

# 10. Hafta Cuma Dersi

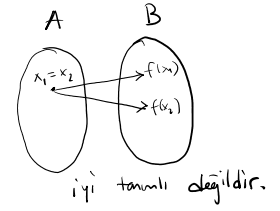
8 Aralık 2023 Cuma 14:28

$$f: A \rightarrow B$$

tanım      görüntü

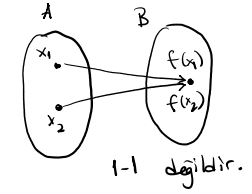
**İyi-tanımlılık:**  $\forall x_1, x_2 \in A, \quad x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$

**değili:**  $\exists x_1, x_2 \in A, \quad x_1 = x_2 \wedge f(x_1) \neq f(x_2)$



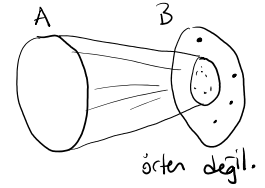
**Birebirlik:**  $\forall x_1, x_2 \in A, \quad f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

**değili:**  $\exists x_1, x_2 \in A, \quad f(x_1) = f(x_2) \wedge x_1 \neq x_2$



**Örtenlik:**  $\forall y \in B \text{ için, } \exists x \in A : f(x) = y$

**değili:**  $\exists y \in B : \forall x \in A, f(x) \neq y$



**örn**  $f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = \frac{x}{x^2 + 1}$

$f$  1-1 midir?  $f$  örten midir?

$f$  1-1 mi?  $x_1, x_2 \in \mathbb{R} \quad f(x_1) = f(x_2) \text{ olsun.}$

$$\Rightarrow \frac{x_1}{x_1^2 + 1} = \frac{x_2}{x_2^2 + 1} \Rightarrow x_1(x_2^2 + 1) = x_2(x_1^2 + 1)$$

$$\Rightarrow x_1 x_2^2 + x_1 = x_2 x_1^2 + x_2$$

$$\Rightarrow x_1 - x_2 = x_2 x_1^2 - x_1 x_2^2$$

$$\Rightarrow (x_1 - x_2) = x_1 x_2 (x_1 - x_2)$$

$x_1 x_2 = 1$  ters örneği bende ara

$\Rightarrow x_1 - x_2$  diyorduk.

ters örnekle

$$f(x_1) = f(x_2) \wedge x_1 \neq x_2$$

$$x_1 = 2 \quad x_2 = \frac{1}{2} \quad x_1 \neq x_2 \checkmark$$

$$f(x_1) = f(2) = \frac{2}{2^2 + 1} = \frac{2}{5} = \checkmark$$

$$f(x_2) = f\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{\left(\frac{1}{2}\right)^2 + 1} = \frac{\frac{1}{2}}{\frac{1}{4} + 1} = \frac{\frac{1}{2}}{\frac{5}{4}} = \frac{2}{5}$$

$\Rightarrow f$  1-1 değildir.

$f$  örten midir?

$$\forall y \in \mathbb{R} \quad \exists x \in \mathbb{R} \quad f(x) = y$$

$f$  örten değil.

$$\exists y \in \mathbb{R}, \quad \forall x \in \mathbb{R} \quad f(x) \neq y$$

Ters örnekle

$$y = 1$$

$$f(x) = 1$$

$$\frac{x}{x^2 + 1} = 1$$

$$ax^2 + bx + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$\Delta < 0 \text{ ise reel kök yok.}$$

$$\Delta = b^2 - 4ac$$

$$\forall y \in \mathbb{R}$$

$$f(x) = y$$

$$\frac{x}{x^2 + 1} = y$$

$$x = yx^2 + y$$

$$yx^2 - x + y = 0$$

$$\Delta = (-1)^2 - 4 \cdot y \cdot y = 1 - 4y^2$$

$$1 - 4y^2 \geq 0$$

$$1 - 4y^2 < 0$$

ters örneği bende ara.

$$\Rightarrow x = x^2 + 1 \Rightarrow x^2 - x + 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{\Delta}}{2} \quad \Delta = 1 - 4 = -3 < 0$$

$x \notin \mathbb{R}$ .

$\Rightarrow f$  örten değil.

15.  $f(x) = \frac{x+1}{x}$ , for all real numbers  $x \neq 0 \rightarrow$

16.  $f(x) = \frac{x}{x^2+1}$ , for all real numbers  $x \checkmark$

17.  $f(x) = \frac{3x-1}{x}$ , for all real numbers  $x \neq 0 \rightarrow$

18.  $f(x) = \frac{x+1}{x-1}$ , for all real numbers  $x \neq 1 \checkmark$

$$f(x) = x \pmod{1}$$

$15 \neq 1 \quad f(15) = f(1)$   
 $\rightarrow \quad (1-1)$

1-1 - örten : hem 1-1  $\wedge$  hem örten  
 bijective injective surjective

Ters Fonksiyon

$f^{-1}$  iyi tanımlı olması için  
 $f$ , 1-1 örten olmalıdır

$f \circ g$  :

$$\frac{f(g(x))}{g: A \rightarrow B \quad f: B \rightarrow C}$$

$$f \circ g: A \rightarrow C$$

Öm  $f$  1-1 ve  $g$  1-1 ise  $f \circ g$  1-1 midir pozitif

ispat.  $f \circ g(x_1) = f \circ g(x_2)$  olsun.

$$\Rightarrow f(g(x_1)) = f(g(x_2))$$

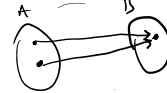
$$x_1 = x_2$$

$$f: A \rightarrow B$$

$$f(x) = y$$

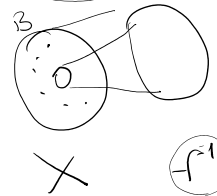
$f$  örten değil

$f$  1-1 değil

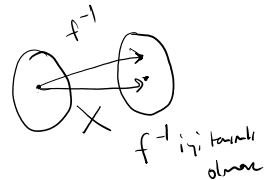


$$f^{-1}: B \rightarrow A$$

$$f^{-1}(y) = x$$



$f^{-1}$  fonksiyon olmaz.



$f^{-1}$  iyi tanımlı olmaz

$$\Rightarrow \underbrace{f}_{1-1}(\underbrace{g(x_1)}) = \underbrace{f}_{1-1}(\underbrace{g(x_2)})$$

$\underbrace{f}_{1-1}$

$$\Rightarrow \underline{g(x_1)} = \underline{g(x_2)}$$

$$\Rightarrow \underbrace{g}_{1-1}$$

$$\boxed{x_1 = x_2} \checkmark$$

$$\Rightarrow f \circ g \text{ 1-1 'dir.}$$