Ejercicios de ITS Tanda 4

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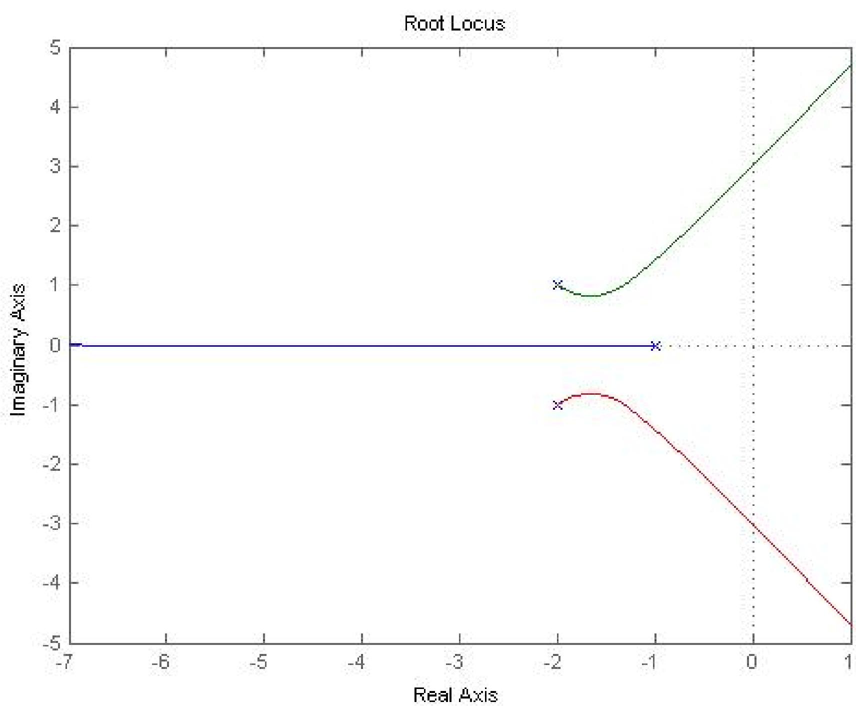
1.- Trazar el lugar de las raíces en función del parámetro K de los sistemas realimentados con función de transferencia en bucle abierto:

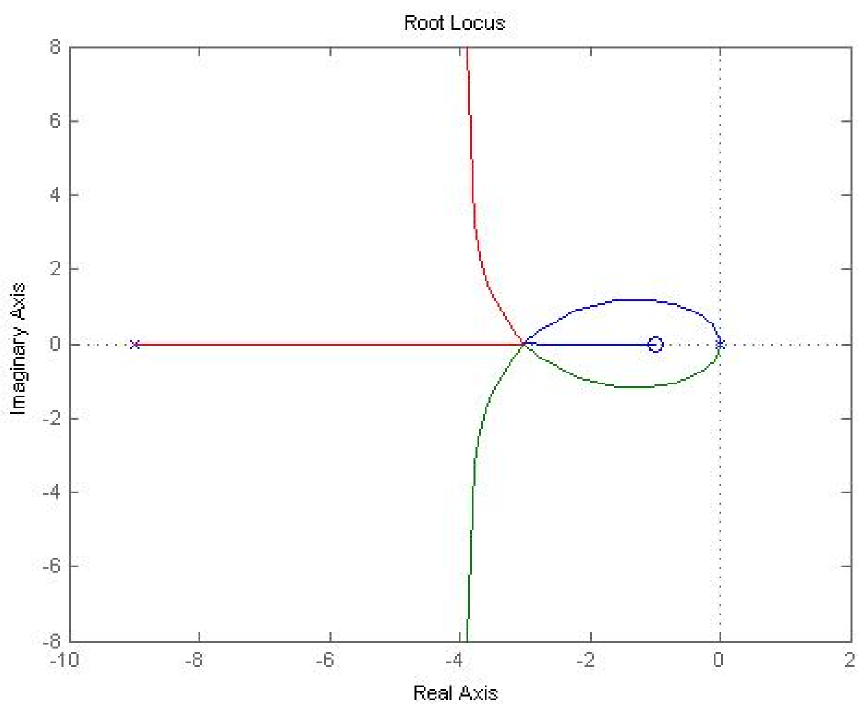
a) $G(s) = \frac{K}{(s+1)*(s+2-j)*(s+2+j)}$

$$polos = [-1, -2 + j, -2 - j]$$
$$ceros = []$$
$$\sigma = \frac{\sum polos - \sum ceros}{np - nz} = \frac{-5}{3}$$
$$\Sigma = [\frac{\pi}{3}, \pi, \frac{5 * \pi}{3}]$$



b) $G(s) = \frac{K*(s+1)}{s^2*(s+9)}$

$$polos = [0, 0, -9]$$
$$ceros = [-1]$$
$$\sigma = \frac{\sum polos - \sum ceros}{np - nz} = \frac{-8}{2} = -4$$
$$\Sigma = [\frac{\pi}{2}, \frac{3*\pi}{2}]$$



c) $G(s) = \frac{s+5}{((s+2)*(s+K)}$

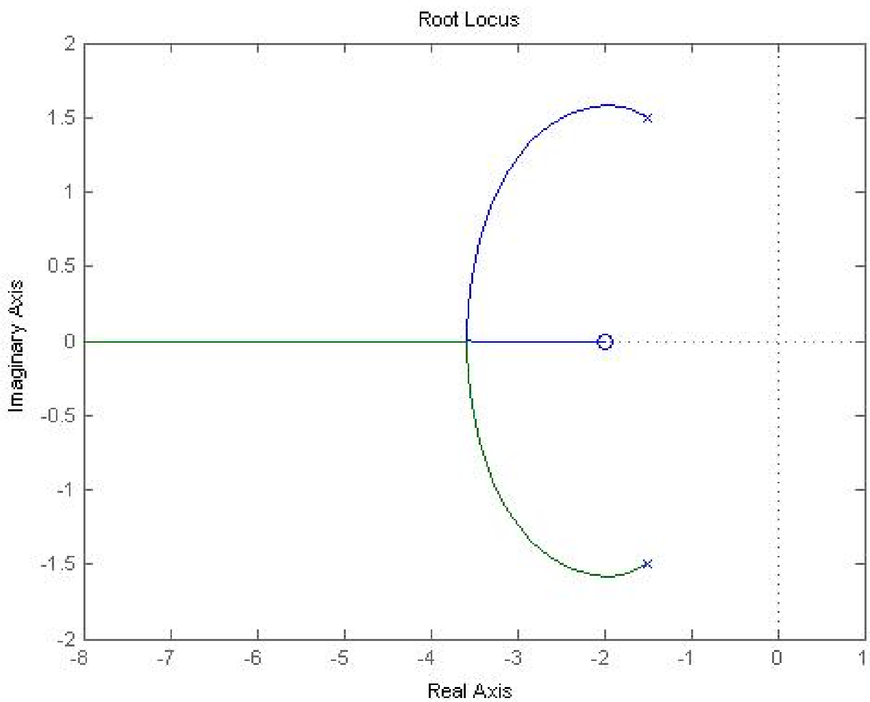
Para poder hallar el lugar de las raíces, debemos sacar K del denominador.

$$1 + K * H(s) = 0 \rightarrow 1 + G(s) = 0 \rightarrow 1 + fracs + 5((s+2)*(s+K) = 0$$
$$s + 5 + ((s+2)*(s+K) = 0 \rightarrow s + 5 + s^2 + s*k + 2*s + 2*K = 0$$
$$K*(s+2) + s^2 + 3*s + 5 = 0 \rightarrow K + \frac{s^2 + 3*s + 5}{s+2} = 0$$
$$1 + \frac{1}{K} * \frac{s^2 + 3*s + 5}{s+2} = 0$$
$$1 + K * \frac{s+2}{s^2 + 3*s + 4} = 0 \rightarrow H(s) = \frac{s+2}{s^2 + 3*s + 4}$$
$$polos = [\frac{-3+3j}{2}, \frac{-3-3j}{2}]$$
$$ceros = [-2]$$

Debido a la situación de los polos y los ceros, hemos de hallar los puntos de convergencia:

$$K = \frac{-s^2 - 3*s - 5}{s+2} \rightarrow \frac{\delta K}{\delta s} = \frac{(-2*s-3)*(s+2) - (-s^2 - 3*s - 5)}{(s+2)^2} = 0$$
$$(-2*s-3)*(s+2) + s^2 + 3*s + 5 = 0 \rightarrow s*2 + 4*s + 1 = 0 \rightarrow [0, -4]$$

Viendo los valores, podemos declarar que el punto de convergencia es aproximadamente -3,73.


Root Locus
Imaginary Axis
Real Axis
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-8
-7
-6
-5
-4
-3
-2
-1
0
1

2.- Analizar la respuesta en régimen permanente de los sistemas del apartado anterior.

a)

$$K_p = \lim_{s \to 0} G(s) * H(s) = \tfrac{1}{5} \to\to e_{ss} = \tfrac{1}{1+K_p} = \tfrac{1}{1+\infty} = 0$$
$$K_v = \lim_{s \to 0} s * G(s) * H(s) = \tfrac{0}{5} = 0 \to\to e_{ss} = \tfrac{A}{K_v} = \tfrac{1}{0} = \infty$$
$$K_a = \lim_{s^2 \to 0} s^2 * G(s) * H(s) = \tfrac{0}{5} = 0 \to\to e_{ss} = \tfrac{A}{K_a} = \tfrac{1}{0} = \infty$$

b)

$$K_p = \lim_{s \to 0} G(s) * H(s) = \tfrac{1}{0} = \infty \to\to e_{ss} = \tfrac{1}{1+K_p} = \tfrac{1}{1+\frac{1}{5}} = \tfrac{5}{6}$$
$$K_v = \lim_{s \to 0} s * G(s) * H(s) = \tfrac{1}{0} = \infty \to\to e_{ss} = \tfrac{A}{K_v} = \tfrac{1}{\infty} = 0$$
$$K_a = \lim_{s^2 \to 0} s^2 * G(s) * H(s) = \tfrac{1}{9} \to\to e_{ss} = \tfrac{A}{K_a} = \tfrac{1}{\frac{1}{9}} = 9$$

c)

$$K_p = \lim_{s \to 0} G(s) * H(s) = \tfrac{2}{5} \to\to e_{ss} = \tfrac{1}{1+K_p} = \tfrac{1}{1+\frac{2}{5}} = \tfrac{5}{7}$$
$$K_v = \lim_{s \to 0} s * G(s) * H(s) = \tfrac{0}{5} = 0 \to\to e_{ss} = \tfrac{A}{K_v} = \tfrac{1}{0} = \infty$$
$$K_a = \lim_{s^2 \to 0} s^2 * G(s) * H(s) = \tfrac{0}{5} \to\to e_{ss} = \tfrac{A}{K_a} = \tfrac{1}{0} = \infty$$