10th Week Wednesday

28 Nisan 2021 Çarşamba 12:38

$\vec{v} \in V$ $\quad \cancel{\vec{v}} = \boxed{B_1 [\vec{v}]_{B_1} = B_2 [\vec{v}]_{B_2}} = \ldots\ldots$

B_1^{-1} $\quad B_2^{-1}$ $\quad I$ $\quad ?$

$B_1, B_2, \ldots$ are different bases of V.

Change of Basis

$$[\vec{v}]_{B_2} = \underbrace{B_2^{-1} B_1}_{\text{the transition matrix from } B_1 \text{ to } B_2.} [\vec{v}]_{B_1}$$

Ex $\quad B_1 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix} \right\} \qquad B_2 = \left\{ \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix} \right\}$

If the coordinate vector of $\vec{v} \in \mathbb{R}^2$ with respect to B_2 is $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$, $\to [\vec{v}]_{B_2}$

a) Find the transition matrix from B_2 to B_1.

b) Find $[\vec{v}]_{B_1}$.

$$\boxed{B_1 \cdot [\vec{v}]_{B_1} = B_2 \cdot [\vec{v}]_{B_2}}$$

$B_1 = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$

$$[\vec{v}]_{B_1} = \underbrace{B_1^{-1} B_2}_{\text{this is the transition matrix from } B_2 \text{ to } B_1.} [\vec{v}]_{B_2}$$

$B_1^{-1} = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$

$$B_1^{-1} B_2 = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}$$

$$[\vec{v}]_{B_1} = \begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix} \cdot [\vec{v}]_{B_2} = \begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

Ex #3.5-3

$\vec{v}_1 = (3,2)^T \quad \vec{v}_2 = (4,3)^T \qquad \mathcal{U} = \{\vec{u}_1, \vec{u}_2\}$

$\vec{u}_1 = (0,1)^T \quad \vec{u}_2 = (2,1)^T \qquad V = \{\vec{v}_1, \vec{v}_2\}$ are two bases for $\mathbb{R}^2$.

Find the transition matrix from V to $\mathcal{U}$. $\to \mathcal{U}^{-1} V$

$$\forall \vec{x} \in \mathbb{R}^2 \qquad [\vec{x}]_{\mathcal{U}} = \mathcal{U}^{-1} V [\vec{x}]_V$$

$-2+3$ $\quad -\frac{3}{2}+2$

$\mathcal{U} = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} \qquad \mathcal{U}^{-1} = \begin{bmatrix} -1/2 & 1 \\ 1/2 & 0 \end{bmatrix} \qquad \mathcal{U}^{-1} V = \begin{bmatrix} -1/2 & 1 \\ 1/2 & 0 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1/2 & 1 \\ 3/2 & 2 \end{bmatrix}$

$$U = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} \qquad U^{-1} = \begin{bmatrix} -1/2 & 1 \\ 1/2 & 0 \end{bmatrix} \qquad U^{-1}V = \begin{bmatrix} -1/2 & 1 \\ 1/2 & 0 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1/2 & 1 \\ 3/2 & 2 \end{bmatrix}$$

$0-2$

Find the transition matrix from U to V. $\longrightarrow V^{-1}U$

$$V = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} \qquad V^{-1} = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix} \qquad V^{-1}U = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$$

$9-8=1$

EX
#3.5-5

$u_1 = (1,1,1)$
$u_2 = (1,2,2)$
$u_3 = (2,3,4)$

$U = \{u_1, u_2, u_3\}$ is a basis for $\mathbb{R}^3$.

a) Find the transition matrix from the standard basis to U.

$U^{-1}E$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix} \dashrightarrow U^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \qquad U^{-1}E = U^{-1}$$

b) $\vec{v} = (3,2,5)$ Find $[\vec{v}]_U$. $\qquad \vec{v} = 1.u_1 - 4u_2 + 3u_3$

$$[\vec{v}]_U = U^{-1}v = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$$

c) $\vec{v}_1 = (4,6,7)$
$\vec{v}_2 = (0,1,1)$
$\vec{v}_3 = (0,1,2)$

$V = \{v_1, v_2, v_3\}$ is another basis for $\mathbb{R}^3$.

Find the transition matrix from V to U. $\rightarrow U^{-1}V$

$$U^{-1}V = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 0 & 0 \\ 6 & 1 & 1 \\ 7 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$-4+12-7$

d) $\vec{x} = 2\vec{v}_1 + 3\vec{v}_2 - 4\vec{v}_3$ Find the coordinates of $\vec{x}$ wrt the basis U.

$$[\vec{x}]_V = \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix} \qquad [\vec{x}]_U = \underbrace{U^{-1}V}_{\text{the transition matrix from } V \text{ to } U} [\vec{x}]_V$$

$\lfloor -4 \rfloor$

the transition matrix from V to $\mathcal{U}$

$$[\vec{x}]_{\mathcal{U}} = \begin{bmatrix} 1 & -1 & -2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ -2 \end{bmatrix}$$

$$\Rightarrow \vec{x} = 7u_1 + 5u_2 - 2u_3$$