(SEF) – Satır Eşelon Form (Row Echelon Form – REF)

$$\begin{array}{c} \text{soldan sıfır sayısı} \\ 2 \\ 1 \\ 3 \\ 2 \end{array} \begin{bmatrix} 0 & 0 & \boxed{1} & 2 & 3 \\ 0 & \boxed{2} & 3 & 0 & 5 \\ 0 & 0 & 0 & \boxed{5} & 2 \\ 0 & 0 & \boxed{1} & 0 & 2 \end{bmatrix}$$

- ✓ * Her satırda soldan, sıfırdan farklı ilk eleman $\boxed{1}$ olmalı. ✓
- ✓ * Eğer k. satır tamamen 0 değilse, k.satırdaki soldan sıfır sayısı $<$ $k+1$.satırdaki soldan sıfır sayısı
- * Eğer tamamen 0 olan satır(lar) varsa en altta olmalı.

$$\begin{array}{c} \text{soldan sıfır sayısı} \\ 1 \\ 2 \\ 4 \end{array} \begin{bmatrix} 0 & \boxed{1} & 0 & 2 & 3 \\ 0 & 0 & \boxed{1} & 4 & 0 \\ 0 & 0 & 0 & 0 & \boxed{1} \end{bmatrix} ✓$$

Örn:

✓ (a) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ SEF ✓ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ SEF değil!

✓ (c) $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ SEF (d) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ → SEF ✓

✗ (e) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ ✗ (f) $\begin{bmatrix} 1 & 4 & 6 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}$ SEF değil. ✗

✓ (g) $\begin{bmatrix} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 2 & 4 \\ 0 & 0 & 1 & 3 & 6 \end{bmatrix}$ (h) $\begin{bmatrix} 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ SEF ✓

Elementer Satır İşlemleri

$r_i \to i.$satır
$r_j \to j.$satır

1) $r_i \leftrightarrow r_j$

1 yapma → 2) $c.r_i \to r_i$ → i. satırı, $c.(i.\text{satır})$ ile değiştir.

0 yapma → 3) $c.r_j + r_i \to r_i$ → i.satırı, $(c.r_j + r_i)$ ile değiştir.

başelemanın 1 olduğu satır

$$\left[\begin{array}{cccc|c} 0 & -1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 6 \\ 2 & 4 & 1 & -2 & -1 \\ 3 & 1 & -2 & 2 & 3 \end{array}\right] \xrightarrow{r_1 \leftrightarrow r_2} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & -1 & -1 & 1 & 0 \\ 2 & 4 & 1 & -2 & -1 \\ 3 & 1 & -2 & 2 & 3 \end{array}\right] \xrightarrow[-3r_1 + r_4 \to r_4]{-2r_1 + r_3 \to r_3} \left[\begin{array}{ccccc} 1 & 1 & 1 & 1 & 6 \\ 0 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & -4 & -13 \\ 0 & -2 & -5 & -1 & -15 \end{array}\right]$$

$r_1 \to 1.$satır

$$\begin{array}{ccccc|l} -2 & -2 & -2 & -2 & -12 & \to -2r_1 \\ 2 & 4 & 1 & -2 & -1 & \to r_3 \\ \hline 0 & 2 & -1 & -4 & -13 & \to \text{yeni } r_3 \end{array}$$

$$\begin{array}{ccccc|l} -3 & -3 & -3 & -3 & -18 & \to -3r_1 \\ 3 & 1 & -2 & 2 & 3 & \to r_4 \\ \hline 0 & -2 & -5 & -1 & -15 & \text{yeni } r_4 \end{array}$$

$$\xrightarrow{-1.r_2 \to r_2} \left[\begin{array}{ccccc} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & -4 & -13 \\ 0 & -2 & -5 & -1 & -15 \end{array}\right] \xrightarrow[2r_2 + r_4 \to r_4]{-2r_2 + r_3 \to r_3} \left[\begin{array}{ccccc} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & -3 & -2 & -13 \\ 0 & 0 & -3 & -3 & -15 \end{array}\right]$$

$r_j \to 2.$satır

$$\begin{array}{ccccc|l} 0 & -2 & -2 & 2 & 0 & \to -2r_2 \\ 0 & 2 & -1 & -4 & -13 & \to r_3 \\ \hline 0 & 0 & -3 & -2 & -13 & \to \text{yeni } r_3 \end{array}$$

$$\begin{array}{ccccc} 0 & 2 & 2 & -2 & 0 \\ 0 & -2 & -5 & -1 & -15 \\ \hline 0 & 0 & -3 & -3 & -15 \end{array}$$

$$\xrightarrow{-\frac{1}{3}.r_3 \to r_3} \left[\begin{array}{ccccc} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2/3 & 13/3 \\ 0 & 0 & -3 & -3 & -15 \end{array}\right] \xrightarrow{3r_3 + r_4 \to r_4} \left[\begin{array}{ccccc} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2/3 & 13/3 \\ 0 & 0 & 0 & -1 & -2 \end{array}\right]$$

$$\begin{array}{ccccc} 0 & 0 & 3 & 2 & 13 \\ 0 & 0 & -3 & -3 & -15 \\ \hline 0 & 0 & 0 & -1 & -2 \end{array}$$

$$\xrightarrow{-1.r_4 \to r_4} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & b \end{array} \\ \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2/3 & 13/3 \end{array}\right] \end{array} \to \text{SEF ✓}$$

$\boxed{x_1} + \overset{-1}{x_2} + \overset{3}{x_3} + \overset{2}{x_4} = 6$ $\boxed{x_1 = 2}$

$\boxed{x_2} + \overset{3}{x_3} - \overset{2}{x_4} = 0$ $\boxed{x_2 = -1}$

$\boxed{x_3} + \frac{2}{3}\overset{2}{x_4} = \frac{13}{3}$ $\boxed{x_3 = 3}$

$$-1 \cdot r_4 \to r_4 \quad \left[\begin{array}{cccc|c} 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2/3 & 13/3 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \to \text{SEF} \checkmark$$

$\boxed{x_2} + x_3 - x_4 = 0 \quad \boxed{x_2 = -1}$

$\boxed{x_3} + \frac{2}{3} x_4 = \frac{13}{3} \quad \boxed{x_3 = 3}$

$\boxed{x_4 = 2}$

$(x_1, x_2, x_3, x_4) = (2, -1, 3, 2) \to$ tek çözüm.

Örn

$$\begin{array}{cc} & \begin{array}{ccc} x_1 & x_2 & b \end{array} \\ \text{(a)} & \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{array}\right] \end{array} \qquad \begin{array}{cc} & \begin{array}{cc} x_1 & x_2 \end{array} \\ \text{(b)} & \left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right] \checkmark \; 0=0 \end{array}$$

$$\text{(c)} \left[\begin{array}{ccc|c} 1 & -2 & 4 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \; 0=0 \checkmark$$

$$\text{(d)} \left[\begin{array}{ccc|c} 1 & -2 & 2 & -2 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right] \to \text{tek çözüm}$$

$$\text{(e)} \left[\begin{array}{ccc|c} 1 & 3 & 2 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{array}\right] \to \text{sistemin çözümü yok!}$$

$0 = 1$ ✗

Örn

$$\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 3 \end{array}\right] \end{array}$$

çözüm yok!

a) $0x_1 + 0x_2 = 1$

$0 = 1$ mümkün değil!

$\Rightarrow$ Sistemin çözümü yoktur.

b) $x_1 + 3x_2 = 1 \quad \boxed{x_1 = 4}$

$\boxed{x_2 = -1}$

$0 = 0 \checkmark$

$(4, -1) \to$ sistemin tek çözümü var.

c) $x_1 - 2x_2 + 4x_3 = 1$

$\boxed{x_3 = 3}$

$0 = 0 \checkmark$

$\to x_1 - 2x_2 = -11$

$x_3 = 3$

$x_2 = r \in \mathbb{R}$

$\Rightarrow x_1 - 2r = -11 \Rightarrow x_1 = 2r - 11$

$(x_1, x_2, x_3) = (2r - 11, r, 3) : r \in \mathbb{R} \to$ sonsuz çözüm

Bir başka gösterim: $x_1 = s \in \mathbb{R}$

$\Rightarrow s - 2x_2 = -11 \Rightarrow x_2 = \dfrac{s+11}{2}$

$(x_1, x_2, x_3) = \left(s, \dfrac{s+11}{2}, 3\right) \to$ aynı çözümü ifade eder

8. Consider a linear system whose augmented matrix is of the form

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ -1 & 4 & 3 & 2 \\ 2 & -2 & a & 3 \end{array}\right] \xrightarrow[-2r_1 + r_3 \to r_3]{r_1 + r_2 \to r_2} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 6 & 4 & 3 \\ 0 & -6 & a-2 & 1 \end{array}\right] \xrightarrow{\frac{1}{6} \cdot r_2 \to r_2} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 2/3 & 1/2 \\ 0 & -6 & a-2 & 1 \end{array}\right]$$

For what values of a will the system have a unique solution?

$$\xrightarrow{6r_2 + r_3 \to r_3} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 2/3 & 1/2 \\ 0 & 0 & a+2 & 4 \end{array}\right]$$

$(a+2)x_3 = 4$

$0 = 4$

$a + 2 = 0 \Rightarrow 0 = 4 \to$ imkansız çözüm yok.

$\boxed{a \neq 2}$ ise sistemin tek çözümü vardır.