Given:

nice!
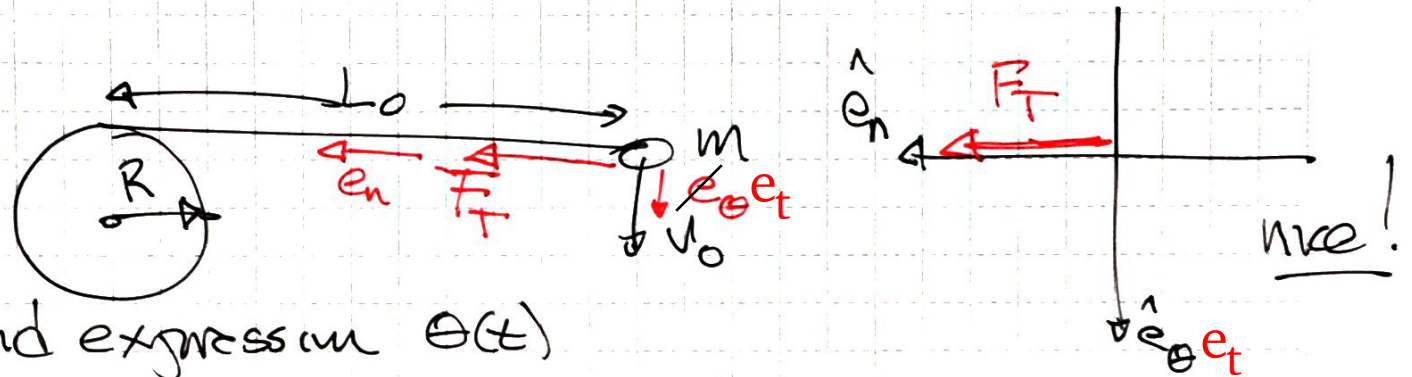
Req'd   Find expression $\theta(t)$

Assume   No gravity, F_T is only force $\Rightarrow a_t = 0$

Estimate   I can only do proportional reasoning w/ no #'s

$\theta(t) \uparrow$ as $v_0 \uparrow$ so v_0 should be in the numerator.

what matters is the relationship between L_0 & R.

which suggest L_0/R is in there somewhere

when L_0 is big I would expect θ to change slowly?

— no I take that back. Wow — I'm not sure!

Strategy = Set components of Forces in $n/t = m\, a(n,t)$

Soln: $F_T = F_{nen} \Rightarrow F_t = 0 \Rightarrow a_t = 0 \Rightarrow v_o = cnst.$

(watch confusion between T & t)!

$$a_n = \frac{F_T}{m} \qquad a_{n/t} = \rho\omega^2 \hat{e}_n + \alpha_t \hat{e}_t = \rho\omega^2 = \frac{v^2}{\rho}$$

$v_o = \rho\omega$ ρ depends on θ

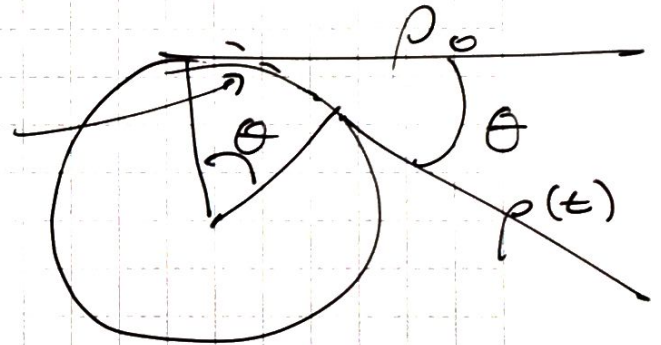$$\Rightarrow \rho(t) = L_o - R\theta(t) \qquad R\theta$$

$$\Rightarrow \omega = \frac{v_o}{\rho} = \frac{v_o}{L_o - R\theta(t)} = \omega(\theta) = \frac{d\theta}{dt}$$

$$\Rightarrow \int_0^t dt = \frac{d\theta}{\omega(\theta)} = \int_0^{\theta(t)} \left(\frac{L_o - R\theta}{v_o}\right) d\theta$$

$$t\Big|_0^t = \left[\frac{L_o\theta}{v_o} - \frac{R\theta^2}{2v_o}\right]\Big|_0^{\theta(t)} \Rightarrow t = \frac{L_o}{v_o}\theta - \frac{R}{2v_o}\theta^2$$

$$\Rightarrow \theta^2 - \frac{2v_o}{R}\frac{L_o}{v_o}\theta + \frac{2v_o t}{R} = 0 \qquad \text{Quadratic} \quad a=1 \quad b = -\frac{2L_o}{R} \quad c = \frac{2v_o t}{R}$$

$$\theta = \frac{\frac{2L_o}{R} \pm \sqrt{4\left(\frac{L_o}{R}\right)^2 - 4\frac{2v_o t}{R}}}{2}$$

Soln: Cont

$$\theta(t) = \frac{L_0}{R} \pm \sqrt{\left(\frac{L_0}{R}\right)^2 - \frac{2v_0 t}{R}}$$

$$\theta(t=0) = 0 = \frac{L_0}{R} \pm \sqrt{\left(\frac{L_0}{R}\right)^2} \quad ? \quad \Rightarrow \quad (-) \text{ sign is correct.}$$

$$\boxed{\theta(t) = \frac{L_0}{R} - \sqrt{\left(\frac{L_0}{R}\right)^2 - \frac{2v_0 t}{R}}} \quad \text{whew!}$$

Discussion: I like it in general, the $\frac{L_0}{R}$ feels good. I am bothered by the $\frac{1}{R}$ dependence I can't quite get that to make sense in my head.