1st Week Wednesday

24 Şubat 2021 Çarşamba 12:42

- * Exchange two eqns. $i \leftrightarrow j$
- * Multiply an eqn. with a constant $c.i \to i$
- * Multiply another eqn with a constant and add this to an eqn. $c.j + \boxed{i \to i}$

Definition A system is said to be in **strict triangular form** if, in the kth equation, the coefficients of the first $k-1$ variables are all zero and the coefficient of x_k is nonzero $(k = 1, \ldots, n)$.

$3x_1 + 8 + 2 = 1$

$$\begin{aligned} 3x_1 + 2x_2 + x_3 &= 1 \\ x_2 - 2x_3 &= 2 \\ 2x_3 &= 4 \end{aligned}$$

$x_1 = -2$

$x_2 = 4$

$x_3 = 2$

Ex

(a)
$$\begin{aligned} 3x_1 + 2x_2 - x_3 &= -2 \\ x_2 &= 3 \checkmark \\ 2x_3 &= 4 \end{aligned}$$

$x_3 = 2$

$3x_1 + 6 - 2 = -2$

$3x_1 + 4 = -2$

$x_1 = -2$

$(x_1, x_2, x_3) = (-2, 3, 2)$

(b)
$$\begin{aligned} 3x_1 + 2x_2 - x_3 &= -2 \\ -3x_1 - x_2 + x_3 &= 5 \\ 3x_1 + 2x_2 + x_3 &= 2 \end{aligned}$$

$\cancel{3x_1} + 2x_2 - \cancel{x_3} = -2$

$+ \; -\cancel{3x_1} - x_2 + \cancel{x_3} = 5$

$\boxed{x_2 = 3}$

$-\cancel{3x_1} - 3 + x_3 = 5$

$+ \; \cancel{3x_1} + 6 + x_3 = 2$

$3 + 2x_3 = 7 \Rightarrow \boxed{x_3 = 2}$

$3x_1 + 2.3 - \cancel{2} = -\cancel{2}$

$\boxed{x_1 = -2}$

$(x_1, x_2, x_3) = (-2, 3, 2)$

* Same variables + same solutions → Equivalent Systems

n columns

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

row

m rows

column

$m \times n$

matrix

System of Linear Eqns ⟷ Matrices

Augmented Matrix

$$\begin{aligned} \text{1st eqn} \to \quad & a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ \text{2nd eqn} \to \quad & a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ & \vdots \\ m\text{th eqn} \to \quad & a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{aligned}$$

$$\begin{array}{l} \\ \text{1st eqn} \to \\ \\ \\ m^{\text{th}}\text{ eqn} \to \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & & x_n & \text{result} \end{array} \\ \left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \ddots & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right] \end{array}$$

m eqns. → m rows

n variables → n+1 columns

T

m eqns. $\longrightarrow$ m rows

n variables $\longrightarrow$ $n+1$ columns

Another representation

matrix multiplication

$$
\begin{array}{c}
\\ \\ m^{th}\text{ eq} \rightarrow
\end{array}
\underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}}_{\text{Coefficient Matrix}}
\underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}}_{\text{column matrix of variables}}
=
\underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}_{m\times 1}}_{\text{column matrix of results}}
$$

$$\underbrace{Ax = b}_{\text{general}}$$ $\longrightarrow$ represents our system of linear eqns.

Homogeneous Systems: $Ax = 0$

$$
\begin{aligned}
a_{11}x_1 + \cdots + a_{1n}x_n &= 0 \\
a_{21}x_1 + \cdots + a_{2n}x_n &= 0 \\
&\vdots \\
a_{m1}x_1 + \cdots + a_{mn}x_n &= 0
\end{aligned}
$$

$(x_1, x_2, \ldots, x_n) = (0, 0, \ldots, 0)$

$\longrightarrow$ The Trivial Solution

* 1 solution (Trivial Solution)
- ~~no solution~~
* infinitely many solutions (trivial soln, ...)

EXAMPLE 4 Solve the system

$$
\begin{aligned}
-1x_2 - 1x_3 + x_4 &= 0 \\
x_1 + x_2 + x_3 + x_4 &= 6 \\
2x_1 + 4x_2 + x_3 - 2x_4 &= -1 \\
3x_1 + x_2 - 2x_3 + 2x_4 &= 3
\end{aligned}
$$

4 eqns, 4 variables

Augmented Matrix $\longrightarrow$ 4×5

$$
\begin{array}{c}
\\ \text{1st eqn} \rightarrow \\ \text{2nd eqn} \rightarrow \\ \text{3rd eqn} \rightarrow \\ \text{4th eqn} \rightarrow
\end{array}
\begin{array}{c}
\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & b \end{array} \\
\left[\begin{array}{cccc|c} 0 & -1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 6 \\ 2 & 4 & 1 & -2 & -1 \\ 3 & 1 & -2 & 2 & 3 \end{array}\right]_{4\times 5}
\end{array}
$$

$r_1 \leftrightarrow r_2$

Apply Elementary Row Operations

* $r_i \leftrightarrow r_j$
* $c.r_i \longrightarrow r_i$
* $c.r_j + \boxed{r_i \longrightarrow r_i}$!