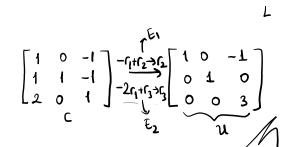
1. Given

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix} B = \begin{bmatrix} 1 & 5 & 0 \\ 3 & -1 & 1 \\ -1 & 0 & -1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix},$$

(a) (4 pts) AB = ?

(b) (4 pts) Find the row echelon form (REF) of the matrix A. Write each row operation clearly.



 \rightarrow (c) (8 pts) Find an LU-decomposition for C.

$$G_{1} = \begin{bmatrix} 4 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} E_{1}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 1 & 1 \end{bmatrix}$$

$$L = E_1^{-1} E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

2. (9 pts) Determine which of the following matrices are in reduced row echelon form (RREF) and which are not Show why

$$A = \begin{bmatrix} 1 & 5 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 7 & 0 & 3 \\ 0 & 1 & 2 & 0 & -4 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

3. Given
$$A = \begin{bmatrix} 1 & -3 & 3 \\ 2 & -1 & 7 \\ -1 & 3 & -2 \end{bmatrix}$$
, $det(A) = 1 \begin{vmatrix} -1 & 7 \\ 3 & -2 \end{vmatrix} + 3 \begin{vmatrix} 2 & 7 \\ -1 & -2 \end{vmatrix} + 3 \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = -19 + 9 + 15 = 5$

(a) (6 pts) Find det(A). ✓

(b) (6 pts) Using $\underline{\det(A)}$ only, find $\underline{\det(A^{-1})}$, $\underline{\det(\underline{A^3})}$, $\underline{\det(\underline{2A})}$.

$$\Rightarrow \det(A, A^{-1}) = \det(I) = I$$

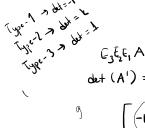
$$= \det(A). \det(A^{-1}) \Rightarrow \det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{5}$$

→(c) (9 pts) Find adj(A).

(d) (3 pts) Find A^{-1} .

→ (e) (6 pts) Assume that we apply the following 3 row operations to A and obtain the new matrix A'. Then what is $\det(A')=?$

(Do not find the matrix A', use elementary matrices)



$$A \xrightarrow{3r_1 + r_2 \to r_2} \dots \xrightarrow{2r_1 \to r_1} \dots \xrightarrow{r_2 \leftrightarrow r_3} A'$$

$$C_{3}E_{2}E_{1}A = A'$$

$$det(A') = det(E_{3}) det(E_{2}) det(E_{1}) det(A) = -10$$

$$A \rightarrow 2r_{2} \rightarrow r_{3}$$

$$\begin{bmatrix} -19 & 3 & 18 \end{bmatrix}$$

det (A.A.A) = det(A) det(A) det(A) = 53 $det (2A) = 2.2.2 \cdot det(A) = 40$

2+2

adj(A) =
$$\begin{bmatrix} -19 & 3 & 19 \\ -3 & 1 & -1 \\ 5 & 0 & 5 \end{bmatrix}$$

$$A_{21} = (-1)^{241} \begin{vmatrix} -3 & 3 \\ 3 & -2 \end{vmatrix} = 3$$

$$A_{22} = (-1)^{341} \begin{vmatrix} 1 & -3 \\ -1 & 3 \end{vmatrix} = 0$$

$$A_{31} = (-1)^{341} \begin{vmatrix} -3 & 3 \\ -1 & 3 \end{vmatrix} = -1$$

$$A_{32} = (-1)^{342} \begin{vmatrix} 1 & 3 \\ -1 & 3 \end{vmatrix} = -1$$

$$A_{33} = (-1)^{343} \begin{vmatrix} 1 & 3 \\ -1 & 3 \end{vmatrix} = -1$$

$$A_{33} = (-1)^{343} \begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix} = -1$$

$$A_{33} = (-1)^{343} \begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix} = -1$$

$$A_{33} = (-1)^{343} \begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix} = -1$$

$$A_{33} = (-1)^{343} \begin{vmatrix} 1 & -3 \\ 2 & -1 \end{vmatrix} = 5$$

$$-1 + 6$$

 (5 pts) Let R² be the set of all ordered pairs of real numbers. Define scalar multiplication and addition on R² by

$$\rightarrow \lambda \odot (x_1, y_1) = (\lambda x_1, \lambda y_1) \times (x_1, y_1) \oplus (x_2, y_2) = (x_1/x_2, 0)$$

Give at least one reason to why \mathbb{R}^2 is not a vector space with these operations.

$$\forall (x_{1},y_{1}), (x_{2},y_{2}) \in \mathbb{R}^{2}, (x_{1},y_{1}) \oplus (x_{2},y_{2}) = (x_{1}/x_{2},0)$$

$$(1,2) \oplus (0,1) = (\underbrace{1/0}_{\notin \mathbb{R}}, 0) \notin \mathbb{R}^{2} \rightarrow$$

$$X_{i} = \frac{\det(A_{i})}{\det(A)}$$

6. (9 pts) For which values of k does the following system of linear equations have

7. (10 pts) Find determinants of the following matrices (Not too many calculations!).

(a)
$$\begin{bmatrix} -1 & 0 & 7 & 5 & -2 \\ 7 & 0 & 12 & 3 & 4 \\ -4 & 0 & -2 & 0 & 0 \\ 5 & 0 & 3 & 0 & 0 \\ 5 & 0 & 3 & 0 & 0 \\ 4 & 0 & -1 & 0 & 3 \end{bmatrix}$$
 (b) $\begin{bmatrix} 3 & -5 & 6 & 5 & -2 \\ 7 & 2 & -12 & -3 & 4 \\ -4 & 1 & -3 & -2 & 1 \\ 5 & 0 & 9 & 0 & -3 \\ 4 & -12 & 0 & -9 & 0 \end{bmatrix}$

$$2 \cdot (-1)^{2+2}$$

$$5 \cdot 3 \quad 0 \quad 0 \quad 3$$

$$4 \cdot -1 \quad 0 \quad 3$$

$$4 \cdot -1 \quad 0 \quad 3$$

$$5 \cdot (-1)^{1+3}$$

$$-4 \cdot -2 \quad 5 \quad 3$$

$$4 \cdot -2 \quad 5 \quad 3$$

$$5 \cdot (-1)^{1+3}$$

$$-4 \cdot -2 \quad 5 \quad 3$$

$$-12 + 10 \quad -2$$

8. (9 pts) Let \mathbb{R}^3 be the vector space of all ordered triples of real numbers. Determine whether the following subsets are subspaces of \mathbb{R}^3 or not, tell why.

2)
$$(x_{1}, y_{1}, \lambda_{1}) + (x_{2}, y_{2}, \lambda_{2}) = (x_{1} + x_{2}, y_{1} + y_{2}, \lambda_{1} + \lambda_{2})$$
 $= (x_{1} + x_{2}, y_{1} + \lambda_{2}) - (x_{1} + \lambda_{2})$
 $= (x_{1} + x_{2}, y_{1} + \lambda_{2}) - (x_{1} + \lambda_{2})$
 $= (x_{1} + x_{2}, y_{1} + \lambda_{2}) - (x_{1} + \lambda_{2})$
 $= (x_{1} + x_{2}, y_{1} + \lambda_{2}) - (x_{1} + \lambda_{2})$
 $= (x_{1} + x_{2}, y_{1} + \lambda_{2}) - (x_{1} + \lambda_{2})$
 $= (x_{1} + x_{2}, y_{1} + \lambda_{2})$
 $= (x_{1} + x_{2}, y_{2} + \lambda_{2})$
 $= (x_{1} + x_{2},$