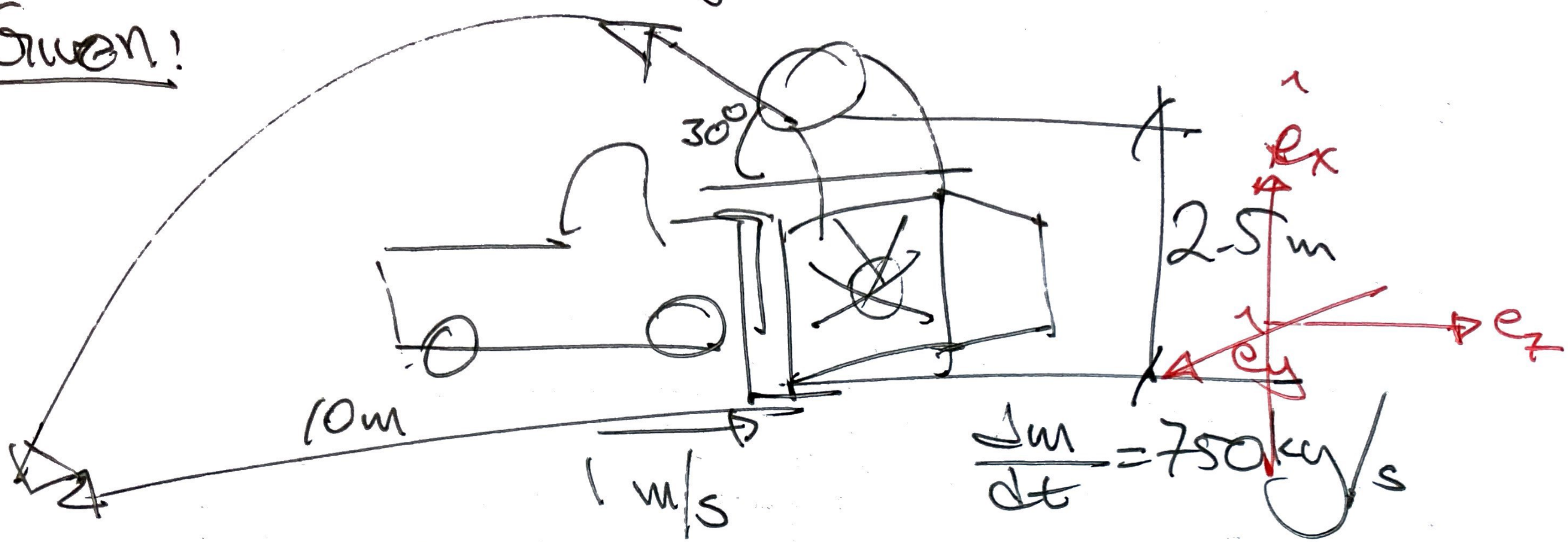
Given:



Req'd: Find force on snowplow

Assump: no air drag on blown snow (bad assumption!)
all speeds constant, ejected in x-y plane

Strategy: $F = -\frac{dm}{dt}\bar{v}_{flow}$ + projectile motion!

Estimate: ejection velocity >> 1 m/s to go 10 m
$\Rightarrow F_x \,\&\, F_y > F_z$, roughly expecting $V_{flow} \approx 10$ m/s or less

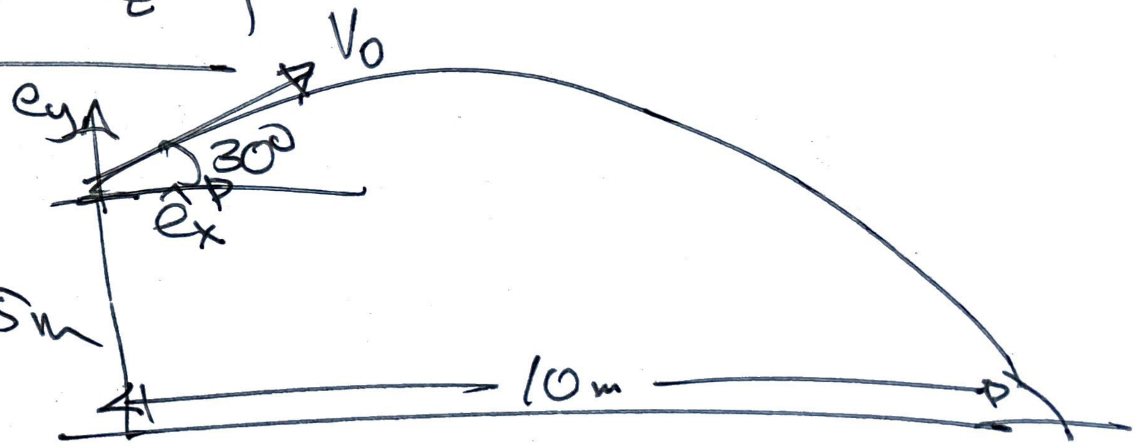
Soln:

(A) $\bar{F}_{in} = -\frac{dm}{dt} v_{flow} = -\left(-\frac{750\,kg}{s}\right)\left(-1\,m/s\,\hat{e}_z\right)$

entering (−) e_z directn

$$\boxed{F_{in} = -750\,N\,\hat{e}_z}$$

(B) Projectile



$a = -g\hat{e}_y = \frac{dv_y}{dt}$

(x)

$10\,m = V_0 \cos 30^\circ\, t_{flight}$

(y) $-\int_0^t g\,dt = \int_{v_{0y}}^{v_{fy}} dv_y \Rightarrow v_{fy} = v_{0y} - gt = \frac{dy}{dt}$

$$\int_0^{t_f}(v_{0y} - gt)\,dt = \int_{2.5m}^{0m} dy \Rightarrow \left(v_{0y}t - g\frac{t^2}{2}\right)\Big|_0^{t_f} = y\Big|_{2.5}^{0}$$

Soln cont

$$0 - 2.5\,m = V_0 \sin\theta\, t_f - \frac{1}{2} g t_f^2$$

from b4 $$v_0 = \frac{10\,m}{\cos 30^\circ\, t_f}$$

$$\Rightarrow -2.5\,m = \left[\frac{10\,m}{\cos 30^\circ \cancel{t_f}}\right] \sin\theta\, \cancel{t_f} - \frac{1}{2} g t_f^2$$

$$-2.5\,m = 10\,m \tan 30^\circ - \frac{1}{2}(9.81\,m/s^2)\, t_f^2$$

$$4.96\, t_f^2 = 10\,m \tan 30^\circ + 2.5\,m = 8.27\,m$$

$$t_f^2 = \frac{8.27\,\cancel{m}}{4.91\,\cancel{m}/s^2} = 1.68\,s^2 \qquad \underline{\underline{t_f = 1.3\,s}}$$

$$V_0 = \frac{10\,m}{\cos 30^\circ (1.3\,s)} = \boxed{8.88\ m/s}$$

mass flow

$$\bar{F}_{out} = -\frac{dm}{dt}\bar{v}_{flow} = -\left(750\,\frac{kg}{s}\right)\left(4.44\,m/s\, e_x + 7.69\, e_y\right)$$

$$\bar{F}_{out} = -3.33\,kN\,\hat{e}_x - 5.77\,kN\,\hat{e}_y$$

Solu: cont

$$\bar{F}_{net} = \bar{F}_{in} + \bar{F}_{out} = (-750 e_z) + (-3.3 \text{kN} e_x - 5.7 \text{kN} e_y)$$

$$\boxed{\hat{F}_{net} = (\underbrace{-3.33 \text{kN} \hat{e}_x}_{\text{down into ground}} \underbrace{- 5.77 \text{kN} \hat{e}_y}_{\text{laterally away from ejected snow}} \underbrace{- 750 \text{N} \hat{e}_z}_{\text{provided by friction in tires}})}$$

(this will produce a moment since its above road level!!)

Discussion: Cool problem a bit long, #s land in line w/ estimate and seem plausible. Downward force will increase normal force and help w/ traction!