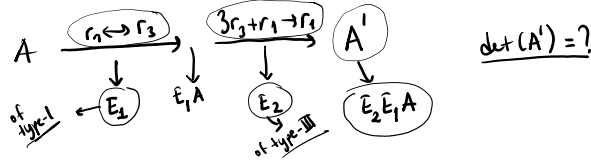


Determinants

Elementary Matrix of	type I $r_i \leftrightarrow r_j$	$\rightarrow \det(E) = -1$
"	type II $k r_i \rightarrow r_i$	$\rightarrow \det(E) = k$
"	type III $k r_j + r_i \rightarrow r_i$	$\rightarrow \det(E) = 1$

Ex/ A , $\det(A) = 5$ Apply $r_1 \leftrightarrow r_3$ and then $3r_3 + r_1 \rightarrow r_1$

what happens to $\det(A)$?



$$\det(A') = ?$$

$$A' = E_2 E_1 A$$

$$\det(A') = \det(E_2 E_1 A) = \det(E_2) \det(E_1) \det(A) = 1 \cdot (-1) \cdot 5 = -5$$

$$\det(AB) = \det(A) \det(B)$$

Ex/ $A = \begin{bmatrix} 2 & 1 & -2 \\ 0 & -3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ $\det(A) = 2 \cdot (-3) \cdot 4 = -24$

diagonal upper lower
 $\det(A) = \prod_{i=1}^n a_{ii}$

Diagram showing the transformation of matrix A to A' using elementary matrices E_1 and E_2 .

$A \xrightarrow{\frac{1}{4} r_3 \rightarrow r_3} E_1 A \xrightarrow{2r_3 + r_1 \rightarrow r_1} E_2 E_1 A = A'$

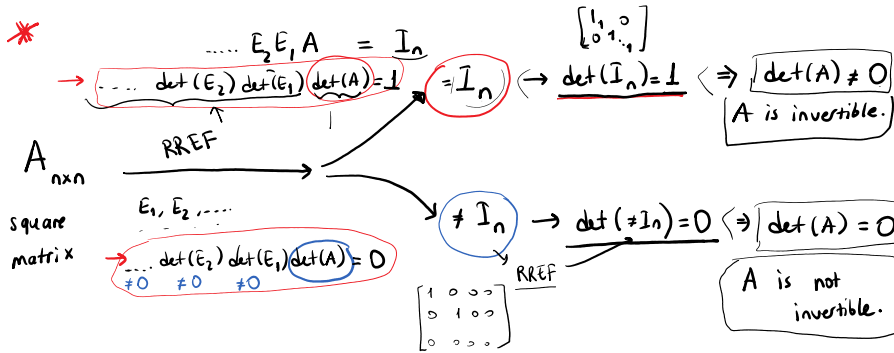
Annotations: E_1 is type 2, E_2 is type 3.

$$A' = E_2 E_1 A$$

$$\det(A') = \det(E_2) \det(E_1) \det(A) = 1 \cdot \frac{1}{4} \cdot (-24) = -6$$

$$\det(A') = 2 \cdot (-3) \cdot 1 = -6$$

*

Adjoint Matrix

$\text{adj}(A)$

square $A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}_{n \times n}$

$\text{adj}(A) = \begin{bmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{bmatrix}$

Annotations: A_{ij} is the cofactor of a_{ji} .

$$A_{ij} = (-1)^{i+j} |M_{ij}|$$

$$A \cdot \text{adj}(A) = \begin{bmatrix} a_{11}A_{11} + a_{12}A_{21} + \dots + a_{1n}A_{n1} & 0 & \dots & 0 \\ 0 & \det(A) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \det(A) \end{bmatrix} = \det(A) \cdot I_n$$

$$A \cdot \text{adj}(A) = \begin{bmatrix} a_{11}A_{11} + a_{12}A_{12} + \dots + a_{1n}A_{1n} & 0 & \dots & 0 \\ 0 & a_{21}A_{11} + \dots + a_{2n}A_{2n} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & a_{n1}A_{11} + a_{n2}A_{21} + \dots + a_{nn}A_{nn} \end{bmatrix} = \begin{bmatrix} \det(A) & 0 & \dots & 0 \\ 0 & \det(A) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \det(A) \end{bmatrix} = \det(A) \cdot I_n$$

$$A \cdot \text{adj}(A) = \det(A) \cdot I_n \rightarrow A \cdot \left(\text{adj}(A) \cdot \frac{1}{\det(A)} \right) = I_n \rightarrow A^{-1} = \frac{1}{\det(A)} \cdot \text{adj}(A)$$

\downarrow number scalar \downarrow matrix

Ex) $A = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

$A_{ij} = (-1)^{i+j} |M_{ij}|$

$\text{adj}(A) = ?$ $\det(A) = ?$ $A^{-1} = ?$

$\text{adj}(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & -2 \\ -7 & 4 & 2 \\ 4 & -3 & 1 \end{bmatrix}$

$\frac{1}{\det(A)} \cdot \text{adj}(A) = A^{-1} = \begin{bmatrix} 2/5 & 1/5 & -2/5 \\ -7/5 & 4/5 & 2/5 \\ 4/5 & -3/5 & 1/5 \end{bmatrix}$

$$A_{11} = (-1)^{1+1} |M_{11}| = 1 \cdot \begin{vmatrix} 2 & 2 \\ 2 & 3 \end{vmatrix} = 6 - 4 = 2$$

$$A_{21} = (-1)^{2+1} |M_{21}| = -1 \cdot \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1 \cdot (3 - 4) = 1$$

$$A_{31} = (-1)^{3+1} |M_{31}| = 1 \cdot \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = 1 \cdot (2 - 4) = -2$$

$$A_{12} = (-1)^{1+2} |M_{12}| = -1 \cdot \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} = -1 \cdot (9 - 2) = -7$$

$$A_{22} = (-1)^{2+2} |M_{22}| = 1 \cdot \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix} = 1 \cdot (6 - 2) = 4$$

$$A_{32} = (-1)^{3+2} |M_{32}| = -1 \cdot \begin{vmatrix} 2 & 2 \\ 2 & 2 \end{vmatrix} = -1 \cdot (4 - 4) = 0$$

$$A_{13} = (-1)^{1+3} |M_{13}| = 1 \cdot \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} = 1 \cdot (6 - 2) = 4$$

$$A_{23} = (-1)^{2+3} |M_{23}| = -1 \cdot \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = -1 \cdot (2 - 1) = -1$$

$$A_{33} = (-1)^{3+3} |M_{33}| = 1 \cdot \begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix} = 1 \cdot (4 - 2) = 2$$

1st row $\det(A) = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = 2 \cdot 2 + 1 \cdot (-7) + 2 \cdot 4 = 4 - 7 + 8 = 5$