7. Hafta Cuma Dersi

17 Kasım 2023 Cuma 14:28

Önermeler :

* $A \subseteq B \Rightarrow \forall x,\ x \in A \Rightarrow x \in B$

$x \in A$ olsun.

$\Rightarrow x \ldots P(x) \ldots \Rightarrow Q(x) \Rightarrow x \in B$ ■

* $A = B$ $(\subseteq)$: $(\supseteq)$:

* $A \not\subseteq B \Rightarrow \exists x : x \in A \wedge x \notin B$

$A = \{x \in E : P(x)\}$ (önerme şartı) $B = \{x \in E : Q(x)\}$ (önerme şartı)

$16n+5$ $-12+12$ $16n-7$

İşlemler :

$A \cap B = \{x \in E : x \in A \wedge x \in B\}$

$A \cup B = \{x \in E : x \in A \vee x \in B\}$

$A - B = \{x \in E : x \in A \wedge x \notin B\}$

$A^c = \{x \in E : x \notin A\}$

$A \times B = \{(x,y) : \forall x \in A, \forall y \in B\}$

Birleşim: $\bigcup_{i=1}^{n} A_i = \{x \in E \mid x \in A_i, \exists i \in \{1,..,n\}\}$
$= A_1 \cup A_2 \cup \ldots \cup A_n$

Kesişim: $\bigcap_{i=1}^{n} A_i = \{x \in E \mid x \in A_i, \forall i \in \{1,..,n\}\}$
$= A_1 \cap A_2 \cap \ldots \cap A_n$

örn $A \cup B \cup C = \{x \in E : x \in A \vee x \in B \vee x \in C\}$

örn $A \cap B \cap C = \{x \in E : x \in A \wedge x \in B \wedge x \in C\}$

Örnek $i \in \mathbb{Z}^+$

- $A_i = \{x \in R \mid -\frac{1}{i} < x < \frac{1}{i}\}$ kümeleri tanımlanıyor.

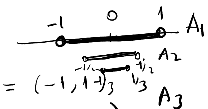
a) $\bigcup_{i=1}^{3} A_i = ?$ b) $\bigcap_{i=1}^{3} A_i = ?$ A_3

$\bigcup_{i=1}^{3} A_i = A_1 \cup A_2 \cup A_3 = A_1$

$A_1 = \{x \in \mathbb{R} : -\frac{1}{1} < x < \frac{1}{1}\}$ $= (-1,1)$

$A_2 = \{x \in \mathbb{R} : -\frac{1}{2} < x < \frac{1}{2}\} = (-1/2, 1/2)$

$A_3 = \{x \in \mathbb{R} : -1/3 < x < 1/3\} = (-1/3, 1/3)$



Ayrık Küme: A ve B ayrık kümelerdir $\Leftrightarrow A \cap B = \emptyset$

Karşılıklı Ayrık Kümeler:
$A_1, A_2, \ldots, A_n$ karşılıklı ayrıktır $\Leftrightarrow A_i \cap A_j = \emptyset, \forall i \neq j.$

Bölmelenme(Partition):
$[A_1, A_2, \ldots, A_n]$, A kümesinin bir bölmelenmesidir $\Leftrightarrow$
$(A_1, A_2, \ldots, A_n$ karşılıklı ayrıktır $\wedge \bigcup_{i=1}^{n} A_i = A)$ ✓ ✓

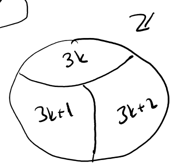
A, B, C karşılıklı ayrık kümeler $\Leftrightarrow$ $A \cap B = \emptyset$, $A \cap C = \emptyset$, $B \cap C = \emptyset$

$\{A, B, C\}$, D için bir bölmelenme $\Leftrightarrow$ A, B, C karşılıklı ayrık $\wedge$ $A \cup B \cup C = D$

örn Bölüm - Kalan 3'e bölünebilme $n \in \mathbb{Z}$, $n = 3k$ (A) $n = 3k+1$ (B) $n = 3k+2$ (C)

$A \cap B = \emptyset$
$B \cap C = \emptyset$
$A \cap C = \emptyset$

$A \cup B \cup C = \mathbb{Z}$ ✓



A, B, C, $\mathbb{Z}$ için bir bölmelenme (partition) belirtir.

4'e bölünebilme $n = 4k$ $n = 4k+1$ $n = 4k+2$ $n = 4k+3$

$\{A_1, A_2, A_3, A_4\} \rightarrow \mathbb{Z}$ için başka bir bölmelenmedir.

A_1 A_2 A_3 A_4

Kuvvet Kümesi: $\wp(A)$ = A 'nın tüm altkümelerinin kümesi

$$|\wp(A)| = 2^n \qquad n = |A|$$

$A = \{1,2,3\}$ $\quad |A| = 3$ ($s(A)$)

$\wp(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Theorem 6.2.2 Set Identities

Let all sets referred to below be subsets of a universal set U.

1. *Commutative Laws:* For all sets A and B, (Değişme)
   (a) $A \cup B = B \cup A$ and (b) $A \cap B = B \cap A$.
2. *Associative Laws:* For all sets A, B, and C, (Birleşme)
   (a) $(A \cup B) \cup C = A \cup (B \cup C)$ and
   (b) $(A \cap B) \cap C = A \cap (B \cap C)$.
3. *Distributive Laws:* For all sets, A, B, and C, (Dağılma)
   (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and
   (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
4. *Identity Laws:* For all sets A, (Birim (etkisiz) eleman)
   (a) $A \cup \emptyset = A$ and (b) $A \cap U = A$. (U: evrensel küme)
5. *Complement Laws:*
   (a) $A \cup A^c = U$ and (b) $A \cap A^c = \emptyset$.
6. *Double Complement Law:* For all sets A,
   $(A^c)^c = A$. ✓
7. *Idempotent Laws:* For all sets A,
   (a) $A \cup A = A$ and (b) $A \cap A = A$.
8. *Universal Bound Laws:* For all sets A,
   (a) $A \cup U = U$ and (b) $A \cap \emptyset = \emptyset$.
9. *De Morgan's Laws:* For all sets A and B,
   (a) $(A \cup B)^c = A^c \cap B^c$ and (b) $(A \cap B)^c = A^c \cup B^c$.
10. *Absorption Laws:* For all sets A and B,
    (a) $A \cup (A \cap B) = A$ and (b) $A \cap (A \cup B) = A$.
11. *Complements of U and $\emptyset$:*
    (a) $U^c = \emptyset$ and (b) $\emptyset^c = U$.
12. *Set Difference Law:* For all sets A and B,
    $A - B = A \cap B^c$.

$(A \cup B)^c = A^c \cap B^c$ ispatlayınız.

İspat: $(\subseteq)$: $(A\cup B)^c \subseteq A^c \cap B^c$

$x \in (A \cup B)^c$ olsun.

$\Rightarrow x \notin A \cup B$ ($\sim(x \in A \vee x \in B)$)

$\Rightarrow x \notin A \wedge x \notin B$

$\Rightarrow x \in A^c \wedge x \in B^c$

$\Rightarrow x \in A^c \cap B^c$.

$(\supseteq)$: $A^c \cap B^c \subseteq (A \cup B)^c$ ($A \cup B$: $x \in A \vee x \in B$)

$x \in A^c \cap B^c$ olsun.

$\Rightarrow x \in A^c \wedge x \in B^c$

$\Rightarrow x \notin A \wedge x \notin B$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \in (A \cup B)^c$.

Sonuç olarak, $(A \cup B)^c = A^c \cap B^c$

Örn/ $(A \cap B)^c = A^c \cup B^c$ ispatlayınız.

$(\subseteq)$: $x \in (A \cap B)^c$ olsun.

$\Rightarrow x \notin A \cap B \rightarrow \sim(x \in A \wedge x \in B)$

$\Rightarrow x \notin A \vee x \notin B$

1.durum: $x \notin A$ olsun. $\Rightarrow x \in A^c \Rightarrow x \in A^c \cup B^c$

2.durum: $x \notin B$ olsun $\Rightarrow x \in B^c \Rightarrow x \in A^c \cup B^c$

$\Rightarrow x \in A^c \cup B^c$ ✓

$(\supseteq)$: $x \in A^c \cup B^c$ olsun.

$\Rightarrow x \in A^c \vee x \in B^c$

1. durum: $x \in A^c$ olsun.

$\Rightarrow x \notin A \Rightarrow x \notin A \cap B$

$\Rightarrow x \in (A \cap B)^c$ ✓

2.durum: $x \in B^c$ olsun

$\Rightarrow x \notin B \Rightarrow x \notin A \cap B$

$\Rightarrow x \in (A \cap B)^c$ ✓

$\Rightarrow x \in (A \cap B)^c$.

$\sim = \emptyset \rightarrow$ çelişki yöntemi

$B = \dots$
$C = \dots$

$B = \{y \in \mathbf{Z} \mid y = 10b - 3$ for some integer $b\}$, and
$C = \{z \in \mathbf{Z} \mid z = 10c + 7$ for some integer $c\}$.

$C \subseteq B$

$x \in C$ olsun $\Rightarrow x = 10k + 7, \exists k \in \mathbb{Z}$

$\Rightarrow x = 10k + \underbrace{7 - 10}_{-3} + 10$

$\Rightarrow x = 10\underbrace{(k+1)}_{\in \mathbb{Z}} - 3$

$B \subseteq C$

$x \in B$ olsun.

$\Rightarrow x = 10k - 3, \exists k \in \mathbb{Z}$

$\Rightarrow x = 10k \underbrace{- 3 + 10}_{+7} - 10$

$\Rightarrow x = 10k + 7 - 10$

$\Rightarrow x = 10\underbrace{(k-1)}_{\in \mathbb{Z}} + 7 \Rightarrow x \in C$.