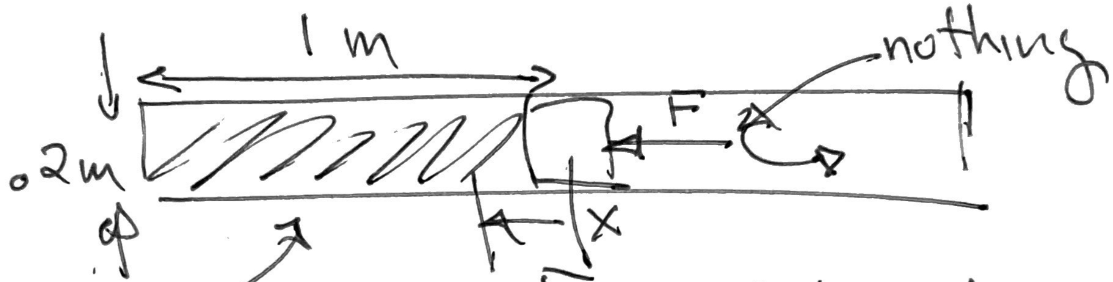
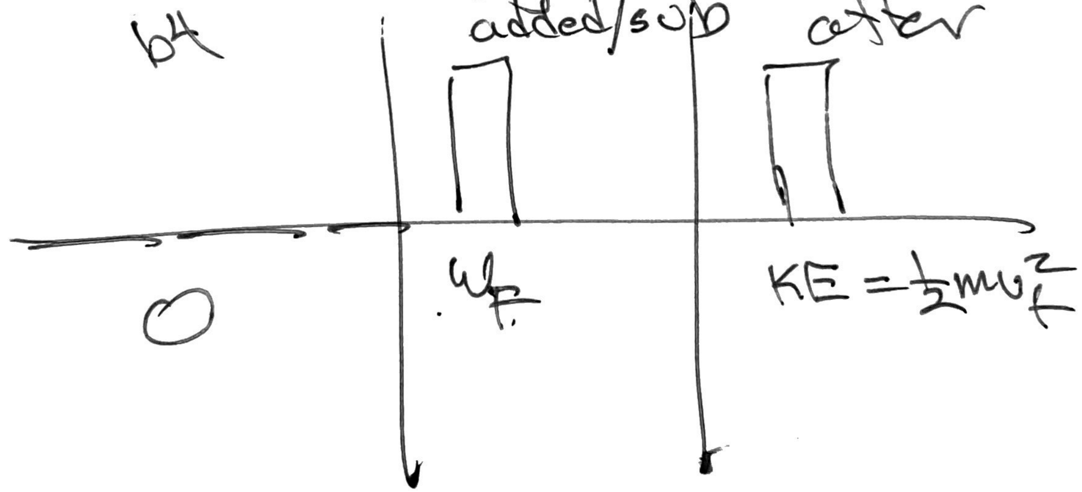
Given



gas $P_0 = 1 \cdot 10^5$ N/m2 (atmospheric pressure)

Assump No friction, no gravity

Strategy Work Energy Bar chart.



$$0 + W_F = \frac{1}{2}mv_f^2$$

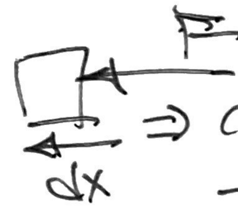
Estimate $F = P\cdot A = 1\cdot 10^5\, N/m^2 \cdot \pi(.1)^2 = \frac{10^5\pi}{100} = \pi\cdot 10^3\, N$

W_F for F = cnst $= F\Delta x = \pi\cdot 10^3\, N \cdot \frac{1}{2} m = 1.5\, kJ$

$\frac{1}{2} m v^2 = 1.5\, kJ \Rightarrow v^2 = \frac{3\, kJ}{8\, kg} = .375\cdot 10^3 \frac{m^2}{s^2} = 375\, m^2/s^2$

$\Rightarrow v_f \cong 20\, m/s$, expect $v_f > 20\, m/s$ since F is greater!

Soln: $\bar{F} \neq$ cnst $\Rightarrow W_F = \int \bar{F}\cdot d\bar{x} = \int F\cos\theta\, dx$



$\Rightarrow \cos\theta = 1$

$W_F = \int_0 F\,dx$ — find F!!

$PA = F$ $\quad PV = \text{cnst} = 1\cdot 10^5 \frac{N}{m^2}\cdot\overbrace{\left(1m\cdot\frac{\pi}{100}m^2\right)}^{L\cdot A}$

$\Rightarrow \text{cnst} = \pi\cdot 10^3\, Nm = J$

$$\Rightarrow F = P\cdot A = \left(\frac{\pi\cdot 10^3 J}{V}\cdot\frac{\pi}{100}m^2\right) = \left(\frac{\pi\cdot 10^3 J\cdot\cancel{\frac{\pi}{100}m^2}}{(1m - x)\cdot\cancel{\frac{\pi}{100}m^2}}\right)$$

Soln: cont

$$W_F = \int F\,dx = \int_0^{.5} \frac{\pi \cdot 10^3 J}{(1-x)}\,dx$$

$$W_F = \pi \cdot 10^3 J \int \frac{dx}{1-x} \qquad u\text{-sub} \quad u = 1-x \quad du = -dx$$

$$\Rightarrow \int \frac{dx}{1-x} = \int -\frac{du}{u} = -\ln(u)$$

$$= \pi \cdot 10^3 J \left[-\ln(1-x) \Big|_0^{.5} \right] = \pi \cdot 10^3 J \Big[\underbrace{-\ln .5}_{+.693} + \underbrace{\ln 1}_{0} \Big]$$

$$W_F = \pi \cdot 10^3 J\,(.693) = 2.18 \cdot 10^3 J = \tfrac{1}{2} m v_f^2$$

$$\frac{2 \cdot 2.18 \cdot 10^3 J}{8\,kg} = v_f^2 = 545\,m^2/s^2 \Rightarrow \boxed{v = 23.3\,m/s}$$

Discussion: Impressive speed — 40 mph! feels consistent w/ estimate. Messy to set up!