3rd Week

23 Ekim 2021 Cumartesi 20:45

10. Consider a linear system whose augmented matrix is of the form

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & a & b \end{array}\right] \to \text{REF}$$

→ **(a)** For what values of a and b will the system have infinitely many solutions? →

→ **(b)** For what values of a and b will the system be inconsistent? for → no solution

$a-5=0$ and $b-4\neq 0$

$\boxed{a=5 \text{ and } b\neq 4}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & a & b \end{array}\right] \begin{array}{l} -r_1+r_2\to r_2 \\ -r_1+r_3\to r_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & a-3 & b-2 \end{array}\right]$$

i) $r_i \leftrightarrow r_j$

ii) $c.r_i \to r_i \to$ leading 1

iii) $c.r_j + r_i \to r_i \to$ make 0

$-2r_2+r_3\to r_3$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & a-5 & b-4 \end{array}\right]$$

$-2+a-3$

$-2+b-2$

zero here. nonzero here

no solution

a) For inf. many soln. case, you should have at least one free variable.

$\Rightarrow a-5=0 \quad b-4=0 \quad \Rightarrow \boxed{a=5 \text{ and } b=4}$

c) For unique soln. case,

$\Rightarrow a-5\neq 0$ b can be anything but $a\neq 5$

ex

$a=8$ olsun.

$b=4$ olsun.

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 3\ 1 & 0 \end{array}\right]$$

$x_2+x_3=1$

$x_3=0$

$\frac{1}{3}r_3\to r_3$

$x_2=1$

$x_1+x_2+3x_3=2$

$\Rightarrow x_1=1$

$(1,1,0)$ → unique.

5. For each of the systems of equations that follow, use Gaussian elimination to obtain an equivalent system whose coefficient matrix is in row echelon form. Indicate whether the system is consistent. If the system is consistent and involves no free variables, use back substitution to find the unique solution. If the system is consistent and there are free variables, transform it to reduced row echelon form and find all solutions.

transform the aug. matrix into REF

(a) $x_1 - 2x_2 = 3$
$2x_1 - x_2 = 9$

(b) $2x_1 - 3x_2 = 5$
$-4x_1 + 6x_2 = 8$

(c) $x_1 + x_2 = 0$
$2x_1 + 3x_2 = 0$
$3x_1 - 2x_2 = 0$

(d) $3x_1 + 2x_2 - x_3 = 4$
$x_1 - 2x_2 + 2x_3 = 1$
$11x_1 + 2x_2 + x_3 = 14$

(e) $2x_1 + 3x_2 + x_3 = 1$
$x_1 + x_2 + x_3 = 3$
$3x_1 + 4x_2 + 2x_3 = 4$

(f) $x_1 - x_2 + 2x_3 = 4$
$2x_1 + 3x_2 - x_3 = 1$
$7x_1 + 3x_2 + 4x_3 = 7$

6. Use Gauss–Jordan reduction to solve each of the following systems.

transform the aug. matrix into RREF

(a) $x_1 + x_2 = -1$
$4x_1 - 3x_2 = 3$

(b) $x_1 + 3x_2 + x_3 + x_4 = 3$
$2x_1 - 2x_2 + x_3 + 2x_4 = 8$
$3x_1 + x_2 + 2x_3 - x_4 = -1$

(c) $x_1 + x_2 + x_3 = 0$
$x_1 - x_2 - x_3 = 0$

(d) $x_1 + x_2 + x_3 + x_4 = 0$
$2x_1 + x_2 - x_3 + 3x_4 = 0$
$x_1 - 2x_2 + x_3 + x_4 = 0$

→ homogeneous system

7. Give a geometric explanation of why a homogeneous linear system consisting of two equations in three unknowns must have infinitely many solutions. What are the possible numbers of solutions of a nonhomogeneous 2×3 linear system? Give a geometric explanation of your answer.

$Ax=0$

trivial soln. $(0,0,\dots,0)$ ← Homogeneous

may also have inf. many

$Ax=b$

Non-homogeneous

RREF

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & -1 & 3 & 0 \\ 1 & -2 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} -2r_1+r_2\to r_2 \\ -r_1+r_3\to r_3 \end{array} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & -3 & 1 & 0 \\ 0 & -3 & 0 & 0 & 0 \end{array}\right] \to$$

$$\xrightarrow{-1r_2\to r_2} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & -3 & 0 & 0 & 0 \end{array}\right] \xrightarrow{3r_2+r_3\to r_3} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 9 & -3 & 0 \end{array}\right]$$

$$\xrightarrow{\frac{1}{9}r_3\to r_3} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 1 & -1/3 & 0 \end{array}\right] \to \text{REF}$$

$c.r_j+r_i\to r_i$ . . .

r_3

$$\begin{array}{l} -3r_3+r_2\to r_2 \\ -1r_3+r_1\to r_1 \end{array} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 4/3 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1/3 & 0 \end{array}\right] \xrightarrow{-r_2+r_1\to r_1} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 4/3 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1/3 & 0 \end{array}\right] \to \text{RREF}$$

pivot!

$x_1 + \frac{4}{3}x_4 = 0$ $\quad \boxed{x_4 = r\in\mathbb{R}}$ free

$\boxed{x_2=0}$

$x_3 + \frac{-1}{3}x_4 = 0 \Rightarrow \boxed{x_3 = \frac{r}{3}}$

$\boxed{x_1 = -\frac{4r}{3}}$

Solution set $\to \left\{ \left(-\frac{4r}{3}, 0, \frac{r}{3}, r\right) : r\in\mathbb{R} \right\}$

inf. many solutions.

13. Given a homogeneous system of linear equations, if the system is overdetermined, what are the possibilities as to the number of solutions? Explain.

$m\geq n$

→ trivial soln ✓ inf. many ✓

14. Given a nonhomogeneous system of linear equations, if the system is underdetermined, what are the possibilities as to the number of solutions? Explain.

Homogeneous

$Ax=0$

$\#$ eqns $= m$

$\#$ unk. $= n$

less unknowns $=n$

m more eqns

$=0$
$=0$
$=0$
$=0$

After REF, if still you have n eqns that become not all zero → only trivial soln.

After REF, if you have less than n eqns that become not all zero. → infinitely many soln

$Ax=b$

$n>m$

~~unique~~ no inf. many ✓

$= b_1$

$= b_m$

After REF $0 = $ non zero → no soln.