8th Week

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$A \subseteq B$: assume / show: $\forall x,\ x \in A \Rightarrow x \in B$ (left / right)

$A = B$: $A \subseteq B \ \wedge\ B \subseteq A$

?
- is it subset relation prob? ✓
- is it equality of sets ✓?
- is it a conditional statement? → direct proof / contrapositive / contradiction depending on the problem.
- is it an empty set prob? → contradiction

if $x \in A \cap B \Rightarrow x \in A \wedge x \in B$

if $x \in A \cup B \Rightarrow x \in A \vee x \in B$

if $(x,y) \in A \times B \Rightarrow x \in A \wedge y \in B$

#6.22 $\forall$ sets A, B show that $(A-B) \cup (A \cap B) \subseteq A$

$(\forall x,\ \underbrace{x \in (A-B) \cup (A \cap B)}_{\text{assume this}} \Rightarrow \underbrace{x \in A}_{\text{try to show this!}})$

Proof: Let $x \in (A-B) \cup (A \cap B)$.

$\Rightarrow x \in (A-B) \vee x \in (A \cap B)$

$\Rightarrow x \in A \wedge x \notin B \ \vee\ x \in A \wedge x \in B \Rightarrow x \in A \wedge (x \notin B \vee x \in B)$

$\Rightarrow x \in A.$ ∎

#6.19 ---- show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

equality ✓ $\subseteq$ $\supseteq$

Proof: $(\subseteq)$: Let $(m,n) \in A \times (B \cap C)$.

$\Rightarrow m \in A \ \wedge\ n \in (B \cap C)$

$\Rightarrow m \in A \ \wedge\ (n \in B \wedge n \in C)$

$\Rightarrow m \in A \wedge n \in B \ \wedge\ m \in A \wedge n \in C$

$\Rightarrow (m,n) \in A \times B \ \wedge\ (m,n) \in A \times C$

$\Rightarrow (m,n) \in (A \times B) \cap (A \times C)$.

$(m,n) \in A \times B$ $\wedge$ $(m,n) \in A \times C$

$(\supseteq)$: Let $(m,n) \in (A \times B) \cap (A \times C)$.

$\Rightarrow (m,n) \in A \times B \ \wedge\ (m,n) \in A \times C$

$\Rightarrow m \in A \wedge n \in B \ \wedge\ m \in A \wedge n \in C$

$\Rightarrow m \in A \ \wedge (n \in B \wedge n \in C)$

$\Rightarrow m \in A \ \wedge\ (n \in B \cap C)$

$\Rightarrow (m,n) \in A \times (B \cap C)$.

$\Rightarrow \therefore A \times (B \cap C) = (A \times B) \cap (A \times C)$ ∎

$(A - C) \cap (B - C) \cap (A - B) = \emptyset.$

Proof (contradiction) Let $x \in (A-C) \cap (B-C) \cap (A-B)$

$\Rightarrow x \in (A-C) \ \wedge\ x \in (B-C) \ \wedge\ x \in (A-B)$

$\Rightarrow (x \in A \wedge x \notin C) \ \wedge\ (\underset{*}{x \in B} \wedge x \notin C) \ \wedge\ (x \in A \wedge \underset{**}{x \notin B})$

$\Rightarrow$ (*), (**) contradicts! ※

$\therefore (A-C) \cap (B-C) \cap (A-B) = \emptyset$ ∎

#6. if $(B \cap C) \subseteq A \Rightarrow (C-A) \cap (B-A) = \emptyset$ $\quad p \Rightarrow q$

Assume that $\underbrace{\forall x,\ (x \in (B \cap C) \Rightarrow x \in A)}_{p\ (*)}$ and let $\underbrace{y \in (C-A) \cap (B-A)}_{q}$

$\Rightarrow y \in (C-A) \ \wedge\ y \in (B-A)$

Assume that $\forall x\ (x \in (B \cap C) \Rightarrow x \in A)$ (*) and let $y \in (C-A) \cap (B-A)$

$\Rightarrow y \in (C-A) \ \wedge \ y \in (B-A)$

$\Rightarrow (y \in C \wedge y \notin A) \ \wedge \ (y \in B \wedge y \notin A)$

$\Rightarrow y \in B \cap C \ \wedge \ y \notin A$ *

$\Rightarrow y \in A$ **

$\times$

Proof:

$A \subseteq B \Rightarrow B^c \subseteq A^c$

Let $A \subseteq B$.

$\Rightarrow \forall x,\ x \in A \Rightarrow x \in B$.

contrapositive

$\Rightarrow \forall x,\ x \notin B \Rightarrow x \notin A$.

$\Rightarrow \forall x,\ x \in B^c \Rightarrow x \in A^c$

$\Rightarrow B^c \subseteq A^c$ ∎

Ex

$\underset{p}{A \subseteq B} \ \checkmark \Rightarrow \underset{q}{A \cap (B \cap C)^c = \emptyset}$

$p \wedge \sim q$

proof:

Assume that $(\forall x,\ x \in A \Rightarrow x \in B)$ $\wedge$ let $y \in A \cap (B \cap C)^c$.

$\Rightarrow y \in A \ \wedge \ y \notin (B \cap C)$

$\Rightarrow y \in B \ \wedge \ y \notin (B \cap C)$

if $y \notin C$

then we can not reach to a contradiction.

Counter Example

$A = \{1, 3\}$ $A \subseteq B$ ✓

$B = \{1, 2, 3\}$

$C = \{1, 4, 5\}$

$(B \cap C)^c = \{2, 3, 4, 5\}$

$A \cap (B \cap C)^c = \{3\}$

$\neq \emptyset$