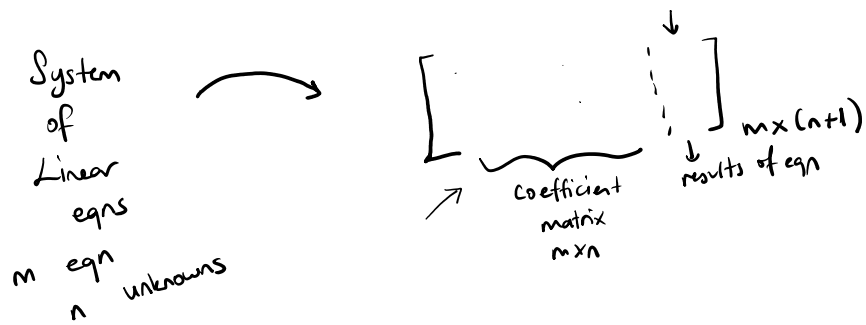


Augmented Matrix of a system of Linear Eqs



Matrix

$A = \begin{bmatrix} \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \end{bmatrix}$

Algebra

dimension $m \times n$

m rows
n columns

$a_{ij}x_j$

ith row, jth column entry = A_{ij}

row column

(Subtraction)

§ **Addition** (Only the matrices with the same dimension can be added)

Ex/

$A = \begin{bmatrix} \textcircled{1} & \textcircled{2} & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2 \times 3}$ $B = \begin{bmatrix} \textcircled{-1} & \textcircled{2} & 0 \\ 2 & 1 & -2 \end{bmatrix}_{2 \times 3}$ $A+B=?$
 2×3 2×3 2×3

$A+B = \begin{bmatrix} 1+(-1) & 2+2 & 3+0 \\ 4+2 & 5+1 & 6+(-2) \end{bmatrix} = \begin{bmatrix} 0 & 4 & 3 \\ 6 & 6 & 4 \end{bmatrix}_{2 \times 3}$

Scalar Multiplication

$c \in \mathbb{R}$ $A \rightarrow$ matrix $m \times n$

$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & & & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}_{m \times n}$

$\textcircled{c} \cdot A = \begin{bmatrix} c \cdot a_{11} & c \cdot a_{12} & \dots & c \cdot a_{1n} \\ c \cdot a_{21} & & & c \cdot a_{2n} \\ \vdots & & & \vdots \\ c \cdot a_{m1} & c \cdot a_{m2} & \dots & c \cdot a_{mn} \end{bmatrix}_{m \times n}$

Ex/

$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 4 & -1 & -2 & 0 \\ 3 & 0 & -4 & -2 \end{bmatrix}_{3 \times 4}$

$\textcircled{-2} \cdot A = \begin{bmatrix} (-2) \cdot 1 & (-2) \cdot 2 & \dots & \dots \\ & & & \end{bmatrix} = \begin{bmatrix} -2 & -4 & -6 & -2 \\ -8 & 2 & 4 & 0 \\ -6 & 0 & 8 & 4 \end{bmatrix}_{3 \times 4}$

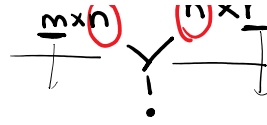
scalar multip.

Matrix Multiplication

$A \cdot B$

$A_{m \times n} \cdot B_{n \times r} = C_{m \times r}$

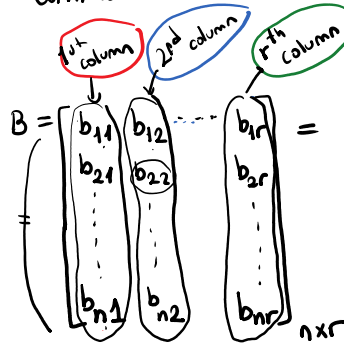
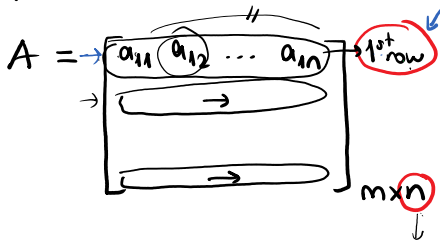
A.B



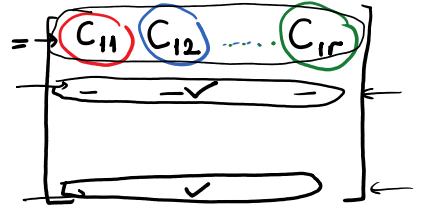
$AB \neq BA$

* Matrix multiplication is not commutative

$A \cdot B =$



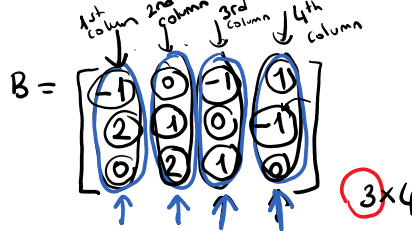
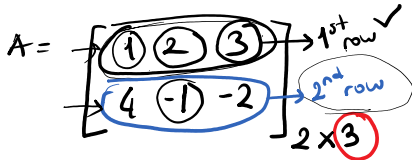
$2.3 = 3.2$



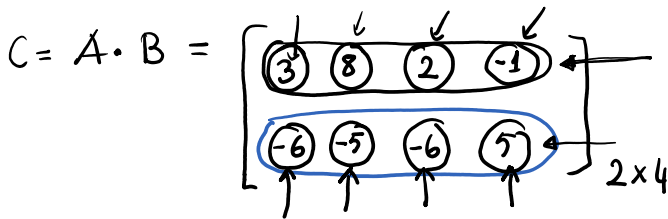
\rightarrow 1st row of A \cdot 1st column of B = $a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + \dots + a_{1n} \cdot b_{n1} = C_{11}$
 \rightarrow 1st row of A \cdot 2nd column of B = $a_{11} \cdot b_{12} + a_{12} \cdot b_{22} + \dots + a_{1n} \cdot b_{n2} = C_{12}$
 \vdots
 \rightarrow 1st row of A \cdot rth column of B = $a_{11} \cdot b_{1r} + a_{12} \cdot b_{2r} + \dots + a_{1n} \cdot b_{nr} = C_{1r}$

repeat this for each row of A in order.

Ex



$A \cdot B = C_{2 \times 4}$



$A \cdot B = \begin{bmatrix} 3 & 8 & 2 & -1 \\ -6 & -5 & -6 & 5 \end{bmatrix}$

$B \cdot A \rightarrow$ can not be multiplied!

$1 \cdot -1 + 2 \cdot 2 + 3 \cdot 0 = 3 \leftarrow C_{11}$
 $1 \cdot 0 + 2 \cdot 1 + 3 \cdot 2 = 8 \leftarrow C_{12}$
 $1 \cdot -1 + 2 \cdot 0 + 3 \cdot 1 = 2 \leftarrow C_{13}$
 $1 \cdot 1 + 2 \cdot -1 + 3 \cdot 0 = -1 \leftarrow C_{14}$

$4 \cdot -1 + -1 \cdot 2 + -2 \cdot 0 = -6 \leftarrow C_{21}$
 $4 \cdot 0 + -1 \cdot 1 + -2 \cdot 2 = -5 \leftarrow C_{22}$
 $4 \cdot -1 + -1 \cdot 0 + -2 \cdot 1 = -6 \leftarrow C_{23}$
 $4 \cdot 1 + -1 \cdot -1 + -2 \cdot 0 = 5 \leftarrow C_{24}$