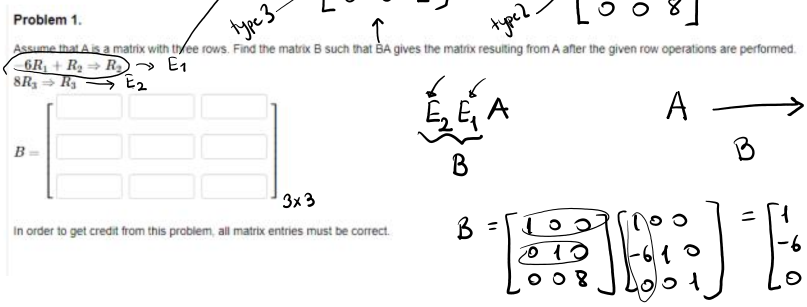
5th Week

06 Kasım 2021 Cumartesi 19:59



Problem 1.

Assume that A is a matrix with three rows. Find the matrix B such that BA gives the matrix resulting from A after the given row operations are performed.

$-6R_1 + R_2 \Rightarrow R_2$ → E_1

$8R_3 \Rightarrow R_3$ → E_2

$B = $ [☐ ☐ ☐ / ☐ ☐ ☐ / ☐ ☐ ☐] 3x3

In order to get credit from this problem, all matrix entries must be correct.

type3 → $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -6 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

type2 → $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

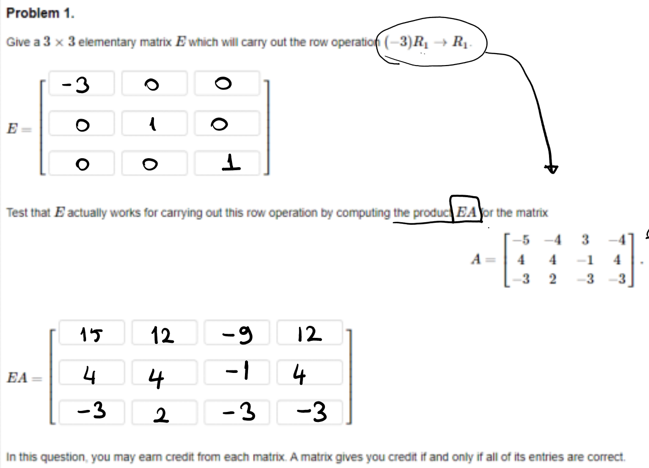
$E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/8 \end{bmatrix}$

$\underbrace{E_2 E_1}_{B} A$

$A \xrightarrow{\quad B \quad}$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -6 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -6 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$



Problem 1.

Give a 3×3 elementary matrix E which will carry out the row operation $(-3)R_1 \to R_1$.

$$E = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E^{-1} = \begin{bmatrix} -1/3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

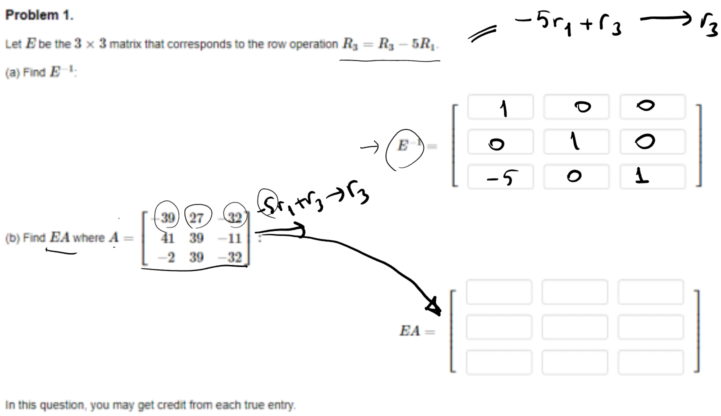
Test that E actually works for carrying out this row operation by computing the product EA for the matrix

$$A = \begin{bmatrix} -5 & -4 & 3 & -4 \\ 4 & 4 & -1 & 4 \\ -3 & 2 & -3 & -3 \end{bmatrix}.$$

$-3r_1 \to r_1$

$$EA = \begin{bmatrix} 15 & 12 & -9 & 12 \\ 4 & 4 & -1 & 4 \\ -3 & 2 & -3 & -3 \end{bmatrix}$$

In this question, you may earn credit from each matrix. A matrix gives you credit if and only if all of its entries are correct.



Problem 1.

Let E be the 3×3 matrix that corresponds to the row operation $R_3 = R_3 - 5R_1$. $= -5r_1 + r_3 \longrightarrow r_3$

(a) Find E^{-1}:

$$E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix}$$

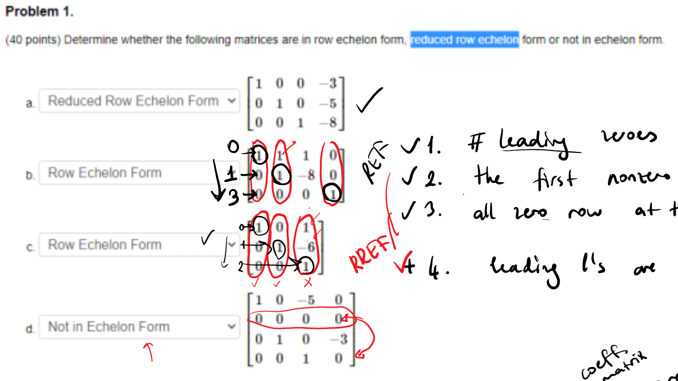
(b) Find EA where $A = \begin{bmatrix} -39 & 27 & -32 \\ 41 & 39 & -11 \\ -2 & 39 & -32 \end{bmatrix}$: $-5r_1 + r_3 \to r_3$

$EA = $ [☐ ☐ ☐ / ☐ ☐ ☐ / ☐ ☐ ☐]

In this question, you may get credit from each true entry.

$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix}$ → type-3

$$\left[\begin{array}{ccc|ccc} \multicolumn{3}{c|}{E} & \multicolumn{3}{c}{I_3} \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -5 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{5r_1 + r_3 \to r_3} \left[\begin{array}{ccc|ccc} \multicolumn{3}{c|}{I_3} & \multicolumn{3}{c}{E^{-1}} \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 5 & 0 & 1 \end{array}\right]$$



Problem 1.

(40 points) Determine whether the following matrices are in row echelon form, reduced row echelon form or not in echelon form.

a. Reduced Row Echelon Form $\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -8 \end{bmatrix}$ ✓

b. Row Echelon Form $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ 0, 1, 3

c. Row Echelon Form ✓ $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$ 0, 1, 2 ✓ ✓ ✗

d. Not in Echelon Form $\begin{bmatrix} 1 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

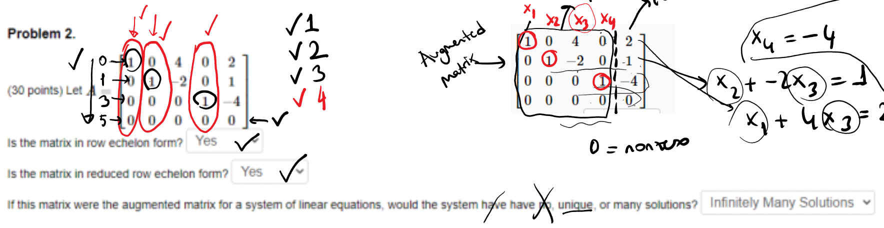
REF

✓ 1. # leading zeroes shouldget greater downside.

✓ 2. the first nonzero elt. in each row = 1

✓ 3. all zero row at the bottom

RREF ✓ 4. leading 1's are the only non zero elt. in their columns.



Problem 2.

(30 points) Let $A = \begin{bmatrix} 1 & 0 & 4 & 0 & 2 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ 0, 1, 3, 5

✓1 ✓2 ✓3 ✓4

Is the matrix in row echelon form? Yes ✓

Is the matrix in reduced row echelon form? Yes ✓

If this matrix were the augmented matrix for a system of linear equations, would the system have ~~no~~, unique, or many solutions? Infinitely Many Solutions

Augmented matrix → coeff matrix, results

$$\left[\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ 1 & 0 & 4 & 0 & 2 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$0 = $ nonzero

$x_4 = -4$

$x_2 + -2x_3 = 1$

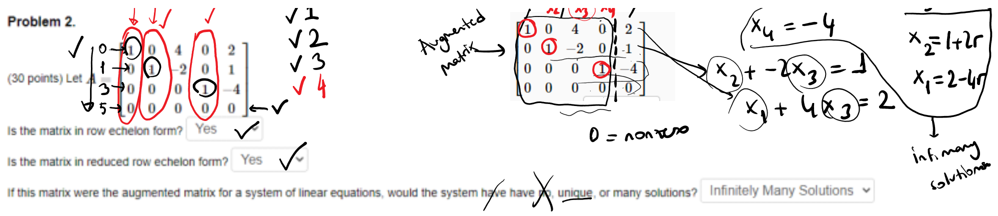
$x_1 + 4x_3 = 2$

$x_3 = r \in \mathbb{R}$

$x_2 = 1 + 2r$

$x_1 = 2 - 4r$

infi. many solutions



Problem 2.

(30 points) Let $A = \begin{bmatrix} 1 & 0 & 4 & 0 & 2 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Is the matrix in row echelon form? Yes ✓

Is the matrix in reduced row echelon form? Yes ✓

If this matrix were the augmented matrix for a system of linear equations, would the system have have no, unique, or many solutions? Infinitely Many Solutions

Inverses of Elementary Matrices

Type-I : $r_i \leftrightarrow r_j$

$$\boxed{E^{-1} = E}$$

Ex/ $E = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad I_3 \xrightarrow{r_1 \leftrightarrow r_2}$

$$E \mid I = \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{r_1 \leftrightarrow r_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$\underbrace{E}_{I}$; $\underbrace{\quad}_{I}$; $\underbrace{\quad}_{E^{-1}}$

Type-II : $c.r_i \longrightarrow r_i$

$$\boxed{E^{-1} : \frac{1}{c} r_i \rightarrow r_i}$$

$$\boxed{E : c \longrightarrow 1/c : E^{-1}}$$

Ex/ $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & ② & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad I \xrightarrow{2r_2 \rightarrow r_2}$

$$[E \mid I] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{2}r_2 \rightarrow r_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$\underbrace{\quad}_{I_3}$; $\underbrace{\quad}_{E^{-1}}$

$$E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \boxed{1/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Type 3 : $E: cr_j + r_i \longrightarrow r_i$

$E^{-1} : -c\, r_j + r_i \longrightarrow r_i$

same

$$\boxed{E : c \longrightarrow -c : E^{-1}}$$

Ex/ $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$ $\quad I_3 \xrightarrow{3r_2 + r_3 \rightarrow r_3}$

$$[E \mid I] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & ① & 0 & 0 & 1 & 0 \\ 0 & ③ & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-3r_2 + r_3 \rightarrow r_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -3 & 1 \end{array}\right]$$

$\underbrace{\quad}_{I}$; $\underbrace{\quad}_{I}$; $\underbrace{\quad}_{E^{-1}}$