4. Hafta Çarşamba Dersi

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$A^{-1} \longrightarrow$ kare matris (satır sayısı = sütun sayısı, $m=n$)

- A^{-1} varsa $\longrightarrow A \xrightarrow{\text{İSEF}} I$
- A^{-1} yoksa $\longrightarrow A \xrightarrow{\text{İSEF}} I$'ya ulaşmaz

Lineer denklem sistemleri ;

$$\begin{aligned} a_{11}x_1 + \dots + a_{1n}x_n &= b_1 \\ &\vdots \\ a_{m1}x_1 + \dots + a_{mn}x_n &= b_n \end{aligned}$$

$$\left[\begin{array}{ccc|c} a_{11} & \dots & a_{1n} & b_1 \\ \vdots & \ddots & \vdots & \vdots \\ a_{m1} & \dots & a_{mn} & b_n \end{array}\right] \rightarrow \text{eklenmiş matris} \quad m\times(n+1)$$

$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \rightarrow$ 1. denklem

$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \rightarrow$ 2. denklem

$\vdots$

katsayılar matrisi

değişkenlerden oluşan sütun matrisi

sonuçlardan oluşan sütun matrisi

$$Ax = b$$

$$A = \underbrace{\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}}_{m\times n} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{n\times 1} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}}_{m\times 1}$$

Homojen lineer denklem sistemi $(b_1 = b_2 = \dots = b_n = 0) \longrightarrow \boxed{Ax = 0}$

A^{-1} kullanarak yapacağımız genelleştirmeler için $(Ax=b \,/\, Ax=0)$ $A \rightarrow$ kare matris olmalı.

$\rightarrow Ax = b \longrightarrow A^{-1}$ varsa ; $\underbrace{A^{-1}A}_{I}x = A^{-1}b \rightarrow \boxed{x = A^{-1}b}$ sistemin tek çözümü bulunur. $\quad \boxed{AA^{-1} = I}$

$\rightarrow Ax = 0 \longrightarrow A^{-1}$ varsa ; $\underbrace{A^{-1}A}_{I}x = A^{-1}0 = 0 \rightarrow \boxed{x = 0} \rightarrow$ sadece trivial çözüm vardır. $\quad \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$

$$\left[\begin{array}{c|c} A & b \end{array}\right] \xrightarrow{\text{İSEF}} \left[\begin{array}{cccc|c} 1 & & & 0 & b'_1 \\ & 1 & & & b'_2 \\ & & 1 & & \vdots \\ 0 & & & 1 & b'_n \end{array}\right] \begin{matrix} \rightarrow x_1 \\ \rightarrow x_2 \\ \\ \rightarrow x_n \end{matrix} \Rightarrow \text{tek çözüm}$$

$$\left[\begin{array}{c|c} A & \begin{matrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{matrix} \end{array}\right] \xrightarrow{\text{İSEF}} \left[\begin{array}{cccc|c} 1 & & & 0 & 0 \\ & 1 & & & 0 \\ & & 1 & & 0 \\ 0 & & & 1 & 0 \end{array}\right] \begin{matrix} \rightarrow x_1 \\ \rightarrow x_2 \\ \rightarrow x_3 \\ \rightarrow x_n \end{matrix} \Rightarrow \text{tek çözüm = trivial çözüm. } (x_1 = x_2 = \dots = x_n = 0)$$

Örn

$$\begin{aligned} x_1 + 4x_2 + 3x_3 &= 12 \\ -x_1 - 2x_2 &= -12 \\ 2x_1 + 2x_2 + 3x_3 &= 8 \end{aligned}$$

A^{-1} kullanarak çözümü bul.

$$A = \begin{bmatrix} 1 & 4 & 3 \\ -1 & -2 & 0 \\ 2 & 2 & 3 \end{bmatrix} \xrightarrow[-2r_1+r_3 \rightarrow r_3]{r_1+r_2 \rightarrow r_2} \begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & 3 \\ 0 & -6 & -3 \end{bmatrix} \xrightarrow{\frac{1}{2}r_2 \rightarrow r_2} \begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 3/2 \\ 0 & -6 & -3 \end{bmatrix}$$

$[A \,|\, I] \dashrightarrow$

$$\xrightarrow{6r_2+r_3 \rightarrow r_3} \begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 3/2 \\ 0 & 0 & 6 \end{bmatrix} \xrightarrow{\frac{1}{6}r_3 \rightarrow r_3} \begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 3/2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{6r_2+r_3\to r_3}\begin{bmatrix} 0 & 1 & 3/2 \\ 0 & 0 & 6 \end{bmatrix}\longrightarrow\begin{bmatrix} 0 & 1 & -1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow[-3r_3+r_1\to r_1]{-\frac{3}{2}r_3+r_2\to r_2}\begin{bmatrix} 1 & 4 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\xrightarrow{-4r_2+r_1\to r_1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I\to\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}\to\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1/2 & 0 \\ -2 & 0 & 1 \end{bmatrix}\longrightarrow\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1/2 & 0 \\ 1 & 3 & 1 \end{bmatrix}\longrightarrow\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1/2 & 0 \\ 1/6 & 1/2 & 1/6 \end{bmatrix}\longrightarrow\begin{bmatrix} 1/2 & -3/2 & -1/2 \\ 1/4 & -1/4 & -1/4 \\ 1/6 & 1/2 & 1/6 \end{bmatrix}$$

$-1/2 \quad -3/2 \quad -1/2$

$$x = A^{-1}b = \begin{bmatrix} -1/2 & -1/2 & 1/2 \\ 1/4 & -1/4 & -1/4 \\ 1/6 & 1/2 & 1/6 \end{bmatrix}\begin{bmatrix} 12 \\ -12 \\ 8 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ -\frac{8}{3} \end{bmatrix} \qquad A^{-1}\longrightarrow\begin{bmatrix} -1/2 & -1/2 & 1/2 \\ 1/4 & -1/4 & -1/4 \\ 1/6 & 1/2 & 1/6 \end{bmatrix}$$

$x_1 = 4 \quad x_2 = 4 \quad x_3 = -8/3 \rightarrow$ tek çözüm //

LU - Çarpanlara Ayırma (Kare matris)

$$\begin{bmatrix} \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot \\ 0 & 0 & \cdot \end{bmatrix}$$

$A \rightarrow A = LU$

L: alt üçgensel

U: üst üçgensel

$A \xrightarrow{SEF} U$

$E_1 E_2 \dots E_n$

$E_n \dots E_2 E_1 A = U$

$A = U \underbrace{E_n^{-1} \dots E_2^{-1} E_1^{-1}}_{L}$

E 1. tipte ise örn

$$E = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]\xrightarrow{r_2\leftrightarrow r_3}\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \Rightarrow \boxed{E^{-1} = E}$$

(sağ blok: E^{-1})

E 2. tipte ise örn

$$E = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & k & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]\xrightarrow{\frac{1}{k}r_2\to r_2}\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/k & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \qquad E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/k & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(sağ blok: E^{-1})

k yerine $1/k$ yaz

E 3. tipte ise örn

$$E \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & k & 1 & 0 & 0 & 1 \end{array}\right]\xrightarrow{-kr_2+r_3\to r_3}\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -k & 1 \end{array}\right]$$

(sol blok: I, sağ blok: E^{-1})

k yerine $-k$ yaz

$$A = \begin{bmatrix} 1 & 4 & 3 \\ -1 & -2 & 0 \end{bmatrix}\xrightarrow[-2r_1+r_3\to r_3]{\overset{E_1}{r_1+r_2\to r_2}}\begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & 3 \\ 0 & -6 & -3 \end{bmatrix}\xrightarrow{\overset{E_3}{\frac{1}{2}r_2\to r_2}}\begin{bmatrix} 1 & 4 & \\ 0 & 1 & 3/2 \\ 0 & -6 & -3 \end{bmatrix}$$

$$A = \begin{bmatrix} \cdots & \cdots & \cdots \\ -1 & -2 & 0 \\ 2 & 2 & 3 \end{bmatrix} \xrightarrow[E_2]{-2r_1+r_3\to r_3} \begin{bmatrix} \cdots & \cdots & \cdots \\ 0 & 2 & 3 \\ 0 & -6 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots \\ 0 & -6 & -3 \end{bmatrix}$$

$[A \,\vdots\, I] \dashrightarrow$

$$\xrightarrow[]{\overset{E_4}{6r_2+r_3\to r_3}} \begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 3/2 \\ 0 & 0 & 6 \end{bmatrix} \xrightarrow{\overset{E_5}{\frac{1}{6}r_3\to r_3}} \begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 3/2 \\ 0 & 0 & 1 \end{bmatrix} \to \text{SEF} = U$$

u'

$$\begin{array}{cc} E_2^{-1} & E_1^{-1} \end{array}$$
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$L = E_5^{-1} E_4^{-1} E_3^{-1} E_2^{-1} E_1^{-1} =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 14 & -12 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 14 & -12 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 84 & -72 & 6 \end{bmatrix} = L$$

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 84 & -72 & 6 \end{bmatrix} \begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 3/2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$