidate entrant quantity compute the response of the cartel (as in Question 8a).

    (b) Entry causes the cartel to collapse, so there is Cournot competition between all firms, taking as given the fringe quantity. To evaluate this scenario, compute the quantities that arise in Cournot equilibrium given a residual demand curve that accounts for the fringe (or derived closed-form solutions, which are available).

12. Provide an economic interpretation of your results in the previous question.

2. Write down the maximization problem that the firms face, and derive their first order conditions.

$$Q_t^D = \alpha_1 P_t + \varepsilon_t \qquad \text{i.e} \qquad P_t = \frac{Q_t^D - \varepsilon_t}{\alpha_1}$$

$$Q_{c,t} = Q_t^D - Q_{F,t}$$

⚠ Here $\alpha_1 < 0$

Firm j in the Cournot oligopoly faces problem:

$$\pi(t) \equiv \max_{q_j} \left[q_j P_t(q_j) - C_j(q_j) \right]$$

with $Q_t^D = Q_{F,t} + Q_{c,t \setminus j} + q_j$

$$P_t = \frac{1}{\alpha_1}\left[q_j + Q_{F,t} + Q_{c,t\setminus j} - \varepsilon_t \right]$$

$$\pi(t) \equiv \max_{q_{j,t}} \left[\frac{1}{\alpha_1} q_{j,t}\left(q_{j,t} + Q_{F,t} + Q_{c,t\setminus j} - \varepsilon_t \right) - C_{j,t}(q_{j,t}) \right]$$

FOC % $q_{j,t}$: $\frac{1}{\alpha_1}\left[2q_{j,t} + Q_{F,t} + Q_{c,t\setminus j} - \varepsilon_t \right] = \frac{dC_{j,t}}{dq_{j,t}}$

$$\frac{q_{j,t}}{\alpha_1} + P_t = \frac{dC_{j,t}}{dq_{j,t}} \qquad (1)$$

Assume the marginal cost of each firm is constant over time and quantity. Further, assume the marginal cost of Roche, $c_{roche} = 5.9$.

3. Use Roche's first order condition (FOC) to calibrate α_1. To do so, first calculate the value of α_1 that satisfies the FOC in each pre-cartel period. Then average over these values to obtain your estimate. Periods during which collusion occurred should be ignored in this calibration because the FOCs do not apply. Report your estimate.

Here $C_{j,t}(q) = c_j \cdot q \qquad \frac{q_{j,t}}{\alpha_1} = c_j - P_t$

$$(1): \quad \alpha_1 = \frac{q_{j,t}}{c_j - P_t} \qquad (2_{j,t})$$

4. Now use your $\hat{\alpha}_1$ and calibrate the other firms' marginal costs. To do so, obtain the costs that satisfy their FOCs in each pre-cartel period, and average over the periods to obtain your estimates. Again, only use time periods when the cartel was not active. Report your estimates.

$$(2_{j',t}) : \quad c_{j'} = P_t + \frac{q_{j',t}}{\alpha_1} \qquad (2'_{j',t})$$

5. Using your $\hat{\alpha}_1$, calculate the effective demand shifter, ϵ_t, in every period. Graph your result.

$$Q_t^D = \alpha_1 P_t + \epsilon_t \quad \Longrightarrow \quad \hat{\varepsilon}_t \equiv Q_t^D - \hat{\alpha}_1 P_t$$

6. Suppose the four non-fringe firms were to merge. Using your calibrated effective demand shifter, determine what the new firm's optimal quantity would be in each time period and calculate the market clearing price. Create a graph comparing this monopoly price to the price in the data. Do the prices in the data approach the monopoly prices during the cartel periods?

$$P_t = \frac{Q_{c,t} + Q_{F,t} - \varepsilon_t}{\alpha_1}$$

After merger, the quasi-monopoly would solve:

$$\max_{Q_t^m} \left[\frac{1}{\alpha_1} Q_t^m \left(Q_t^m + Q_{F,t} - \varepsilon_t \right) - C_{cong}(Q_t^m) \right]$$

$$\text{FOC } \% \ Q_t^m : \quad \frac{1}{\alpha_1}\Big(Q_t^m + \underbrace{Q_t^D}_{= Q_t^m + Q_{F,t}} - \varepsilon_t \Big) = \frac{Q_t^m}{\alpha_1} + P_t(Q_t^m) = \frac{dC_t^m}{dQ_{c,t}}$$

Assuming no synergies in production costs, still assuming constant MC for all member firms,

$$C_t^m(Q_{c,t}) = \min_{j \in C} [c_j] \cdot Q_{c,t} = c_{Roche} \cdot Q_{c,t}$$

So

$$\boxed{Q_t^m = \frac{1}{2}\left[\alpha_1 c_{Roche} + \varepsilon_t - Q_{F,t} \right]}$$

$$P_t^m = \frac{Q_t^m + Q_{F,t} - \varepsilon_t}{\alpha} = \frac{1}{2}\Big[$$

Collusion

Suppose the four non-fringe firms agree to collude with each other starting in 1991. More specifically, suppose that Roche is the cartel's leader, and they choose the cartel's production target in every year to maximize their own profit. Each firm in the cartel then produces their share of this target, and these shares are set at the market shares in the last pre-cartel period (1990). Assume that if any firm deviates from its production quota, the cartel reverts to Cournot competition in the next period and forevermore.

7. Write down Roche's maximization problem as the cartel leader.

$$\max_{Q_{c,t}} \left[s_{Roche}\, Q_{c,t} \left(P_t(Q_{c,t}) - c_{Roche} \right) \right]$$

FOC % $Q_{c,t}$:

$$P_t(Q_{c,t}) + Q_{c,t} \frac{dP_t}{dQ_{c,t}} = c_{Roche}$$

i.e

$$\frac{1}{\alpha_1} \left[2Q_{c,t} + Q_{f,t} - \varepsilon_t \right] = c_{Roche}$$

i.e

$$\boxed{\begin{aligned} Q_{c,t} &= \frac{1}{2}\left[\alpha_1 c_{Roche} + \varepsilon_t - Q_{f,t} \right] \\ P_t &= \frac{Q_{c,t} + Q_{f,t} - \varepsilon_t}{\alpha_1} = \frac{1}{2\alpha_1}\left[\alpha_1 c_{Roche} + Q_{f,t} - \varepsilon_t \right] = \frac{1}{2}\left[c_{Roche} + \frac{Q_{f,t} - \varepsilon_t}{\alpha_1} \right] \end{aligned}}$$

8. In each period from 1991 to 1998, compute:

   (a) The cartel level of output, the market clearing price, and the profit of each firm.

   (b) The profit each firm would earn if they were to deviate from their prescribed output. Assume that the deviating firm produces the quantity that maximizes its profit, conditional on the its competitors producing according to the collusive scheme, and also on the fringe production.

DEVIATION OF FIRM j :

$$Q_t^D = Q_{F,t} + (1 - s_j) Q_{c,t} + q_{j,t}$$

FOC :

$$\frac{1}{\alpha_1}\left[2q_{j,t} + Q_{F,t} + (1 - s_j) Q_{c,t} - \varepsilon_t \right] = c_j$$

i.e

$$q_{j,t}^{dev} = \frac{1}{2}\left[\alpha_1 c_j + \varepsilon_t - Q_{F,t} - (1 - s_j) Q_{c,t} \right]$$

$$\pi^{dev}_{j,t} = q^{dev}_{j,t}\left[\frac{Q_{F,t} + (1-s_j)Q_{c,t} + q^{dev}_{j,t} - \varepsilon_t}{\alpha_1} - c_j\right]$$

(c) The profit each firm would earn under Cournot competition.

$$q_{j,t}\frac{dP_t}{dq_{j,t}} + P_t = c_j$$

$$\forall j \in C, \qquad 2q^*_{j,t} + \sum_{j'\neq j} q^*_{j',t} = \alpha_1 c_j + \varepsilon_t - Q_{F,t}$$

$$\text{i.e} \quad \underbrace{\begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{pmatrix}}_{\equiv M} Q^* = \alpha_1 c + (\varepsilon_t - Q_{F,t})\mathbb{1}$$

$$\text{i.e} \quad Q^* = M^{-1}\left[\alpha_1 c + (\varepsilon_t - Q_{F,t})\mathbb{1}\right]$$

In order for collusion to be sustainable, it must be true that each firm is better off continuing to cooperate with the cartel than they are deviating. This means that for each firm the net present value of profits from continued cooperation must exceed the net present value of profits from deviation. We call this the incentive compatibility constraint (ICC).

9. Write down the ICC faced by the firms in the coalition.

Let

$$\pi^{coalition}_{j,t} \equiv s_j Q_{c,t}\left(P_t(Q_{c,t}) - c_j\right)$$

$$\pi^{cournot}_{j,t} \equiv q^*_j\left(\frac{Q_{f,t} + \sum_{j'\in C} q^*_{j'} - \varepsilon_t}{\alpha_1} - c_j\right)$$

$$\mathrm{ICC}_{j,t}: \quad \sum_{t'\geq t} \beta_j^{t'-t}\,\mathbb{E}\left[\pi^{coalition}_{j,t'} \,\middle|\, I_{j,t}\right] \geqslant \pi^{dev}_{j,t} + \sum_{t'>t} \beta^{t'-t}\,\mathbb{E}\left[\pi^{cournot}_{j,t'} \,\middle|\, I_{j,t}\right]$$

where $I_{j,t}$ is a filtration of the information of firm j at time t.

In the case of linear demand, as each firm knows its MC and has data on pre-coalition activity, it can follow the lines of q1 to q5 to figure out, under the assumption of constant MC for all firms,

$$\hat{\alpha}_1(c_j) \equiv \hat{\alpha}_{1,j}, \qquad \hat{\varepsilon}_{t,j} \quad \text{and} \quad (\widehat{c_{j'}})_{j'\in C\setminus\{j\}}$$

10. Assume firms have a discount factor of 0.8, and that they assume the effective demand shifter and fringe production will not change in future periods ($\epsilon_{t+1} = \epsilon_t$, $Q_{F,t+1} = Q_{F,t}$). Calculate the ICC for each firm at each time period in 1991-1998. Graph your results.

Let ε the demand shifter in 1991, Q_F the corresponding fringe supply.

$\forall t \in [\![1991, 1998]\!]$, firms estimate

- $$\widehat{\pi}^{\text{coalition}}_{j,t} \equiv s_j \widehat{Q}_{c,t} \left(\widehat{P}_t - c_j\right)$$

  with $$\begin{cases} \widehat{Q}_{c,t} = \frac{1}{2}\left[\alpha_1 c_{\text{Roche}} + \varepsilon - Q_f\right] = \widehat{Q}_c \\ \widehat{P}_t = \frac{1}{2}\left[c_{\text{Roche}} + \dfrac{Q_f - \varepsilon}{\alpha_1}\right] = \widehat{P} \end{cases}$$

  i.e $\widehat{\pi}^{\text{coalition}}_{j,t} \equiv \widehat{\pi}^{\text{coalition}}_j$ is time-invariant

- $$\widehat{\pi}^{\text{cournot}}_{j,t} \equiv \widehat{q}^*_j \left(\frac{Q_f + \sum_{j' \in C} \widehat{q}^*_{j'} - \varepsilon}{\alpha_1} - c_j\right)$$

  with $$\widehat{Q}^* = M^{-1}\left[\alpha_1 c + (\varepsilon - Q_F)\mathbb{1}\right]$$

  $\widehat{\pi}^{\text{Cournot}}_{j,t} = \widehat{\pi}^{\text{cournot}}_j$ also time-invariant

- $$\widehat{\pi}^{\text{dev}}_{j,t} = \widehat{q}^{\text{dev}}_{j,t}\left[\frac{\widehat{Q}_{F,t} + (1-s_j)\widehat{Q}_{c,t} + \widehat{q}^{\text{dev}}_{j,t} - \widehat{\varepsilon}_t}{\alpha_1} - c_j\right] = \widehat{\pi}^{\text{dev}}_j$$

  with $$\widehat{q}^{\text{dev}}_{j,t} = \frac{1}{2}\left[\alpha_1 c_j + \varepsilon - Q_F - (1-s_j)\widehat{Q}_{c,t}\right] = \widehat{q}^{\text{dev}}_j$$

  $$\widehat{Q}_{c,t} = \frac{1}{2}\left[\alpha_1 c_{\text{Roche}} + \varepsilon - Q_f\right] = \widehat{Q}_c$$

We define $ICC_{j,t}$ in general as :

$$ICC_{j,t} = \sum_{t'=t+1}^{\infty} \beta^{t'-t} \mathbb{E}\left[\pi_{j,t'}^{(coll)} - \pi_{j,t'}^{(cournot)} \mid I_t \right] - \left(\pi_{j,t}^{(dev)} - \pi_{j,t}^{(coll)} \right)$$

Due to the time invariance implied by the assumptions above, we have :

$$\widehat{ICC_{j,t}} = \frac{\beta}{1-\beta} \left(\widehat{\pi_j^{coll}} - \widehat{\pi_j^{cournot}} \right) - \left(\widehat{\pi_j^{dev}} - \widehat{\pi_j^{coll}} \right)$$

$$= \frac{\widehat{\pi_j^{coll}}}{1-\beta} - \frac{\beta}{1-\beta} \widehat{\pi_j^{cournot}} - \widehat{\pi_j^{dev}}$$

Entry

Suppose it is 1995 and a new firm is considering entering the Vitamin C market. If it enters, it would have a marginal cost equal to that of Roche.

11. In a static entry game, under what fixed cost would the new firm be willing to enter the market? Consider two scenarios:

    (a) The incumbent firms continue to operate as a cartel, in the sense that they set quantities taking as given the production of the entrant and the fringe. In this scenario, the entrant enters without joining the coalition. To evaluate the scenario, search for the quantity that maximizes the entrant's profit; for each candidate entrant quantity compute the response of the cartel (as in Question 8a).

Entrant firm solves, assuming a fully fixed setting:

$$\max_{q_{e,t}} \left[\left(P(q_{e,t} \mid Q_{c,t} ; \varepsilon_t) - c_e \right) q_{e,t} \right]$$

<u>FOC:</u>

$$P(q_{e,t} \mid Q_{c,t} ; \varepsilon_t) + q_{e,t} \frac{dP}{dq_{e,t}} = c_e$$

$$P(q_{e,t} \mid Q_{c,t} ; \varepsilon_t) = \frac{Q_{c,t} + q_{e,t} + Q_{f,t} - \varepsilon_t}{\alpha_1} \qquad \frac{dP}{dq_{e,t}} = \frac{1}{\alpha_1}$$

$$\alpha_1 c_e = Q_{c,t} + Q_{f,t} + 2 q_{e,t} - \varepsilon_t \longrightarrow q_{e,t} = \frac{1}{2} \left[\varepsilon_t + \alpha_1 c_e - Q_{c,t} - Q_{f,t} \right]$$

Corresponding ceteris paribus prices and profits are:

$$P_t = \frac{1}{2} \left[c_e + \frac{Q_{c,t} + Q_{f,t} - \varepsilon_t}{\alpha_1} \right]$$

$$\pi_{e,t}(c_e) = \frac{1}{4} \left[\varepsilon_t + \alpha_1 c_e - Q_{c,t} - Q_{f,t} \right] \left[\frac{Q_{c,t} + Q_{f,t} - \varepsilon_t}{\alpha_1} - c_e \right]$$

$$= -\frac{\alpha_1}{4} \left[\frac{Q_{c,t} + Q_{f,t} - \varepsilon_t}{\alpha_1} - c_e \right]^2$$

The entrant chooses to enter only if $\pi_{e,t}(c_e) \geqslant 0$

Notice that profits are minimal when $c_e = P_{le}$ the price if the cartel holds and before/without entry.

CARTEL´S RESPONSE