2nd Week Wednesday

03 Mart 2021 Çarşamba 12:36

make 1's: 1.) $r_i \leftrightarrow r_j$

2.) $cr_i \to r_i$

make 0's: 3.) $cr_j + r_i \to r_i$

8. Consider a linear system whose augmented matrix is of the form

$r_j = r_1$

$$\left[\begin{array}{ccc|c} \boxed{1} & 2 & 1 & 1 \\ -1 & 4 & 3 & 2 \\ 2 & -2 & a & 3 \end{array}\right] \xrightarrow[-2r_1+r_3 \to r_3]{1.r_1+r_2 \to r_2} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & \boxed{6} & 4 & 3 \\ 0 & -6 & -2+a & 1 \end{array}\right] \xrightarrow{\frac{1}{6}r_2 \to r_2} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & \boxed{1} & 2/3 & 1/2 \\ 0 & \boxed{-6} & -2+a & 1 \end{array}\right]$$

For what values of a will the system have a unique solution?

$$\xrightarrow{6r_2+r_3 \to r_3} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 2/3 & 1/2 \\ 0 & 0 & 2+a & 4 \end{array}\right] \quad \begin{array}{l} x_1 = \ldots \\ x_2 = \ldots \end{array}$$

$2+a$

$(2+a)x_3 = 4$

$x_3 = \dfrac{4}{2+a}$ ✓

If $a=-2 \Rightarrow 0 \ 0 \ 0 \ | \ 4$ $\quad 0=4 \to$ impossible!

$\Rightarrow$ NO SOLUTION!

If $a \neq -2 \Rightarrow$ unique solution.

Ex

$$x_1 + x_2 - x_3 = 2$$
$$x_1 + 2x_2 + x_3 = 3$$
$$x_1 + x_2 + (a^2-5)x_3 = a$$

For what values of a, the system has what type of soln?

The augmented matrix $\to$ REF

$$\left[\begin{array}{ccc|c} \boxed{1} & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & a^2-5 & a \end{array}\right] \xrightarrow[-r_1+r_3 \to r_3]{-r_1+r_2 \to r_2} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & \boxed{1} & 2 & 1 \\ 0 & \boxed{0} & a^2-4 & a-2 \end{array}\right] \end{array} \to (a^2-4)x_3 = (a-2)$$

REF

$$(a-2)(a+2)x_3 = (a-2)$$

If $a=2 \Rightarrow 0 \ 0 \ 0 \ | \ 0 \to$ infinitely many solutions.

If $a=-2 \Rightarrow 0 \ 0 \ 0 \ | \ -4 \to$ NO SOLUTION!

If $a \neq \mp 2 \Rightarrow x_3 = \dfrac{1}{a+2} \in \mathbb{R}$ ✓ x_2 ✓ x_1 ✓ $\to$ unique solution.

Reduced Row Echelon Form

A matrix is said to be in **reduced row echelon form** if

(i) The matrix is in row echelon form.

(ii) The first nonzero entry in each row is the only nonzero entry in its column.

✓ ✓ 1) leading nonzero in row $= 1 \rightarrow k^{th} + 0\,0\,0 \ldots 0$

✓ ~~×~~ → 2) # leading zeros of k^{th} row < # leading zeros of $k{+}1^{th}$ row

✓✓ × → 3) if all zero row(s) → bottom (it may be more than one)

→ REF

4) leading 1's are the only nonzero elts in their column. →

→ RREF

3. The augmented matrices that follow are in reduced row echelon form. In each case, find the solution set to the corresponding linear system.

(a) $\left[\begin{array}{ccc|c} \boxed{1} & 0 & 0 & -2 \\ 0 & \boxed{1} & 0 & 5 \\ 0 & 0 & \boxed{1} & 3 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} \boxed{1} & 4 & 0 & 2 \\ 0 & 0 & \boxed{1} & 3 \\ 0 & 0 & 0 & \mathbf{0} \end{array}\right]$ ✓ — RREF

(c) $\begin{array}{c} 0 \\ 2 \\ 4 \end{array}\left[\begin{array}{ccc|c} \boxed{1} & -3 & 0 & 2 \\ 0 & 0 & \boxed{1} & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(d) $\begin{array}{c} 0 \\ 2 \end{array}\left[\begin{array}{cccc|c} \boxed{1} & 2 & 0 & 1 & 5 \\ 0 & 0 & \boxed{1} & 3 & 4 \end{array}\right]$

(e) $\begin{array}{c} 0 \\ 3 \\ \\ \\ \end{array}\left[\begin{array}{cccc|c} \boxed{1} & 5 & -2 & 0 & 3 \\ 0 & 0 & 0 & \boxed{1} & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ ✓

(f) $\begin{array}{c} 1 \\ 2 \\ \\ \end{array}\left[\begin{array}{ccc|c} 0 & \boxed{1} & 0 & 2 \\ 0 & 0 & \boxed{1} & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$ ✓

$$\left[\begin{array}{cccc|c} \boxed{1} & 1 & 1 & 1 & 6 \\ 0 & \boxed{1} & 1 & -1 & 0 \\ 0 & 0 & \boxed{1} & 2/3 & 13/3 \\ 0 & 0 & 0 & \boxed{1} & 2 \end{array}\right] \begin{array}{c} \\ \\ \\ r_4 \end{array}$$

✓ REF ✓

$$\xrightarrow[-\frac{2}{3} r_4 + r_3 \rightarrow r_3]{-1 r_4 + r_1 \rightarrow r_1 \quad 1 r_4 + r_2 \rightarrow r_2} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 4 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & \boxed{1} & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

0 0 0 −1 −2
1 1 1 1 6

0 0 0 $-\frac{2}{3}$ $-\frac{4}{3}$
0 0 1 $\frac{2}{3}$ $\frac{13}{3}$
$\frac{9}{3}$

$$\xrightarrow[-1 r_3 + r_2 \rightarrow r_2]{-1 r_3 + r_1 \rightarrow r_1} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & \boxed{1} & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

0 0 −1 0 −3
1 1 1 0 4

0 0 −1 0 −3
0 1 1 0 2

$$\xrightarrow{-1 r_2 + r_1 \rightarrow r_1} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \rightarrow \text{RREF}$$

$x_1 = 2$, $x_2 = -1$, $x_3 = 3$, $x_4 = 2$

HW Consider the linear system

$$\begin{array}{rcrcrcrcrcl} x_1 & + & x_2 & + & 3x_3 & + & x_4 & & & = & 3 \\ & & & & 2x_3 & + & x_4 & & & = & 7 \\ -x_1 & - & x_2 & & & & & - & 2x_5 & = & 4 \end{array}$$

1) Write the augmented matrix
2) Make it RREF
3) Find the solution(s).