Augmented Matrix of a system of Linear Eqns

System of Linear eqns → $\Big[\underbrace{\quad\quad}_{\substack{\text{coefficient}\\ \text{matrix}\\ m\times n}} \Big| \; \Big]_{m\times(n+1)}$ ↓ results of eqn

m eqn
n unknowns

Matrix Algebra

$a_{ij} x_j$

$A = \Big[\quad \Big]$ $\underset{\uparrow}{\text{dimension}}$ $m\times n$

m rows
n columns

i^{th} row, j^{th} column entry $= A_{ij}$ (row, column)

(Subtraction)
Addition (Only the matrices with the same dimension can be added)

Ex/

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times3} \quad B = \begin{bmatrix} -1 & 2 & 0 \\ 2 & 1 & -2 \end{bmatrix}_{2\times3} \quad \underbrace{A+B}_{2\times3} = ?$$

$$A+B = \begin{bmatrix} 1+(-1) & 2+2 & 3+0 \\ 4+2 & 5+1 & 6+(-2) \end{bmatrix} = \begin{bmatrix} 0 & 4 & 3 \\ 6 & 6 & 4 \end{bmatrix}_{2\times3}$$

Scalar Multiplication

$c \in \mathbb{R}$ $\quad A_{m\times n} \to$ matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & \cdots & & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

$$c\,A = \begin{bmatrix} c.a_{11} & c.a_{12} & \cdots & c.a_{1n} \\ c.a_{21} & \cdots & & c.a_{2n} \\ \vdots & & & \\ c.a_{m1} & c.a_{m2} & \cdots & c.a_{mn} \end{bmatrix}_{m\times n}$$

Ex/

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 4 & -1 & -2 & 0 \\ 3 & 0 & -4 & -2 \end{bmatrix}_{3\times4}$$

$$\underbrace{(-2).A}_{\text{scalar multip.}} = \begin{bmatrix} (-2).1 & (-2).2 & \cdots \\ & & \end{bmatrix} = \begin{bmatrix} -2 & -4 & -6 & -2 \\ -8 & 2 & 4 & 0 \\ -6 & 0 & 8 & 4 \end{bmatrix}_{3\times4}$$

Matrix Multiplication

A.B

$$A_{m\times n} \cdot B_{n\times r} = C_{m\times r}$$

$m \times n$ $n \times r$ $m \times r$

$A.B$

$AB \neq \cancel{BA}$

* Matrix multiplication is not commutative

$2.3 = 3.2$

$A.B =$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ & & & \\ & & & \end{bmatrix}_{m \times n} \cdot B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1r} \\ b_{21} & b_{22} & & b_{2r} \\ \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & & b_{nr} \end{bmatrix}_{n \times r} = C = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1r} \\ & & & \\ & & & \end{bmatrix}$$

1st row, 2nd column, r th column

1st row of A · 1st column of B $= a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + \dots + a_{1n} b_{n1} = C_{11}$

1st row of A · 2nd column of B $= a_{11} \cdot b_{12} + a_{12} \cdot b_{22} + \dots + a_{1n} \cdot b_{n2} = C_{12}$

$\vdots$

1st row of A · r th column of B $= a_{11} \cdot b_{1r} + a_{12} \cdot b_{2r} + \dots + a_{1n} \cdot b_{nr} = C_{1r}$

repeat this for each row of A in order.

Ex

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -1 & -2 \end{bmatrix}_{2 \times 3}$$

1st row, 2nd row

$$B = \begin{bmatrix} -1 & 0 & -1 & 1 \\ 2 & 1 & 0 & -1 \\ 0 & 2 & 1 & 0 \end{bmatrix}_{3 \times 4}$$

1st column, 2nd column, 3rd column, 4th column

$A \cdot B = C_{2 \times 4}$

$$C = A \cdot B = \begin{bmatrix} 3 & 8 & 2 & -1 \\ -6 & -5 & -6 & 5 \end{bmatrix}_{2 \times 4}$$

$1.-1 + 2.2 + 3.0 = 3 \leftarrow C_{11}$

$1.0 + 2.1 + 3.2 = 8 \leftarrow C_{12}$

$1.-1 + 2.0 + 3.1 = 2 \leftarrow C_{13}$

$1.1 + 2.-1 + 3.0 = -1 \leftarrow C_{14}$

$4.-1 + -1.2 + -2.0 = -6 \leftarrow C_{21}$

$4.0 + -1.1 + -2.2 = -5 \leftarrow C_{22}$

$4.-1 + -1.0 + -2.1 = -6 \leftarrow C_{23}$

$4.1 + -1.-1 + -2.0 = 5 \leftarrow C_{24}$

$$A \cdot B = \begin{bmatrix} 3 & 8 & 2 & -1 \\ -6 & -5 & -6 & 5 \end{bmatrix}$$

$B \cdot A \rightarrow$ can not be multiplied!