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Design and analysis of hash functions

Coding and Crypto Course

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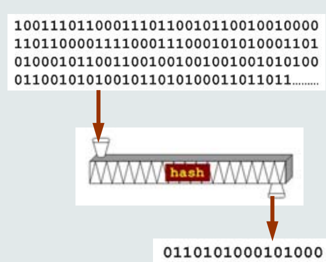
what is a hash function?

- $h : \{0,1\}^* \rightarrow \{0,1\}^n$
  (general: $h : S \rightarrow \{0,1\}^n$ for some set S)
- input: bit string m of arbitrary length
  - length may be 0
  - in practice a very large bound on the length is imposed, such as 2^{64} (≈ 2.1 million TB)
  - input often called the *message*
- output: bit string $h(m)$ of fixed length n
  - e.g. n = 128, 160, 224, 256, 384, 512
  - *compression*
  - output often called *hash value*, *message digest*, *fingerprint*
- $h(m)$ is easy to compute from m
- no secret information, no key

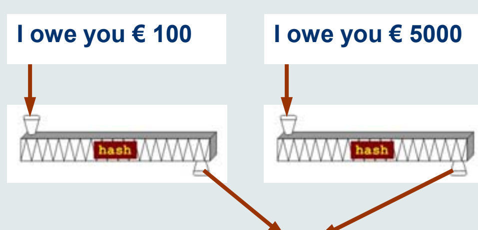


non-cryptographic hash functions

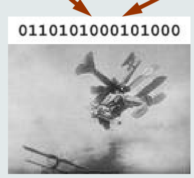
- hash table
  - index on database keys
  - use: efficient storage and lookup of data
- checksum
  - Example: CRC – Cyclic Redundancy Check
    - CRC32 uses polynomial division with remainder
    - initialize:
      - p = 1 0000 0100 1100 0001 0001 1101 1011 0111
      - append 32 zeroes to m
    - repeat until length (counting from first 1-bit) ≤ 32:
      - left-align p to leftmost nonzero bit of m
      - XOR p into m
  - use: error detection
    - but only of unintended errors!
- non-cryptographic
  - extremely fast
  - not secure at all

hash collision

- m_1, m_2 are a *collision* for h if
  $h(m_1) = h(m_2)$ while $m_1 \neq m_2$



different documents



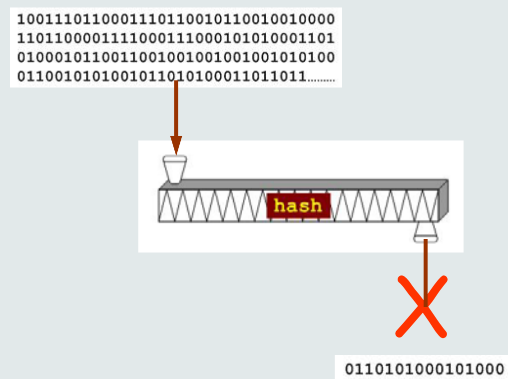
identical hash = collision

- there exist a lot of collisions
  - pigeonhole principle (a.k.a. Schubladensatz)

preimage

- **given h_0, then m is a *preimage* of h_0 if**

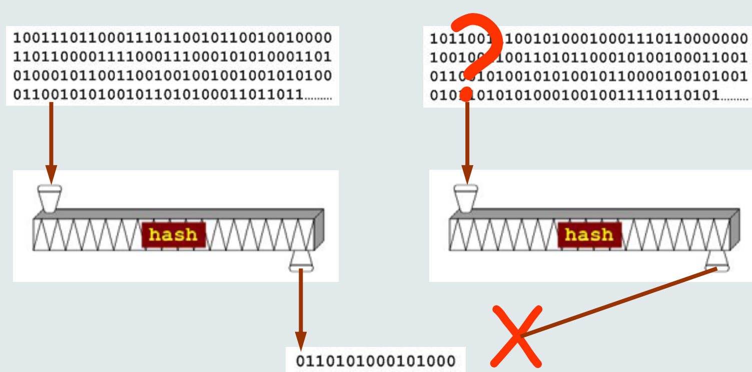
  $h(m) = h_0$



second preimage

- **given m_0, then m is a *second preimage* of m_0 if**

  $h(m) = h(m_0)$ **while** $m \neq m_0$



cryptographic hash function requirements

- *collision resistance*: it should be computationally infeasible to find a collision m_1, m_2 for h
  - i.e. $h(m_1) = h(m_2)$
- *preimage resistance*: given h_0 it should be computationally infeasible to find a preimage m for h_0 under h
  - i.e. $h(m) = h_0$
- *second preimage resistance*: given m_0 it should be computationally infeasible to find a second preimage m for m_0 under h
  - i.e. $h(m) = h(m_0)$
- more formal definitions exist, but we'll keep things practical

other terminology

- *one-way* = preimage + second preimage resistant
  - sometimes only preimage resistant
- *weak collision resistant* = second preimage resistant
- *strong collison resistant* = collision resistant
- OWHF – one-way hash function
  - preimage and second preimage resistant
- CRHF – collision resistant hash function
  - second preimage resistant and collision resistant

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other requirements

- *target collision resistance (TCR)* (Bellare-Rogaway)
  - attacker chooses m_0
  - attacker is given random r
  - attacker not able to compute m such that $h(r,m) = h(r,m_0)$
- is in between (full) collision resistance and second preimage resistance

- *random oracle property*
  - output of a hash function indistinguishable from random bit string

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relations between requirements

- Theorem: If h is collision resistant then it is second preimage resistant
  - Proof: a second preimage is a collision.
- Non-theorem: If h is second preimage resistant then it is preimage resistant
  - Non-proof:

    suppose that for any h_0 one can compute a preimage m. Then, given m_0, one can certainly do that for $h_0 = h(m_0)$.
  - problem: to guarantee that $m \neq m_0$
- in practice:

  collision resistant → second preimage resistant
  second preimage resistant → preimage resistant

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pathologic counterexamples

- if $g : \{0,1\}^* \rightarrow \{0,1\}^n$ is collision resistant, then take

  $h(m) = 1 \,||\, m$ if m has length n,

  $h(m) = 0 \,||\, g(m)$ otherwise,

  then h is collision resistant but not preimage resistant

- the *identity function* $id : \{0,1\}^n \rightarrow \{0,1\}^n$ is second preimage resistant but not preimage resistant

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how are hash functions used?

- asymmetric digital signature
- integrity protection
  - strong checksum
  - for file system integrity (Tripwire) or software downloads
- one-way ‘encryption’
  - for password protection
- MAC – message authentication code
  - symmetric ‘digital signature’
- confirmation of / commitment to knowledge
  - e.g. in hash chain based payment systems (‘hashcash’)
- key derivation
- pseudo-random number generation
- …

trivial (brute force) attacks

- assume: hash function behaves like random function
- preimages and second preimages can be found by random guessing search
  - search space: ≈ n bits, ≈ 2^n hash function calls
- collisions can be found by birthdaying
  - search space: ≈ ½n bits, ≈ $2^{½n}$ hash function calls
- this is a big difference
  - MD5 is a 128 bit hash function
  - (second) preimage random search: ≈ 2^{128} ≈ 3×10^{38} MD5 calls
  - collision birthday search: only ≈ 2^{64} ≈ 2×10^{19} MD5 calls

rainbow table attack

- assume messages are taken from a fixed set
  - e.g. 8 bit printable ASCII
- define a reduction function *red* that transforms a hash value back into some message
- build hash chains: $h_{i+1} = h(red(h_i))$
- for each chain only store e.g. every k^{th} element
- do a one time brute force computation on all possible chains
- storage (the 'rainbow table') reduced by factor k
- to find one preimage only k hash calls required
- time-memory tradeoff
- used for password recovery

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Merkle time-memory tradeoff

- if you have computed 2^t hashes, cost to find a second preimage for one of them is only 2^{n-t}
  - trivial: sort computed hashes and do table lookups

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birthday paradox

- birthday paradox
  given a set of t (≥ 10) elements
  take a sample of size k (drawn with repetition)
  in order to get a probability $\geq$ ½ on a collision
  (i.e. an element drawn at least twice)
  k has to be $> 1.2 \sqrt{t}$
- consequence
  if $F : A \rightarrow B$ is a surjective random function
  and $\#A >> \#B$
  then one can expect a collision after about $\sqrt{(\#B)}$ random function calls

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proof of birthday paradox

- probability that all k elements are distinct is

$$\prod_{i=0}^{k-1}\frac{t-i}{t} = \prod_{i=0}^{k-1}\left(1-\frac{i}{t}\right) \le \prod_{i=0}^{k-1} e^{-\frac{i}{t}} = e^{-\sum_{i=0}^{k-1}\frac{i}{t}} = e^{-\frac{k(k-1)}{2t}}$$

and this is > ½ when $k(k-1) > (2 \log 2)t$

($\approx k^2$) ($\approx 1.4\, t$)

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meaningful birthdaying

- random birthdaying
  - do exhaustive search on ½n bits
  - messages will be 'random'
  - messages will not be 'meaningful'
- Yuval (1979)
  - start with two meaningful messages m_1, m_2 for which you want to find a collision
  - identify ½n independent positions where the messages can be changed at bitlevel without changing the meaning
    - e.g. tab ←→ space, space ←→ newline, etc.
  - do random search on those positions

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implementing birthdaying

- **naïve**
  - store $2^{½n}$ possible messages for m_1 and $2^{½n}$ possible messages for m_2 and check all 2^n pairs
- **less naïve**
  - store $2^{½n}$ possible messages for m_1 and for each possible m_2 check whether its hash is in the list
- **smart: Pollard-ρ with Floyd's cycle finding algorithm**
  - computational complexity still $O(2^{½n})$
  - but only constant small storage required

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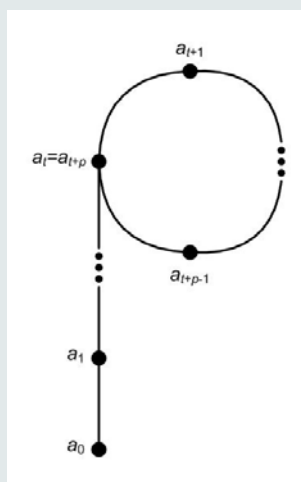
Pollard-ρ and Floyd cycle finding

- **Pollard-ρ**
  - iterate the hash function:
    a_0, $a_1 = h(a_0)$, $a_2 = h(a_1)$, $a_3 = h(a_2)$, …
  - this is ultimately periodic:
    - there are minimal t, p such that $a_{t+p} = a_t$
    - theory of random functions: both t, p are of size $2^{½n}$
- **Floyd's cycle finding algorithm**
  - Floyd: start with (a_1,a_2) and compute (a_2,a_4), (a_3,a_6), (a_4,a_8), …, (a_q,a_{2q}) until $a_{2q} = a_q$; this happens for some $q < t + p$



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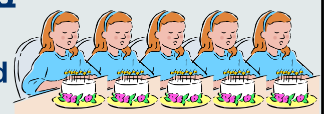
parallel birthdaying



- birthdaying can easily be parallellized
  - Van Oorschot – Wiener 1999
  - kind of time-memory tradeoff
- define *distinguished points* by some condition
  - e.g. the first 16 bits must all be 0
- give all processors random a_0 and let them iterate until a distinguished point a_d is reached
- centrally store pairs (a_0,a_d) until two a_d's collide
  - storage: O(#distinguished points)
- to find the actual collision you only have to recompute the two trails from the two a_0's
- it can be shown that the time needed with m processors is $O(2^{½n}/m)$
  - though 'total cost' remains $O(2^{½n})$

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meet in the middle attack

- assume a hash function design works with intermediate values and allows you to compute backwards halfway
  - given target hash value h_0
  - first half: $IV = h_1(m_1)$
  - second half: $h(m_1||m_2) = h_2(IV,m_2)$ where h_2 is easily invertible in the sense that $IV = h_2^{-1}(h_0,m_2)$ can be computed for any m_2
- then a birthday type attack on (second) preimage resistance is possible
  - birthday for collision $h_1(m_1) = h_2^{-1}(h_0,m_2)$
- this reduces the search space from 2^n to $2^{n/2}$
  - but only for badly designed hash functions
  - note: birthdaying for two functions: iterate them alternatingly

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security parameter

- *security parameter n*: resistant against (brute force / random guessing) attack with search space of size 2^n
  - complexity of an n-bit exhaustive search
  - n-bit *security level*
- nowadays 2^{80} computations deemed impractical
  - security parameter 80 seen as sufficient in most cases
- but 2^{64} computations should be about possible
  - though a.f.a.i.k. nobody has done it yet
  - security parameter 64 now seen as insufficient in most cases
- in the future: security parameter 128 will be required

- for collision resistance hash length should be $2n$ to reach security with parameter n

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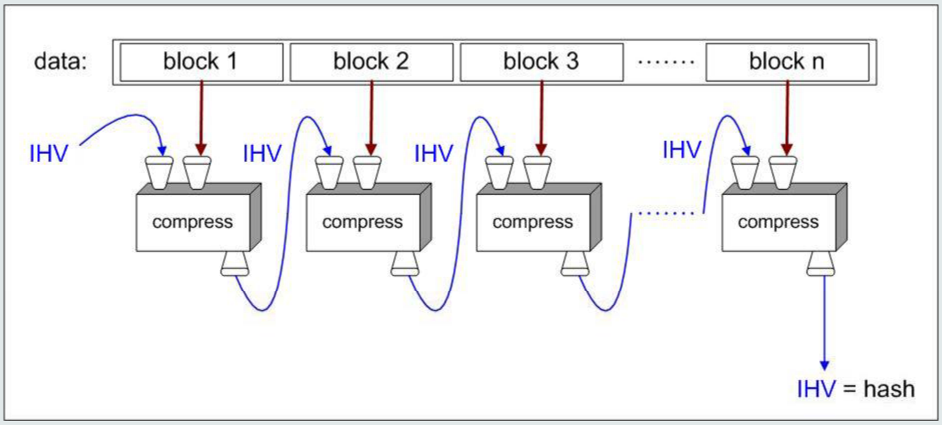
hash function design - iterated compression



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hash function designs

- other designs exist, e.g. sponge functions
- but we can't do everything in just 2 hours

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Merkle-Damgård construction

- assume that message m can be split up into blocks $m_1, \ldots, m_s$ of equal block length r
  - most popular block length is $r = 512$
- *compression function*: $CF : \{0,1\}^n \times \{0,1\}^r \rightarrow \{0,1\}^n$
- *intermediate hash values* (length n) as CF input *and* output
- *message blocks* as second input of CF
- start with fixed initial IHV_0 (a.k.a. IV = *initialization vector*)
- iterate CF : $IHV_1 = CF(IHV_0, m_1)$, $IHV_2 = CF(IHV_1, m_2)$, …, $IHV_s = CF(IHV_{s-1}, m_s)$,
- take $h(m) = IHV_s$ as hash value
- advantages:
  - this design makes ***streaming*** possible
  - hash function analysis becomes compression function analysis
  - analysis easier because domain of CF is finite

avoiding meet in the middle attacks

- compression function should not be invertible
- usually done by *feed-forward* technique
  - use input *IHV* also at the very end of the compression function

padding

- *padding*: add dummy bits to satisfy block length requirement
- non-ambiguous padding: add one 1-bit and as many 0-bits as necessary to fill the final block
  - when original message length is a multiple of the block length, apply padding anyway, adding an extra dummy block
  - any other non-ambiguous padding will work as well

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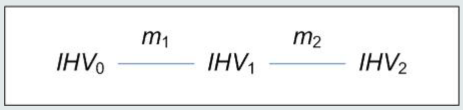
Merkle-Damgård strengthening

- **let padding leave final 64 bits open**
- **encode in those 64 bits the original message length**
  - that's why messages of length ≥ 2^{64} are not supported
- **reasons:**
  - needed in the proof of the Merkle-Damgård theorem
  - prevents some attacks such as
    - trivial collisions for random *IHV*



  - now $h(IHV_0, m_1||m_2) = h(IHV_1, m_2)$
- see next slide for more

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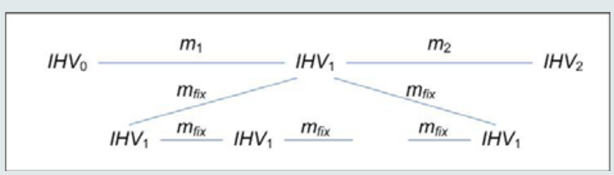
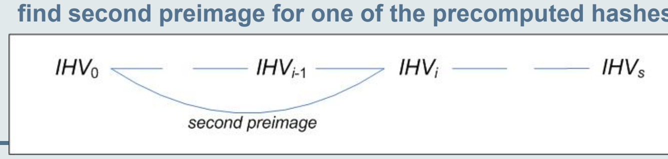
continued

- fixpoint attack
  fixpoint: *IHV*, *m* such that $CF(IHV, m) = IHV$



- long message attack
  message length *s*, so *s* hashes precomputed, cost $2^n/s$
  Merkle time-memory tradeoff on intermediate hash values to find second preimage for one of the precomputed hashes



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compression function collisions

- *collision* for a compression function: m_1, m_2, *IHV* such that $CF(IHV,m_1) = CF(IHV,m_2)$
- *pseudo-collision* for a compression function: m_1, m_2, IHV_1, IHV_2 such that $CF(IHV_1,m_1) = CF(IHV_2,m_2)$
- Theorem (Merkle-Damgård): If the compression function *CF* is pseudo-collision resistant, then a hash function *h* derived by Merkle-Damgård iterated compression is collision resistant.
  - Proof: easy, locate the iteration where the collision occurs
- Note:
  - a method to find pseudo-collisions does not lead to a method to find collisions for the hash function
  - a method to find collisions for the compression function is almost a method to find collisions for the hash function, we ‘only’ have a wrong *IHV*

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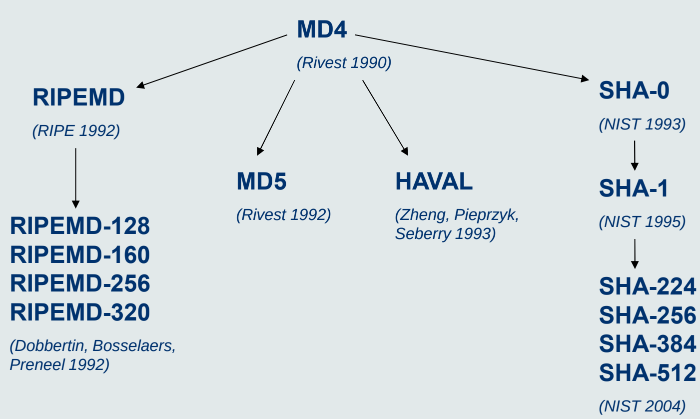
the MD4 family of hash functions



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design of MD4 family compression functions

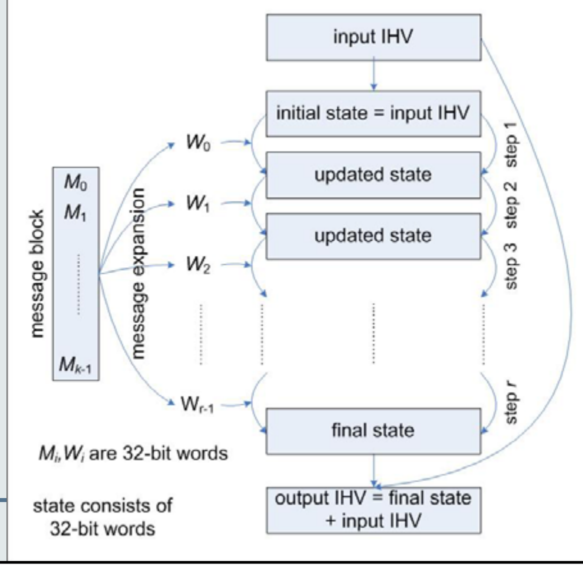
message block
- split into words

message expansion
- input words for each step

IHV → initial state

each step updates
- state with an input word

final state 'added'
- to *IHV*
  - (feed-forward)



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design details

- **MD4, MD5, SHA-0, SHA-1 details:**
  - 512-bit message block split into 16 32-bit words
  - state consists of 4 (MD4, MD5) or 5 (SHA-0, SHA-1) 32-bit words
  - MD4: 3 rounds of 16 steps each, so 48 steps, 48 input words
  - MD5: 4 rounds of 16 steps each, so 64 steps, 64 input words
  - SHA-0, SHA-1: 4 rounds of 20 steps each, so 80 steps, 80 input words
  - message expansion and step operations use only very easy to implement operations:
    - bitwise Boolean operations
    - bit shifts and bit rotations
    - addition modulo 2^{32}
  - proper mixing believed to be cryptographically strong

message expansion

- MD4, MD5 use *roundwise permutation*, for MD5:
  - $W_0 = M_0$, $W_1 = M_1$, …, $W_{15} = M_{15}$,
  - $W_{16} = M_1$, $W_{17} = M_6$, …, $W_{31} = M_{12}$, (jump 5 mod 16)
  - $W_{32} = M_5$, $W_{33} = M_8$, …, $W_{47} = M_2$, (jump 3 mod 16)
  - $W_{48} = M_0$, $W_{49} = M_7$, …, $W_{63} = M_9$ (jump 7 mod 16)
- SHA-0, SHA-1 use *recursivity*
  - $W_0 = M_0$, $W_1 = M_1$, …, $W_{15} = M_{15}$,
  - SHA-0: $W_i = W_{i-3}$ XOR W_{i-8} XOR W_{i-14} XOR W_{i-16} for i = 17, …, 80
  - problem: k^{th} bit influenced only by k^{th} bits of preceding words, so not much diffusion
  - SHA-1: $W_i = (W_{i-3}$ XOR W_{i-8} XOR W_{i-14} XOR W_{i-16})<<<1 (additional rotation by 1 bit, this is the *only* difference between SHA-0 and SHA-1)

step operations in MD4

- in each step only one state word is updated
- the other state words are *rotated* by 1
- state (A,B,C,D) in step i updated to (D,A',B,C), where
  $A' = (A + f_i(B,C,D) + W_i + K_i) \lll s_i$
  K_i, s_i step dependent constants,
  $+$ is addition mod 2^{32},
  f_i round dependend boolean functions:
  $f_i(x,y,z) = xy \text{ OR } (\neg x)z$ for $i = 1, \ldots, 16$,
  $f_i(x,y,z) = xy \text{ OR } xz \text{ OR } yz$ for $i = 17, \ldots, 32$,
  $f_i(x,y,z) = x \text{ XOR } y \text{ XOR } z$ for $i = 33, \ldots, 48$,
  these functions are nonlinear, balanced, and have an *avalanche effect*

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step operations in MD5

- very similar to MD4
- state update:

  $A' = B + ((A + f_i(B,C,D) + W_i + K_i) \lll s_i)$

  K_i, s_i chosen differently (more variation),

  one boolean function changed,

  one more boolean function f_i needed for 4th round:

  $f_i(x,y,z) = xz \text{ OR } y(\neg z)$ for $i = 17, \ldots, 32$,

  $f_i(x,y,z) = y \text{ XOR } (y \text{ OR } (\neg z))$ for $i = 49, \ldots, 64$,

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some constants in MD4 and MD5

- initial *IHV*:
  - 0x67452301, 0xefcdab89, 0x98badcfe, 0x10325476
- MD4: $K_i = 0$ (1st round),

  0x5a827999 (2nd round, this is $\sqrt{2}$),

  0x6ed9eba1 (3rd round, this is $\sqrt{3}$)
- MD5: K_i = first 32 bits of binary value of $|\sin(i+1)|$

visualisation of MD5 compression

step operations in SHA-0 and SHA-1

- different constants, boolean functions used in different order
- big-endian byte ordering in stead of little-endian
- state update: from (A,B,C,D,E) to $(E',A,B>>>2,C,D)$

$$E' = (A<<<5 + f_i(B,C,D) + E + W_i + K_i) <<< s_i$$

visualisation of SHA-1 compression

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RIPEMD design

- **basic idea of RIPEMD:**
  two *parallel* MD4-like compression functions
  with different constants
  and different message schedules
- **RIPEMD-128 and RIPEMD-160**
  - final states of two compressions ***added together***
- **RIPEMD-256 and RIPEMD-320**
  - main difference with RIPEMD-128 resp. RIPEMD-160 is:
    final states of two compressions ***concatenated***

SHA-2 family design

- SHA-224 is SHA-256 with different *IV* and output truncation to 224 bits
- complexity of step operation increased
  - state of 8 words (A,B,C,D,E,F,G,H) updated to ($T_1+T_2,A,B,C,D+T_1,E,F,G$) where
    $T_1 = H + ((E{>}{>}{>}6) \text{ XOR } (E{>}{>}{>}11) \text{ XOR } (E{>}{>}{>}25)) + f(E,F,G) + W_i + K_i$
    $T_2 = ((A{>}{>}{>}2) \text{ XOR } (A{>}{>}{>}13) \text{ XOR } (E{>}{>}{>}22)) + g(A,B,C)$
    for fixed boolean functions f, g
  - extra rotations should provide much more diffusion
- SHA-384 is SHA-512 with different *IV* and output truncation to 384 bits
- SHA-512 uses 64-bit words, is very similar to SHA-256

performance comparison

- for what it's worth
  performance measured on a 2.1 GHz Pentium 4:

hash function	MB/sec
MD5	217
SHA-1	68
RIPEMD-160	53
SHA-256	44

finding fixpoints

- in the MD4 family finding fixpoints is easy
- given message *m*, the compression function is

  $CF(IHV,m) = E(IHV,m) + IHV$

  (*feed-forward* technique) where $E(x,m)$ is invertible: given y it's easy to compute $x = E^{-1}(y,m)$ such that $y = E(x,m)$
- the fixpoint is $E^{-1}(0,m)$

differential cryptanalysis

- attacking only collision resistance
- two stages:
  - choose *differential path*
    - until recently done by hand
    - De Cannière-Rechberger (2006): automated for SHA-1
    - Stevens (TU/e, 2006-2009): automated for MD5
    - Stevens (CWI, 2012): new results for SHA-1
  - *brute force search* for message pair *m*, *m'* that 'follows the path'

differential path

- e.g. in MD5, let *m*, *m'* differ in one word M_{14} only
  - message expansion: difference propagates only to W_{14}, W_{25}, W_{35}, W_{50}, so collision to be found in steps 15 – 51
- look for *inner collisions*
  - collision after step 26 propagates to step 35
- look for *inner almost-collisions*
  - prescribed small bit difference vectors may be found
- distinguish between *additive differences* (because of additions modulo 2^{32}) and *XOR-differences*
- differential path describes *conditions* on the inputs
- differential path is good if it has only a few conditions

finding the collision

- conditions have a certain *probability*
- leads to probability for the differential path
  - this should be >> 2^{-64}
- brute force search on message pair *m*, *m'*
- all kinds of improvements and tricks are possible

Wang's attack on MD5

- ***two-block*** **collision**
  - for any input *IHV*, identical for the two messages
    i.e. $IHV_0 = IHV_0'$, $\Delta IHV_0 = 0$
  - ***near-collision*** after first block:
    $IHV_1 = CF(IHV_0,m_1)$, $IHV_1' = CF(IHV_0,m_1')$,
    with ΔIHV_1 having only a few carefully chosen ±1s
  - full collision after second block:
    $IHV_2 = CF(IHV_1,m_2)$, $= CF(IHV_1',m_2')$,
    i.e. $IHV_2 = IHV_2'$, $\Delta IHV_2 = 0$
- **with IHV_0 the standard *IV* for MD5, and a third block for padding and MD-strengthening, this gives a collision for the full MD5**

example

IHV_0 FE2BB52F2807AC73BE5191B597442F78

m_1	m_1'
CAB9E742C4B626871AB9A524846B05C1	CAB9E742C4B626871AB9A524846B05C1
8895FB9365E9A69F480392FF2C3B3F79	8895FB1365E9A69F480392FF2C3B3F79
41AD3406FFADB4034BDF847A4D37014F	41AD3406FFADB4034BDF847A4DB7014F
DB3283CB19D46FA8A765C6B3F016BF30	DB3283CB19D46FA8A765C633F016BF30

IHV_1	IHV_1'
2F98389AC366091 1D10870 3372E84679	2F98381AC3660993D10870B572E846FB

m_2	m_2'
6AFF7C2E5773689B3319B81564ABE7F5	6AFF7C2E5773689B3319B81564ABE7F5
B9CF66C5E4FE790CEE047D36CC77B0AE	B9CF6645E4FE790CEE047D36CC77B0AE
5D087F30B560EB8872B34D406778662D	5D087F30B560EB8872B34D4067F8652D
D88464677DBD9B80989EF24FB82E0EA3	D88464677DBD9B80989EF2CFB82E0EA3

IHV_2 461A448DCF403F043DBADCD87214F197

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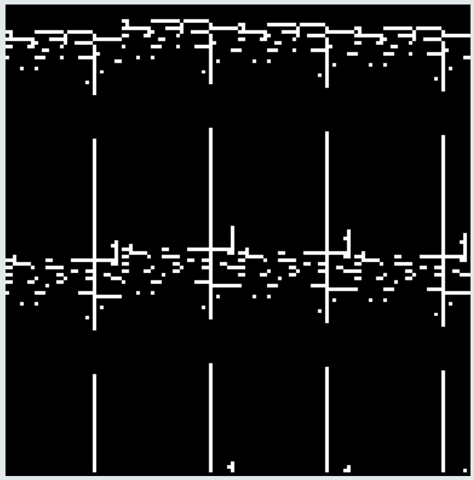
visualisation of the collision



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new ideas in Wang's construction

- **two-block collision**
- **describes precise differential path**
  - previous attacks described only partial paths
- **set of sufficient conditions**
- ***message modification* techniques**
  - modify message bits to satisfy conditions
  - speeds up collision search

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recent results on MD5 and SHA-1

- MD4: broken
  - Dobbertin 1995, collision found
  - Wang 2004, complexity: only 2^6
  - Wang 2005, even a preimage attack, complexity: 2^{56}
- MD5: broken
  - Den Boer-Bosselaers 1993: pseudo-collision in the compression function
  - Wang 2004, collision found, complexity: 2^{39}
  - Klima 2006, Stevens 2006-9, complexity: 2^{16} (matter of seconds on a PC)
- SHA-0: broken
  - Biham-Chen 2004 and Joux et al. 2004, complexity: 2^{51}
- SHA-1: weakened
  - Wang 2005, complexity: 2^{63}, no collisions found yet
  - reduced to 64 steps: broken by De Cannière-Rechberger 2006, complexity: 2^{35}
  - Stevens 2012, complexity: $2^{??}$, no collisions found yet
- RIPEMD and HAVAL: some versions affected / broken

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complexiteiten van bekende aanvallen

	MD5		SHA-1		SHA-2(256)	
jaar	identical prefix	chosen prefix	identical prefix	chosen prefix	identical prefix	chosen prefix
– 2003	64	64	80	80	128	128
2004	40		69			
2005	37		63			
2006	32	49		80 - ε		
2007	25	42	61			
2008	21					
2009	16	41	???			

cijfers voor optimale snelheid, niet voor optimale blokgrootte

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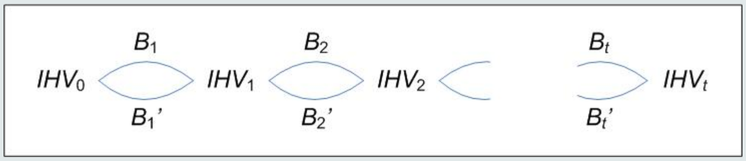
Joux' multicollision attack

- ***k-collision***: k-tuple $m_1, \ldots, m_k$ with $h(m_i)$ all equal
- Joux (2004): 2^t-collision costs only t times as much as 2-collision



- this is trivial, but it has interesting consequences

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hash function concatenation

- let h_1 be an n_1-bit *iterative* hash function, and let h_2 be an n_2-bit hash function (not necessarily iterative)
- let h be the *concatenation*, i.e. $h(m) = h_1(m) \,||\, h_2(m)$
- naïve expectation: collision resistance security level of h is $\frac{1}{2}(n_1+n_2)$-bit
- this is wrong, Joux showed that it is essentially at most $\frac{1}{2}\max(n_1,n_2)$-bit
- very simple argument
  - compute 2^t-collision for h_1 at cost $t\,2^{\frac{1}{2}n_1}$
  - do birthday attack on these 2^t messages for h_2 at cost 2^t
  - collision for h_2 will be found if $t > \frac{1}{2}n_2$
- total cost is $O(n_2\, 2^{\frac{1}{2}n_1} + 2^{\frac{1}{2}n_2})$

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Joux's preimage attack

- easy exercise: show that a preimage attack on $h = h_1 \| h_2$ is possibe with a security level of $\max(n_1,n_2)$-bit
- in fact the complexity is $O(n_2\, 2^{½n_1} + 2^{n_1} + 2^{n_2})$
- conclusion: *concatenation of iterative hash functions gives almost no extra security above that of the strongest component*

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Kelsey-Schneier attack

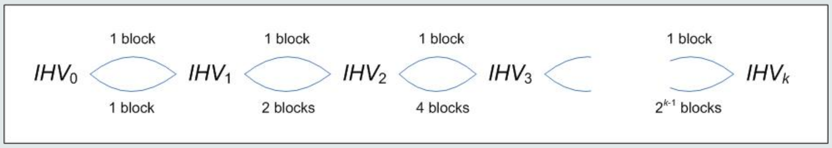
- second preimage: should have cost 2^n
- can we do better than Merkle time-memory tradeoff?
  - if you have computed 2^t hashes, cost to find a second preimage for one of them is only 2^{n-t}
- Kelsey-Schneier (2006) for iterative hash functions:

  for a message of 2^t blocks the cost drops to $t\, 2^{½n+1} + 2^{n-t+1}$

  for many hash functions even to $3x2^{½n+1} + 2^{n-t+1}$
- uses *expandable messages*, i.e. multi-collisions of many different lengths

expandable messages

- *generic method*, starting from given IHV_0
  finding collision between message of length 1 and message of any given length α takes $\alpha + 2^{½n+1}$
  do this for $\alpha = 1, 2, 4, 8, \ldots, 2^{k-1}$ as follows:



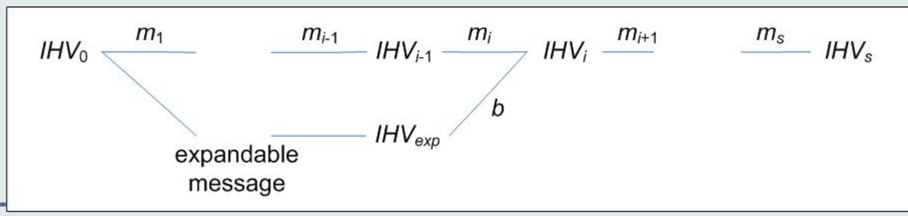
  this gives 2^k messages, all of different length covering the range from k to $2^k + k - 1$, that all have the same final *IHV* (before padding and MD-strengthening)
- cost: about $k\ 2^{½n+1}$

method with fixpoints

- *better method* for many hash functions
- when *fixpoints* are easy to compute, expandable messages can be found faster
  starting from given IHV_0
  choose $2^{½n}$ random blocks and compute their IHV_1s
  generate $2^{½n}$ random fixpoints (IHV,m), i.e. such that $IHV = h(IHV,m)$
  there will be a colliding $IHV = IHV_1$
  repeat the fixpoint as many times as required
- cost: about $2^{½n+1}$
- remember: finding fixpoints is easy in the MD4-family

how to generate second preimages

- given *very long message m*, with $2^t + t + 1$ blocks
- this gives $2^t + t + 1$ intermediate *IHV*s
- make an expandable message with parameter t
- let IHV_{exp} be its output *IHV*
- find a block b that connects IHV_{exp} to one of the message *IHV*s
  - cost: 2^{n-t+1} (second preimage attack with time-memory tradeoff)



continued

- from the expandable message choose the proper message length to fit the length of m
- total cost: $t\,2^{\frac{1}{2}n+1} + 2^{n-t+1}$
  - with fixpoints even $3 \mathrm{x} 2^{\frac{1}{2}n+1} + 2^{n-t+1}$
- with $t = \frac{1}{2}n$ this gives second preimages at the cost of collisions
- not very realistic: with $t = 32$ for MD5 ($n = 128$) we get second preimages for messages of 2^{32} blocks (= 256 GB) in 2^{97} compression function calls

herding attack

- Kelsey-Kohno 2005
- a.k.a. *Nostradamus attack*
- commitment to bit string by publishing hash
  - Nostradamus makes claim about predictions
  - does not publish predictions, but only a hash h_{pred}
  - when time of predicted event has been reached, Nostradamus publishes document describing actual events, that hashes to h_{pred}
- attack: you can commit by a hash to a bit string before you know the string
- this is done by *herding*

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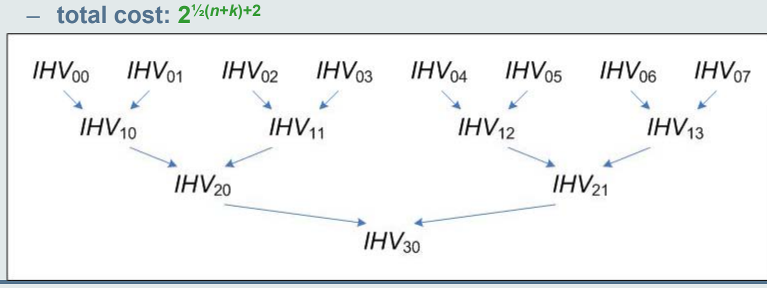
how to herd a hash

- build a tree of depth k and width 2^k
- start with 2^k random *IHV*s
- find 2^{k-1} pairs of them, such that for each pair a collision is found (cost: $2^{½(n+k+1)}$)
- repeat k times until one final collision is found
  - total cost: $2^{½(n+k)+2}$



continued

- publish the final hash
- when known what string m_0 to hash, compute its hash IHV_{-1}
- make a linking block b to connect IHV_{-1} to any of the 2^k initial IHVs
  - cost: 2^{n-k} (preimage attack with time-memory tradeoff)
- path m_1 to final hash already known (in the tree)
- append suffix $b||m_1$ to message m_0
- use Yuval's trick to hide suffix in meaningful message

- total cost of attack: $2^{n-k} + 2^{½(n+k)+2} = 2^{n-k}$

faster herding

- the preimage in the herding attack is not necessary when you commit to one of a set of known messages
  - complexity drops from 2^{n-k} to $2^{½(n+k)+2}$

repairing – message preprocessing

- repair proposals to be able to continue using MD5 and SHA-1 without changing implementations
- Szydlo-Yin 2005:

  - message whitening: use only 384 message bits per hash input, and append 128 0-bits
    in 32-bit words: $M_1, M_2, \ldots, M_{12}, 0,0,0,0$
  - self-interleaving: use only 256 message bits per hash input, doubling each 32-bit word
    in 32-bit words: $M_1, M_1, M_2, M_2, \ldots, M_8, M_8$
  - make up your own variant
- imposes many more conditions on differential paths that are probably very hard to fulfill

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repairing – randomized hashing

- Halevi-Krawczyk 2005:
- randomize input
- random 512-bit r called *salt*
- change hash function h to h_r by
  $h_r(M_1||\ldots||M_k) = h_r(r||M_1 \text{ XOR } r||\ldots||M_k \text{ XOR } r)$
- salt prepended inside so that it's automatically signed
- salt r has to be sent / stored with the data

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applications of hash collisions

- assumption: attacker can make collisions for arbitrary *IHV*, but he has no control over how the collisions look like; they're a few random looking 512-bit blocks
- Mikle 2004, Kaminsky 2004: use collision to change program flow
  - files good.exe and bad.exe collide, program looks for specific bit in the colliding blocks that differs in both files, and shows different behaviour
  - can mislead software integrity protection systems, e.g. Tripwire

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more applications

- Daum-Lucks 2005: similar idea for PostScript documents
- file 1:

macro	coll.blk. 1	document 1	document 2

have this signed by trusted party

- file 2:

macro	coll.blk. 2	document 1	document 2

has identical signature

- relies on superficial inspection by signer and verifier
- fraud easily detected by code inspection of one file only
  - two complete documents in there
  - strange block of random looking data

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colliding certificates

- hide collision in public key inside X.509 certificate
  - by Lenstra, Wang, de Weger (Mar. 2005)
- two different certificates with identical CA signature
- cert. 1

name	public key	coll.blk. 1	CA signature

- cert. 2

name	public key	coll.blk. 2	CA signature

- code inspection of only one certificate reveals nothing
  - cryptographic key is random-looking anyway
- drawbacks
  - control over CA needed
  - identical user names limits possible abuse scenarios

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chosen-prefix collisions

- latest development on MD5
- Marc Stevens (TU/e MSc student) 2006
  - paper by Marc, Arjen Lenstra and BdW, EuroCrypt 2007
- Marc Stevens (CWI PhD student) 2009
  - paper by Marc, Alex Sotirov, Jacob Appelbaum, David Molnar, Dag Arne Osvik, Arjen Lenstra and BdW, Crypto 2007
  - rogue CA attack

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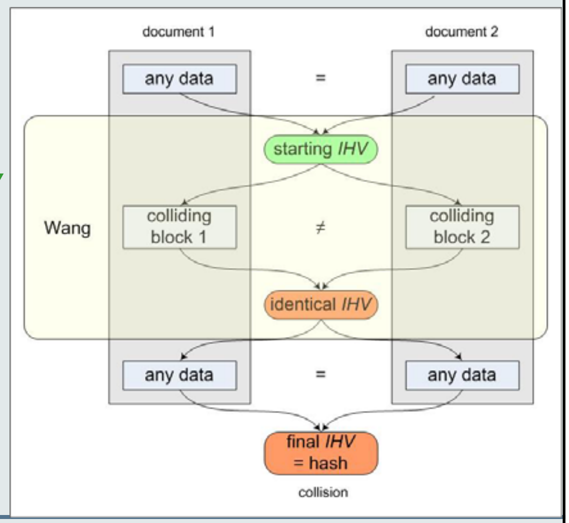
MD5: identical IV attacks

- all attacks following Wang's method, up to recently
- MD5 collision attacks work for any starting *IHV*
  - data before and after the collision can be *chosen at will*
- but starting *IHV*s must be identical
  - data before and after the collision *must be identical*
- called ***random collision***



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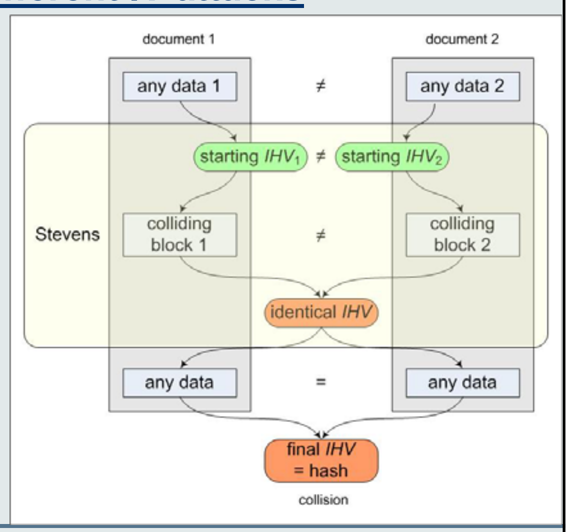
MD5: different IV attacks

- new attack
  - Marc Stevens, TU/e
  - Oct. 2006
- MD5 collisions for any starting pair {IHV_1, IHV_2}
  - data before the collision *needs not to be identical*
  - data before the collision can still be chosen at will, for each of the two documents
  - data after the collision still must be identical
- called ***chosen-prefix collision***
- one example produced so far



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how to make chosen-prefix collisions

- random collision (Wang): two MD5 input blocks
  - 1024 bits, looking random
  - nowadays: few seconds on a PC
  - executable can be downloaded (www.win.tue.nl/hashclash)
- chosen-prefix collisions (Stevens): larger number of MD5 input blocks, depending on computation effort
  - our example: 96 bits + 8 MD5 input blocks
  - 4192 bits, still looking random
  - requires massive parallel computation
  - we used a cluster at TU/e and a grid of volunteer home computers (up to 1200 machines) running BOINC
  - peak performance 400 GigaFLOPS
  - took 6 months in total

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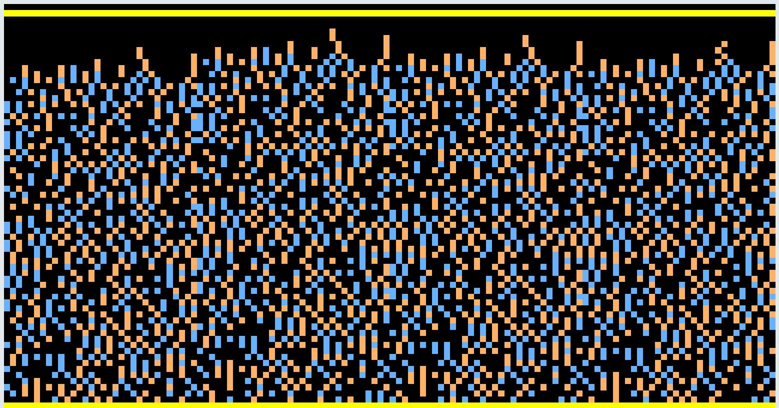
chosen-prefix collision finding method

- chosen prefix pair
  - in our example: each consisting of 4 input blocks, the last one missing 96 bits
  - containing two different certificate owner names
- 96 bits computed by birthdaying method to prepare "smooth" pair of IV's
  - differing only in 8 triples of bits
  - complexity: 2^{48}
- fully automated construction of "differential paths" for MD5 compression function
  - each path is able to eliminate one triple of bit differences
  - note: original Wang construction has one manually found differential path

visualizing the collision



chosen-prefix collision in certificate

- **allows X.509 certificates with identical signatures but *different owner names***
  - `http://www.win.tue.nl/hashclash/ChosenPrefixCollisions/`
- **cert 1**

Alice	public key	coll.blk. 1	CA signature

- **cert 2**

Bob	public key	coll.blk. 2	CA signature

- **apparently higher risk**
  - still control over CA needed
- **drawback: complexity**
  - took 6 months to find one example
- **this will not be the end…**

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indeed that was not the end
in 2008 the ethical hackers came by

observation: commercial certification authorities still use MD5

idea: proof of concept of realistic attack as wake up call
→ attack a real, commercial certification authority

purchase a web certificate for a valid web domain
but with a "little spy" built in
prepare a rogue CA certificate with identical MD5 hash
the commercial CA's signature also holds for the rogue CA certificate

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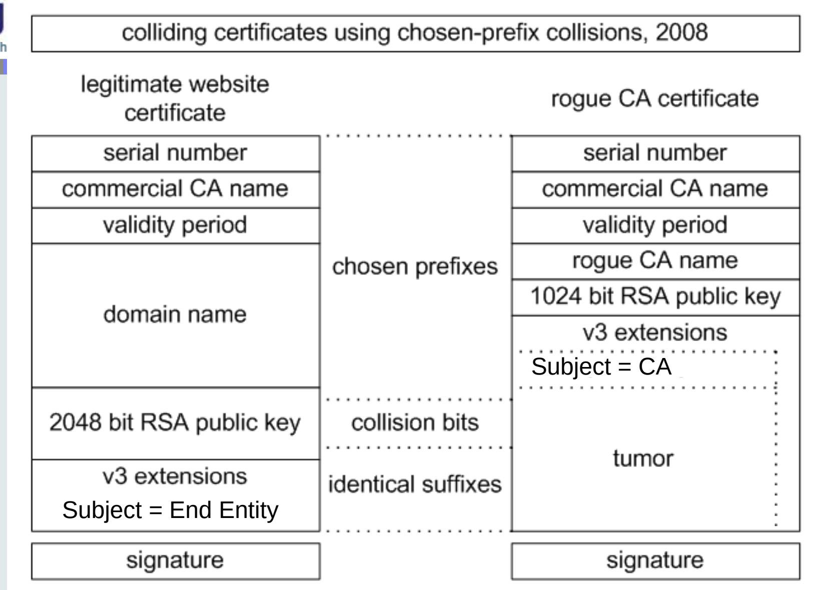

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problems to be solved

predict the serial number
predict the time interval of validity
    at the same time
    a few days before
more complicated certificate structure
    “Subject Type” after the public key
small space for the collision blocks
    is possible but much more computations needed
not much time to do computations
    to keep probability of prediction success reasonable

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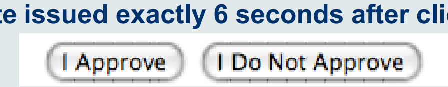
how difficult is predicting?

time interval:
    CA uses automated certification procedure
    certificate issued exactly 6 seconds after click



serial number :

Nov 3 07:44:08 2008 GMT	643006
Nov 3 07:45:02 2008 GMT	643007
Nov 3 07:46:02 2008 GMT	643008
Nov 3 07:47:03 2008 GMT	643009
Nov 3 07:48:02 2008 GMT	643010
Nov 3 07:49:02 2008 GMT	643011
Nov 3 07:50:02 2008 GMT	643012
Nov 3 07:51:12 2008 GMT	643013
Nov 3 07:51:29 2008 GMT	643014
Nov 3 07:52:02 2008 GMT	have a guess…

the attack at work

estimated: 800-1000 certificates issued in a weekend

procedure:

1. buy certificate on friday, serial number *S*-1000
2. predict serial number *S* voor time *T* Sunday evening
3. make collision for serial number *S* and time *T*: 2 days time
4. short before *T* buy additional certificates until *S*-1
5. buy certificate on time *T-6*
   hope that nobody comes in between and steals our serial number *S*

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to let it work

cluster of >200 PlayStation3 game consoles (1 PS3 = 40 PC's)

complexity: 2^{50}
memory: 30 GB

→ collision in 1 day

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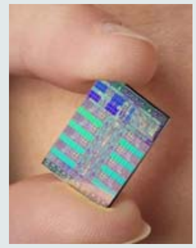
why PlayStation3s?



cell-processor on PlayStation3:
- small instruction set
- 8 very fast parallel processors
  - identical instruction on different data
- 128 bit registers
- ideal for MD5

more modern alternatives:
- cloud (BOINC, Amazon EC2)
- grafical cards (NVidia GTX285)

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result

success after 4th attempt (4th weekend)

purchased a few hundred certificates
- (promotion action: 20 for one price)
- total cost: < US$ 1000

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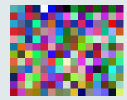
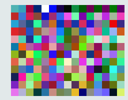
other attack ideas for chosen-prefix collisions

- **hide collision in image (not macro)**
  - inside document (MS Word, Adobe pdf, …)
- **file 1:** document 1 | image | coll.blk. 1

  have this signed by trusted party
- **file 2:** document 2 | image | coll.blk. 2

  has identical signature

can you find the hidden images on this slide?

- **code inspection of one document reveals almost nothing**
  - collision covers only a few pixels in the image
  - macro features not needed anymore

code signing example

- **Win32 executable still runs normally when random bits attached to it**
- **assumption (example)**
  - Microsoft publishes `Word.exe` on download site
  - comes with MD5-based signature (Authenticode)
- **abuse scenario**
  - attacker prepares `Worse.exe` (doing whatever he wants)
  - attacker computes bitstrings b1 and b2 such that MD5(`Word.exe||b1`) = MD5(`Worse.exe||b2`)
    - we can do that!
  - attacker gets a Microsoft Authenticode signature on `Word.exe||b1` (same functionality as `Word.exe`)
  - attacker renames `Worse.exe||b2` to `Word.exe` and publishes on Microsoft's download site

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faster herding

- chosen-prefix collisions make the herding attack faster
- predict whether Ajax or Feyenoord will win their next match
  - IHV_1 = MD5-CF(IHV_0,"my prediction is: Ajax wins")
  - IHV_2 = MD5-CF(IHV_0,"my prediction is: Feyenoord wins")
  - IHV_3 = MD5-CF(IHV_0,"my prediction is: it's a draw")
  - produce a chosen-prefix collision m_1, m_2 for IHV_1 and IHV_2: IHV_4 = MD5-CF(IHV_1,m_1) = MD5-CF(IHV_2,m_2)
  - produce a chosen-prefix collision m_3, m_4 for IHV_3 and IHV_4: IHV_5 = MD5-CF(IHV_3,m_3) = MD5-CF(IHV_4,m_4)
  - publish IHV_5 before the match
  - after the match:
    - if Ajax won, publish: "my prediction is: Ajax wins" || m_1 || m_4
    - if Feyenoord won, publish: "my prediction is: Feyenoord wins" || m_2 || m_4
    - if it's a draw, publish: "my prediction is: it's a draw" || m_3
    - (hide suffixes e.g. in image, Yuval's trick won't work now)
  - only 2 chosen-prefix collisions required → practical attack!

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the "meaningful message" argument

- colliding data cannot be chosen at will, but follow from Wang's (Stevens') construction method
  - indistinguishable from random data
  - two colliding data differ in a few bit positions only

  → will most probably not constitute a "meaningful message" as input
- this makes attacks more difficult
  - but not impossible, as we've seen
  - meaningful message argument can be weakened by hiding collisions inside the bit level structure of a document

conclusion on collisions

- at this moment, 'meaningful' hash collisions are
  - easy to make
  - but also easy to detect
  - still hard to abuse realistically
- with chosen-prefix collisions we come close to realistic attacks
  - especially herding
- to do *real* harm, second pre-image attack needed
  - real harm is e.g. forging digital signatures
  - this is not possible yet, not even with MD5

provable hash functions

- people don't like that one can't prove much about hash functions
- reduction to established 'hard problem' such as factoring is seen as an advantage
- Chaum-Van Heijst-Pfitzmann:
  - DLP is a collision problem:
    - a collision x_1, x_2 for $F(x) = a^x$ and $G(x) = (a^x b)^{-1}$ solves $a^x = b$
  - let $p = 2q+1$ for p, q prime, and a, b generators in Z_p^*
  - define hash function
    h: {0, …, q-1} x {0, …, q-1} → {0, …, p-1}
    $h(x,y) = a^x b^y \bmod p$
  - Theorem: h is collision resistant if and only if DLP in Z_p^* is hard

provable hash functions - VSH

- Contini-Lenstra-Steinfeld 2006
- VSH – Very Smooth Hash
- collision resistance provable under assumption that a problem directly related to factoring is hard
- also DLP-variant exists
- much more efficient than Chaum-Van Heijst-Pfitzmann
- but still far from ideal
  - bad performance compared to SHA-256
  - all kinds of multiplicative relations between hash values exist

SHA-3 competition

- NIST started in 2007 an open competition for a new hash function to replace SHA-256 as standard
- more than 50 candidates in 1st round
- now 5 finalists left
- decision in 2012

literature and web resources

- Menezes-Van Oorschot-Vanstone: Handbook of Applied Cryptography, Chapter 9
  - downloadable
  - bit out of date
- Daum-Dobbertin - Chapter 109 of the Handbook of Information Security
  - pretty recent, readable
- NIST website: http://csrc.nist.gov/pki/HashWorkshop
- our website on chosen-prefix collisions: http://www.win.tue.nl/hashclash/