

ith Week Monday

2 Mart 2021 Pazartesi 09:33

The Deferminant of a Matrix

> det(M) = |M|

$$M \rightarrow 1 \times L$$
 matrix $\rightarrow a$ number $\rightarrow det(M) = M_{11}$

$$M \rightarrow 2 \times 2$$
 matrix $\rightarrow M = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11} a_{22} - a_{12} a_{21}$

$$M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$
 det $(M) = 1.4 - 2.3 = -2$

$$A \rightarrow n \times$$

$$h = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

matrix

(n/3)

matrix
$$(n)^3$$
 determinant of the minor: $|M_{ij}| \rightarrow \frac{(n-1) \times (n-1)}{\text{obtained from delethy}}$ the ith row and jth column of the matrix A.

$$|M_{24}| = \det \left(\begin{bmatrix} 2 & 3 \\ 8 & 9 \end{bmatrix}\right) = 2.9 - 3.8$$
delete the
$$2^{n_1} \text{ row}$$

$$1^{st} \text{ column}$$
of A.

$$\underbrace{\text{cofactor}}: A_{ij} \longrightarrow (-1)^{i+j} \mid M_{ij} \mid \underbrace{A_{21}} = (-1)^{2+1} \mid M_{21} \mid$$

$$e^{+}/A_{21} = (-1)^{2+1}/M_{21}$$

$$= -1.-6 = 6$$

$$A = \begin{bmatrix} a_{12} & a_{12} & a_{10} \\ a_{21} & a_{22} & a_{2n} \\ a_{n2} & a_{n2} & a_{n3} \end{bmatrix}_{n \times n}$$

$$dut(A) = \underbrace{a_{11} \cdot A_{11}}_{a_{12}} + \underbrace{a_{12} \cdot A_{12}}_{a_{12}} + \dots + \underbrace{a_{1n} \cdot A_{1n}}_{a_{1n}} \rightarrow \underbrace{a_{1n} \cdot A_{1n}}_{a_{1n} \cdot a_{1n}} \rightarrow \underbrace{a_{1n} \cdot A_{1n}}_{a_{1n} \cdot a_{2n} \cdot a_{2n}} \rightarrow \underbrace{a_{1n} \cdot A_{2n}}_{a_{1n} \cdot a_{2n} \cdot a_{2n} \cdot a_{2n}}$$

$$\frac{1^{th} \text{ row}}{\text{cofactor expansion}}: \quad a_{i1} \quad A_{i1} \quad + \quad a_{i2} \quad A_{i2} \quad + \quad \dots \quad + \quad a_{in} A_{in} \quad (= \text{ det}(A))$$

$$\frac{1^{th} \text{ column}}{1^{th} \text{ column}}: \quad a_{i1} \quad A_{i2} \quad + \quad a_{i2} \quad A_{i2} \quad + \quad \dots \quad + \quad a_{in} A_{in} \quad (= \text{ det}(A))$$

a 1 j A 1 j + a 2 j A 2 j + ---- + a n j A n j (=det(A))

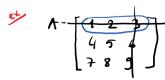
Choose any row or column

Wrong cofactor

a 1 j A 1 j + a 2 j A 2 j + ----- + a n j A n j (=det(A))

Choose any row or column

Lift row what for them =



$$A_{11} = \begin{pmatrix} -1 \\ + \end{pmatrix}^{1+1} \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} = -3$$

$$\frac{\det(A)}{(-1)} = \underbrace{(A_{11})}_{1+1} \underbrace{(A_{12})}_{1+1} \underbrace{(A_{12})}_{(-1)^{1+2}} \underbrace{(A_{13})}_{1+2} \underbrace{(A_{13})}_{(-1)^{1+3}} \underbrace{(A_{13})}_{(-1)^{1+3}}$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} = -3$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 34 - 42 \\ 7 - 43 \end{vmatrix} = 6$$

$$A_{13} = (-1)^{1+2} \begin{vmatrix} 34 - 42 \\ 7 - 3 \end{vmatrix} = 6$$

$$A_{14} = (-1)^{1+2} \begin{vmatrix} 34 - 42 \\ 7 - 3 \end{vmatrix} = 6$$

$$A_{15} = (-1)^{1+2} \begin{vmatrix} 34 - 42 \\ 7 - 3 \end{vmatrix} = 6$$

$$A_{16} = (-1)^{1+2} \begin{vmatrix} 34 - 42 \\ 7 - 3 \end{vmatrix} = 6$$

$$A_{17} = (-1)^{1+2} \begin{vmatrix} 34 - 42 \\ 7 - 3 \end{vmatrix} = 6$$

$$A_{18} = (-1)^{1+2} \begin{vmatrix} 34 - 42 \\ 7 - 3 \end{vmatrix} = 6$$

$$A_{19} = (-1)^{1+2} \begin{vmatrix} 34 - 42 \\ 7 - 3 \end{vmatrix} = 6$$

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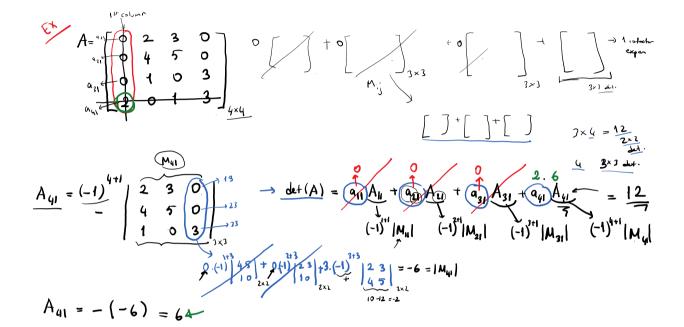
$$A_{19} = (-1)^{1+2} \begin{vmatrix} 34 - 42 \\ 7 - 3 \end{vmatrix} = 6$$

$$A_{19} = (-1)^{1+2}$$

$$A_{13} = \underbrace{(-1)^{1+3}}_{+} \begin{vmatrix} \frac{31}{4} & \frac{35}{5} \\ 7 & 8 \end{vmatrix} = -3$$

$$= -3 + 12 - 9 = 0$$

Break HW: Try choosing another row/column and see it will give the same result.



If A is a diagonal matrix;

$$A = \begin{bmatrix} d_1 & d_2 & 0 \\ 0 & d_1 \end{bmatrix} \implies det(A) = d_1 d_2 \dots d_n$$

$$A = \begin{bmatrix} d_1 & d_2 & 0 \\ 0 & d_1 \end{bmatrix} \implies det(A) = 2(-1) \begin{vmatrix} 1+1 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} + 0 + 0 + 0$$

$$0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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= 2. (-1).3.6

* If A is an upper/lower triangular matrix; det(A) = did2 ... dn

$$de+(A) = 2(-1)^{1/2} + 0 + 0 + 0$$

$$2^{2} + 0$$

$$-|3| -12| + 0 + 0$$

$$3|(-1)^{1/2} + 0 + 0$$

$$-|2| + 0 + 0$$

$$3|(-1)^{1/2} + 0 + 0$$

$$-|2| + 0 + 0$$

The Properties of Determinants

$$\frac{1}{A}$$
 det $(A) = \det(A^T)$

If A has an all zero row/column.
$$\rightarrow$$
 det(A) = 0

$$\rightarrow *$$
 det $(AB) = det(A) \cdot det(B)$

E₁: Type-I elementary matrix $(r_i \leftrightarrow r_j)$ De+ $(E_1) = -1$ E_2 : Type-II " " $(kr_i \rightarrow r_i)$ De+ $(E_2) = k$ E_3 : Type-II " " $(kr_j + r_i \rightarrow r_i)$ De+ $(E_3) = 1$

Let A be any matrix.

$$\det(E_1A) = \underbrace{\det(E_1)}_{-1} \det(A) = \underbrace{-\det(A)}_{-1}$$

$$\rightarrow$$
 If you apply a 2° type row operation to A; E_2A

$$\det(E_2A) = \underbrace{\det(E_1)}_{k} \det(A) = \underbrace{k}_{k} \det(A)$$

$$\det (\underline{E_3}A) = \underbrace{\det (\overline{E_3})}_{1}. \det (A) = \underbrace{\det (A)}_{1}$$

Get a random square matrix A.

Find its determinant.

Agely one row operation to A. See what happens to the determinant. Apply another ,, 4 " " " " " " "