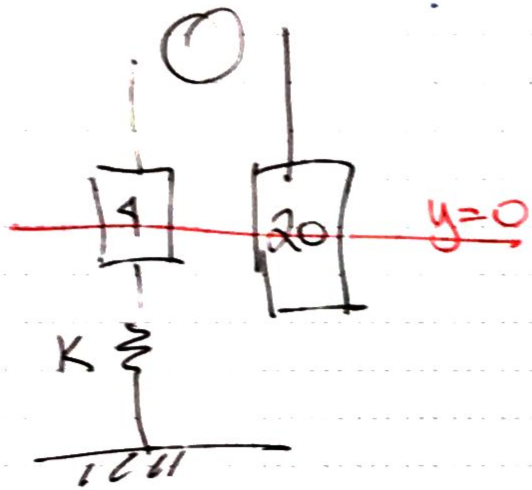
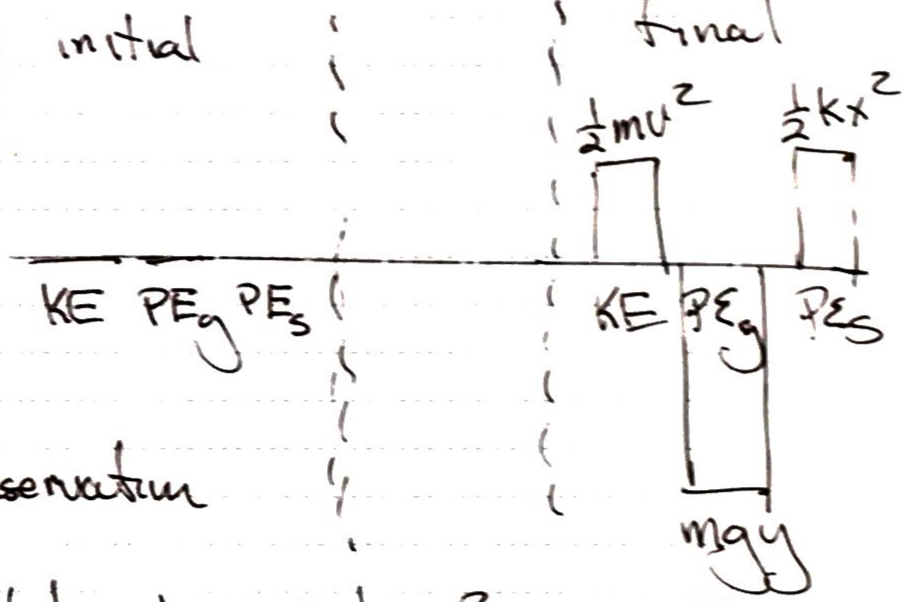
Sample: ①



$k = 200\,N/m$



Req'd v_{max}

Assume massless pulleys, $\mu = 0$

Strategy: Bar Graph! Energy conservation

Estimate:

if the spring stretchs 1m $\Rightarrow \frac{1}{2}kx^2 = 100J$

if mass falls 1m $\Rightarrow mgy = (20-4)10\cdot 1 = 160J$

$\Rightarrow$ falls $\simeq .6m$ (a little over half) until stops.

$\Rightarrow$ halfway @ .3m might be V_{max} $\simeq$ 50 J from F_g

and 10 J stored by spring $(\frac{1}{2}k(.3)^2$

$\Rightarrow \sim 40J = \frac{1}{2}mv^2 \Rightarrow v^2 \simeq 4\,m^2/s^2 \Rightarrow V_{max} \simeq 1.5-2\,m/s$

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Soln:

$0 = \frac{1}{2} m_t v^2 + "m_{eq}" g y + \frac{1}{2} k y^2$ Δx for spring $= y$

$m_t = (4+20)\,kg$ 4 kg up & 20 kg down

$m_{eq} = 16\,kg$ going down.

while I would normally avoid putting #'s in this early it actually seems like it might simplify things

$0 = \frac{1}{2}(24\,kg)v^2 + (16\,kg)(10\,m/s^2)(-y) + \frac{1}{2}\,200\,N/m \cdot y^2$

$0 = 12v^2 - 160y + 100y^2$

$\frac{dv}{dy} = 0$ (flat of plot)

$\frac{d}{dy}(12v^2) = \frac{d}{dy}(160y - 100y^2)$

$24v\frac{dv}{dy} = 160 - 200y$

$\frac{dv}{dy} = \frac{160-200y}{24v} \Rightarrow 160 - 200y = 0$

$y = \frac{160}{200} = \boxed{.8\,m}$

$\frac{dv}{dt} = a = 0 \Rightarrow v_{max}$ when $a=0$

$0 = \frac{d}{dt}\left[12v^2 - 160y + 100y^2\right]$

$0 = \cancel{24v\frac{dv}{dt}} - 160\frac{dy}{dt} + 200y\frac{dy}{dt}$

$\hookrightarrow 0$

$\Rightarrow 0 = -160\cancel{\frac{dy}{dt}} + 200y\cancel{\frac{dy}{dt}}$

$\Rightarrow y = \frac{160}{200} = \underline{.8\,m}$

Soln: (cont) $y = .8\,m$ when $v = v_{max}$

$$0 = 12\,v_{max}^2 - 160(.8m) + 100(.8m)^2$$

$$0 = 12\,v_{max}^2 - 128J + 64J$$

$$\Rightarrow 12\,v_{max}^2 = 128J - 64J = 64J$$

$$v_{max}^2 = \frac{64J}{12kg} = 5.33\,m^2/s^2 \Rightarrow \boxed{v_{max} = 2.31\,m/s}$$

Quick check @ $y = .7$ & $.9m$ suggests this is indeed the max!

Discussion: I was a little surprised to see this so close to 1m so I went back and looked at my estimate. Turns out I had my logic reversed @ 1m the spring has NOT absorbed all the energy yet @ 1.5m $PE_g = 240J$ $PE_s = 100(2.25) = 225J$ so a better estimate would have be $y_{max} = 1.6m \Rightarrow V_{max}$ @ $y \cong .8m$

Whoo hoo!

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