Gauss Yok Etme Metodu:

Eklenmiş Matris $\longrightarrow$ SEF   (sistemi çözmek için)

Gauss - Jordan Yok Etme Metodu:

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Ex/

$$\begin{aligned}
x_1 + x_2 + 3x_3 + x_4 &= 3 \\
2x_3 + x_4 &= 7 \\
-x_1 - x_2 \qquad\qquad - 2x_5 &= 4
\end{aligned}$$

$$\longrightarrow
\begin{array}{c}
\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & b \end{array} \\
\left[\begin{array}{ccccc|c}
1 & 1 & 3 & 1 & 0 & 3 \\
0 & 0 & 2 & 1 & 0 & 7 \\
-1 & -1 & 0 & 0 & -2 & 4
\end{array}\right]
\end{array}$$

3×6

Elementer Satır İşlemleri

$r_i \leftrightarrow r_j$

$(c.r_i) \rightarrow r_i$ → i. satırı, c(i.satır) ile değiştir.

$(c.r_j) + (r_i) \rightarrow (r_i)$ → i. satırı, $(c.r_j + r_i)$ ile değiştir.

$1r_1 + r_3 \rightarrow r_3$
$$\left[\begin{array}{ccccc|c}
1 & 1 & 3 & 1 & 0 & 3 \\
0 & 0 & 2 & 1 & 0 & 7 \\
0 & 0 & 3 & 1 & -2 & 7
\end{array}\right]$$

$\frac{1}{2}r_2 \rightarrow r_2$
$$\left[\begin{array}{ccccc|c}
1 & 1 & 3 & 1 & 0 & 3 \\
0 & 0 & 1 & 1/2 & 0 & 7/2 \\
0 & 0 & 3 & 1 & -2 & 7
\end{array}\right]$$

$$\begin{array}{cccccc}
0 & 0 & -3 & -3/2 & 0 & -21/2 \\
+ & 0 & 0 & 3 & 1 & -2 & 7
\end{array}$$

$-3r_2 + r_3 \rightarrow r_3$
$$\left[\begin{array}{ccccc|c}
1 & 1 & 3 & 1 & 0 & 3 \\
0 & 0 & 1 & 1/2 & 0 & 7/2 \\
0 & 0 & 0 & -1/2 & -2 & -7/2
\end{array}\right]$$

$-2r_3 \rightarrow r_3$
$$\left[\begin{array}{ccccc|c}
1 & 1 & 3 & 1 & 0 & 3 \\
0 & 0 & 1 & 1/2 & 0 & 7/2 \\
0 & 0 & 0 & 1 & 4 & 7
\end{array}\right] \rightarrow \underline{\text{SEF}} \checkmark$$

- - - - - -

$$\begin{array}{cccccc}
0 & 0 & 0 & -1/2 & -2 & -7/2 \\
0 & 0 & 1 & 1/2 & 0 & 7/2
\end{array}$$

$-1r_3 + r_1 \rightarrow r_1$
$-\frac{1}{2}r_3 + r_2 \rightarrow r_2$
$$\left[\begin{array}{ccccc|c}
1 & 1 & 3 & 0 & -4 & -4 \\
0 & 0 & 1 & 0 & -2 & 0 \\
0 & 0 & 0 & 1 & 4 & 7
\end{array}\right]$$

$-3r_2 + r_1 \rightarrow r_1$
$$\left[\begin{array}{ccccc|c}
1 & 1 & 0 & 0 & 2 & -4 \\
0 & 0 & 1 & 0 & -2 & 0 \\
0 & 0 & 0 & 1 & 4 & 7
\end{array}\right] \rightarrow \text{İSEF} \checkmark$$

Farklı çözüm kümesi

$x_3 = r \in \mathbb{R}$

$x_5 = \frac{r}{2}$

$x_4 = 7 - 2r$

$x_1 = s$

$x_1 + x_2 + 2 \cdot \frac{r}{2} = -4$

$x_1 + x_2 = -4 - r$

$\underline{x_1 + x_2} = -4 - r - s$

$x_2 = -4 - r - s$

$r - 2x_5 = 0$

$$\begin{cases}
x_1 + x_2 + 2x_5 = -4 \\
x_3 - 2x_5 = 0 \\
x_4 + 4x_5 = 7
\end{cases}$$

$x_1 + x_2 + 2r = -4$

$x_5 = r \in \mathbb{R} \rightarrow$ bağımsız değişken

$\Rightarrow x_3 = 2r$ } bağımlı

$\Rightarrow x_4 = 7 - 4r$

$x_1 = s \in \mathbb{R} \rightarrow$ bağımsız değişken

$x_2 = -4 - 2r - s$ } bağımlı

$(x_1, x_2, x_3, x_4, x_5) = (s, -4-2r-s, 2r, 7-4r, r) \rightarrow$ Sonsuz çözümü vardır.

$(x_1, x_2, x_3, x_4, x_5) = (s, -4-2r-s, 2r, 7-4r, r) \rightarrow$ Sonsuz çözümü vardır.

Örn

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & | & 1 \\ -1 & -1 & 0 & 0 & 1 & | & -1 \\ -2 & -2 & 0 & 0 & 3 & | & 1 \\ 0 & 0 & 1 & 1 & 3 & | & -1 \\ 1 & 1 & 2 & 2 & 4 & | & \end{bmatrix}$$

$r_1 + r_2 \rightarrow r_2$
$2r_1 + r_3 \rightarrow r_3$
$-1r_1 + r_5 \rightarrow r_5$

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & | & 1 \\ 0 & 0 & 1 & 1 & 2 & | & 0 \\ 0 & 0 & 2 & 2 & 5 & | & 3 \\ 0 & 0 & 1 & 1 & 3 & | & -1 \\ 0 & 0 & 1 & 1 & 3 & | & 0 \end{bmatrix}$$

$-2r_2 + r_3 \rightarrow r_3$
$-1r_2 + r_4 \rightarrow r_4$
$-1r_2 + r_5 \rightarrow r_5$

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & | & 1 \\ 0 & 0 & 1 & 1 & 2 & | & 0 \\ 0 & 0 & 0 & 0 & 1 & | & 3 \\ 0 & 0 & 0 & 0 & 1 & | & -1 \\ 0 & 0 & 0 & 0 & 1 & | & 0 \end{bmatrix}$$

$-1r_3 + r_4 \rightarrow r_4$
$-1r_3 + r_5 \rightarrow r_5$

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & | & 1 \\ 0 & 0 & 1 & 1 & 2 & | & 0 \\ 0 & 0 & 0 & 0 & 1 & | & 3 \\ 0 & 0 & 0 & 0 & 0 & | & -4 \\ 0 & 0 & 0 & 0 & 0 & | & -3 \end{bmatrix}$
SEF ✓

$\left.\begin{array}{l} 0 = -4 \\ 0 = -3 \end{array}\right\}$ imkansız!

$\Rightarrow$ Sistemin çözümü yoktur.

Örn

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & | & 1 \\ -1 & -1 & 0 & 0 & 1 & | & -1 \\ -2 & -2 & 0 & 0 & 3 & | & 1 \\ 0 & 0 & 1 & 1 & 3 & | & 3 \\ 1 & 1 & 2 & 2 & 4 & | & 4 \end{bmatrix} \rightarrow \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & | & 1 \\ 0 & 0 & 1 & 1 & 2 & | & 0 \\ 0 & 0 & 0 & 0 & 1 & | & 3 \\ 0 & 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$0 = 0$ ✓   SEF

Gauss

$x_2 = s \in \mathbb{R} \rightarrow$ bağımsız değişken

$x_4 = r \in \mathbb{R} \rightarrow$ bağımsız değişken

$x_1 + x_2 + x_3 + x_4 + x_5 = 1$
$x_3 + x_4 + 2x_5 = 0 \rightarrow x_3 + x_4 + 6 = 0 \Rightarrow x_3 = -6 - r$ (bağımlı)
$x_5 = 3$

$x_1 + s - 6 - r + r + 3 = 1 \Rightarrow x_1 = 4 - s$

$x_1 + x_3 + x_5 = 1 - x_2 - x_4$
$x_3 + 2x_5 = -x_4$
$x_5 = 3$

$(4-s, s, -6-r, r, 3) : r, s \in \mathbb{R} \rightarrow$ sonsuz çözümü vardır.

$m = $ Denklem Sayısı $n = $ Değişken Sayısı

$m > n$

★ Sistem SEF'e çevrilmelidir.

$m = 3 \quad n = 2$

(a): $\begin{bmatrix} 1 & 1 & | & 1 \\ 1 & -1 & | & 3 \\ -1 & 2 & | & -2 \end{bmatrix} \xrightarrow{SEF} \begin{bmatrix} 1 & 1 & | & 1 \\ 0 & 1 & | & -1 \\ 0 & 0 & | & 1 \end{bmatrix} \rightarrow$ Çözüm yok. $0 = 1$!

$m = 4 \quad n = 3$

(b): $\begin{bmatrix} 1 & 2 & 1 & | & 1 \\ 2 & -1 & 1 & | & 2 \\ 4 & 3 & 3 & | & 4 \\ 2 & -1 & 3 & | & 5 \end{bmatrix} \xrightarrow{SEF} \begin{bmatrix} 1 & 2 & 1 & | & 1 \\ 0 & 1 & x_3 & | & 0 \\ 0 & 0 & 1 & | & \frac{3}{2} \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \rightarrow$ tek çözüm vardır.

$m = 4 \quad n = 3$

(c): $\begin{bmatrix} 1 & 2 & 1 & | & 1 \\ 2 & -1 & 1 & | & 2 \\ 4 & 3 & 3 & | & 4 \\ 3 & 1 & 2 & | & 3 \end{bmatrix} \xrightarrow{SEF} \begin{bmatrix} 1 & 2 & 1 & | & 1 \\ 0 & 1 & \frac{1}{3} & | & \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \rightarrow$ sonsuz çözüm vardır.

$m < n$

★ Kesinlikle tek çözüm olamaz!
sonsuz / hiç ? $\rightarrow$ Sistem SEF'e çevrilmelidir.

$m = 2 \quad n = 3$

(a): $\begin{bmatrix} 1 & 2 & 1 & | & 1 \\ 2 & 4 & 2 & | & 3 \end{bmatrix} \xrightarrow{SEF} \begin{bmatrix} 1 & 2 & 1 & | & 1 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \rightarrow$ Çözüm yok. $0 = 1$!

$m = 3 \quad n = 5$

b): $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & | & 2 \\ 1 & 1 & 1 & 2 & 2 & | & 3 \\ 1 & 1 & 1 & 2 & 3 & | & 2 \end{bmatrix} \xrightarrow{SEF} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & | & 2 \\ 0 & 0 & 0 & 1 & 1 & | & 1 \\ 0 & 0 & 0 & 0 & 1 & | & -1 \end{bmatrix} \rightarrow$ sonsuz çözüm var.

$x_1 + x_2 + x_3 + x_4 + x_5 = 2$ $x_2 = r$ $x_3 = s$ $r, s \in \mathbb{R}$
$x_4 + x_5 = 1 \rightarrow x_4 = 2$ $x_1 = 1 - r - s$
$x_5 = -1$

$(1-r-s, r, s, 2, -1) : r, s \in \mathbb{R} \rightarrow$ sonsuz çözüm

$m = n$, $Ax = 0$ / $Ax = b$

Matrislerde İşlemler

Matrislerde İşlemler

Toplama- Çıkarma : $A_{m \times n} \pm B_{m \times n} = C_{m \times n}$ $\qquad [a_{ij} \pm b_{ij}] = [c_{ij}]$

(Skaler) Çarpma : $c \in \mathbb{R}$
$\qquad c \cdot matris = matris$
$\qquad c \cdot A_{m \times n} = [c \cdot a_{ij}]_{m \times n}$
$\qquad \qquad \qquad \searrow [a_{ij}]$

reel sayı ↗

örn $-2 \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -4 \\ -6 & -8 \end{bmatrix}$

$-\frac{1}{2} \cdot \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\frac{3}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$

Matris Çarpımı : $A_{\underline{m \times \boxed{n}}} \cdot B_{\boxed{n} \times \underline{k}} = C_{\boxed{m \times k}}$ ⟶ sıralama önemli

örn $A_{2 \times 3} \cdot B_{3 \times 5} = C_{2 \times 5}$ ama $B_{3 \times \boxed{5}} \cdot A_{\boxed{2} \times 3}$ → çarpma yapılamaz.

$A_{3 \times \boxed{3}} \cdot B_{\boxed{3} \times 3} = C_{3 \times 3}$ $\qquad B_{3 \times \boxed{3}} \cdot A_{\boxed{3} \times 3} = D_{3 \times 3}$ $\qquad C \neq D$ (bazı özel durumlar hariç)

örn
$\overset{A}{\underset{1}{2}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}}_{2 \times \boxed{3}} \cdot \overset{B}{\begin{bmatrix} -1 & 0 & -1 & 1 \\ 2 & 1 & 2 & 2 \\ 3 & 2 & 0 & -3 \end{bmatrix}}_{\boxed{3} \times 4} = \overset{C}{\begin{bmatrix} 12 & 8 & 3 & -4 \\ 24 & 17 & 6 & -4 \end{bmatrix}}_{2 \times 4}$

$\begin{array}{l} -4+15+18 \\ 0+5+12 \\ -4+10+0 \\ 4+10-18 \end{array}$

Satırlar $\qquad\qquad$ Sütunlar

→ A'nın 1.satırı · B'nin 1.sütunu

iç çarpım → $(1, 2, 3) \cdot (-1, 2, 3) = 1 \cdot -1 + 2 \cdot 2 + 3 \cdot 3 = 12 \to C_{11}$

B'nin 2.sütunu
$(1, 2, 3) \cdot (0, 1, 2) = 1 \cdot 0 + 2 \cdot 1 + 3 \cdot 2 = 8 \to C_{12}$

B'nin 3.sütunu
$(1, 2, 3) \cdot (-1, 2, 0) = 1 \cdot -1 + 2 \cdot 2 + 3 \cdot 0 = 3 \to C_{13}$

B'nin 4.sütunu
$(1, 2, 3) \cdot (1, 2, -3) = 1 \cdot 1 + 2 \cdot 2 + 3 \cdot -3 = -4 \to C_{14}$

A'nın 2.satırı · B'nin 1.sütunu $\to C_{21}$
$\qquad\qquad\qquad$ B'nin 2.sütunu $\to C_{22}$
$\qquad\qquad\qquad$ B'nin 3.sütunu $\to C_{23}$
$\qquad\qquad\qquad$ B'nin 4.sütunu $\to C_{24}$