Moodle Ders Kayıt Anahtarı : sb1234e
MAT 104E Sp'21

1.1 Systems of Linear Equations

→ linear eqn → $3x_1 + 4x_2 - x_3 = 2$ ←
$3x_1^2 - 2x_2 = 3$ → Not linear!
linear eqn ← $3x_1 - x_3 + x_4 = 4$

A *linear equation in* *n unknowns* is an equation of the form

1 single linear eqn → $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$ → linear eqn

where $a_1, a_2, \ldots, a_n$ and b are real numbers and $x_1, x_2, \ldots, x_n$ are variables. A *linear system* of m equations in n unknowns is then a system of the form

a system of linear eqns.

$$\begin{aligned}
\text{1st eqn} &\rightarrow a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \rightarrow \text{the result of the 1st eqn} \\
\text{2nd eqn} &\rightarrow a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\
&\vdots \\
m^{th}\text{ eqn} &\rightarrow a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \rightarrow
\end{aligned} \qquad (1)$$

m eqns
n unknowns
$\boxed{m \times n \text{ system}}$
#of eqn # of unknown

where the a_{ij}'s and the b_i's are all real numbers. We will refer to systems of the form (1) as $m \times n$ linear systems. The following are examples of linear systems:

Ex/
1 ✓ $2x_1 - x_3 + x_4 = 4$
2 ✓ $x_1 - x_5 = 2$
} $m \times n$ system $m = 2$, $n = 5$ → 2×5 linear system

→ $(x_1, x_2, x_3, x_4, x_5)$ ← solution

No solution!

solution. If a linear system has no solution, we say that the system is *inconsistent*. If the system has at least one solution, we say that it is *consistent*. Thus system (c) is

2 × 2 Systems

Let us examine geometrically a system of the form

$$\begin{aligned}
-2/ \quad a_{11}x_1 + a_{12}x_2 &= b_1 \\
+ \quad a_{21}x_1 + a_{22}x_2 &= b_2
\end{aligned}$$

$-2a_{11}x_1 - 2a_{12}x_2 = -2b_1$

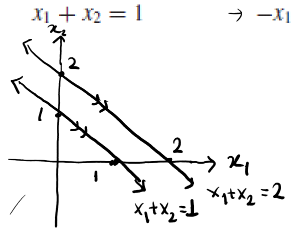
Each equation can be represented graphically as a line in the plane. The ordered pair (x_1, x_2) will be a solution of the system if and only if it lies on both lines. For example,

(i) $x_1 + x_2 = 2$
$x_1 - x_2 = 2$

Soln ⇒ $(x_1, x_2) = (2, 0)$; $x_1 = 2$, $x_2 = 0$

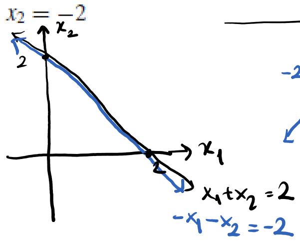
A unique solution.

(ii) $x_1 + x_2 = 2$
$x_1 + x_2 = 1$

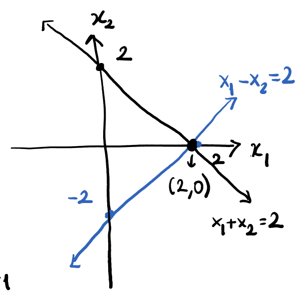


No solution (x_1, x_2)

(iii) $x_1 + x_2 = 2$
→ $-x_1 - x_2 = -2$



Infinitely many solutions.



In general, there are three possibilities: the lines intersect at a point, they are parallel, or both equations represent the same line. The solution set then contains either one, zero, or infinitely many points.

- ✓ 1 solution
- ✓ No solution
- ✓ Infinitely many solutions

no other possibilities exist about the number of solutions of a system of linear eqns.

Allowed operations on eqns in a system

1) [i] <-> [j] exchange the place
2) c. [i]
   constant
3) c. [j] + [i]

1) the idea of elimination

(i)

$$x_1 + \cancel{x_2} = 2$$
$$+ \quad x_1 - \cancel{x_2} = 2$$
$$2x_1 = 4$$
$$\Rightarrow \boxed{x_1 = 2}$$

2) substitution

$$2 + x_2 = 2$$
$$\boxed{x_2 = 0}$$

the unique soln. $(x_1, x_2) = (2, 0)$

(ii)

$$x_1 + x_2 = 2$$
$$-1 / \; x_1 + x_2 = 1$$
$$\rightarrow \cancel{x_1} + \cancel{x_2} = 2$$
$$+ \quad -\cancel{x_1} - \cancel{x_2} = -1$$
$$0 + 0 = 1$$
$$\boxed{0 = 1}$$

impossible!

No solution!

(iii)

$$\overset{2}{\cancel{x_1}} + \overset{0}{x_2} = 2$$
$$+ \quad -\cancel{x_1} - \cancel{x_2} = -2$$
$$0 + 0 = 0$$
$$\boxed{0 = 0}$$

always true!

Infinitely many solutions.

$(1,1)$
$(2,0)$
$(-2,4)$
$\vdots$