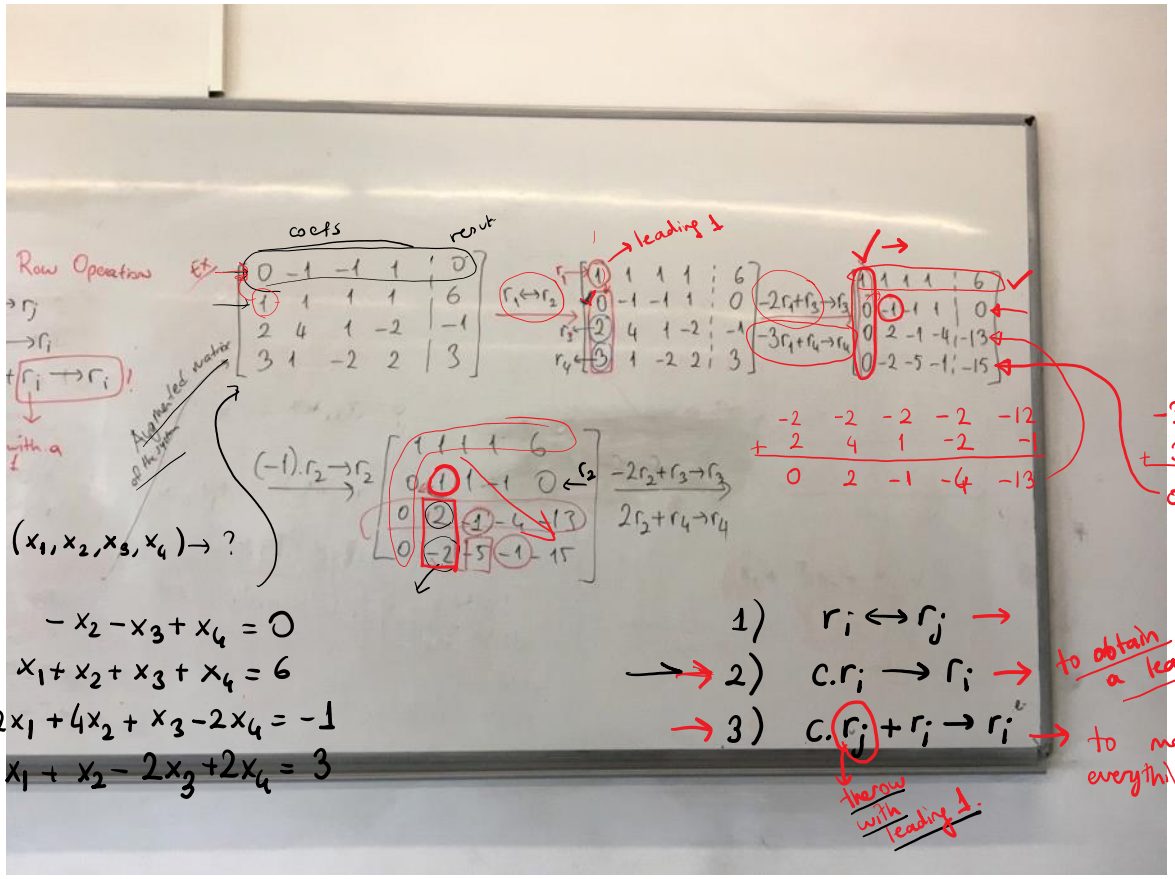
2nd Week

16 Ekim 2021 Cumartesi 13:16



$-2r_2 + r_3 \to r_3$

$2r_2 + r_4 \to r_4$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & -3 & -2 & -13 \\ 0 & 0 & -3 & -3 & -15 \end{bmatrix} \xrightarrow{-\frac{1}{3}r_3 \to r_3} \begin{bmatrix} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & \frac{2}{3} & \frac{13}{3} \\ 0 & 0 & -3 & -3 & -15 \end{bmatrix}$$

$$\begin{array}{r} 0\ -2\ -2\ 2\ 0 \\ +\ 0\ 2\ -1\ -4\ -13 \end{array}$$

$$\begin{array}{r} 0\ 2\ 2\ -2\ 0 \\ +\ 0\ -2\ -5\ -1\ -15 \end{array}$$

$3r_3 + r_4 \to r_4$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & \frac{2}{3} & \frac{13}{3} \\ 0 & 0 & 0 & -1 & -2 \end{bmatrix} \xrightarrow{(-1)r_4 \to r_4} \begin{bmatrix} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & \frac{2}{3} & \frac{13}{3} \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \to \text{REF} \checkmark$$

$$x_1 + x_2 + x_3 + x_4 = 6$$
$$x_2 + x_3 - x_4 = 0$$
$$x_3 + \frac{2}{3}x_4 = \frac{13}{3}$$

$x_1 = 2$ ✓

$x_2 = -1$ ✓

$x_3 = 3$ ✓

$x_4 = 2$ ✓

Solution Set $= \{(2, -1, 3, 2)\}$

→ unique solution.

Ex

$$x_1 + x_2 + 3x_3 + x_4 = 3$$
$$2x_3 + x_4 = 7$$
$$-x_1 - x_2 - 2x_5 = 4$$

5 unknowns.

3 eqns.

→ 3 rows

5 + 1 → 6

coeffs ↓ results columns

$$\begin{bmatrix} 1 & 1 & 3 & 1 & 0 & | & 3 \\ 0 & 0 & 2 & 1 & 0 & | & 7 \\ -1 & -1 & 0 & 0 & -2 & | & 4 \end{bmatrix} \xrightarrow{1\cdot r_1 + r_3 \to r_3} \begin{bmatrix} 1 & 1 & 3 & 1 & 0 & | & 3 \\ 0 & 0 & 2 & 1 & 0 & | & 7 \\ 0 & 0 & 3 & 1 & -2 & | & 7 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}r_2 \to r_2} \begin{bmatrix} 1 & 1 & 3 & 1 & 0 & | & 3 \\ 0 & 0 & 1 & 1/2 & 0 & | & 7/2 \\ 0 & 0 & 3 & 1 & -2 & | & 7 \end{bmatrix} \xrightarrow{-3r_2 + r_3 \to r_3} \begin{bmatrix} 1 & 1 & 3 & 1 & 0 & | & 3 \\ 0 & 0 & 1 & 1/2 & 0 & | & 7/2 \\ 0 & 0 & 0 & -1/2 & -2 & | & -7/2 \end{bmatrix}$$

$0 \quad 0 \quad -3 \quad -3/2 \quad 0 \quad -21/2$

$$\xrightarrow{-2r_3 \to r_3} \begin{bmatrix} 1 & 1 & 3 & 1 & 0 & | & 3 \\ 0 & 0 & 1 & 1/2 & 0 & | & 7/2 \\ 0 & 0 & 0 & 1 & 4 & | & 7 \end{bmatrix} \to \text{REF}$$

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$

$x_1 + x_2 + 3x_3 + x_4 = 3$ ($3x_3 = 2r$, $x_4 = 7-4r$)

$x_3 + \frac{1}{2}x_4 = 7/2$

$x_4 + 4x_5 = 7$

$x_2 = s \in \mathbb{R}$ — independent variable.

$x_5 = r \in \mathbb{R}$ — independent variable

$x_1 + s + 6r + 7 - 4r = 3$

$\Rightarrow x_1 = -4 - 2r - s$ — dependent on x_2 and x_5

$x_3 = \frac{7-7+4r}{2} = 2r$

$x_3 = 2r$

$x_4 = 7 - 4r$ ✓ — dependent to x_5

infinitely many solutions $\leftarrow \left\{ \left(\underbrace{-4-2r-s}_{x_1}, \underbrace{s}_{x_2}, \underbrace{2r}_{x_3}, \underbrace{7-4r}_{x_4}, \underbrace{r}_{x_5} \right) : r, s \in \mathbb{R} \right\}$

(a) $x_1 - 2x_2 = 3$
$2x_1 - x_2 = 9$

(b) $2x_1 - 3x_2 = 5$
$-4x_1 + 6x_2 = 8$

(c) $x_1 + x_2 = 0$
$2x_1 + 3x_2 = 0$
$3x_1 - 2x_2 = 0$

(d) $3x_1 + 2x_2 - x_3 = 4$
$x_1 - 2x_2 + 2x_3 = 1$
$11x_1 + 2x_2 + x_3 = 14$

(e) $2x_1 + 3x_2 + x_3 = 1$
$x_1 + x_2 + x_3 = 3$
$3x_1 + 4x_2 + 2x_3 = 4$

(f) $x_1 - x_2 + 2x_3 = 4$
$2x_1 + 3x_2 - x_3 = 1$
$7x_1 + 3x_2 + 4x_3 = 7$