8th Week

27 Kasım 2021 Cumartesi 20:36

1 - Solving system of LE. (2-3)
→ what about the solutions?
→ Use Cramer's Rule
→ Use Gauss Gauss-Jordan

2 - Operations on matrices
→ multiplication, addition, transpoz....
→ det
→ inverse → $[A \mid I]$
→ adjoint

3 - Elementary Matrices, ___ ___ .
↓
determinant
Inverses

4 - Vector space (why not ?)

5 - Subspace or not

$$A = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{bmatrix} 1 & 2 & -2 \\ 1 & 0 & 4 \\ 2 & 1 & 3 \end{bmatrix} \end{array}$$ is given.

! $\boxed{\det(AB) = \det(A)\det(B)}$

a) $\boxed{\det(A) = ?}$

→ b) Using only $\det(A)$, find $\det(A^{-1})$, $\det(A^3)$, $\det(A^T A)$. $\boxed{\det(3A)}$.

$\det(A^{-1}) = \frac{1}{\det(A)}$

→ c) Find A^{-1} using row operations.

$$A \cdot A^{-1} = I$$

$$\det(A \cdot A^{-1}) = \det(I) = 1$$

$$\rightarrow \underbrace{\det(A)}_{4} \cdot \underbrace{\det(A^{-1})}_{= \frac{1}{4}} = 1$$

$$\det(A) = 1 \underbrace{\begin{vmatrix} 0 & 4 \\ 1 & 3 \end{vmatrix}}_{-4} - 2 \underbrace{\begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix}}_{3-8} + (-2) \cdot \underbrace{\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix}}_{-2} = 4$$

$+10$

$$\det(A^3) = \underbrace{\det(A)}_{4} \cdot \underbrace{\det(A)}_{4} \cdot \underbrace{\det(A)}_{4} = 4^3 = 64$$

$$\det(A^T A) = \underbrace{\det(A^T)}_{4} \underbrace{\det(A)}_{4} = 16$$

$$\det(3A) = !$$

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & j \end{bmatrix} \qquad 3A = \begin{bmatrix} 3a & 3b & 3c \\ 3d & 3e & 3f \\ 3g & 3h & 3j \end{bmatrix}$$

$3r_1 \to r_1$ → $3r_2 \to r_2$ → $3r_3 \to r_3$

E_3

Type-1 $(r_i \leftrightarrow r_j)$

Ex $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$\det(E) = -1$

Type-2 $(kr_i \to r_i)$ $k \neq 0$

Ex $\begin{bmatrix} 1 & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\boxed{\det(E) = k}$

Type-3 $(kr_j + r_i \to r_i)$

Ex $\begin{bmatrix} 1 & 0 & 0 \\ k & 1 & 0 \end{bmatrix}$

$3r_1 \to r_1$ $\quad$ $3r_2 \to r_2$ $\quad$ $3r_3 \to r_3$

E_1 $\quad$ E_2 $\quad$ E_3

Ex/ Type-3 $(kr_j + r_i \to r_i)$

$$\begin{bmatrix} 1 & 0 & 0 \\ k & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\det(E) = 1$

$$\underbrace{E_3 E_2 E_1 A} = \underline{3A}$$

$$\longrightarrow \det(3A) = \det(E_3 E_2 E_1 A) = \overbrace{\det(E_3)}^{3} \cdot \overbrace{\det(E_2)}^{3} \cdot \overbrace{\det(E_1)}^{3} \cdot \overbrace{\det(A)}^{4}$$

$$= 27 \cdot 4$$

$$\boxed{\begin{array}{l} A \to 3\times 3 \text{ matrix}, \ k \in \mathbb{R} \\ \det(kA) = k \cdot k \cdot k \cdot \det(A) \end{array}}$$

$$\boxed{\begin{array}{l} A \to 4\times 4 \text{ matrix} \\ \det(kA) = k \cdot k \cdot k \cdot k \cdot \det(A) \end{array}}$$

$A \to n\times n$ matrix $\to \det(kA) = k^n \cdot \det(A)$

c) $[A \mid I] \to [I \mid A^{-1}]$

1)

$$\begin{bmatrix} 2 & 7 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 1 & 5 & -9 & 8 \\ 5 & 18 & 4 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 2 & 1 & 0 \\ 2 & -1 & 1 \\ 0 & -1 & 1 \\ -2 & 2 & 2 \end{bmatrix}}_{4\times 3} \underbrace{\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}}_{3\times 1} = 0.$$

4×1

a) Write the system in the form of $Ax = b$.

b) Transform the augmented matrix into RREF.

c) Solve the system.

$$\begin{bmatrix} 2 & 1 & 0 \\ 2 & -1 & 1 \\ 0 & -1 & 1 \\ -2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 1 \\ 12 \end{bmatrix}$$

$2.1 + 1.3 + 0.4$

$2.1 + -1.3 + 1.4 = 3$

$-2.1 + 2.3 + 2.4 = 12$

$6 + 8$

$$\to \begin{bmatrix} 2 & 7 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 1 & 5 & -9 & 8 \\ 5 & 18 & 4 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -1 \\ -12 \end{bmatrix}$$

Gauss $\to$ REF

Gauss-Jordan $\to$ RREF

$$\underset{A}{\begin{bmatrix} 2 & -4 & 4 & 10 \\ 0 & 1 & 2 & 0 \\ 1 & -2 & 2 & 5 \\ 4 & 4 & 5 & 6 \end{bmatrix}} \xrightarrow[E]{-2r_3 + r_1 \to r_1} \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & -2 & 2 & 5 \\ 4 & 4 & 5 & 6 \end{bmatrix}}_{A'}$$

$\det(A') = 0$

$\neq 0$

$EA = A'$ $\to \det(A') = 0$

$$\det(EA) = \underbrace{\det(E)}_{1} \underbrace{\det(A)}_{?} = \underbrace{\det(A')}_{0}$$

$\det(A) = 0$

$$\begin{aligned} x_1 + x_2 &= 3 \\ x_1 + (a^2 - 8)x_2 &= a \end{aligned}$$

augmented matrix

$$\longrightarrow \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & a^2-8 & a \end{array}\right] \xrightarrow[-r_1 + r_2 \to r_2]{\sim \text{REF}} \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & a^2-9 & a-3 \end{array}\right]$$

$$\begin{aligned} x_1 + x_2 &= 3 \\ x_1 + (a^2-8)x_2 &= a \end{aligned} \longrightarrow \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & a^2-8 & a \end{array}\right] \xrightarrow{-r_1+r_2 \to r_2} \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & a^2-9 & a-3 \end{array}\right]$$

$0 = $ başka sayı $\rightarrow$ no soln.

$0 = 0 \rightarrow$ inf. many soln.

bunlar dışında kalan durumlar $\rightarrow$ unique.

$$(a^2-9)\,x_2 = (a-3)$$

$a = 3 \longrightarrow$ inf. many soln.

$a \neq 3$;

$$\cancel{(a-3)}(a+3)\,x_2 = \cancel{(a-3)}$$

$$(a+3)\,x_2 = 1 \qquad a=-3 \rightarrow \text{no soln.}$$

$a \neq 3, -3 \Rightarrow$ unique soln. $\quad x_2 = \dfrac{1}{a+3}$ ✓ $\longrightarrow$ substitute find x_1