6th Week Wednesday

31 Mart 2021 Çarşamba 12:28

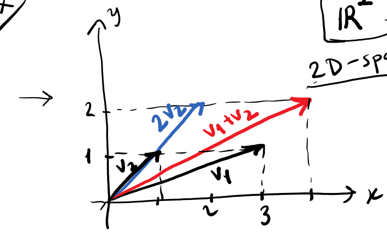
# VECTOR SPACES

a mathematical world
members + operations.

$\mathbb{Z}$ 2, −1, 2, 3, 0
+, −, ·, ÷, %

Ex

$\mathbb{R}^2 \to$ ✓ +, ·
2D-space




$\vec{v}_1 = (3,1)$

$\vec{v}_2 = (1,1)$

✓ $\vec{v}_1 + \vec{v}_2 = (4,2)$

✓ $2 \in \mathbb{R}$ $\quad 2\vec{v}_2 = (2.1, 2.1) = (2,2)$

$(V, +, \cdot)$

$V \to$ n-dim

$\vec{v}_1, \vec{v}_2, \dots,$

$(v_{11}, v_{12}, \dots, v_{1n})$ n-tuple

$\boxed{\alpha \in \mathbb{R}}$ scalars

✓ $\to$ $\oplus$ : vector addition

✓ $\to$ $\odot$ : scalar multiplication

$\alpha \cdot v$ = vector; $\alpha$ a scalar, $v$ vector

$(S, \oplus, \odot) \to$ a vector space or not ?

$\langle 3, 1 \rangle \to$ webwork notation

8 properties + 2 properties ✓✓

$(V, \oplus, \odot)$

* closed under vector addition
$v_1, v_2 \in V \quad v_1 \oplus v_2 \in V$

* closed under scalar multiplication
$\alpha \in \mathbb{R} \quad v \in V \quad \alpha v \in V$

1) $\underbrace{\vec{v}_1 \oplus \vec{v}_2}_{LHS} \overset{?}{=} \underbrace{\vec{v}_2 \oplus \vec{v}_1}_{RHS}$

2) $(\vec{v}_1 \oplus \vec{v}_2) \oplus \vec{v}_3 = \vec{v}_1 \oplus (\vec{v}_2 \oplus \vec{v}_3)$

3) The identity element of vector addition $\in V$

4) Inverse element of vector addition $\in V$

Ex $\quad \mathbb{R}^2 \quad \vec{0} = (0,0)$

$v = (v_1, v_2) \in \mathbb{R}^2 \quad -\vec{v} = (-v_1, -v_2)$

a notation for the identity elt.

$\vec{v}_1 \oplus \boxed{\vec{0}} = \vec{0} \oplus \vec{v}_1 = \vec{v}_1$

$\vec{v}_1 \oplus \boxed{-\vec{v}_1} = \vec{0}$ additive inverse

5) $\alpha \in \mathbb{R}$ (scalar) $\quad \alpha \odot (\vec{v}_1 \oplus \vec{v}_2) = \alpha \odot \vec{v}_1 \oplus \alpha \odot \vec{v}_2$

6) $r, s \in \mathbb{R}$ (scalars) $\quad \vec{v} \in V \quad (r+s) \odot \vec{v} = r \odot \vec{v} \oplus s \odot \vec{v}$

7) $r, s \in \mathbb{R}$ (scalars) $\quad \vec{v} \in V \quad (r.s) \odot \vec{v} = r \odot (s \odot \vec{v})$

8) $1 \in \mathbb{R} \quad \vec{v} \in V \quad 1 \odot \vec{v} = \vec{v}$

Ex $\quad \mathbb{R}^2$ ✓

$\vec{v}_1 = (2,1) \quad \vec{v}_2 = (3,5)$

let $\alpha = 3$

LHS: $3(\vec{v}_1 + \vec{v}_2) = 3.(5,6) = (15,18)$ ✓

RHS: $3\vec{v}_1 = 3(2,1) = (6,3)$

$+ \; 3\vec{v}_2 = 3(3,5) = (9,15)$

$(15,18)$ ✓

**10.** Let $S$ be the set of all ordered pairs of real numbers. $\mathbb{R}^2$
Define scalar multiplication and addition on $S$ by

$$\alpha(x_1, x_2) = (\alpha x_1, \alpha x_2) \to \text{the same scalar multiplication } \mathbb{R}^2$$

$$(x_1, x_2) \oplus (y_1, y_2) = (x_1 + y_1, 0)$$

3) $(x_1, x_2) \oplus (e_1, e_2) = (x_1, x_2) \quad \Rightarrow (e_1, e_2) = (0,0)$ ✓

3) $(x_1,x_2) \oplus \underline{(e_1,e_2)} = (x_1,x_2)$ $\Rightarrow$ $(e_1,e_2)=(0,0)$ ✓

$(x_1,x_2) \oplus (\underset{0}{e_1}, \underset{0}{e_2}) = (\ x_1+e_1\ ,0)$

4) $(x_1,x_2) \oplus (\underbrace{y_1,y_2}_{\in V}) = (0,0)$

$(\underline{x_1,x_2}) \oplus (y_1,y_2) = (x_1+y_1\ ,0) \quad = (0,0)$

$x_1+y_1=0$

$y_1=-x_1$

$y_2=?$

infinitely many inverses X

$\mathbb{R}^+ = (0,\infty)$

**12.** Let $R^+$ denote the set of positive real numbers. Define the operation of scalar multiplication, denoted $\circ$, by

1-dimensional

$$\boxed{\alpha \circ x = x^\alpha}$$

for each $x \in R^+$ and for any real number $\alpha$. Define the operation of addition, denoted $\oplus$, by

$$\boxed{x \oplus y = x \cdot y \qquad \text{for all} \qquad x, y \in R^+}$$

Thus, for this system, the scalar product of $-3$ times $\frac{1}{2}$ is given by

$$-3 \circ \frac{1}{2} = \left(\frac{1}{2}\right)^{-3} = 8$$

and the sum of 2 and 5 is given by

$$2 \oplus 5 = 2 \cdot 5 = 10$$

Is $R^+$ a vector space with these operations? Prove your answer.

5) 6) 7) 8)

$3,5 \in R^+$

$3 \oplus 5 = 3.5 = 15$

$-2 \in \mathbb{R} \rightarrow$ scalar

$\underset{\text{scalar}}{-2} \circ \underset{\text{vector}}{3} = \underset{\text{vector}}{3^{-2}} \in \mathbb{R}^+$

LHS $\overset{?}{=}$ RHS

1) $\underset{xy}{x \oplus y} \overset{?}{=} \underset{yx}{y \oplus x}$ ✓

$xy = yx$

2) $x \oplus (\underbrace{y \oplus z}_{yz}) = (\underbrace{x \oplus y}_{xy}) \oplus z$ ✓

$xyz \overset{✓}{=} xyz$

3) $x \oplus e = x$

$x.e = x$

the identity elt of vector addition $=1$ ✓

4) $x \oplus \underset{\in \mathbb{R}^+}{y} = 1$

$xy = 1$

$y = \frac{1}{x} \in \mathbb{R}^+$ ✓