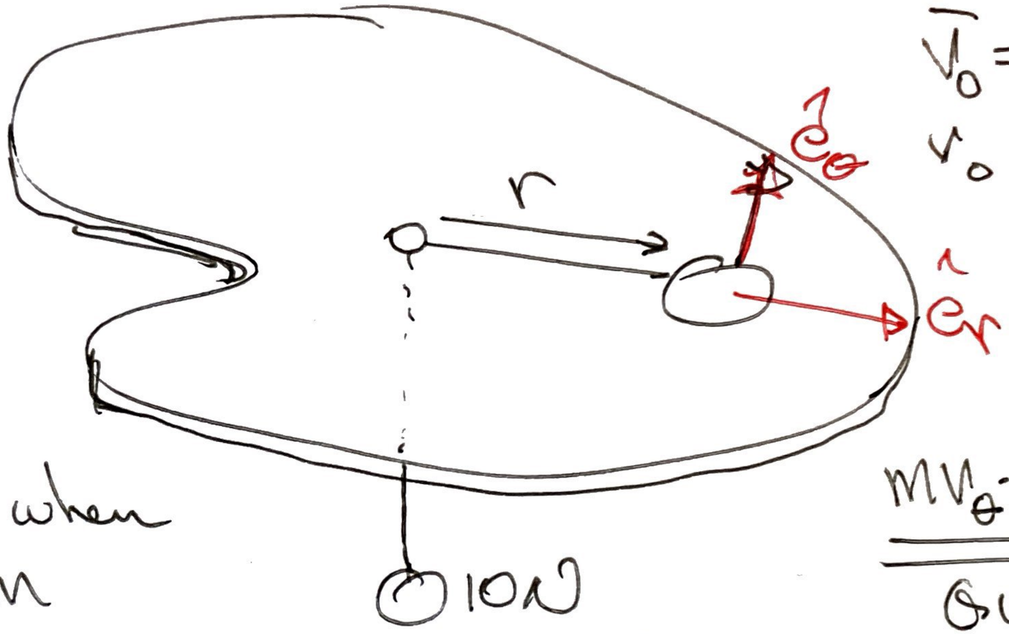
Given:



$\bar{V}_0 = 8\,\hat{e}_\theta$

$r_0 = 1\text{ m}$

Req'd find v when $r = 2\text{ m}$

$m v_\theta \cdot r = \text{cnst}$

Given.

Assump frictionless + $m v_\theta r = \text{cnst}$

Strategy Energy bar chart + math!

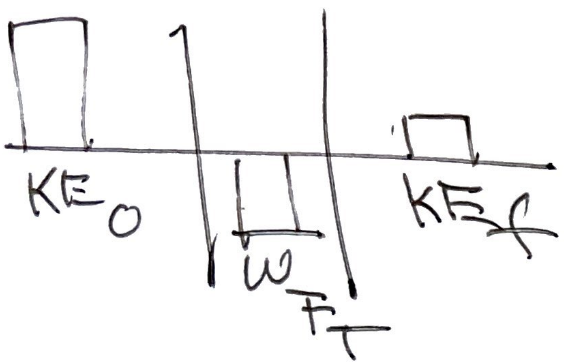
Estimate initial energy $= \frac{1}{2} m v_0^2 = \frac{1}{2}(1)(8^2) = 32\text{ J}$

$10\text{N}\cdot 1\text{m}$ removed $= -10\text{ J} \Rightarrow$ energy final $= 22\text{ J}$

$22\text{ J} = \frac{1}{2} m v_f^2 \Rightarrow 44 = v_f^2 \therefore |v_f| \cong 6.2\text{ m/s}$ but $v_r = ?$ $v_\theta = ?$

both cmp $< 6\text{ m/s}$

Soln: Bar Chart



$$\Rightarrow \tfrac{1}{2} m v_0^2 - W_{F_T} = \tfrac{1}{2} m v_f^2$$

$$W_{F_T} = F_T \cos\theta \Delta x \quad (\text{cnst } F)$$

$$= 10N(\cos 180^\circ) 1m = -10J$$

$$\vec{v}_0 = 8 \hat{e}_\theta \, m/s \Rightarrow v_0^2 = 64 \, m^2/s^2$$

$$\vec{v}_f = v_{r_f} \hat{e}_r + v_{\theta_f} \hat{e}_\theta$$

$$\Rightarrow v_f^2 = v_{r_f}^2 + v_{\theta_f}^2 \quad \text{but 1.}$$

1 eqn but 2 unknowns

$$m v_\theta r = \text{cnst} = 1kg \cdot 8 \, m/s \cdot 1m = 8 J\cdot s$$

$$\Rightarrow m v_{\theta_f} 2m = 8J\cdot s \Rightarrow v_{\theta_f} = \frac{8J\cdot s}{1kg \cdot 2m} = 4 \, m/s$$

Now only 1 unknown!

Soln: cont

$$\frac{1}{2}mv_0^2 - 10J = \frac{1}{2}m(v_{r_f}^2 + v_{\theta_f}^2)$$

$$\underbrace{\frac{1}{2}(1\,kg)64\frac{m^2}{s^2}}_{32J} - 10J = \frac{1}{2}(1\,kg)(v_{r_f}^2 + (4\,m/s)^2)$$

$$22J = \frac{1}{2}v_{r_f}^2 + 8\,kg\,m^2/s^2 \to J$$

$$\frac{28J}{1\,kg} = v_{r_f}^2 = 28\,m^2/s^2 \Rightarrow \underline{v_{r_f} = 5.29\,m/s}$$

$$\Rightarrow \underline{\bar{v}_f = 5.29\,m/s\,\hat{e}_r + 4\,m/s\,\hat{e}_\theta} \quad \underline{|v_f| = 6.63\,m/s < 8\,m/s}$$

Discussion: all works reasonably (and matches answer)

Need the conservation of angular momentum clue. Next week!