10. Hafta Çarşamba Dersi

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$\dim(V) = n$

$\vec{v} \in V \qquad B = \{b_1, b_2, \dots, b_n\} \rightarrow V$ için bir baz

$$\vec{v} = \alpha_1 b_1 + \alpha_2 b_2 + \dots + \alpha_n b_n$$

$$\underbrace{\begin{bmatrix} | \\ \vec{v} \\ | \end{bmatrix}}_{\vec{v}} = \underbrace{\begin{bmatrix} | & | & & | \\ b_1 & b_2 & \cdots & b_n \\ | & | & & | \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}}_{[\vec{v}]_B} \qquad \vec{v} = B.[\vec{v}]_B$$

$C = \{c_1, c_2, \dots, c_n\} \rightarrow V$ için başka bir baz

$$\vec{v} = \beta_1 c_1 + \beta_2 c_2 + \dots + \beta_n c_n$$

$$\underbrace{\begin{bmatrix} | \\ \vec{v} \\ | \end{bmatrix}}_{\vec{v}} = \underbrace{\begin{bmatrix} | & | & & | \\ c_1 & c_2 & \cdots & c_n \\ | & | & & | \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix}}_{[\vec{v}]_C} \qquad \vec{v} = C.[\vec{v}]_C$$

$\vdots$

$$B^{-1} \quad \underbrace{\vec{v} = B.[\vec{v}]_B}_{} = C.[\vec{v}]_C = D.[\vec{v}]_D = \dots$$

$D^{-1} \qquad D^{-1} \qquad ?$

$$[\vec{v}]_B = B^{-1}\vec{v}$$

$$[\vec{v}]_D = \underbrace{D^{-1}C}_{}\,[\vec{v}]_C$$

C bazından D bazına geçiş matrisi

Örn #3.5-3

$\vec{v}_1 = (3,2)^T, \quad \vec{v}_2 = (4,3)^T \qquad \mathcal{U} = \{u_1, u_2\} \rightarrow \mathbb{R}^2$ için 2 baz olm. üzere,

$\vec{u}_1 = (0,1)^T, \quad \vec{u}_2 = (2,1)^T \qquad V = \{v_1, v_2\}$

V bazından $\mathcal{U}$ bazına geçiş matrisini bulunuz. $\rightarrow \mathcal{U}^{-1}V$

$$\vec{v} = \mathcal{U}.\underbrace{[\vec{v}]_{\mathcal{U}}}_{?} = V.[\vec{v}]_V \qquad [\vec{v}]_{\mathcal{U}} = \underbrace{\mathcal{U}^{-1}V}_{}\,[\vec{v}]_V$$

$\mathcal{U}^{-1}$

$$\mathcal{U} = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} \quad 0-2 \qquad \mathcal{U}^{-1} = \begin{bmatrix} 1/2 & 1 \\ -1/2 & 0 \end{bmatrix}$$

$$\mathcal{U}^{-1}V = \begin{bmatrix} 1/2 & 1 \\ -1/2 & 0 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 7/2 & 5 \\ 3/2 & -2 \end{bmatrix}$$

$$V = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} \quad \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$$

$9-8 \qquad 0$

$\mathcal{U}$ bazından V bazına geçiş matrisini bulunuz. $\rightarrow V^{-1}\mathcal{U}$

$$V^{-1} = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix} \qquad U = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} \qquad V^{-1}U = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$$

Örn

#3.5-5

$u_1 = (1,1,1)$ $\quad$ U R^3'ün bir bazı olmak üzere.

$u_2 = (1,2,2)$ $\quad$ Standart bazdan U bazına geçiş matrisini bulunuz $\rightarrow U^{-1}E$

$u_3 = (2,3,4)$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix} \dashrightarrow U^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \qquad U^{-1}E = U^{-1}$$

$\vec{v} = (3,2,5)$ vektörünün U bazına göre koordinatlarını bulunuz.

$$\vec{v} = U \cdot [\vec{v}]_U \qquad [\vec{v}]_U = U^{-1}v$$

$\checkmark \quad \checkmark \quad ?$

$$[\vec{v}]_U = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}_{3\times3} \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}_{3\times1} = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}_{3\times1}$$

$-3+4-5$

$$\vec{v} = 1.u_1 - 4.u_2 + 3.u_3 \quad \checkmark$$

$v_1 = (4,6,7) \quad v_2 = (0,1,1) \quad v_3 = (0,1,2) \quad V$ başka bir baz $(\mathbb{R}^3)$

a) V bazından U bazına geçiş matrisi ? $\quad U^{-1}V$

$$U^{-1}V = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 0 & 0 \\ 6 & 1 & 1 \\ 7 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

b) $\vec{x} = 2\vec{v}_1 + 3\vec{v}_2 - 4\vec{v}_3$ $\quad$ x'in U bazındaki koordinatlarını bulunuz.

$$[\vec{x}]_V = (2,3,-4) \qquad [\vec{x}]_U = \underbrace{U^{-1}V}\,[\vec{x}]_V = \begin{bmatrix} 1 & -1 & -2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ -2 \end{bmatrix}$$

$$\vec{x} = 7u_1 + 5u_2 - 2u_3 \quad \checkmark$$