3rd Week Wednesday

10 Mart 2021 Çarşamba 12:26

* $AB \neq BA$ (except some unusual cases) $m=n$

✓ ✓

$A_{n\times n}, B_{n\times n} \Rightarrow AB \to$ defined, $BA \to$ defined

Ex/

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3} \qquad B = \begin{bmatrix} 1 & 3 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & 1 \end{bmatrix}_{3\times 3}$$

$AB = \checkmark$

$BA = \checkmark$

$$AB = \begin{bmatrix} -5 & 7 & 7 \\ -8 & 19 & 22 \\ -11 & 31 & 37 \end{bmatrix}_{3\times 3}$$

dot products

$1.1 + 2.0 + 3.-2 = -5$

$1.3 + 2 -1 + 3.2 = 3-2+6 = 7$

$1.4 + 2.0 + 3.1 = 7$

$4.1 + 5.0 + 6.-2 = -8$

$4.3 + 5.-1 + 6.2 = 12-5+12 = 19$ (24-5)

$4.4 + 5.0 + 6.1 = 16+6 = 22$

$7.1 + 8.0 + 9.-2 = 7+-18 = -11$

$7.3 + 8.-1 + 9.2 = 21-8+18 = 31$

$7.4 + 8.0 + 9.1 = 28+9 = 37$

≠

$$B = \begin{bmatrix} 1 & 3 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$BA = \begin{bmatrix} 41 & 49 & 57 \\ -4 & -5 & -6 \\ 13 & 14 & 15 \end{bmatrix}$$

$1.1 + 3.4 + 4.7 = 1+12+28 = 41$

$1.2 + 3.5 + 4.8 = 2+15+32 = 49$

$1.3 + 3.6 + 4.9 = 3+18+36 = 57$

$0.1 + -1.4 + 0.7 = -4$

$0.2 + -1.5 + 0.8 = -5$

$0.3 + -1.6 + 0.9 = -6$

$-2.1 + 2.4 + 1.7 = -2+8+7 = 13$

$-2.2 + 2.5 + 1.8 = -4+10+8 = 14$

$-2.3 + 2.6 + 1.9 = -6+12+9 = 15$

$\Rightarrow \quad AB \neq BA$

Transpose

$$A = [a_{ij}]_{m\times n} \longrightarrow A^T = [a_{ji}]_{n\times m}$$

Ex/ $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}_{3\times 2}$ $\qquad A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}_{2\times 3}$ $\qquad A_{ij} = A^T_{ji}$

Square Matrix :

\# rows = \# columns $\qquad A_{n\times n}$

Diagonal : $\begin{bmatrix} d_1 & & & \\ & d_2 & & \\ & & \ddots & \\ & & & d_n \end{bmatrix}_{n\times n}$ $\qquad (d_1, d_2, \dots, d_n)$

Symmetric Matrix :

(square)

$$A^T = A$$

$\underset{n\times n}{A} \qquad \underset{n\times n}{A^T}$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ $\qquad A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & 6 & 9 \end{bmatrix} = A$

Anti-symmetric Matrix :

$$-A^T = A$$

$A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix} = -A^T = \begin{bmatrix} 0 & & \\ -2 & 0 & \\ -3 & -6 & 0 \end{bmatrix}$

Ex/ $A = \begin{bmatrix} 0 & 1 & -2 & 3 \\ -1 & 0 & 4 & -5 \\ 2 & -4 & 0 & 6 \\ -3 & 5 & -6 & 0 \end{bmatrix} = -A^T = \begin{bmatrix} 0 & +1 & -2 & +3 \\ -1 & 0 & +4 & -5 \\ +2 & -4 & 0 & +6 \\ -3 & +5 & -6 & 0 \end{bmatrix}$

Triangular Matrix :

(square)

Upper-triangular

$$\begin{bmatrix} d_1 & & \cdots \\ & d_2 & \\ 0 & & \ddots \\ & & & d_n \end{bmatrix}$$

Lower-triangular

$$\begin{bmatrix} d_1 & & 0 \\ & d_2 & \\ \vdots & & \ddots \\ & & & d_n \end{bmatrix}$$

Ex

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 3 \end{bmatrix} \rightarrow \text{upper triangular}$$

Identity Matrix :

the identity element of matrix multiplication

(square)

For all A

$$A_{n\times n}\, I_n = A_{n\times n}$$

$$I_n\, A_{n\times n} = A_{n\times n}$$

$$I_n = \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 \cdots & \\ 0 & & & 1 \end{bmatrix}_{n\times n}$$

The Multiplicative Inverse of a Matrix

$$\left\{ \begin{aligned} A\,B &= I_n \\ B\,A &= I_n \end{aligned} \right.$$

$B = A^{-1}$ → the multiplicative inverse of A.

($A = B^{-1}$ also)

HW

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \qquad AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad B = ?$$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \overbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}^{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$1.a + 2.c = 1$

$1.b + 2.d = 0$

$3.a + 4.c = 0$

$3b + 4d = 1$

$$\begin{array}{c} \\ - \\ - \\ - \\ - \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 \\ 3 & 0 & 4 & 0 & 0 \\ 0 & 3 & 0 & 4 & 1 \end{array}\right] \end{array} \rightarrow \text{REF}$$

$a = ?$

$b = ?$

$c = ?$

$d = ?$

3x3

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