6. Hafta Çarşamba Dersi

31 Mart 2021 Çarşamba 08:32

VEKTÖR UZAYLARI

$\dots -3, -2, -1, 0, 1, 2, \dots$

$\mathbb{Z}$ $+, \cdot$ $\mathbb{Q}$ $\mathbb{R}$

$v_1, v_2, \dots$

$(v_1, v_2, \dots, v_n)$

$(\mathbb{R}^n, \oplus, \odot)$ vektörel toplama, skaler çarpma

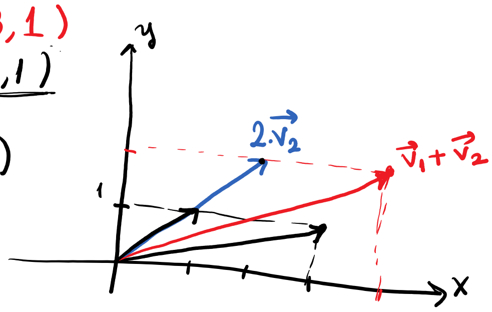
$(3,1) + (1,1) = (4,2)$

$v_1 = \langle 3, 1 \rangle \rightarrow$ webwork

Ör: $\mathbb{R}^2$ $\vec{v_1} = (3,1)$ $\vec{v_2} = (1,1)$

$2.\vec{v_2} = (2,2)$

skaler $\in \mathbb{R}$, vektör



$(V, +, \cdot)$ $r \in \mathbb{R}$ ✓ $\vec{v_1} + \vec{v_2} \in V$ ✓ $r\vec{v_1} \in V$ → kapalılık

V $\oplus \rightarrow$ $\odot \rightarrow$

$\mathbb{R}^2$ $(3,1)$ $(1,2)$

1) $\vec{v_1} + \vec{v_2} = \vec{v_2} + \vec{v_1}$ (+ değişme)

2) $(\vec{v_1} + \vec{v_2}) + \vec{v_3} = \vec{v_1} + (\vec{v_2} + \vec{v_3})$ (+ birleşme)

$\vec{0} = (0,0)$

→ 3) $\vec{0} \in V$ ✓ $\vec{v_1} + \vec{0} = \vec{0} + \vec{v_1} = \vec{v_1}$ (+ etkisiz (birim eleman)) 0

→ 4) $\forall \vec{v_1} \in V$ $-\vec{v_1} \in V$ $\vec{v_1} + (-\vec{v_1}) = \vec{0}$ (+ ters eleman)

5) $r \in \mathbb{R}$ $r(\vec{v_1} + \vec{v_2}) = r\vec{v_1} + r\vec{v_2}$ (skaler çarpmanın toplama üzerine dağılma özelliği)

$\mathbb{R}^2, +, \cdot$

6) $r, s \in \mathbb{R}$ $(r+s).\vec{v_1} = r\vec{v_1} + s\vec{v_2}$

$3, 5 \in \mathbb{R}$ $15(1,2) = (15,30)$ ✓

7) $r, s \in \mathbb{R}$ $(r.s).\vec{v_1} = r.(s.\vec{v_1})$

$5(1,2) = (5,10)$ $3(5,10) = (15,30)$

8) $1 \in \mathbb{R}$ $1.\vec{v_1} = \vec{v_1}$ (skaler çarpmanın birim elemanı)

⇒ $(V, \underline{+}, \underline{\cdot})$ $\mathbb{R}$ üzerinde vektör uzayıdır, denir.

$5(1,2) = (5,10)$

10. Let S be the set of all ordered pairs of real numbers. Define scalar multiplication and addition on S by

S $\alpha \in \mathbb{R}$

$$\alpha(x_1, x_2) = (\alpha x_1, \alpha x_2) \checkmark$$

$$\boxed{(x_1, x_2) \oplus (y_1, y_2) = (x_1 + y_1, 0)}$$

S bir vektör uzayı mıdır?

$(x_1, x_2) \oplus (0,0) = (x_1 + 0, 0) = (x_1, 0)$

3. özellik sağlanmıyor.

Ör: 1, 2, 3, 4, 5, 5, 5

Let R^+ denote the set of positive real numbers. Define the operation of scalar multiplication, denoted $\circ$, by

$3 \oplus 1 = 3.1 = 3$

$$\boxed{\alpha \circ x = x^\alpha} \rightarrow \text{skaler çarpma}$$

for each $x \in R^+$ and for any real number α. Define the operation of addition, denoted $\oplus$, by

1) $x \oplus y \stackrel{?}{=} y \oplus x$

$x.y = \checkmark \ y.x$

2) $x \oplus (y \oplus z) \stackrel{?}{=} (x \oplus y) \oplus z$ ✓

yz xy xyz

for each $x \in R^+$ and for any real number α. Define the operation of addition, denoted $\oplus$, by

$$x \oplus y = x \cdot y \quad \text{for all} \quad x, y \in R^+$$ → vektörel toplama

Thus, for this system, the scalar product of -3 times $\frac{1}{2}$ is given by

$$-3 \circ \frac{1}{2} = \left(\frac{1}{2}\right)^{-3} = 8$$

and the sum of 2 and 5 is given by

$$2 \oplus 5 = 2 \cdot 5 = 10$$

Is R^+ a vector space with these operations? Prove your answer.

2) $x \oplus (y \oplus z) \stackrel{?}{=} (x \oplus y) \oplus z$

$x\,yz = xy\,z$ ✓

3) $2 \oplus e = 2e = 2$ $\quad$ $e = 1$ ✓

4) $x \oplus y = xy = 1$

$\forall x \in \mathbb{R}^+$ için $y = 1/x \in \mathbb{R}^+$ ✓

5) $\underbrace{r(\vec{v_1} + \vec{v_2})}_{\text{sol taraf}} = r\vec{v_1} \oplus r\vec{v_2}$

$(v_1 v_2)^r \qquad v_1^r + v_2^r$

$r \in 2$ $\quad$ $v_1 = 3$ $\quad$ $v_2 = 5$ $\quad$ $3^2 \quad 5^2$

$3 \oplus 5 = 15$

$2 \circ 15 = 15^2$

sol taraf

$2 \circ 3 = 3^2 \qquad 2 \circ 5 = 5^2$

$3^2 + 5^2 = 3^2 . 5^2 = 15^2$ ✓

2, 4 ∈ skaler $\quad$ 5

✓ 6) $(r + s).\vec{v_1} = \overset{2}{r\vec{v_1}} + \overset{3}{s\vec{v_1}}$

$+ \; 5 \quad = 5^2 . 5^4$

$v_1^{r+s} = v_1^r \oplus v_1^s = v_1^{r+s}$

2, 4

✓ 7) $(r.s).\vec{v_1} = r.(s.\vec{v_1})$

$v_1^{rs} = (v_1^s)^r$

✓ 8) $r = ?$ $\quad$ $v_1^r = v_1$ $\quad$ $r = 1$