

# An analytical approach for evaluating the collusion possibility between generation companies and transmission companies

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## ABSTRACT

The performance of transmission companies (TransCos) and their profits and losses are critical from the network operator's point of view, as they provide the infrastructure for competition between players in the electricity market. Therefore, it is necessary to identify the factors affecting the profitability of TransCos. In this paper, the factors affecting the income of TransCos are evaluated using a new and powerful analytical method that consists of two lemmas. In Lemma 1, the contribution of the five factors to the flow of lines and the locational marginal price (LMP) of buses is calculated using structural decomposition. In Lemma 2, the impact of five factors on changes in the congestion rent of lines, zones, and the entire network is evaluated. Finally, six new indices are proposed to evaluate the impact of generating companies (GenCos) on the income of TransCos. The proposed method is tested on the IEEE 24-bus network, and the simulation results are analyzed. By using this proposed tool, the system operator can quickly and accurately identify potential deviations and possible collusion between GenCos and TransCos in the electricity market.

## 1. Introduction

### 1.1. Motivation

One of the main parts of the power grid is the transmission system [1–3]. The transmission system has a vital role in facilitating competition and improving the economic conditions of the electricity market [4, 5]. Transmission companies (TransCos) must provide the necessary platform for high-volume power exchange, cost-effectiveness, independence, and fairness for all market participants [6–8]. Ideally, where there is no congestion in the network, the locational marginal price (LMP) of buses is slightly different, and with zero losses, the price of all buses will be the same. However, in most power systems, there is congestion in some lines that causes damage to access. Market power is formed for some market participants when they are free [9].

An important issue in electricity market studies is the evaluation of factors affecting the revenue of TransCos. In most electricity markets, the most important part of TransCo's revenue is the congestion rent (CR). Congestion rent equals the difference between payments received from actors on the consumption side and the costs paid to actors on the production side. The greater the difference (due to congestion), the greater the profit of the TransCos [10]. The development of the network

in the future can be predicted by the revenue of TransCos from electricity transit on different lines. Therefore, analyzing the revenue of a TransCo is important for both the TransCo and the operator. Despite research in the field of economic issues related to TransCos (for instance, pricing and allocation of transmission costs, congestion management, etc.), there has been no detailed and analytical study on the parameters affecting the income of TransCos, especially factors related to research production actors. The purpose of this paper is to propose a new analytical method to accurately determine the contribution of various factors, especially the factors available to generation companies (GenCos), to the CR of TransCos.

### 1.2. Literature review

Various articles have addressed the issue of transmission cost allocation and transmission line congestion management methods. For instance, the relative sharing method tracks the path of power flow in transmission lines from the injection/output of active power in each bus according to Kirchhoff's law [11,12]. According to the postage stamp method, electricity companies divide the costs of TransCos equally between users of the service, but the distance and route of power transmission are not considered [13–15]. In the MW-Mile method, users pay or receive a fee depending on their network usage. The main feature of

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Nomenclature		
<b>TransCo</b>	Transmission company	$dp_G^{\text{Min}}$ Variation of the minimum generation capacity of expensive units
<b>GenCo</b>	Generation company	$dp_G^{\text{Max}}$ Variation of the maximum generation capacity of cheap units
<b>CR</b>	Congestion rent	$dp_D$ Variation of the load consumption
<b>TCR</b>	Total congestion rent	$dp_L^{\text{Max}}$ Variation of the capacity of congested lines
<b>LMP</b>	Locational marginal price	$N_c$ The total number of factors affecting the LMP of buses and line flows
<b>EMUCL</b>	The effect of marginal unit on the <b>CR</b> of line	$N_{\text{mrg}}$ Number of marginal units
<b>EAMUCL</b>	The effect of all marginal units on the <b>CR</b> of line	$N_{\text{gmin}}$ Number of expensive units
<b>SMUCL</b>	The share of marginal unit on the <b>CR</b> of line	$N_{\text{gmax}}$ Number of cheap units
<b>EMUCZ</b>	The effect of marginal unit on the <b>CR</b> of zone	$N_b$ Number of bus networks
<b>EAMUCZ</b>	The effect of all marginal units on the <b>CR</b> of zone	$N_l$ Number of normal lines
<b>SMUCZ</b>	The share of marginal unit on the <b>CR</b> of zone	$N_{l\text{cong}}$ Number of congested lines
<b>FLMP</b>	<b>LMP</b> difference matrix of two buses connected to each other	$M_L$ Cross-matrix of lines and buses
$C_{LMP}, C_{flow}$	The matrix of coefficients affecting the changes in the <b>LMP</b> of busses and line flows	$M_G$ Cross-matrix of generators and buses
$c_{LMP}^{\text{mu}}, c_{LMP}^{\text{eu}}, c_{LMP}^{\text{cu}}$	Coefficients of the effect of marginal, expensive, cheap units on the <b>LMP</b> of buses	<b>CR</b> <b>CR</b> matrix of network lines
$c_{flow}^{\text{mu}}, c_{flow}^{\text{eu}}, c_{flow}^{\text{cu}}$	Coefficients of the effect of marginal, expensive, cheap units on line flows	<b>dCR</b> Grid line <b>CR</b> variation matrix
$c_{LMP}^l, c_{flow}^l$	Coefficients of the effect of load on the <b>LMP</b> of buses and line flows	$CR_{\text{zone}i}$ <b>CR</b> matrix of lines located in zone $i$
$c_{LMP}^{cl}, c_{flow}^{cl}$	Coefficients of the effect of congested line on the <b>LMP</b> of buses and line flows	$dCR_{\text{zone}i}$ The <b>CR</b> change matrix of lines located in zone $i$
$c_{j,i}^{\text{mu}}$	The coefficient of influence of marginal unit $j$ on the flow changes of line $i$	<b>TCR</b> Network <b>TCR</b> matrix
$z_j^i$	The influence factor of marginal unit $j$ on the flow changes of zone $i$	<b>dTCR</b> Matrix of network <b>TCR</b> changes
$a_{\text{mrg}}$	Bid price of the marginal unit	$I_{j,i}^{\text{mu}}$ Variation of <b>CR</b> rate of line $i$ in exchange for the bid change of marginal unit $j$
$p_G^{\text{Min}}$	Minimum generation capacity of expensive units	$I_{j,i}^{\text{eu}}$ Variation of <b>CR</b> rate of line $i$ in exchange for changing the capacity of expensive unit $j$
$p_G^{\text{Max}}$	Maximum generation capacity of cheap units	$I_{j,i}^{\text{cu}}$ Variation of <b>CR</b> rate of line $i$ in exchange for changing the minimum generation capacity of cheap units $j$
$p_D$	Load consumption	$I_i^{\text{ln}}$ Variation of <b>CR</b> rate of line $i$ in exchange for changing load of bus $n$
$p_L^{\text{Max}}$	The capacity of congested lines	$I_i^{\text{cl}}$ Variation of <b>CR</b> rate of line $i$ in exchange for the bid change of marginal unit $j$
$da_{\text{mrg}}$	Variation of the bid price of the marginal unit	$W_{\text{zone}i}$ Matrix of coefficients affected by the <b>CR</b> changes in zone $i$
		<b>W</b> Matrix of coefficients of variables affecting <b>TCR</b>

this method is to obtain the share of generators and loads on each line [16]. The Zbus approach distributes costs based on the node impedance matrix and the current injected into each bus of the system [17]. The method of Conejo et al. is to find the share of network users in allocating transmission costs using the Zbus method [18]. Some methods of allocating transmission costs may be fair and equitable but do not provide a good economic signal to network users [19]. Only the coverage of

transmission fixed costs is mentioned in articles [20–24].

The importance of congestion management, especially to increase competition in the electricity market and enhance network security, is discussed in [25,26]. There are different ways to manage congestion, such as load reduction methods [27], production reprogramming [28–30], reactive power management [31,32], regional congestion management [32,33], voltage stability [34], and use of FACTS devices

**Table 1**  
Comparison of the related research works.

Ref.	Topic	Method	Considering Network zone	Considering Transmission constraints	Proposing new index
[52]	Evaluating market power	Analytical	x	x	✓
[53]	Evaluating market power	Simulation	x	✓	✓
[40]	Evaluating market power	Analytical	✓	✓	✓
[54]	Evaluating market power	Simulation	x	x	x
[55]	Evaluating market power	Analytical	✓	✓	✓
[56]	Evaluating market power	Analytical	x	✓	✓
[45]	Assessment of collusion	Analytical	x	✓	x
[57]	Assessment of collusion	Simulation	x	✓	x
[58]	Assessment of collusion	Analytical	x	✓	x
[59]	Assessment of collusion	Simulation	x	✓	x
[35]	Evaluating transmission network revenue	Simulation	x	x	x
[58]	Evaluating transmission network revenue	Simulation	x	✓	x
[59]	Evaluating transmission network revenue	Simulation	x	✓	x
Our method	Determining the contribution of factors affecting the revenue of transmission companies	Analytical	✓	✓	✓

[35]. Most of these methods use the approach of increasing production to reduce congestion. Therefore, an accurate and comprehensive evaluation of the impact of production on line flow is central. There are a few articles on the revenue and profit of TransCos and the determination of the parameters affecting it [35,36].

Another important topic related to the subject of our paper is assessing the potential impact of a company's behavior on the profit of other companies in an electricity market. In this regard, several articles have addressed the issue of market power analysis and collusion in the electricity market. Market power occurs when producers use methods to increase supply or reduce production to influence market prices in the desired direction [37,38]. David and Van presented an overview of market power, focusing on different equilibrium models and methods of reducing market power [39].

Several indicators are presented to assess market power. An assessment of regional market power with mustard ratio (MRR) is presented in [40]. Also, two indicators of location and network market are presented to evaluate the behaviours of GenCos that may use transmission congestion [41]. Collusion is another destructive distortion of competition in the electricity market. Collusion is defined as an agreement between GenCos' competitors to raise prices for higher profits [42]. Collusion means the agreement of two or more market players to raise profits [43]. The concept of collusion does not necessarily mean an explicit agreement between network participants. Rather, participants can benefit by considering the actions of other users and taking correct and timely actions [44]. This type of collusion is called implicit collusion. So, companies are interdependent in determining price, so the actions of companies are interdependent. Covert collusion increases consumer prices and reduces competition. Hidden collusion is difficult to detect because there is no agreement between the GenCos. In general, detecting collusion is difficult for the network operator [45]. Liu and Hobbes introduce a structure for collusion in which network congestion is considered in modeling [46]. References [47,50,51], methods and indicators to evaluate the potential of collusion in the electricity market to increase profits are presented in which the method of structural analysis is used. In Table 1, the related articles in the literature are compared. In each article, a method and its zone for the study network, transmission constraints, and the presentation of new indices are employed.

### 1.3. Contribution

In [42,45–50], the structural analysis method has been introduced and is used to evaluate the potential for deviation in the market. Due to the strong mathematical support and high efficiency of this method, in this paper, the structural analysis method is adopted to evaluate the effect of the behavior of GenCos on the revenue of TransCos.

Major research innovations are:

- (1) Providing an analytical method to determine the exact contribution of each of the following factors to the LMP of buses, line flows, and the revenue of the TransCo without the need to perform multiple load flows and only by performing one power flow.
  - Factors related to GenCos (including bid prices, expressed production capacity, and minimum production capacity of each generation unit)
  - The amount of load consumed by each network bus
  - The capacity of congested network lines
- (2) Evaluating the behavior of each GenCo on the profits of each Transco in a multi-zone system.
- (3) Determine the contribution of each factor to the total congestion rent (TCR).
- (4) Proposing several new analytical indicators to evaluate the impact of each unit on the network line and also the impact on the revenue of each TransCo (owner of lines in a network zone).

### 1.4. Paper organization

The whole paper is divided into five sections. Section 2 deals with the basic definitions of the problem and its formulation. Section 3 deals with the formulation of the proposed method, so the formulations of the proposed method are defined in this section. In Section 4, the simulation results are discussed, and the final section is devoted to the conclusion of the paper.

## 2. Problem definition

The mathematical basis of the method proposed in this paper is the structural analysis of the variables of a QP problem. Therefore, to express the proposed model, it is necessary to first introduce the assumed basic concepts. The general form of quadratic programming [60–63] is given by (1):

$$\begin{aligned} & \text{Min } \frac{1}{2} X^T . H . X + F^T . X \\ & \text{Subject to} \\ & A^{eq} . X = B^{eq} : \Lambda \\ & A . X \leq B : \Omega \\ & LB \leq X \leq UB : (\Gamma^-, \Gamma^+) \end{aligned} \quad (1)$$

In order to present the proposed formulas in the next section, it is necessary to present the basic formulas. So that some Eqs. (2)–(8) are referred to in [47]. After solving the optimization equation, marginal, minimum, and maximum values, as well as Lagrange coefficients, are obtained for equality and inequality constraints.  $X_{min}$  and  $X_{max}$  are the low and high limits of the variables, respectively, and the other variables represent the marginal value of the variables. Therefore, according to (2), all parameters are divided into three parts:

$$\begin{aligned} X &= \begin{bmatrix} X_{min} \\ X_{max} \\ X_{mrg} \end{bmatrix}, LB = \begin{bmatrix} LB_{min} \\ LB_{max} \\ LB_{mrg} \end{bmatrix}, UB = \begin{bmatrix} UB_{min} \\ UB_{max} \\ UB_{mrg} \end{bmatrix} \\ \Gamma^- &= \begin{bmatrix} \Gamma_{min}^- \\ \Gamma_{max}^- \\ \Gamma_{mrg}^- \end{bmatrix}, \Gamma^+ = \begin{bmatrix} \Gamma_{min}^+ \\ \Gamma_{max}^+ \\ \Gamma_{mrg}^+ \end{bmatrix}, F = \begin{bmatrix} F_{min} \\ F_{max} \\ F_{mrg} \end{bmatrix} \end{aligned} \quad (2)$$

The inequality  $g(X) \leq 0$  is regarded as active if the value of  $g(X^*)$  is zero at the optimal point  $X^*$  and inactive when the value of  $g(X^*)$  is less than zero. Therefore, we show the active and inactive inequalities as *act* and *pas*, respectively. In (3), matrices  $A$ ,  $B$ , and  $\Omega$  are divided as follows:

$$A = \begin{bmatrix} A^{act} \\ A^{pas} \end{bmatrix}, B = \begin{bmatrix} B^{act} \\ B^{pas} \end{bmatrix}, \Omega = \begin{bmatrix} \Omega^{act} \neq 0 \\ \Omega^{pas} = 0 \end{bmatrix} \quad (3)$$

Knowing that passive constraints have no multiplication, the Lagrange function for the optimization problem is as (4):

$$\begin{aligned} L(X, \Lambda, \Omega, \Gamma) &= \frac{1}{2} X^T . H . X + F^T . X + \Lambda^T . (A^{eq} . X - B^{eq}) \\ &+ (\Omega^{act})^T . (A^{act} . X - B^{act}) + (\Gamma_{min}^-)^T . (LB_{min} - X_{min}) \\ &+ (\Gamma_{max}^+)^T . (X_{max} - UB_{max}) \end{aligned} \quad (4)$$

Lemma1: “For the QP problem (1) with a Lagrangian function (4), the output of the optimization including marginal variables and Lagrangian multipliers can be decomposed as the matrix form (5) into structural factors of the QP problem [47].” According to Lemma 1, the general form of the coefficient matrix to calculate the effect of variables on the desired variables follows Eq. (5).

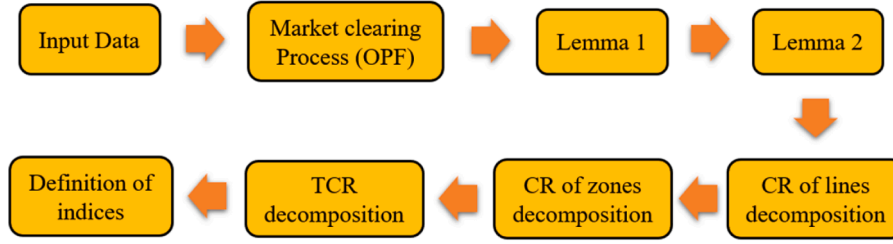


Fig. 1. General diagram of the proposed method.

$$\begin{bmatrix} X_{mrg} \\ \Lambda \\ \Omega^{act} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} \end{bmatrix} \begin{bmatrix} F_{mrg} \\ LB_{min} \\ UB_{max} \\ B^{eq} \\ B^{act} \end{bmatrix} \quad (5)$$

Lemma2: "Let vector  $U$  be a nonlinear function of the variable vector  $Q$  as (6):

$$[U]_{m \times 1} = [K \cdot Q]_{m \times 1} \odot [W \cdot Q]_{m \times 1} \quad (6)$$

Then, the Jacobian matrix of  $U$ , which is a function of the coefficient matrices  $K$  and  $W$  as well as  $Q$ , is calculated as (7a)–(7d): "

$$[J(K, W, Q)]_{m \times n} = \frac{dU}{dQ} = (2(K \odot W)_{m \times n} \cdot \text{Diag}(Q_{n \times n}) + Y_{m \times n}) \quad (7a)$$

Where:

$$Y_{m \times n} = [Y_1 \ Y_2 \ \dots \ Y_i \ \dots \ Y_n] \quad (7b)$$

$$Y_i = \sum_{j=1, j \neq i}^n \{ (K_i \odot W_j + K_j \odot W_i) \odot Q_j \} \quad (7c)$$

$$K_i = i^{th} \text{columnof}[K], W_j = j^{th} \text{columnof}[W], Q_j = (q_j) \cdot \text{ones}(m, 1) \quad (7d)$$

In summary:

$$[J(K, W, Q)]_{m \times n} = \left( 2(K \odot W) \cdot \text{Diag}(Q) + \left[ \dots \sum_{j \neq i}^n \{ (K_i \odot W_j + K_j \odot W_i) \odot Q_j \} \dots \right] \right) \quad (7e)$$

The proof of Lemmas 1 & 2 is provided in [47]. The objective function of the market with equation and inequality constraints is given in (8). The answer to the optimization problem includes unit output, line flow, and unit price.

$$\begin{aligned} & \text{Min}_{P_G, P_L} \sum_{i=1}^{N_g} \left( a_i \cdot P_{G_i} + \frac{1}{2} b_i \cdot P_{G_i}^2 \right) \\ & M_G \cdot P_G - M_L \cdot P_L = P_D : LMP \\ & P_l - (\theta_a - \theta_b) x_{ab}^{-1} = 0 \\ & P_G^{Min} \leq P_G \leq P_G^{Max} \\ & -P_L^{Max} \leq P_L \leq P_L^{Max} \\ & S.t. : \\ & P_L = [P_1 \ \dots \ P_l \ \dots \ P_{Nl}]^T \end{aligned} \quad (8)$$

In (8), the objective function of the problem, the equation constraints include  $a_i$ ,  $b_i$ -objective function coefficients,  $M_G$  is the cross-matrix of generators and buses,  $P_G$  is the generating power of generators,  $M_L$  is the cross-matrix of lines and buses, and  $P_L$  is the flow matrix of network

lines.  $P_l$  is the flow of line  $l$  in the network, which is one of the elements of the matrix  $P_L$ .  $P_D$  is the network load matrix, and  $x_{ab}^{-1}$  is the line admittance matrix between the two buses  $a$  and  $b$ . Also,  $\theta_a$ ,  $\theta_b$  are the voltage angle of buses  $a$  and  $b$ . By executing the market-clearing problem on the test network, the output of the problem, including line flow, LMP of buses, and production power of units, is obtained. According to the results, generators that produce power at their maximum capacity are known as cheap units, while those that produce power at their minimum capacity are known as expensive units. In addition, generators that produce power between their minimum and maximum capacity are known as marginal units.

### 3. Proposed formulation

The main purpose of this article is to provide a model to evaluate the impact of GenCos' behaviours on the revenue of TransCos. For this purpose, a structure for the analytical and accurate calculation of the impact of each user on the LMP of buses and line flows, CR changes, and TCR changes using the structural analysis method is proposed. In this regard, the Lagrange method and KKT conditions are used. The general diagram of the proposed method can be seen in Fig. 1.

As shown in Fig. 1, the general steps of the proposed method are as follows:

- 1 The necessary parameters for the market settlement problem are given as input. Using the QP problem solver, the line flow and LMP of each bus are obtained.
- 2 The variables obtained from the first step are applied as input to Lemma1. In this method, we analyze the two variables, flow and LMP, and the contribution of each variable to flow and LMP is obtained.
- 3 In Lemma 2, the amount of CR change in lines is obtained in exchange for changes in variables. By using this method, the operator can have a good view of network users' collusion with transmission companies to increase income.
- 4 After calculating the contribution of each variable to the CR changes of each line, the contribution of each variable to the CR of each network zone is obtained.
- 5 In this step, after calculating the CR of each zone in the previous step, the contribution of each variable to the TCR of the network is determined.
- 6 In the final stage, the proposed indicators evaluating the impact of factors on the CR changes of network lines and zones are calculated.

The formulation of the proposed method is presented in several sections, which include the calculation of coefficients affecting LMP and flow, CR of lines, CR of zones (user collusion with transmission companies), and TCR.

#### 3.1. LMP and flow decomposition based on Lemma 1

In this section, the contribution of effective parameters to the LMP of buses and line flows is calculated. The parameters include five general

categories of marginal units, cheap units, expensive units, loads, and congested lines that have effects on LMP and flow. The effect of each parameter will be examined below. In (9),  $C_{flow}$  and  $C_{LMP}$  are parameters that are obtained by decomposing the two matrices, flow and LMP.

$$[X] = [C] \cdot [O] \Rightarrow \begin{bmatrix} Flow \\ LMP \end{bmatrix} = \begin{bmatrix} C_{flow} \\ C_{LMP} \end{bmatrix} \cdot \begin{bmatrix} a_{mrg} \\ P_G^{Min} \\ P_G^{Max} \\ P_D \\ P_L^{Max} \end{bmatrix} \quad (9)$$

As can be seen in (10), from the analysis of the LMP and Flow matrices, several coefficients are obtained, each of which shows the separate effect of the related parameter on the variable in question. These coefficients are related to the bid price of marginal units ( $c_{flow}^{mu}$ ,  $c_{LMP}^{mu}$ ), expensive units ( $c_{flow}^{eu}$ ,  $c_{LMP}^{eu}$ ), cheap units ( $c_{flow}^{cu}$ ,  $c_{LMP}^{cu}$ ), load power consumption ( $c_{flow}^l$ ,  $c_{LMP}^l$ ) and congested network lines ( $c_{flow}^{cl}$ ,  $c_{LMP}^{cl}$ ).

$$\begin{bmatrix} Flow \\ LMP \end{bmatrix} = \begin{bmatrix} c_{flow}^{mu} & c_{flow}^{eu} & c_{flow}^{cu} & c_{flow}^l & c_{flow}^{cl} \\ c_{LMP}^{mu} & c_{LMP}^{eu} & c_{LMP}^{cu} & c_{LMP}^l & c_{LMP}^{cl} \end{bmatrix} \cdot \begin{bmatrix} a_{mrg} \\ P_G^{Min} \\ P_G^{Max} \\ P_D \\ P_L^{Max} \end{bmatrix} \quad (10)$$

Eq. (10) shows the effect of marginal units, expensive units, cheap units, loads, and congested lines on line flows and the LMP of buses. These coefficients provide a good opportunity for new and accurate analyses that will be used in the next section. According to (9)-(10), the coefficients of factors affecting the LMP of buses and line flows are determined. Next, to determine the effect of each variable on the CR of network lines, the coefficients of each variable based on Lemma 2 will be extracted. Therefore, the flow matrix is defined as follows:

$$Flow_{N_l \times 1} = [C_{flow}]_{N_l \times N_c} [O]_{N_c \times 1} \quad (11)$$

Where the matrix  $O$  and  $C_{flow}$  are equal to:

$$C_{flow} = \begin{bmatrix} c_{flow}^{mu} & c_{flow}^{eu} & c_{flow}^{cu} & c_{flow}^l & c_{flow}^{cl} \end{bmatrix} \quad (12a)$$

$$O^T = \begin{bmatrix} a_{mrg} & P_G^{Min} & P_G^{Max} & P_D & P_L^{Max} \end{bmatrix} \quad (12b)$$

And  $N_c$  is equal to:

$$N_c = (N_{gmrg} + N_{gmin} + N_{gmax}) + N_b + N_{lcong} = N_g + N_b + N_{lcong} \quad (13)$$

### 3.2. Congestion rent decomposition based on Lemma 2

In this section, with the help of Lemma 2, the nonlinear CR citizen-ship of Flow and LMP will be linearized. For this purpose, the Hadamard definition between two matrices is used, which is denoted by  $A \odot B$  and is related to the multiplication of two matrices according to (14) [64,65]:

$$(A \odot B)_{ij} = a_{ij} \cdot b_{ij} \quad (14)$$

To calculate the CR, you need the price difference between the buses, which is indicated by FLMP. It should be noted that the difference in LMP of buses is the presence of congested lines in the network. FLMP can be calculated via (15):

$$FLMP_{N_l \times 1} = ([M_L]^T)_{N_l \times N_b} \times [LMP]_{N_b \times 1} \quad (15)$$

Where  $M_L$  is the intersection matrix of network lines and buses. In other words, this matrix represents the position of each line within its desired bus. The CR of each line is the product of the flow of that line multiplied by the difference between the prices of double-line buses. So, the CR of each line is calculated based on (16):

$$CR_{N_l \times 1} = [Flow]_{N_l \times 1} \odot [FLMP]_{N_l \times 1} \quad (16)$$

Using (16), CR is obtained for each line in the system. Also, the evaluation of the effect of the change in the variables of matrix  $O$  in (13b) on the CR matrix is investigated. A CR change of network lines is defined by  $dCR$ . Eq. (17) expresses the relationship between CR changes in lines and variable changes.

$$dCR_{N_l \times 1} = J(K, Y, O) \cdot [dO] = [I] \cdot [dO] \Rightarrow dCR_{N_l \times 1} = \begin{bmatrix} I_{ji}^{mu} & I_{ji}^{eu} & I_{ji}^{cu} & I_i^l & I_i^{cl} \end{bmatrix} \cdot \begin{bmatrix} da_{mrg} \\ dP_G^{Min} \\ dP_G^{Max} \\ dP_D \\ dP_L^{Max} \end{bmatrix} \quad (17)$$

The components of matrix  $dO$  are as follows:  $da_{mrg}$  is the bid changes of marginal units;  $dP_G^{Min}$  is the alterations in the capacity of expensive units;  $dP_G^{Max}$  is the changes in the minimum generation capacity of cheap units;  $dP_D$  is the changes in load consumption; and  $dP_L^{Max}$  is the alterations in the capacity of congested network lines. Thus, matrix  $I$  in (17) is a matrix of the coefficients of variables affecting the CR changes of network lines, which can be positive or negative. In other words, changing a variable can positively or negatively alter the CR lines. In a transmission network, network ownership can be the responsibility of several TransCos. Therefore, the network can include several zones, the ownership of which is the responsibility of the relevant TransCo. The network studied in this paper includes several zones, which are further defined by the coefficients affecting the CR change of each zone.

### 3.3. Decomposition of Transco's CR

From the owner's point of view, the sum of the CR of lines in its own area is important. In other words, the profit and loss of the owner of the zone  $i$  are important to the owner of that zone. Therefore,  $CR_{zonei}$  is defined by this description. As defined in (18), from the sum of the CR of lines in zone  $i$ ,  $CR_{zonei}$  is calculated.

$$CR_{zonei} = \sum_{l \in zonei} CR_l \quad (18)$$

After calculating  $CR_{zonei}$ , the coefficients affecting it, which are defined by  $dCR_{zonei}$ , are calculated. In other words, these coefficients determine how much the CR of each zone changes for each variable in the network (collusion of users with transmission companies to increase income). The amount of CR changes of zone  $i$  is calculated according to (19a)-(19b).

$$dCR_i = J_i(K, Y, O) \cdot dO = \sum_{i=1}^{N_c} J_{i,i} \cdot dO_i \quad (19a)$$

$$CR_{zonei} = \sum_{l \in zonei} CR_l \Rightarrow dCR_{zonei} = \sum_{l \in zonei} dCR_l \quad (19b)$$

By placing relation (19a) in relation (19b), we have:



**Table 2**  
Proposed indices.

Index	Definition
<b>EMUCL</b>	The effect of marginal unit on CR of line
<b>EAMUCL</b>	The effect of all marginal units on CR of line
<b>SMUCL</b>	The share of marginal unit on CR of line
<b>EMUCZ</b>	The effect of marginal unit on CR of zone
<b>EAMUCZ</b>	The effect of all marginal units on CR of zone
<b>SMUCZ</b>	The share of marginal unit on CR of zone

$$\begin{aligned}
 (19a) \& (19b) \Rightarrow dCR_{zone_i} &= \sum_{l \in zone_i} \sum_{k=1}^{N_c} J_{l,k} \cdot dO_k \\
 &= \sum_k \sum_{l \in zone_i} J_{l,k} \cdot dO_k \Rightarrow dCR_{zone_i} = [W_{zone_i}]_{1 \times N_c} \cdot [dO]_{N_c \times 1} \\
 [W_{zone_i}]_{1 \times N_c} &= [1 \quad 1 \quad \dots \quad 1]_{1 \times N_{l_i}} \times \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_{N_{l_i}} \end{bmatrix}_{N_{l_i} \times N_c} \quad (20) \\
 J_l &= [j_{l,1} \quad j_{l,2} \quad \dots \quad j_{l,N_c}]_{1 \times N_c}
 \end{aligned}$$

Where matrix  $W_{zone_i}$  is the matrix of coefficients affected by the CR changes in zone  $i$ .  $N_{l_i}$  is the number of lines in zone  $i$ . Matrix  $J_l$  is the coefficient affected by the CR change of line  $l$ . Therefore, the effect of each variable on  $dCR_{zone_i}$  can be calculated as a number in (20). The upstream operator of TransCos is the operator of the entire network. From the point of view of the whole network operator, all the variables that affect the profit and loss of the network are important. Therefore, the coefficients affecting the TCR changes of the network are defined.

### 3.4. TCR decomposition

The sum of the CR of zones is required to calculate the network TCR. Therefore, by calculating the sum of the CR of each zone, the network TCR is obtained according to (21):

$$TCR = \sum_i CR_{zone_i} = \sum_i \sum_{l \in zone_i} CR_l \quad (21)$$

The effect of each variable on the TCR of the network is obtained from the sum of the effects of the same variable on all zones.

$$\begin{aligned}
 dTCR &= \sum_i dCR_{zone_i} \Rightarrow dTCR = [W]_{1 \times N_c} \cdot [dO]_{N_c \times 1} \\
 W_{1 \times N_c} &= [1 \quad 1 \quad \dots \quad 1]_{1 \times N_{zone}} \times \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_{N_{zone}} \end{bmatrix}_{N_{zone} \times N_c} \quad (22)
 \end{aligned}$$

The matrix  $W$  is the matrix of coefficients of variables affecting TCR.  $dTCR$  is a scalar and indicates the change in TCR of the network, while  $N_{zone}$  is the number of zones in the network. According to the modeling based on the proposed method and the developed mathematics, the effect of all variables on the CR change of lines and zones has been obtained. To facilitate inference and a general conclusion, six different indicators are suggested below.

### 3.5. Proposed indices

In this section, in order to facilitate the inference and general conclusion, the indices of the contribution of marginal units on the CR corresponding to each CR of the line related to the TransCo of the owner of each zone are expressed. In other words, these indices indicate how

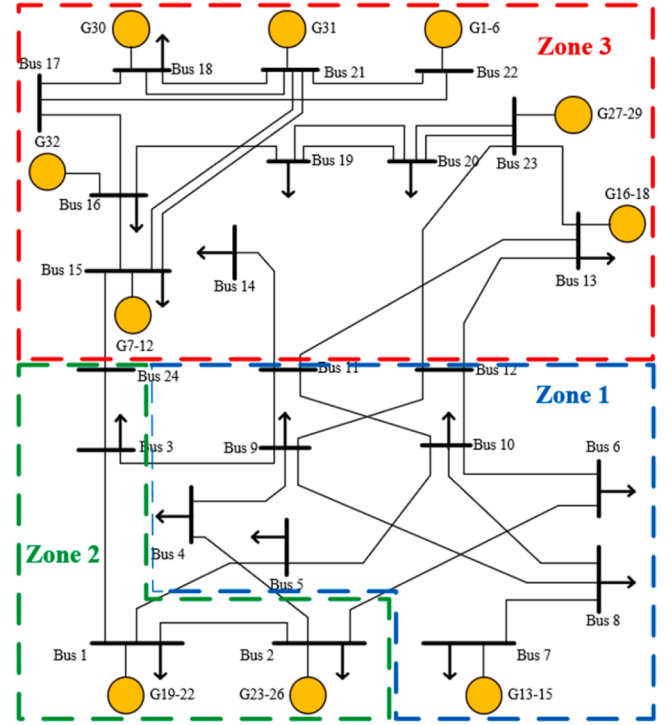


Fig. 2. IEEE standard 24-bus network.

much each unit affects the CR of each network line and the CR of each zone. These indices are introduced in Table 2:

According to Table 2, the **EMUCL**, **EAMUCL**, and **SMUCL** indices are related to the effect of marginal units on the CR of network lines, and the **EMUCZ**, **EAMUCZ**, and **SMUCZ** indices address the effect of marginal units on the CR of network zones, which will be defined in the following. The  $EMUCL_j^i$  is an index that evaluates the effect of the marginal unit  $j$  bid on the CR of line  $i$  in the network according to (23), so that it shows which unit has the greatest impact on the CR of each network line.

$$EMUCL_j^i = \frac{|c_{j,i}^{mu} \times a_j|}{CR_i} \quad (23)$$

Where  $c_{j,i}^{mu}$  is the coefficient of the marginal unit  $j$  on the line  $i$  and  $a_j$  is the bid price of the unit  $j$ . Also, by extending (23) to all marginal units, the  $EAMUCL_{mrg}^i$  index is defined as follows:

$$EAMUCL_{mrg}^i = \sum_{j=1}^{N_{mrg}} EMUCL_j^i = \frac{\sum_{j=1}^{N_{mrg}} |c_{j,i}^{mu} \times a_j|}{CR_i} \quad (24)$$

As a result, the contribution of the marginal unit  $j$  on the CR of line  $i$  in the network as a percentage, denoted by  $SMUCL_j^i$ , can be obtained according to (25).

$$SMUCL_j^i = \frac{EMUCL_j^i}{EAMUCL_{mrg}^i} \times 100 = \frac{|c_{j,i}^{mu} \times a_j|}{\sum_{j=1}^{N_{mrg}} |c_{j,i}^{mu} \times a_j|} \times 100 \quad (25)$$

After calculating the CR-line indices, the bid effect of each marginal unit on the CR of each zone can be obtained. According to (26), we have:

$$EMUCZ_j^i = \frac{|z_j^i \times a_j|}{CR_{zone_i}} \quad (26)$$

$EMUCZ_j^i$  is the index evaluating the effect of bid unit  $j$  on the CR of zone  $i$ , and  $z_j^i$  is the coefficient of marginal unit  $j$  on zone  $i$ . Therefore, by

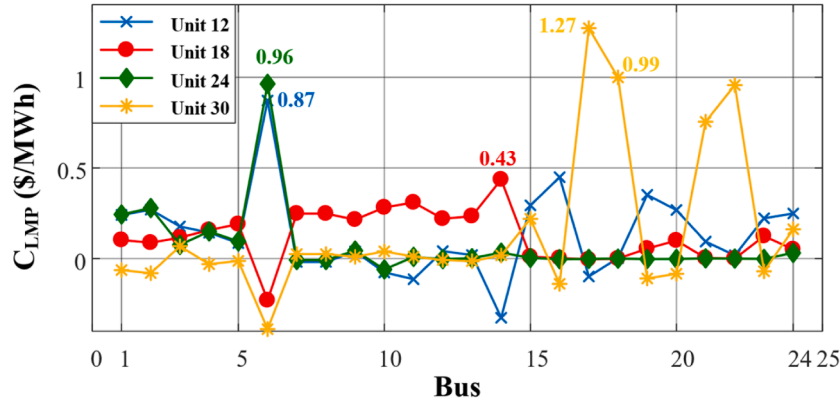


Fig. 3. Impact of selected marginal units on LMP network buses.

extending (26) to all marginal units, the  $EAMUCZ_{mrg}^i$  index, which indicates the effect of all units on the CR of zone  $i$ , is proposed as follows:

$$EAMUCZ_{mrg}^i = \sum_{j=1}^{N_{g_{mrg}}} EMUCZ_j^i = \frac{\sum_{j=1}^{N_{g_{mrg}}} |z_{ij}^i \times a_j|}{CR_{zone_i}} \quad (27)$$

As a result, the contribution of the marginal unit  $j$  (bid) on the CR of zone  $i$  is given as a percentage according to (28).

$$SMUCZ_j^i = \frac{EMUCZ_j^i}{EAMUCZ_{mrg}^i} \times 100 = \frac{|z_{ij}^i \times a_j|}{\sum_{j=1}^{N_{g_{mrg}}} |z_{ij}^i \times a_j|} \times 100 \quad (28)$$

The proposed indicators give an overview of the impact of each unit on the CR of the line and the CR of the TransCo owner of each network zone. In the following, the simulation results of the proposed method are presented and analysed.

#### 4. Simulation result

The test network is the standard 24-bus IEEE network (as shown in Fig. 2). The results of the market clearing, including the generating capacity of generators ( $P_G$ ), the LMP of buses, and line flows, are attached in Table A.1 in Appendix A. The total output power and power consumption are the same due to DC load flow, so there are no losses. In this paper, there are 13 marginal units with numbers 12 to 20, 23, 24, 30, and 32. Also, information about the flow of network lines and their CRs is attached in Table B.1 in Appendix B. Lines whose throughput is equal to the maximum capacity of that line are called congested lines. In this paper, three lines, 10, 23, and 28, are congested.

In general, the results are categorized and presented in five sections as follows:

- 1 Flow & LMP analysis results
- 2 Decomposition of CR of lines
- 3 Decomposition of CR of zones (TransCos)
- 4 TCR decomposition result
- 5 Calculation of the proposed indices

In the following, each section will be presented.

##### 4.1. Flow & LMP analysis results

###### 4.1.1. Impact of marginal units on LMPs

In determining the LMP of each bus, several factors such as bid units, loads, and congested lines are central. The units themselves are divided into three categories: marginal, expensive, and cheap units. For example, Fig. 3 shows the effect of selected marginal units on the LMP of buses. Increasing the bid often leads to a larger LMP, although in some cases the opposite is true. So, increasing the bid by unit 18 or 30 decreases the LMP at bus 6. Meanwhile, unit 30 has the greatest impact on buses 17 and 18 among selected marginal units, with values of 1.27 and 0.99, respectively. It can be said that the greatest impact of this unit is on the adjacent buses. Also, it can be determined which LMP network bus has the greatest impact on each unit.

###### 4.1.2. Line flow analysis

As mentioned, several parameters affect the LMP of buses. These parameters affect the flow of network lines. As the power consumption of the load increases, the line flow naturally changes many times. Fig. 4 shows the effects of loads on network lines. Load 3 has the greatest effect on enhancing the flow of line 7 by a factor of 0.38. In other words, by changing consumption load 3, the flow of line 7 has the maximum positive change. This is because increasing the power consumption of load 6 has the greatest effect on decreasing line flow 7. Also, the most effective load on line 15 is load 9, and the most effective load on line 22

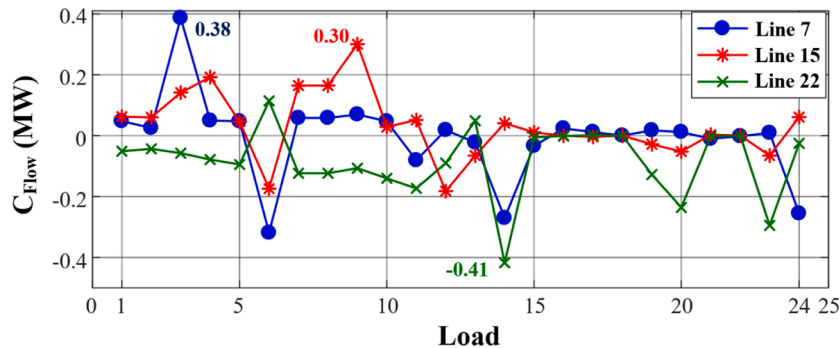


Fig. 4. The effect of network loads on the flow of selected lines.

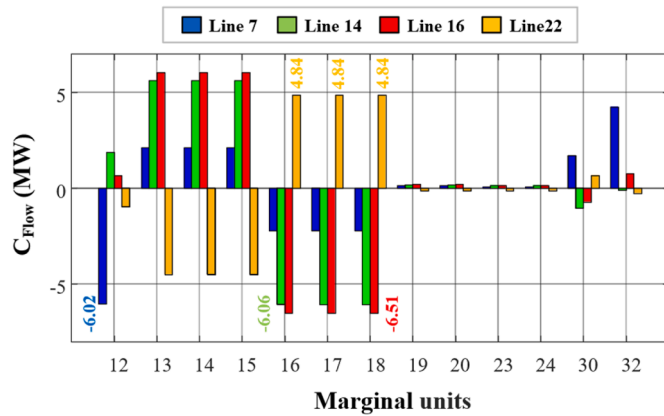


Fig. 5. Impact of marginal units on flow of selected lines.

is load 14. It can be inferred that the maximum effect of the mentioned loads is on the adjacent lines.

Another factor affecting the flow of lines is the marginal units in the network. Each unit, by changing its bid, indirectly creates different effects on the flow of each network line, and the proposed method according to Lemma 1 can accurately determine this effect. In Fig. 5, the effect of marginal units on selected lines 7, 14, 16, and 22 can be seen. Different inferences can be drawn from Fig. 5. For instance, elevating the bid from 13 to 15 units has a positive effect on the flow of lines 7, 14, and 16, but a negative effect on line 22. From another point of view, the most positive effect on line 22 among the marginal units belongs to units 16 to 18, with a coefficient of 4.84. By comparing this result with the 24-bus network, it can be inferred that the greatest effect of 16 to 18 units is on the line connected to the bus of these units. Also, these units have the most negative impact on lines 14 and 16, but unit 12 has the most negative impact on the flow of line 7.

In general, considering the effect of all variables, the contribution of each variable to line flow (in terms of MW) is shown in Fig. 6, where the factors affecting the flow of each line and the amount of flow of each line are plotted. As shown in Fig. 6, changes in power consumption often have a significant effect on the flow of lines, so they have a positive effect on most lines. Thus, it increases the flow of many lines, and on some lines, it has a negative effect, so that the increased load has a negative effect on the flow of these lines. Cheap units produced at their

maximum capacity are called “cheap units”. The effect of these units is on the flow of more positive lines because, by enhancing the generation power of network lines, they are more congested and increase the transmission flow of lines. The effect of congested lines has a significant contribution, and congestion in some lines increases and, in other lines, decreases flow.

#### 4.2. Decomposition of CR of lines

Various parameters influence the CR change of network lines. These variables include bid units, power consumption of network loads, and congested lines. According to (18), the effect of each variable on the CR change of each line can be calculated. In the following, the effect of each change of each parameter on the CR change of each line will be investigated.

##### 4.2.1. The effect of loads on line CR changes

From the analysis of the CR coefficient matrix, we can understand the effect of each load on the network on the CR of lines. Increasing the load on some lines elevates the CR of some lines, and on some lines, it reduces the CR of the line. The effect of loads 10 and 15 on the CR change of grid even lines is shown in Fig. 7. The greatest effect of load 10 between the even lines of the network is on line 16 and then on line 4 of the network.

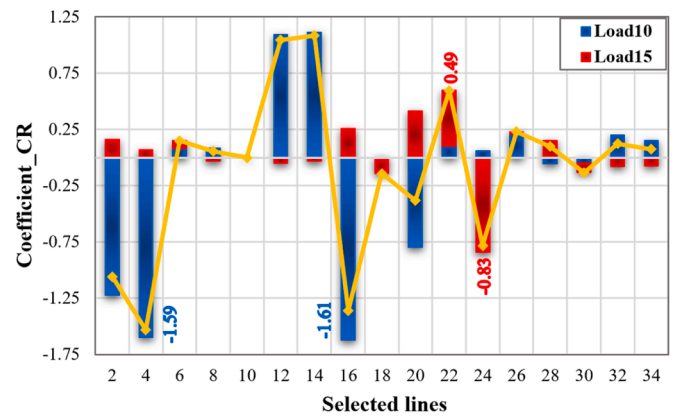


Fig. 7. The effect of selected loads on CR changes of selected lines.

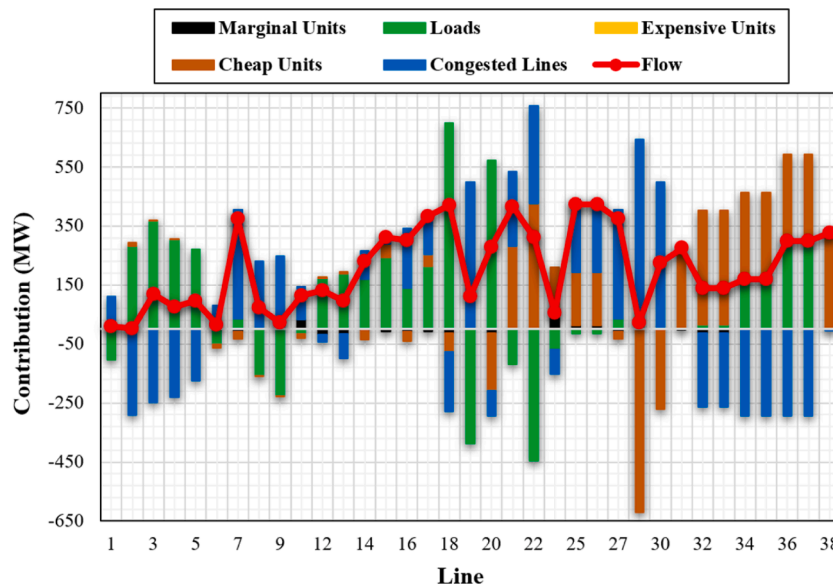


Fig. 6. Contribution of variables on Flow (MW).



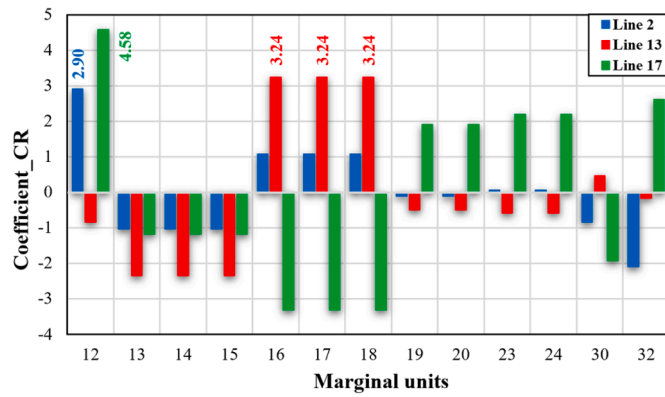


Fig. 8. The Coefficient of marginal units on CR of selected lines.

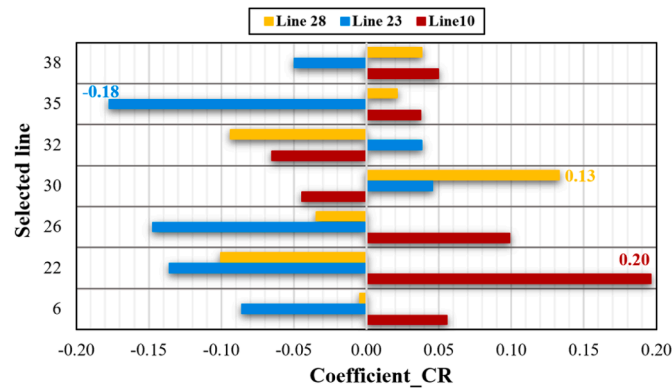


Fig. 9. The Coefficient of congested lines on CR of selected lines.

In other words, increasing the power consumption of load 10 reduces the CR of lines 4 and 16. Also, load 15 has the greatest effect on line 24 and then on line 22 between even lines of the network. Comparing Fig. 7 with the 24-bus network, it can be concluded that the greatest effect of these two loads is on the lines connected to them.

#### 4.2.2. The effect of marginal units on line CR changes

Normally, changing the bid of marginal units alters the CR of network lines. These changes are small in some lines and large in others. Knowing this can be helpful for transferring business owners. The increase of the marginal units (bid) on the selected lines 2, 13, and 17 of the network is shown in Fig. 8. These units have different effects on the three-line CR change. According to Fig. 8, among the marginal units, unit 12 has the greatest effect on the CR change of lines 2, 17. The effect of this unit on line 2 is 2.90, and on line 17 is 4.58. Also, units 16 to 18 have the greatest effect among the selected lines on the CR change of line 13, with a value of 3.24. Furthermore, increasing the bid from 13 to 15 units has the same effect on the CR change of lines 2, 13, and 17 and reduces congestion on these lines. On the other hand, it can be inferred that the greatest impact on the CR change of line 13 belongs to units 13–18, which can be seen from the comparison with the network structure that these units are the closest units to the line.

#### 4.2.3. The effect of congested lines on line CR changes

The effect of each congested line on the CR of other lines can be derived from matrix analysis. Lines 10, 23, and 28 are congested in the network. Fig. 9 shows the effect of congested lines on the CR change of the selected lines 6, 22, 26, 30, 32, 35, and 38. It shows that increasing the capacity of congested line 10 can positively affect the CR change of lines 6, 22, 26, 35, and 38. This line also has a negative effect on the CR change of lines 30 and 32. Similarly, the effects of congested lines 23 and

Table 3

. Lines located in each zone.

Zone	Line
1	5,8,9,10,11,12,13,14,15,16,17,20
2	1,2,3,4,6,7,27
3	18,19,21,22,23,24,25,26,28,29,30,31,32,33,34,35,36,37,38

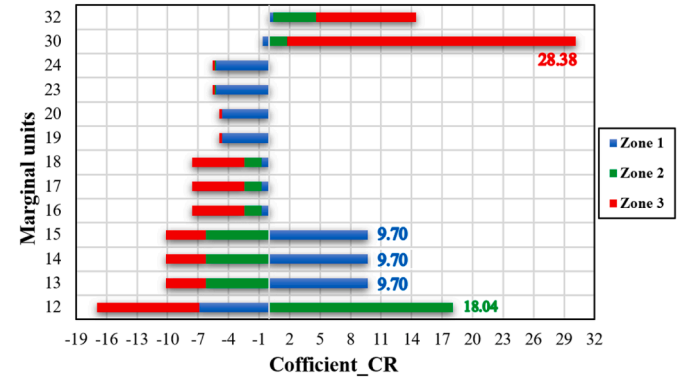


Fig. 10. Impact of marginal units on CR change of network zones.

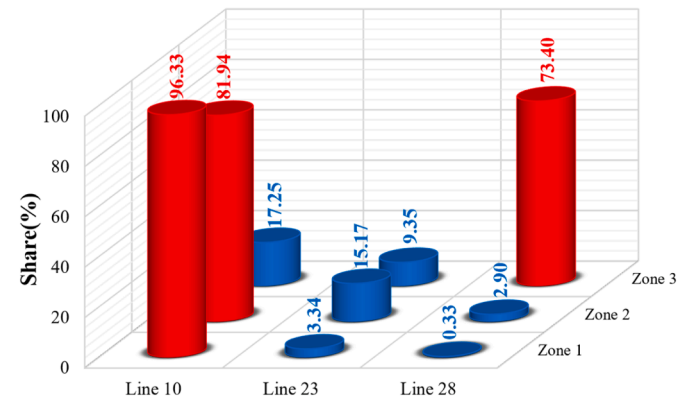


Fig. 11. The share of congested lines on zones CR change.

28 can be seen on other lines. Notably, congested line 23 has a negative effect on the CR changes of most lines, and also, line 10 has a positive effect on the CR changes of most other lines.

#### 4.3. Decomposition of CR of zones (TransCos)

It is assumed that the network has three zones, and the revenue of TransCo, the owner of each zone, is obtained based on the sum of the CR of the lines in each zone. The network zoning is shown in Fig. 2, and the line numbers of each zone are demonstrated in Table 3.

##### 4.3.1. The effect of marginal units on the CR changes of zones

The results of evaluating the collusion of marginal units on the CR of each network zone are presented. In Fig. 10, the largest collusion of CR change in zone 1 is related to the marginal units 13 to 15, which have a coefficient of 9.7 and are located in zone 1. In fact, the marginal units located in this zone have the greatest collusion on the CR changes in their zone. Similarly, the highest collusion of CR change in zone 2 is related to unit 12 with a coefficient of 18.04, which is located in zone 3. It can be seen that the marginal unit located in zone 3 has the greatest collusion on the CR change of zone 2 among the marginal units. Also, the largest collusion to the CR change related to zone 3 is related to unit 30, which is located in zone 3. Therefore, units can have the greatest impact

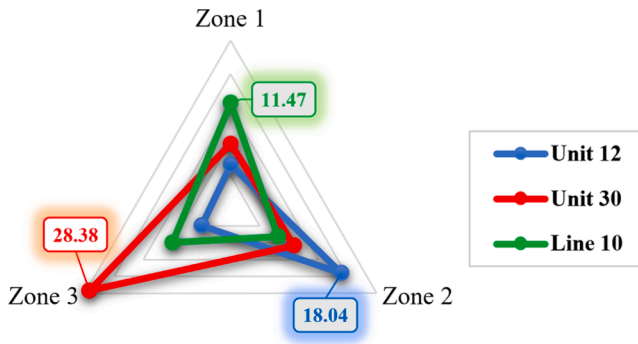


Fig. 12. The most influential elements on each CR change of each zone.

in one zone or another.

#### 4.3.2. The effect of congested lines on CR changes of zones

Congested lines, such as marginal units and network loads, have an impact on each zone's CR change. As shown in Fig. 11, these lines have a different effect on the change of CR in zones. For example, line 10 in zone 1 has a significant contribution to the increase of CR in zone 1, so the other two lines do not have many contributions to changing the CR of zone 1. In addition, line 10 contributes 81.94 percent of the change in CR of zone 2, which is higher than the other lines. Therefore, it can be inferred that among the congested lines, there is a possibility that a line is not in the desired zone but has the greatest effect on the CR change in

that zone. From another point of view, line 28, which does not have many contributions to changing the CR of zones 1 and 2, has the highest contribution to altering the CR of zone 3 with 73.40 percent.

#### 4.3.3. The most effective parameters on CR of zones

Fig. 12 shows the parameter that has the largest collusion to the CR changes in each zone. In zone 1, the variable that has the greatest effect on all CR variables (marginal units, cheap units, expensive units, loads, congested lines) in zone 1 is congested line 10, with a value of 11.47, which is located in zone 1. The variable that has the largest collusion to CR changes in zone 2 is unit 12, with a value of 18.04, which is located in zone 3. That is, one variable in another zone has the largest contribution to CR changes in another zone. Also, the variable that has the largest contribution to CR changes in zone 3 is unit 30, with a value of 28.38, which is located in zone 3. Therefore, in some cases, the variable that greatly introduces alterations in the CR of a zone is related to the variable of the same zone, and in some cases, it is related to the variable of another zone in the network.

From the sum of the CR changes in each of the three zones, the network TCR changes are obtained, which will be discussed below.

#### 4.4. TCR decomposition result

In this section, the share of each parameter in the TCR of the network is determined and evaluated. From the analysis of TCR, the matrix  $W$  is obtained, which specifies the contribution of each variable to the change in TCR. According to Fig. 13, the marginal units with 74.38% have the greatest effect on the CR change of the whole network. The most

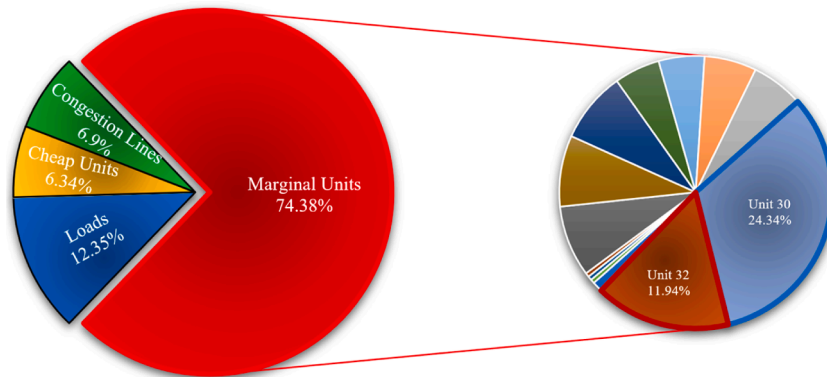


Fig. 13. The effect of main components of structural analysis on TCR change.

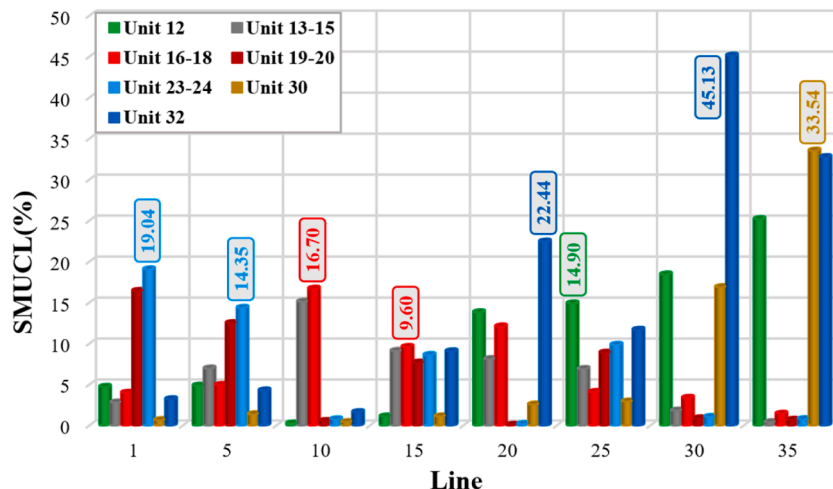


Fig. 14. The effect of marginal units on selected lines.

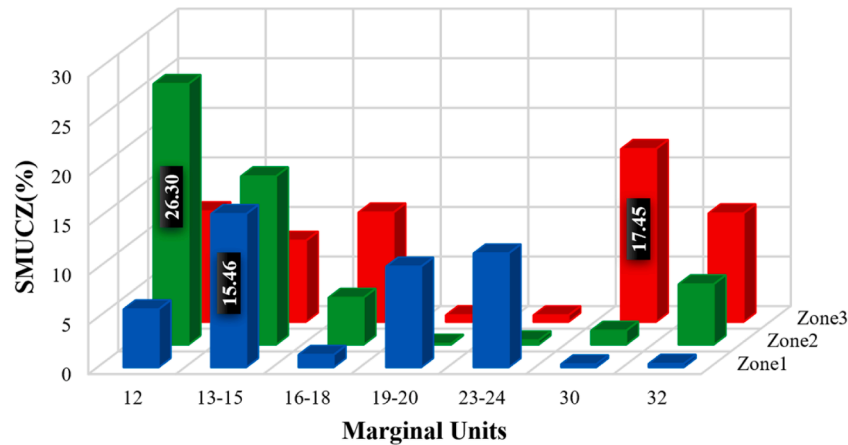


Fig. 15. The effect of marginal units on each zone.

influential variable on TCR change is unit 30 in bus 18, which has an effect of 24.34%. After that, unit 32 is on bus 16, which has an effect of 11.94%. The effects of other units can be observed in Fig. 13. Other units have smaller impacts on TCR. Also, the contribution of loads, cheap units, and congested lines can be seen. The effect of expensive units on their total congestion cost has been neglected due to their small contribution. In the following, the proposed indicators are calculated and analysed to evaluate the effect of the unit's bid on CR lines and network zones.

#### 4.5. Calculation of the proposed indices

Based on the coefficients obtained from the proposed decomposition method, the values of all six proposed indices have been calculated. To avoid exaggeration and focus more on the concept, only the results of the two most important indices, *SMUCL* and *SMUCZ*, are presented. According to (26) and (29), the *SMUCL* and *SMUCZ* indices to determine the effect of the marginal unit  $j$  on the line  $i$  can be seen in Figs. 14 and 15.

Fig. 14 shows the effects of each unit on the selected lines. For instance, unit 32 has the highest impact, on line 30 with 45.13%. Also, this unit has the highest impact, with 22.44% on line 20, but has a small impact on lines 1, 5, and 10. Viewed differently, on lines 1 and 5, units 23 and 24 have the greatest impact, so a significant impact of these units is not detected on lines 10, 20, 30, and 35. Marginal units 16 to 18 have the greatest impact on lines 10 and 15. Also, lines 25 and 35 have the most changes due to marginal units 12 and 30, respectively.

In Fig. 15, the indices of the effect of marginal units on the CR of each zone can be seen. Importantly, the 13 to 15 units located in zone 1 have the greatest effect, with a value of 15.46% on the CR change of zone 1. Similarly, unit 12, which is located in zone 3, has the greatest impact, with a value of 26.30% of the CR change in zone 2. From another point of view, unit 12 has the most impact on zone 2 among zones 1 to 3. Finally, unit 30, which is located in zone 3, has the greatest impact, with a value of 17.45% on the CR change of its zone. This unit has the largest impact on zone 3 between zones 1 to 3. In general, it can be inferred that the most influential variables on the CR change in one zone are not the variables in the same zone and may be due to variables in other zones.

## 5. Conclusion

It is important to identify and quantify the contribution of each of the parameters affecting the income of transmission companies from the perspective of network operators. In addition to identifying the potential for deviations in competitive market performance, it will play an effective role in directing network development in the future. In this paper, using the analytical method of structural decomposition, a new

Table A.1

Information of buses.

Bus	$P_G(\text{MW})$	$P_D(\text{MW})$	LMP(\$/MWh)
1	346.42	216	32.76
2	356.6	194	34.63
3	0	360	20.89
4	0	148	28.55
5	0	142	26.14
6	0	272	69.67
7	364.2	250	21.40
8	0	342	21.40
9	0	350	23.57
10	0	390	19.23
11	0	0	25.51
12	0	0	20.59
13	914.34	530	21.65
14	0	388	32.99
15	144.74	634	10.21
16	252.84	200	11.19
17	0	0	3.68
18	600.78	666	5.37
19	0	362	13.51
20	0	256	15.49
21	800	0	6.88
22	600	0	5.63
23	1320	0	16.58
24	0	0	14.29
Total	5700	5700	–

method is proposed to evaluate the contribution of all parameters affecting the income of transmission companies. Also, new indices are proposed to evaluate the contribution of marginal units (bid) to the CR of lines in a zone. In general, among the five variables of marginal units, cheap units, expensive units, loads, and congested lines, marginal units had the greatest impact on TCR network change, which shows the significant effect of the pricing behavior of generation companies on the income of transmission companies. This can be used as a tool to measure the potential for collusion between a transmission company and a generation company. One of the important results obtained from this proposed method is that the most effective change in changing the CR of a TransCo is not only due to factors in the zone of the same TransCo but may also be a factor in other TransCos. The proposed method can be a suitable tool to predict the network status from the network operator's point of view.

#### CRedit authorship contribution statement

**Mohammad Hasan Nikkhah:** Conceptualization, Methodology, Software, Visualization, Writing – original draft. **Mahdi Samadi:**

**Table B.1**  
Information of buses.

Line	From bus	To bus	Flow(MW)	CR(\$/h)
1	1	2	9.38	17.39
2	1	3	1.56	20.66
3	1	5	119.82	792.82
4	2	4	75.38	458.73
5	2	6	97.00	3399.10
6	3	9	15.28	40.23
7	24	3	373.72	2464.20
8	9	4	72.61	361.38
9	10	5	22.17	153.38
10	10	6	175.00	8827.50
11	7	8	113.19	0
12	9	8	132.62	287.30
13	10	8	96.18	208.33
14	11	9	229.76	445.49
15	12	9	310.18	923.62
16	11	10	301.47	1892.20
17	12	10	381.88	519.95
18	13	11	419.23	1615.90
19	14	11	112.00	837.64
20	13	12	277.49	294.49
21	23	12	414.58	1664.40
22	23	13	310.28	1575.20
23	16	14	500.00	10,897
24	16	15	51.34	55.65
25	21	15	422.63	1408.70
26	21	15	422.63	1408.70
27	15	24	373.72	1524.40
28	17	16	500.00	3756.20
29	16	19	22.86	52.99
30	18	17	225.29	380.29
31	22	17	274.70	535.03
32	21	18	140.01	210.62
33	21	18	140.01	210.62
34	20	19	169.56	336.54
35	20	19	169.56	336.54
36	23	20	297.56	322.14
37	23	20	297.56	322.14
38	22	21	325.29	408.39
Total				48,965.86

Supervision, Conceptualization, Methodology, Validation, Writing – review & editing.

### Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

### Appendix A

#### Table A1

### Appendix B

#### Table B1

### References

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